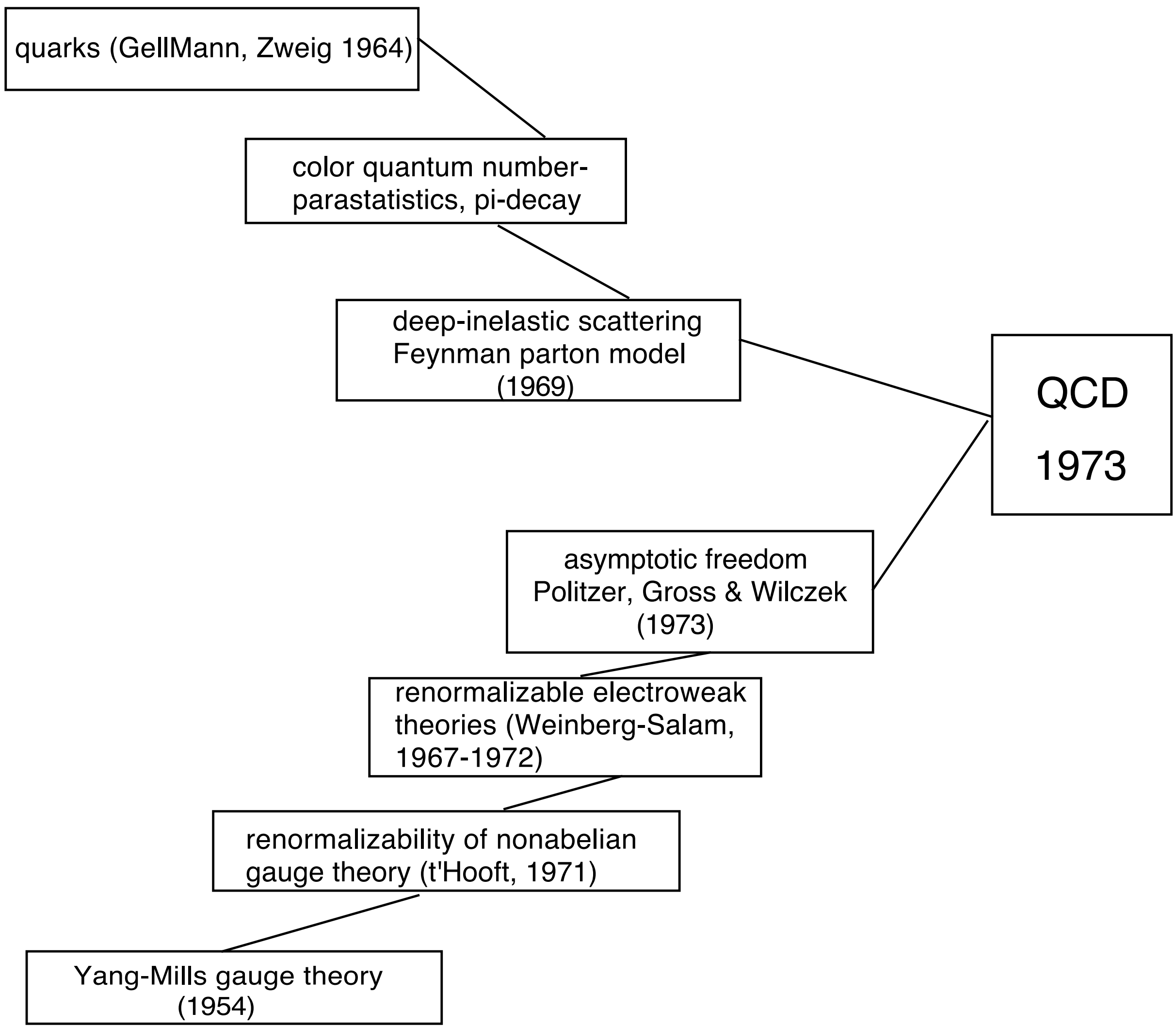
Perturbative QCD in the late 1970's

- **Prehistory- from the mid 60's to 1973.**
- **The "Classic Tests" of QCD.**
- **Technical Advances- spacelike to timelike, inclusive to exclusive.**
- **A Flood of Processes.....**

The QCD Family Tree



Asymptotic Freedom is discovered (better, *recognized*)

Reliable Perturbative Results for Strong Interactions?*

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(Received 3 May 1973)

An explicit calculation shows perturbation theory to be arbitrarily good for the deep Euclidean Green's functions of any Yang-Mills theory and of many Yang-Mills theories with fermions. Under the hypothesis that spontaneous symmetry breakdown is of dynamical origin, these symmetric Green's functions are the asymptotic forms of the physically significant spontaneously broken solution, whose coupling could be strong.

Ultraviolet Behavior of Non-Abelian Gauge Theories*

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(Received 27 April 1973)

It is shown that a wide class of non-Abelian gauge theories have, up to calculable logarithmic corrections, free-field–theory asymptotic behavior. It is suggested that Bjorken scaling may be obtained from strong-interaction dynamics based on non-Abelian gauge symmetry.

and then, in rapid succession.....

Non-Abelian Gauge Theories of the Strong Interactions*

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(Received 30 May 1973)

A class of non-Abelian gauge theories of strong interactions is described, for which parity and strangeness are automatically conserved, and for which the nonconservations of parity and strangeness produced by weak interactions are automatically of order α / m_W^2 rather than of order α. When such theories are "asymptotically free," the order-α weak corrections to natural zeroth-order symmetries may be calculated ignoring all effects of strong interactions. Speculations are offered on a possible theory of quarks.

Asymptotically Free Gauge Theories. I*

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(Received 23 July 1973)

Asymptotically free gauge theories of the strong interactions are constructed and analyzed. The reasons for doing this are recounted, including a review of renormalization-group techniques and their application to scaling phenomena. The renormalization-group equations are derived for Yang-Mills theories. The parameters that enter into the equations are calculated to lowest order and it is shown that these theories are asymptotically free. More specifically the effective coupling constant, which determines the ultraviolet behavior of the theory, vanishes for large spacelike momenta. Fermions are incorporated and the construction of realistic models is discussed. We propose that the strong interactions be mediated by a "color" gauge group which commutes with SU(3) $\times$ SU(3). The problem of symmetry breaking is discussed. It appears likely that this would have a dynamical origin. It is suggested that the gauge symmetry might not be broken and that the severe infrared singularities prevent the occurrence of noncolor singlet physical states. The deep-inelastic structure functions, as well as the electron-positron total annihilation cross section are analyzed. Scaling obtains up to calculable logarithmic corrections, and the naive light-cone or parton-model results follow. The problems of incorporating scalar mesons and breaking the symmetry by the Higgs mechanism are explained in detail.

ADVANTAGES OF THE COLOR OCTET GLUON PICTURE☆

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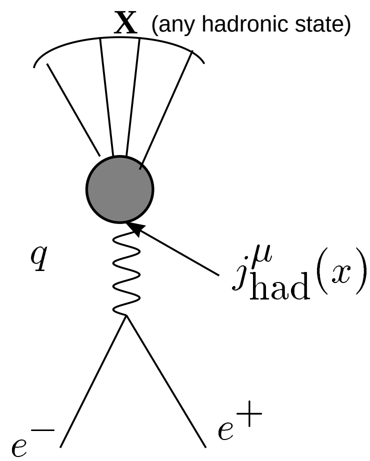
Received 1 October 1973

It is pointed out that there are several advantages in abstracting properties of hadrons and their currents from a Yang–Mills gauge model based on colored quarks and color octet gluons.

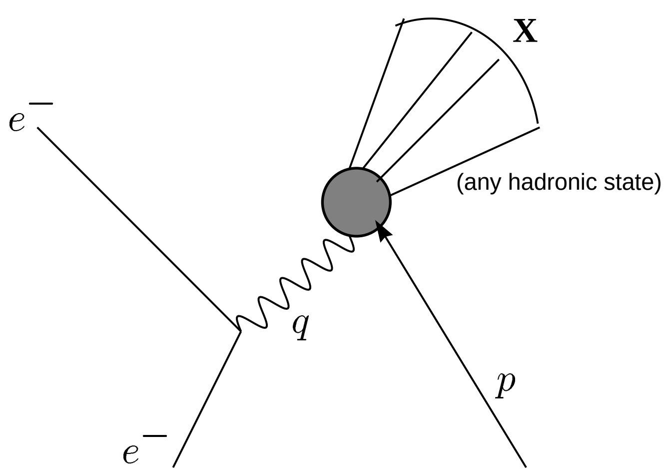
The Classic Tests of QCD

Hadronic $e^+ - e^-$ Annihilation cross-section



$$\sigma(\gamma(q) \to X)$$
$$\propto \mathrm{Im} \int e^{iqx} < 0|T(j^\mu_{\rm had}(x) j^\nu_{\rm had}(0))|0 >$$

Deep-inelastic Scattering



$$\sigma(\gamma(q) + p \to X)$$
$$\propto \mathrm{Im} \int e^{iqx} < p|T(j^\mu_{\rm had}(x) j^\nu_{\rm had}(0)|p >$$

Technical Tools

1. Short-distance ("Operator Product") Expansions (Wilson, 1969):

   $e^+e^- \to X$: spectral representation of $\int e^{iq\cdot x} < 0|T(j(x)j(0))|0 > d^4x$ relates integrals of $\sigma(e^+e^- \to X)$ for *timelike* q to the deep spacelike region $\Rightarrow x \to 0$ spacelike expansion of $T(j(x)j(0)) \simeq C_0(x)1 + \sum C_n(x)O_n$.

   $\gamma(q) + p \to X$: in Bjorken limit $-q^2 \equiv Q^2 \to \infty,\ Q^2/p\cdot q$ fixed, we probe light-cone singularities, $x^2 \to 0$ (but $x \neq 0!$) behavior of $\int e^{iq\cdot x} < p|T(j(x)j(0))|p > d^4x$.

   However, rigorous proofs of the Wilson expansion (to all orders of perturbation theory) were only available (Zimmermann 1973) in the spacelike domain.

2. Renormalization Group equations (Stueckelberg,Petermann, GellMann, Low, Wilson, Callan, Symanzik,.....):
   Once the large momentum (Q^2) dependence has been isolated ("factorized") in a "hard part amplitude" $\mathcal{M}(q^2; m, g, \mu)$, an exact renormalization equation obtains:

$$(\mu\frac{\partial}{\partial\mu} + \beta\frac{\partial}{\partial g} - \gamma_m m\frac{\partial}{\partial m} + \gamma)\mathcal{M}(q^2; m, g, \mu) = 0$$

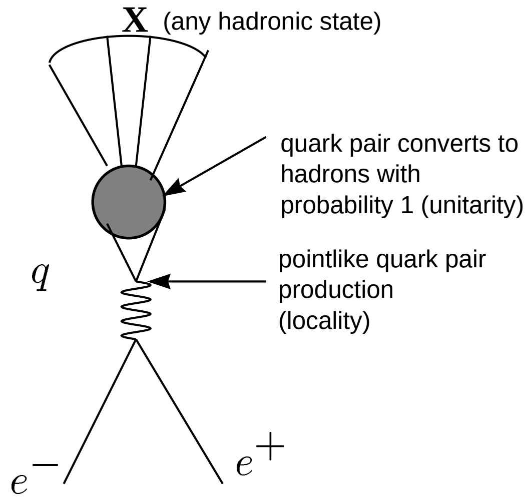
   This translates (in an asymptotically free theory) to a precise prediction for the large q^2 behavior *only for negligible mass dependence of the hard part*, in which case the zero mass limit can be taken and the renormalization scale μ dependence traded for q^2 behavior. The general expectation was that this mass insensitivity would only hold for *inclusive* processes.

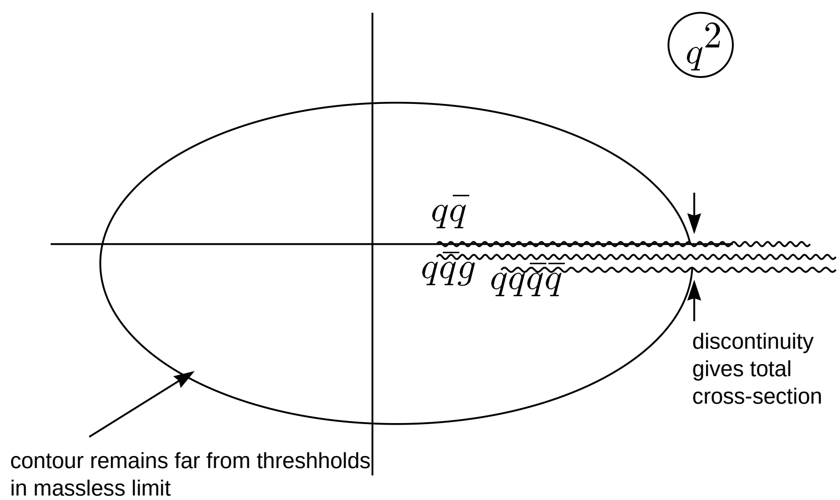
The main obstacles to further progress (ca 1975) were

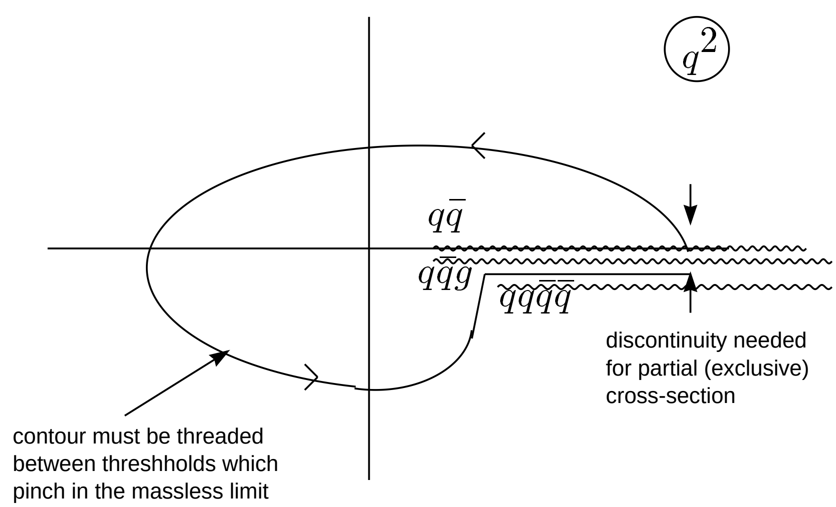
1. The need to extend the operator product expansion to timelike processes.

2. The need for more powerful methods for studying mass sensitivity: in particular, allowing extension to *exclusive* processes, if possible.

Mass insensitivity a la Al (see Phys.Repts. 73, 237)







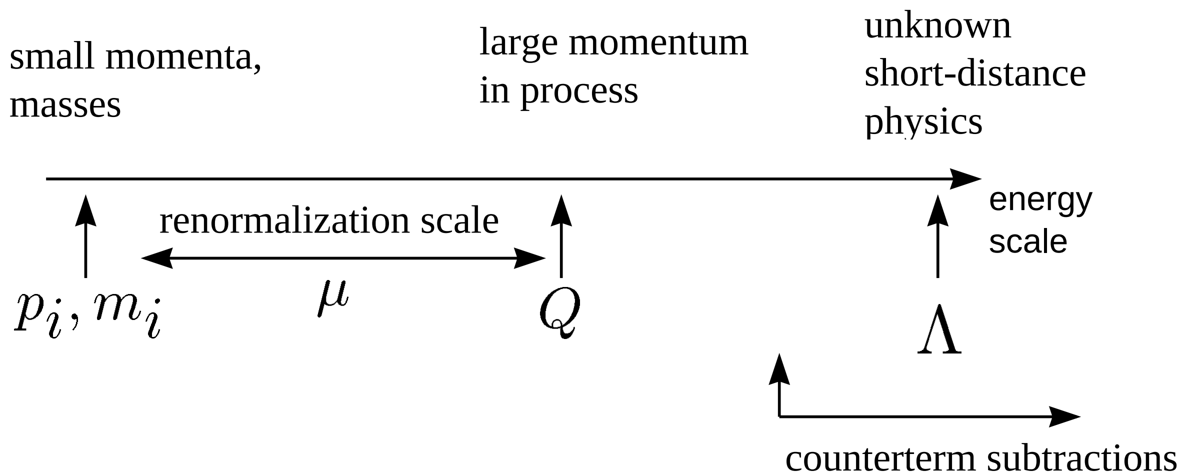
The Mueller Strategy

Beginning ca. 1976 (with a study of inclusive annihilation in ϕ^4 theory) Al pioneered an extremely fruitful approach to the renormalization group study of high energy processes in QCD. In essence, the new approach involved:

1. An escape from the straitjacket of coordinate-space approaches to factorization via Wilson expansions in local operators: instead the factorization was studied directly in momentum space using oversubtraction techniques. The power counting methods used were applicable both in the space-like and the time-like domain. In some cases, the factorization produced intrinsically nonlocal operators ("cut vertices").

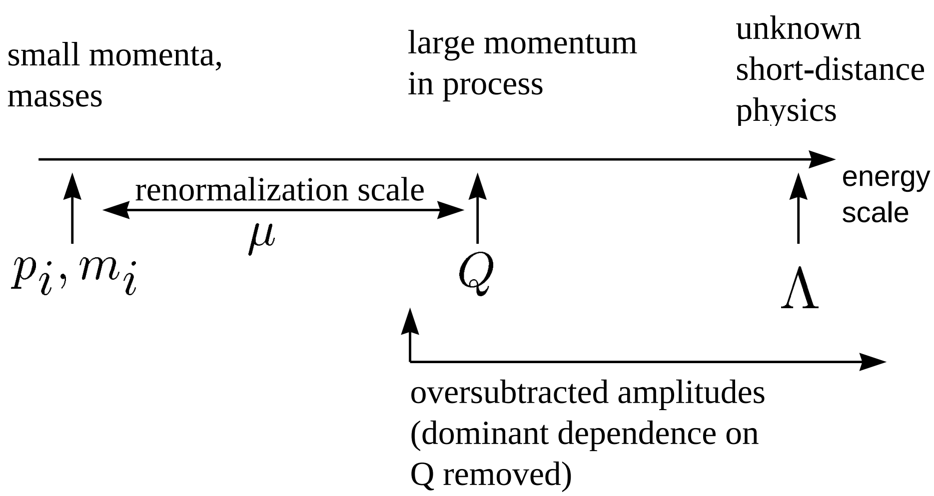
2. In amplitudes of more complicated analyticity, the mass sensitivity of the factorized hard parts could also be reduced to an oversubtraction analysis, involving the mass-inserted (i.e. bare mass differentiated) amplitudes.

This new approach greatly increased the number and variety of high energy processes which could be treated by renormalization group techniques: in other words, processes in which the asymptotic freedom of the theory could be shown to lead reliably to calculable asymptotic behavior.



Schematically:

$$\mathcal{M}(p, m, Q, \mu, \Lambda) \to \mathcal{M}_R(p, m, Q, \mu) + O(\frac{p^2, m^2, Q^2, \mu^2}{\Lambda^2})$$



Schematically:

$$\mathcal{M}_R(p, m, Q, \mu) \to \sum_i \hat{\mathcal{M}}_{R,i}(p, m, \mu) C_i(Q, m, \mu) + O(\frac{p^2, m^2, \mu^2}{Q^2})$$

A Flood of Processes...

A careful application of oversubtraction techniques ($\Rightarrow$ analysis of dominant momentum flows) led to a considerable expansion in the number of processes reliably described by renormalization group equations. Just some of the cases (in addition to the "classics", such as deep-inelastic) discussed in Al's 1980 Physics Reports review:

1. Inclusive annihilation ($e^+ + e^- \to p + X$)
2. Drell-Yan (inclusive μ-pair production, integrated cross-section)
3. Meson Form Factors
4. Baryon Form Factors (*)
5. Heavy Quarkonium Exclusive Decays
6. Wide-angle elastic scattering (*)
7. Drell-Yan (fixed $\vec{\mathbf{q}}$) (*)
8. $x \to 1$ Structure Functions (*)

In some cases (indicated by * above), the analysis revealed the presence of Sudakov double logarithmic contributions, stimulating a rich new vein of enquiry....