Factorization in the non-linear small-x regime of QCD

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Outline

1. **Gluon saturation at small x**
2. **Factorization in Deep Inelastic Scattering**
3. **Nucleus-Nucleus collisions**

1. **Gluon saturation at small x**
   - Gluon evolution
   - Saturation domain
   - Multiple scatterings
   - Color Glass Condensate

2. **Factorization in Deep Inelastic Scattering**
   - Leading Order
   - Next to Leading Order
   - Leading Log resummation

3. **Nucleus-Nucleus collisions**
   - Stages of AA collisions
   - Energy-Momentum tensor
   - Glasma fields

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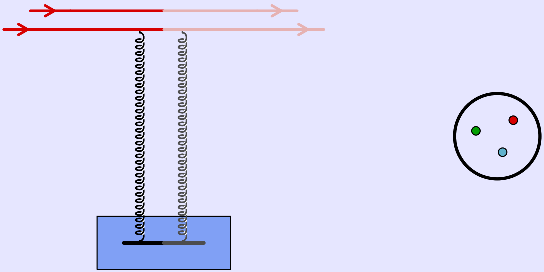
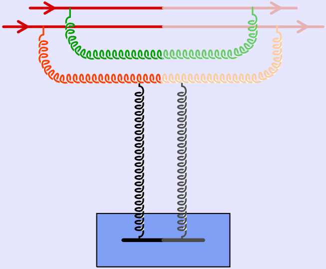
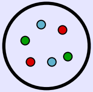
Gluon saturation



- assume that the projectile is big, e.g. a nucleus, and has many valence quarks (only two are represented)
- on the contrary, consider a small probe, with few partons
- at low energy, only valence quarks are present in the hadron wave function

Gluon saturation





- when energy increases, new partons are emitted
- the emission probability is $\alpha_s \int \frac{dx}{x} \sim \alpha_s \ln(\frac{1}{x})$, with x the longitudinal momentum fraction of the gluon
- at small-x (i.e. high energy), these logs need to be resummed

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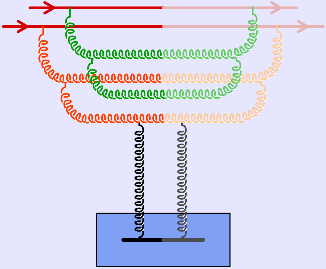
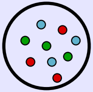
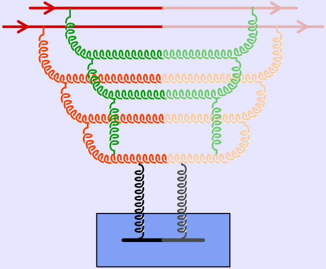
Gluon saturation

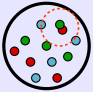




- as long as the density of constituents remains small, the evolution is linear: the number of partons produced at a given step is proportional to the number of partons at the previous step (BFKL)

Gluon saturation





- eventually, the partons start overlapping in phase-space
- parton recombination becomes favorable
- after this point, the evolution is non-linear:
  the number of partons created at a given step depends non-linearly on the number of partons present previously

Criterion for gluon recombination

Gribov, Levin, Ryskin (1983)

Number of gluons per unit area :

$$\rho \sim \frac{xG_A(x, Q^2)}{\pi R_A^2}$$

Recombination cross-section :

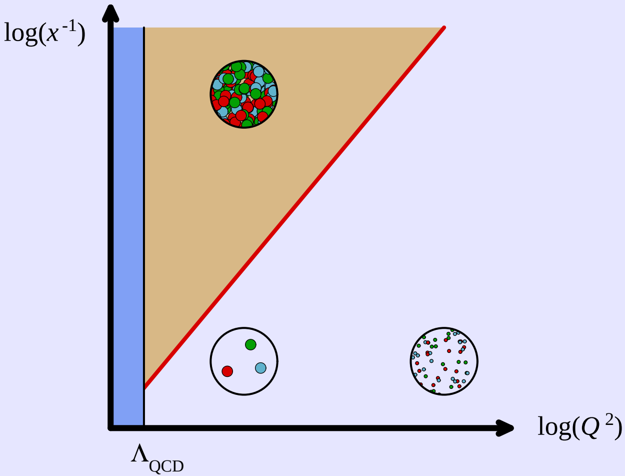
$$\sigma_{gg\to g} \sim \frac{\alpha_s}{Q^2}$$

Recombination happens if $\rho\sigma_{gg\to g} \gtrsim 1$, i.e. $Q^2 \lesssim Q_s^2$, with :

$$Q_s^2 \sim \frac{\alpha_s xG_A(x, Q_s^2)}{\pi R_A^2} \sim A^{1/3}\frac{1}{x^{0.3}}$$

Note: At a given energy, the saturation scale is larger for a nucleus (for $A = 200$, $A^{1/3} \approx 6$)

Saturation domain



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Multiple scatterings

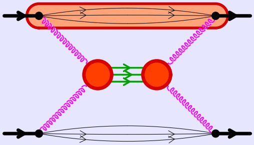
- Power counting :

$$\frac{\text{2 scatterings}}{\text{1 scattering}} \sim \frac{Q_s^2}{M_\perp^2} \quad \text{with} \quad Q_s^2 \sim \alpha_s \frac{xG(x,Q_s^2)}{\pi R^2}$$

- When this ratio becomes ~ 1, all the rescattering corrections become important
  - ▷ one must resum all $\left[Q_s/P_\perp\right]^n$
- These effects are not accounted for in DGLAP or BFKL

Multiple scatterings

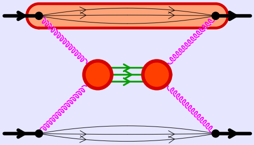
Single scattering :



▷ 2-point function in the projectile ▷ gluon number

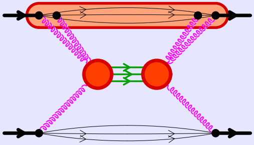
Multiple scatterings

Single scattering :



▷ 2-point function in the projectile ▷ gluon number

Double scattering :



▷ 4-point function in the projectile ▷ higher correlations

CGC: Degrees of freedom

CGC = effective theory of small x gluons

- The fast partons ($k^+ > \Lambda^+$) are frozen by time dilation
  $\triangleright$ described as static color sources on the light-cone :

$$J^\mu = \delta^{\mu+} \rho(x^-, \vec{\boldsymbol{x}}_\perp) \qquad\qquad (0 < x^- < 1/\Lambda^+)$$

- Slow partons ($k^+ < \Lambda^+$) cannot be considered static over the time-scales of the collision process
  $\triangleright$ they must be treated as standard gauge fields

  Eikonal coupling to the current J^μ : $A_\mu J^\mu$

- The color sources ρ are random, and described by a distribution functional $W_{\Lambda^+}[\rho]$, with Λ^+ the longitudinal momentum that separates “soft” and “hard”

CGC: renormalization group evolution

Evolution equation (JIMWLK) :

$$\frac{\partial W_{\Lambda^+}}{\partial \ln(\Lambda^+)} = \mathcal{H}\; W_{\Lambda^+}$$

$$\mathcal{H} = \frac{1}{2}\int_{\vec{\boldsymbol{x}}_\perp,\vec{\boldsymbol{y}}_\perp} \frac{\delta}{\delta\alpha(\vec{\boldsymbol{y}}_\perp)}\eta(\vec{\boldsymbol{x}}_\perp,\vec{\boldsymbol{y}}_\perp)\frac{\delta}{\delta\alpha(\vec{\boldsymbol{x}}_\perp)}$$

where $-\partial_\perp^2\,\alpha(\vec{\boldsymbol{x}}_\perp) = \rho(1/\Lambda^+,\vec{\boldsymbol{x}}_\perp)$

- $\eta(\vec{\boldsymbol{x}}_\perp,\vec{\boldsymbol{y}}_\perp)$ is a non-linear functional of ρ
- This evolution equation resums all the powers of $\alpha_s\ln(1/x)$ and of $Q_s/p_\perp$ that arise in loop corrections
- This equation simplifies into the BFKL equation when the source ρ is small (one can expand η in powers of ρ)

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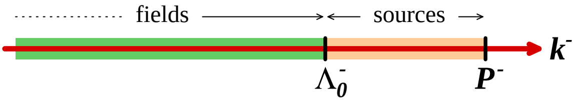
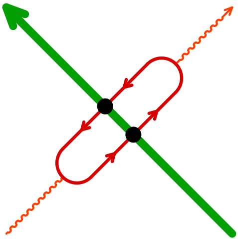
Inclusive DIS at Leading Order

- CGC effective theory with cutoff at the scale Λ_0^- :



- At Leading Order, DIS can be seen as the interaction between the target and a $q\bar{q}$ fluctuation of the virtual photon :



Inclusive DIS at Leading Order

- Forward dipole amplitude at leading order:

$$T_{\rm LO}(\vec{\boldsymbol{x}}_\perp,\vec{\boldsymbol{y}}_\perp) = 1-\frac{1}{N_c}\,{\rm tr}\,\big(\underbrace{U(\vec{\boldsymbol{x}}_\perp)U^\dagger(\vec{\boldsymbol{y}}_\perp)}_{\text{Wilson lines}}\big)$$

$$U(\vec{\boldsymbol{x}}_\perp) = {\rm P}\,\exp i g\int^{1/xP^-} dz^+\,\mathcal{A}^-(z^+,\vec{\boldsymbol{x}}_\perp)$$

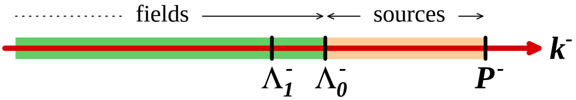
$$[\mathcal{D}_\mu,\mathcal{F}^{\mu\nu}] = \delta^{\nu-}\,\rho(x^+,\vec{\boldsymbol{x}}_\perp)$$

$\triangleright$ at LO, the scattering amplitude on a saturated target is entirely given by classical fields

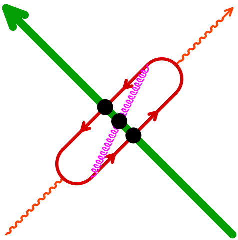
- Note: the $q\bar{q}$ pair couples only to the sources up to the longitudinal coordinate $z^+ \lesssim (xP^-)^{-1}$. The other sources are too slow to be seen by the probe

Inclusive DIS at NLO

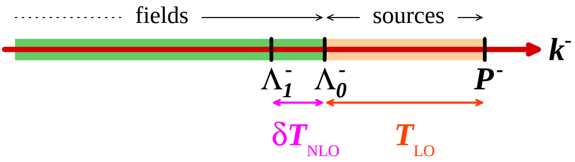
- Consider now quantum corrections to the previous result, restricted to modes with $\Lambda_1^- < k^- < \Lambda_0^-$ (the upper bound prevents double-counting with the sources):



- At NLO, the $q\bar{q}$ dipole must be corrected by a gluon, e.g. :



Inclusive DIS at NLO



- At leading log accuracy, the contribution of the quantum modes in that strip is :

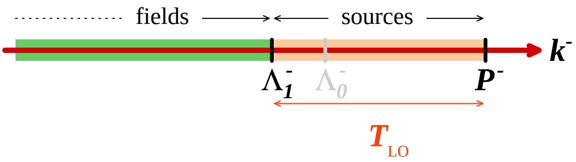
$$\delta \boldsymbol{T}_{\rm NLO}(\vec{\boldsymbol{x}}_\perp, \vec{\boldsymbol{y}}_\perp) = \ln\left(\frac{\Lambda_0^-}{\Lambda_1^-}\right)\ \mathcal{H}\ \boldsymbol{T}_{\rm LO}(\vec{\boldsymbol{x}}_\perp, \vec{\boldsymbol{y}}_\perp)$$

Inclusive DIS at NLO

- These NLO corrections can be absorbed in the LO result,

$$\left\langle T_{\rm LO} + \delta T_{\rm NLO} \right\rangle_{\Lambda_0^-} = \left\langle T_{\rm LO} \right\rangle_{\Lambda_1^-}$$

provided one defines a new effective theory with a lower cutoff Λ_1^- and an extended distribution of sources $W_{\Lambda_1^-}[\rho]$:



$$W_{\Lambda_1^-} \equiv \left[1 + \ln\left(\frac{\Lambda_0^-}{\Lambda_1^-}\right)\ \mathcal{H}\right] W_{\Lambda_0^-}$$

(JIMWLK equation for a small change in the cutoff)

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Inclusive DIS at Leading Log

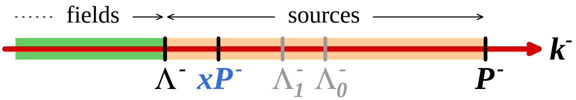
- Iterate the previous process to integrate out all the slow field modes at leading log accuracy:

Inclusive DIS at Leading Log accuracy

$$\sigma_{\gamma^* T} = \int_0^1 dz \int d^2\vec{\boldsymbol{r}}_\perp \left|\psi(\boldsymbol{q}|z,\vec{\boldsymbol{r}}_\perp)\right|^2 \sigma_{\rm dipole}(x,\vec{\boldsymbol{r}}_\perp)$$

$$\sigma_{\rm dipole}(x,\vec{\boldsymbol{r}}_\perp) \equiv 2\int d^2\vec{\boldsymbol{X}}_\perp \int [D\rho]\, W_{xP^-}[\rho]\, \boldsymbol{T}_{\rm LO}(\vec{\boldsymbol{x}}_\perp,\vec{\boldsymbol{y}}_\perp)$$

- One does not need to evolve down to $\Lambda^- \to 0$: the DIS amplitude becomes independent of Λ^- when $\Lambda^- \lesssim xP^-$



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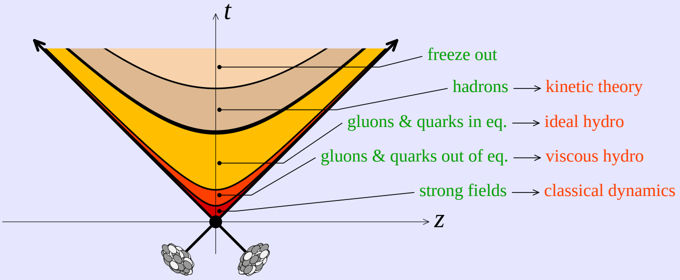
Stages of a nucleus-nucleus collision



- The Color Glass Condensate provides a framework to describe nucleus-nucleus collisions up to a time $\tau \sim {Q_s}^{-1}$

Reminder on hydrodynamics

Equations of hydrodynamics :

$$\partial_\mu T^{\mu\nu} = 0$$

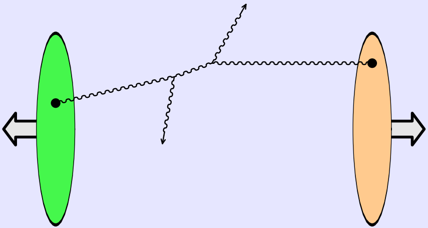
Additional inputs :

$$\text{EoS}: \quad p = f(\epsilon) \quad , \quad \text{Transport coefficients}: \quad \eta, \zeta, \cdots$$

- Required initial conditions : $T^{\mu\nu}(\tau = \tau_0, \eta, \vec{x}_\perp)$

Initial conditions from CGC: power counting

$$\mathcal{L} = -\frac{1}{2}\,\mathrm{tr}\,F_{\mu\nu}F^{\mu\nu} + (J_1^\mu + J_2^\mu)A_\mu$$



- Dilute regime : one parton in each projectile interact

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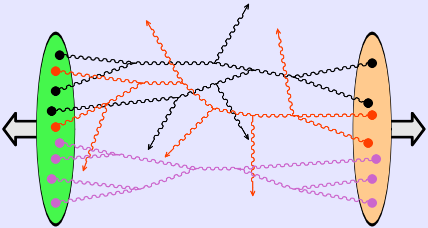

- Dilute regime : one parton in each projectile interact
- Saturated regime : multiparton processes become crucial

Initial conditions from CGC: Leading Order

- In the saturation regime, $\rho_{1,2} \sim g^{-1}$, and we have the following expansion for $T^{\mu\nu}$:

$$T^{\mu\nu} = \frac{Q_s^4}{g^2} \left[c_0 + c_1\, g^2 + c_2\, g^4 + \cdots \right]$$

- The Leading Order contribution is given by classical fields :

$$T^{\mu\nu}_{\rm LO} \equiv c_0 \frac{Q_s^4}{g^2} = \frac{1}{4} g^{\mu\nu}\, \mathcal{F}^{\lambda\sigma}\mathcal{F}_{\lambda\sigma} - \mathcal{F}^{\mu\lambda}\mathcal{F}^{\nu}{}_{\lambda}$$

$$\text{with} \quad \underbrace{\left[\mathcal{D}_\mu, \mathcal{F}^{\mu\nu}\right] = J^\nu}_{\rm Yang-Mills\ equation} \quad , \quad \lim_{t\to-\infty} \mathcal{A}^\mu(t,\vec{\boldsymbol{x}}) = 0$$

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Initial conditions from CGC: Leading Log resummation

- The previous power counting implicitly assumes that the coefficients c_n are numbers of order one. However, large logarithms of the CGC cutoff appear at NLO
- Like in DIS, the coefficients of the logs are given by the action of the JIMWLK Hamiltonian on the LO observable:

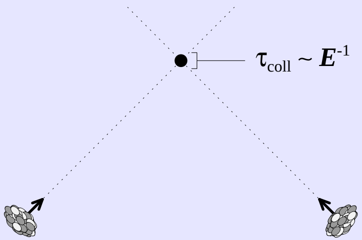
$$\delta T^{\mu\nu}_{\rm NLO} = \left[\ln\left(\frac{\Lambda_0^-}{\Lambda_1^-}\right) \mathcal{H}_1 + \ln\left(\frac{\Lambda_0^+}{\Lambda_1^+}\right) \mathcal{H}_2 \right] T^{\mu\nu}_{\rm LO}$$

- By iterating this process, one arrives at:

$$\left\langle T^{\mu\nu}(\tau,\eta,\vec{\boldsymbol{x}}_\perp)\right\rangle_{\rm LLog} = \int \left[D\rho_1\, D\rho_2\right]\, W_1\left[\rho_1\right]\, W_2\left[\rho_2\right]\, \underbrace{T^{\mu\nu}_{\rm LO}(\tau,\vec{\boldsymbol{x}}_\perp)}_{\text{for fixed } \rho_{1,2}}$$

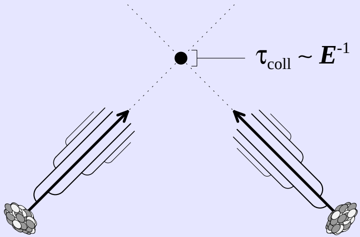
(FG, Lappi, Venugopalan (2008))

Factorization and causality



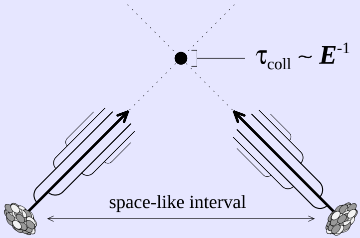
- The duration of the collision is very short: $\tau_{\rm coll} \sim E^{-1}$

Factorization and causality



- The duration of the collision is very short: $\tau_{\rm coll} \sim E^{-1}$
- The logarithms we want to resum arise from the radiation of soft gluons, which takes a long time
  $\triangleright$ it must happen (long) before the collision

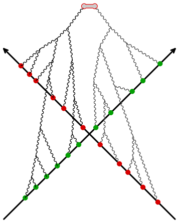
Factorization and causality



- The duration of the collision is very short: $\tau_{\rm coll} \sim E^{-1}$
- The logarithms we want to resum arise from the radiation of soft gluons, which takes a long time
  $\triangleright$ it must happen (long) before the collision
- The projectiles are not in causal contact before the impact
  $\triangleright$ the logarithms are intrinsic properties of the projectiles, independent of the measured observable

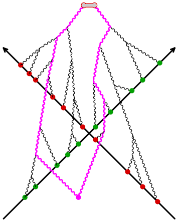
Factorization and causality

$$T^{\mu\nu}_{\rm LO} = \sum_{\rm trees}$$



(all propagators retarded)

$$\delta T^{\mu\nu}_{\rm NLO} = \sum_{\rm trees}$$



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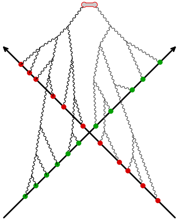

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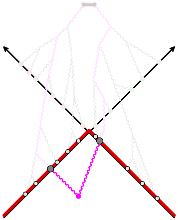
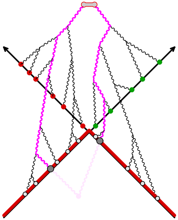


$$\otimes \sum_{\rm trees}$$



$$\underbrace{\qquad\qquad}_{\ln\left(\frac{\Lambda_0^-}{\Lambda_1^-}\right)\mathcal{H}_1+\ln\left(\frac{\Lambda_0^+}{\Lambda_1^+}\right)\mathcal{H}_2} \qquad \underbrace{\qquad\qquad}_{T^{\mu\nu}_{\rm LO}}$$

- Note : this would not work if the graphs were made of Feynman propagators instead of retarded ones

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Correlations in η and $\vec{x}_\perp$

- The factorization valid for $\langle T^{\mu\nu}\rangle$ can be extended to multi-point correlations :

$$\left\langle T^{\mu_1\nu_1}(\tau,\eta_1,\vec{x}_{1\perp})\cdots T^{\mu_n\nu_n}(\tau,\eta_n,\vec{x}_{n\perp})\right\rangle_{\rm LLog} =$$

$$= \int \left[D\rho_1\, D\rho_2\right]\; W_1\left[\rho_1\right]\; W_2\left[\rho_2\right]$$

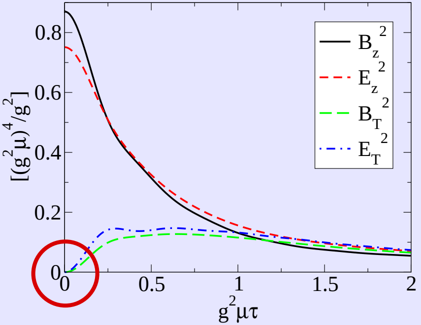
$$\times\; T^{\mu_1\nu_1}_{\rm LO}(\tau,\vec{x}_{1\perp})\cdots T^{\mu_n\nu_n}_{\rm LO}(\tau,\vec{x}_{n\perp})$$

▷ For each $\rho_{1,2}$, solve the Yang-Mills equations to get the classical field $\mathcal{A}^\mu$, then compute $T^{\mu\nu}_{\rm LO}$ from $\mathcal{A}^\mu$. By sampling the distributions $W_{1,2}[\rho_{1,2}]$, one gets all the correlations at leading log accuracy

Initial classical fields, Glasma

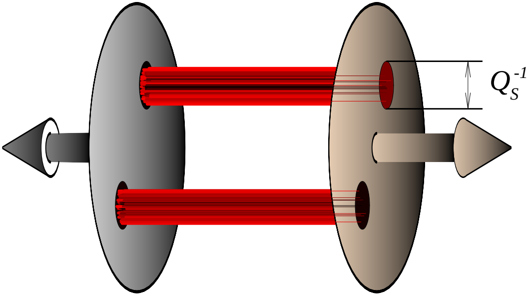
Lappi, McLerran (2006)

- Immediately after the collision, the chromo-$\vec{E}$ and $\vec{B}$ fields are purely longitudinal and boost invariant :



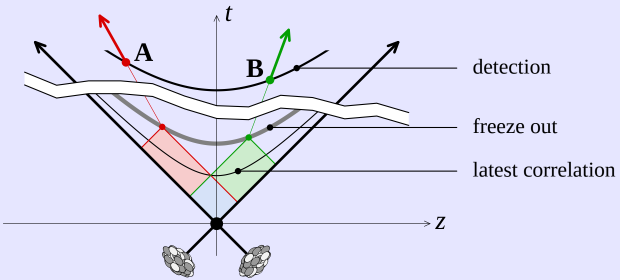
Glasma flux tubes

- The initial chromo-$\vec{E}$ and $\vec{B}$ fields form longitudinal "flux tubes" extending between the projectiles:



- The color correlation length in the transverse plane is Q_S^{-1}
  $\triangleright$ flux tubes of diameter Q_S^{-1}, filling up the transverse area
- The correlation length in the η direction is $\Delta\eta \sim \alpha_s^{-1}$
  $\triangleright$ long range rapidity correlations expected in the data

Importance of initial rapidity correlations



Long range rapidity correlations must be created early

$$t_{\rm correlation} \le t_{\rm freeze\ out}\;\; e^{-\frac{1}{2}|y_A - y_B|}$$

▷ it is impossible to explain the long range η-correlation seen at RHIC by phenomena that occur later than this limit (see R. Venugopalan's talk)

Summary

- Gluon saturation is enhanced in nuclei, and can be reached at higher x (compared to nucleons)
- Saturation plays an important role in the description of the initial stages of nucleus-nucleus collisions
- In the saturated non-linear regime, there exist some universal distributions $W[\rho]$ that describe the dense projectiles both in DIS and AA collisions
  - Resums the logs of $\sqrt{s}$ at leading log accuracy
  - Applies to sufficiently inclusive observables
  - Causality plays an important role in this factorization
  - Ordinary k_t-factorization is broken in AA collisions
- Outstanding issue in AA collisions: the energy-momentum tensor obtained at early times is far from local equilibrium. (How) does thermalization occur?