# The first fermi of a heavy ion collision: progress and open questions

**Raju Venugopalan**
**Brookhaven National Laboratory**

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TIFF (Uncompressed) decompressor
are needed to see this picture.

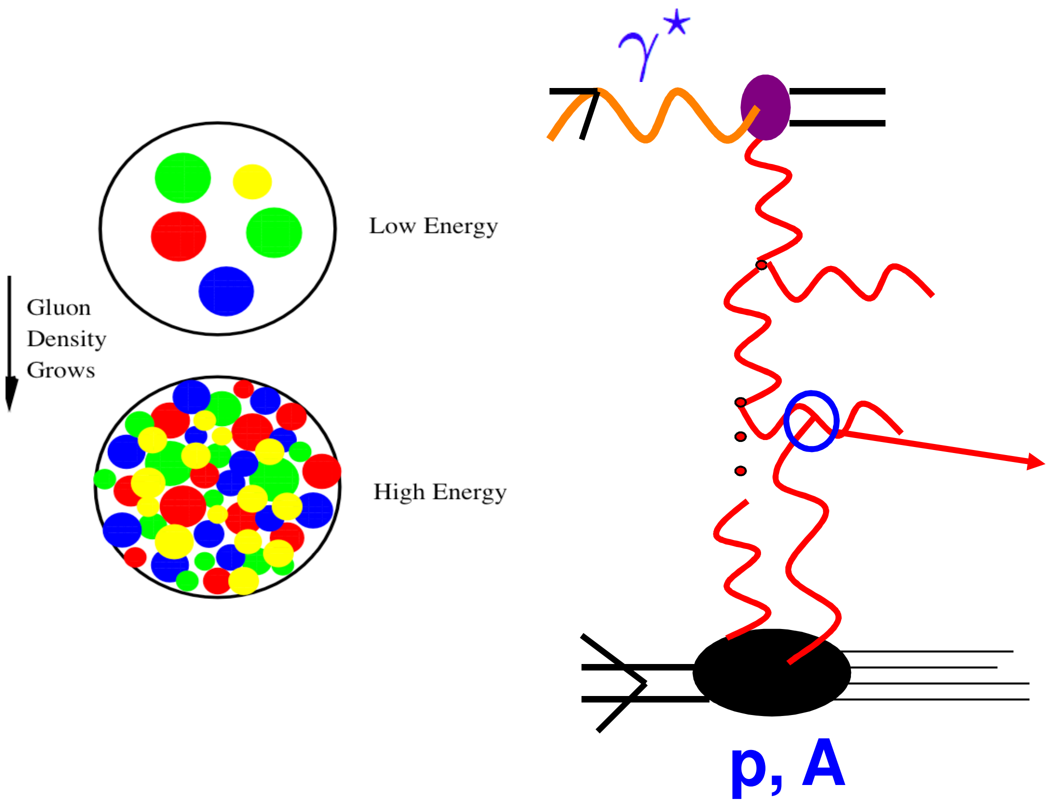
**Fest for Al Mueller's 70th, Columbia Univ. October 23rd-25th, 2009**

# Talk Outline

- High energy QCD as a "many body" theory
- *Ab initio* approach to heavy ion collisions
- Long Range Rapidity Correlations
- The problem of Thermalization

# Gluon saturation in QCD

Gribov,Levin,Ryskin
Mueller,Qiu




Large x - bremsstrahlung
linear evolution (DGLAP/BFKL)

Small x -gluon recombination
non-linear evolution
(BK/JIMWLK)

**Saturation scale $Q_S(x)$ - dynamical scale below which non-linear ("higher twist") QCD dynamics is dominant**

**In IMF, occupation # $f = 1/\alpha_S$ => hadron is a dense, many body system**

Nuclear Physics B307 (1988) 34–60
North-Holland, Amsterdam

# VIRTUAL PAIR CREATION IN A STRONG BREMSSTRAHLUNG FIELD: A QED model for parton saturation

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Received 10 March 1988

Virtual pair creation in a strong, virtual, bremsstrahlung field is considered in QED as a model for parton saturation In a weak field the virtual pair density increases quadratically in the external field, however, at large values of the field the number density becomes independent of the strength of that field A similar effect is found in scalar electrodynamics

## 1. Introduction

At small values of the Bjorken-$x$-variable parton (quark and gluon) number densities are expected to grow rapidly [1] However, when, say, the gluon distribution in a hadron, $xG(x, Q^2)$, reaches a value as large as $Q^2r^2/\alpha$, with $r$ the radius of the hadron, these gluons are so densely packed that one expects scattering and annihilation of partons to become important, thus limiting the ultimate number density to be of the size indicated above [1, 3]

This high density quark-and-gluon system is a most fascinating regime of QCD On the one hand, if $Q^2 \geq 1$ GeV$^2$ the coupling, $\alpha(Q^2)$, is small and the usual non-perturbative condensates are unimportant while, on the other hand, the system is strongly interacting because of the high parton densities. That is, this regime of weak coupling but large numbers of partons is a new regime of QCD Such a high-density parton system occurs in a number of different high-energy processes (i) In deeply inelastic scattering one can directly measure such high-density systems at small $x$ using the virtual photon as a probe [1, 3]. (ii) In the very early stages of a heavy ion collision such a system is produced over a large transverse area [4]. (iii) Two-jet correlations in high-energy reactions can trigger on local hot spots [5], high parton density regions which are smaller than the radius of a normal hadron

So far, it has not been possible to theoretically study this high density, non-equilibrium, regime of QCD directly. Lowest order gluon recombinations have been

[1] Work supported in part by the Department of Energy and NSF Grant PHY82-17853, supplemented by NASA

High energy QCD as a many body system

# Effective Field Theory on Light Front

Susskind
Bardacki-Halpern

Poincare group on LF ⟷ Galilean sub-group of 2D Quantum Mechanics

isomorphism

Eg., LF dispersion relation $P^- = \frac{P_\perp^2}{2P^+}$

Energy ($P^-$), Momentum ($P_\perp^2$), Mass ($2P^+$)

**Large x ($P^+$) modes: static LF (color) sources $\rho^a$**

**Small x ($k^+ << P^+$) modes: dynamical fields $A_\mu^a$**

McLerran, RV

## CGC: Coarse grained many body EFT on LF

$$< P|\mathcal{O}|P > \longrightarrow \int [d\rho^a][dA^{\mu,a}]\, W_{\Lambda^+}[\rho]\, e^{iS_{\Lambda^+}[\rho,A]}\, \mathcal{O}[\rho, A]$$

$W_{\Lambda^+}[\rho]$ non-pert. gauge invariant "density matrix" defined at initial scale $\Lambda_0^+$

RG equations describe evolution of W with x

JIMWLK, BK

# Classical field of a large nucleus

$$< AA >_\rho = \int [d\rho] A_{\rm cl.}(\rho) A_{\rm cl.}(\rho) W_{\Lambda^+}[\rho]$$

**For a large nucleus, A >>1,**

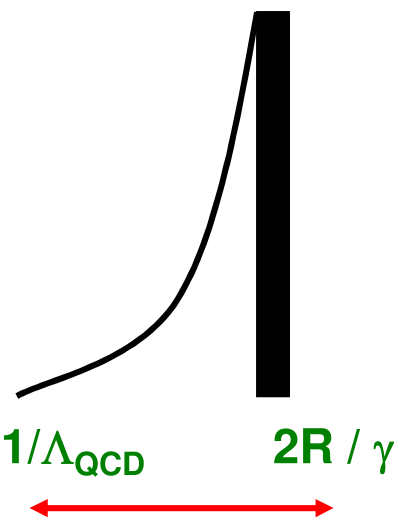
"Pomeron" excitations

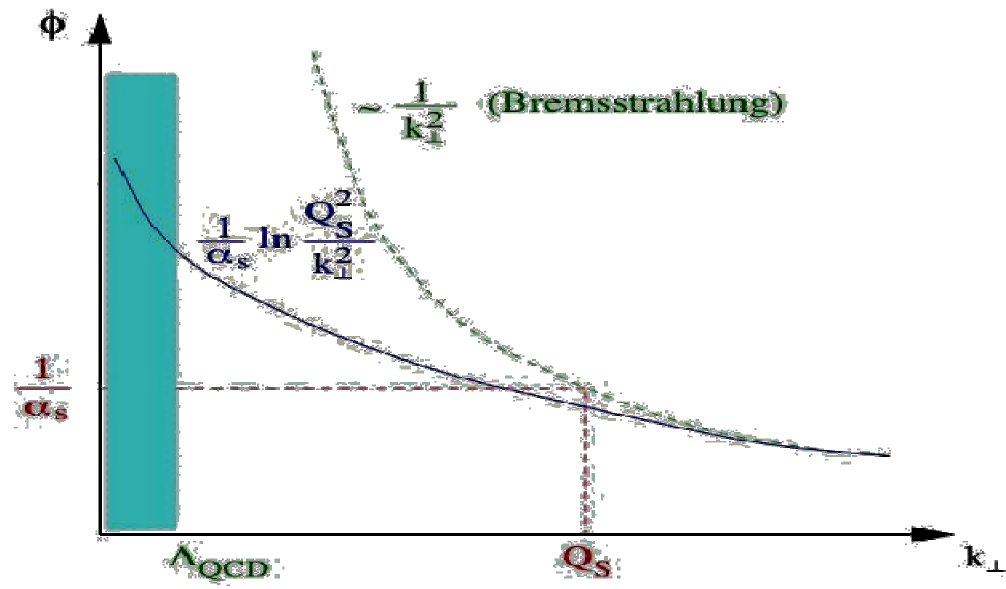
"Odderon" excitations

$$W_{\Lambda^+} = \exp\left(-\int d^2x_\perp \left[\frac{\rho^a\rho^a}{2\,\mu_A^2} - \frac{d_{abc}\,\rho^a\rho^b\rho^c}{\kappa_A}\right]\right)$$

McLerran,RV
Kovchegov
Jeon, RV

$A_{cl}$ from $\longrightarrow$ $(D_\mu F^{\mu\nu})^a = J^{\nu,a} \equiv \delta^{\nu +}\,\delta(x^-)\,\rho^a(x_\perp)$



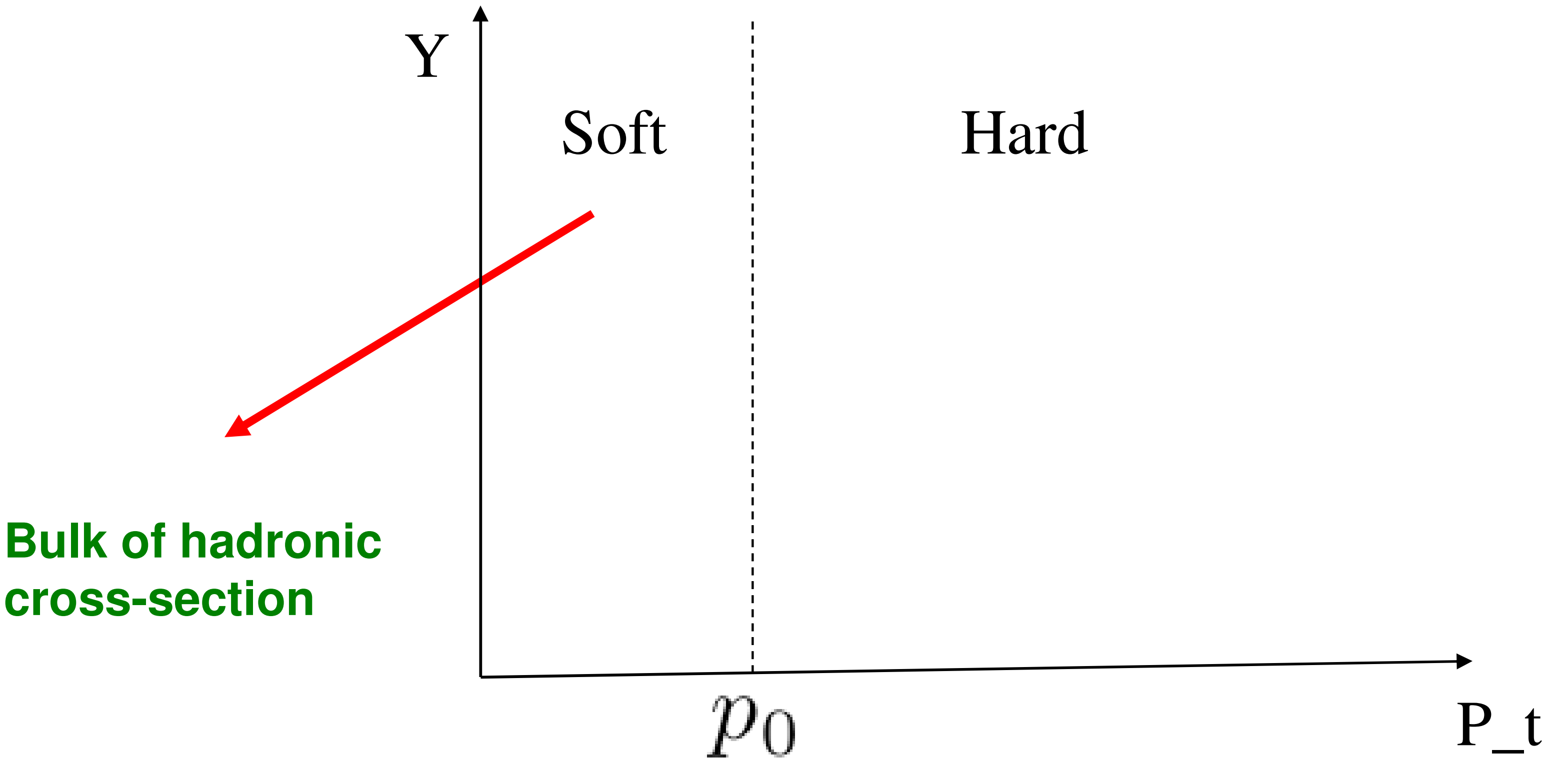




**Wee parton dist. :** $\dfrac{1}{\Lambda_{\rm QCD}} e^{-\lambda \Delta Y/2}$

determined from RG

# What does a heavy ion collision look like ?

- **Traditional view of heavy ion collisions**



# What does a heavy ion collision look like ?

D. Nucleus-Nucleus Collisions at Fantastic Energies

Before leaving this subject it is fun to consider the collision of two nuclei at energies sufficiently high so that in addition to the fragmentation regions, a central plateau region can develop. Let us consider a central collision of a relatively small nucleus, say carbon, with a big one, say lead. Let us look at this collision in a center-of-mass frame for which the rapidities of both of the nucleus projectiles exceeds the critical rapidity. In such a frame they both possess the fur coat of wee-parton vacuum fluctuations. In such a central collision we see that the collision initially occurs between the fur of wee partons in each of the projectiles. Therefore the number of independent collisions will be of order of the area of overlap of the two projectiles; namely the cross-sectional area of the smaller nucleus.

**ract and produce**

idity distribution which is shown in Fig. 9. Much more professional studies ng the same line of initial assumptions can be found in the work of Kancheli,[32] E. Lehman and G. Winbow,[33] J. Koplik and A. Mueller,[34] and Goldhaber.[35]

**Bj, DESY lectures (1975)**

Nuclear Physics B289 (1987) 847–860
North-Holland, Amsterdam

# THE EARLY STAGE OF ULTRA-RELATIVISTIC HEAVY ION COLLISIONS

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Received 20 February 1987

We investigate the properties of the system of partons produced in the very beginning of ultra-relativistic heavy ion collisions. We propose simple criteria for characterizing the partons which get freed during the collision and which give the dominant contribution to the initial energy density. These partons are found to have an average transverse momentum which grows with the size of the colliding nuclei. Numerical estimates of their initial energy density are given.

In order to get numerical estimates, let's take $A^{1/3} = 6$, $R = 1.2A^{1/3}$ fm, $xG = 3$ and $\alpha = \frac{1}{3}$, i.e. $\alpha C_A = 1$ (the value $xG = 3$ is reasonable, even traditional, but at this time it is not a well determined quantity, experimentally). Then eq. (3.15) gives $p_T \approx 0.94$ GeV, i.e. $\tau_0 \sim 0.2$ fm/$c$, and one finds:

$$\frac{dN}{dy} \approx 1300\,, \tag{4.3a}$$

$$\frac{dE_T}{dy} \approx 1.2\ \text{TeV}\,, \tag{4.3b}$$

$$n \approx 37/\text{fm}^3\,, \tag{4.3c}$$

$$\epsilon \approx 35\ \text{GeV}/\text{fm}^3\,. \tag{4.3d}$$

**First estimate in saturation framework**

# What does a heavy ion collision look like ?

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are needed to see this picture.

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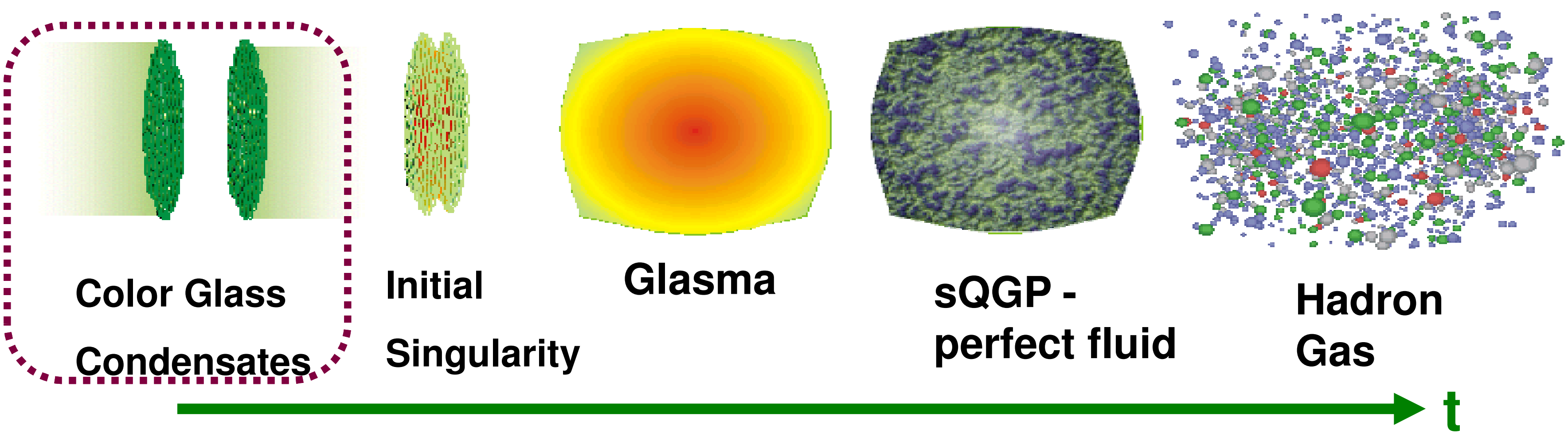

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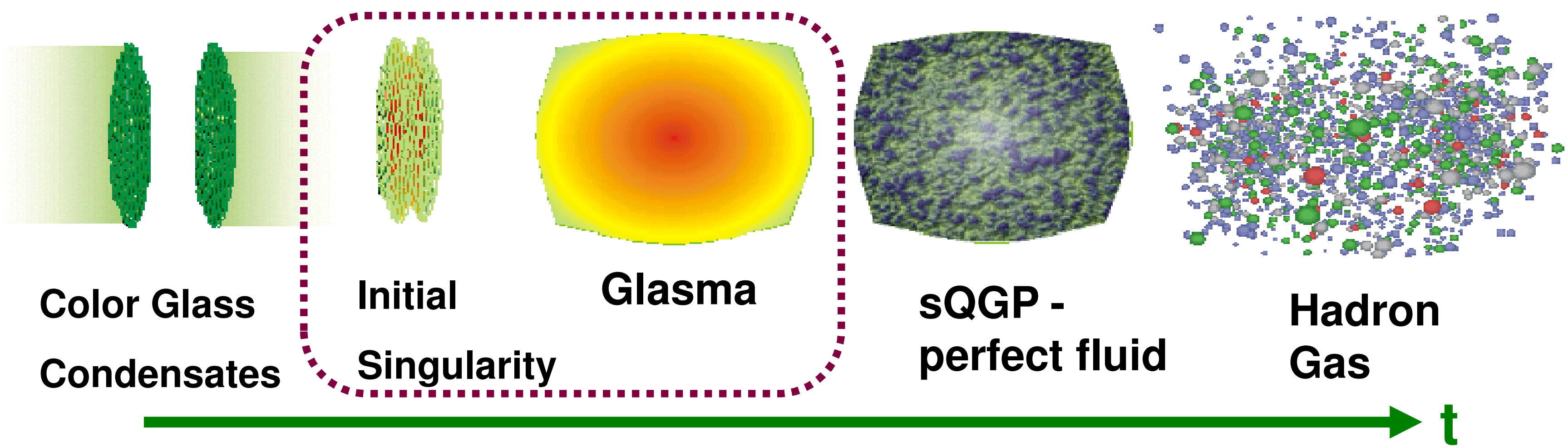
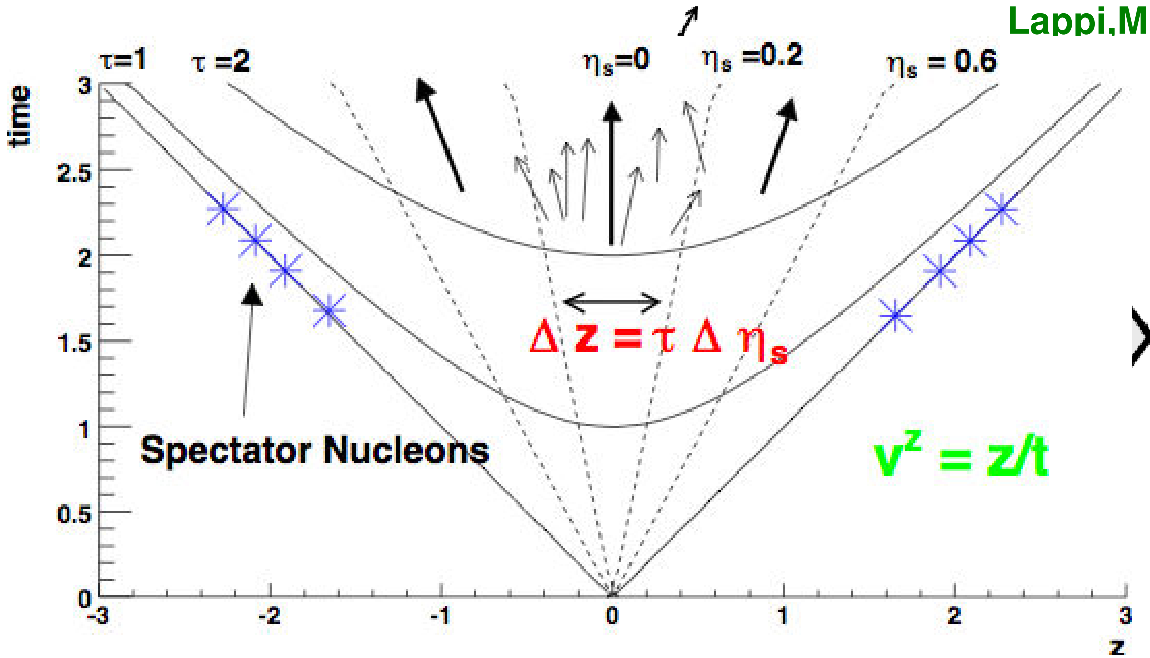


# Forming a Glasma in the little Bang

**Glasma (\Glahs-maa\):** ***Noun:*** **non-equilibrium matter between Color Glass Condensate (CGC)& Quark Gluon Plasma (QGP)**

Lappi,McLerran (2006)




- **Problem: Compute particle production in QCD with** ***strong time dependent*** **sources**
- **Solution: for early times ($t \leq 1/Q_S$) -- n-gluon production computed in A+A to all orders in pert. theory to leading log accuracy**

Gelis, Lappi, RV; arXiv : 0804.2630; 0807.1306; 0810.4829

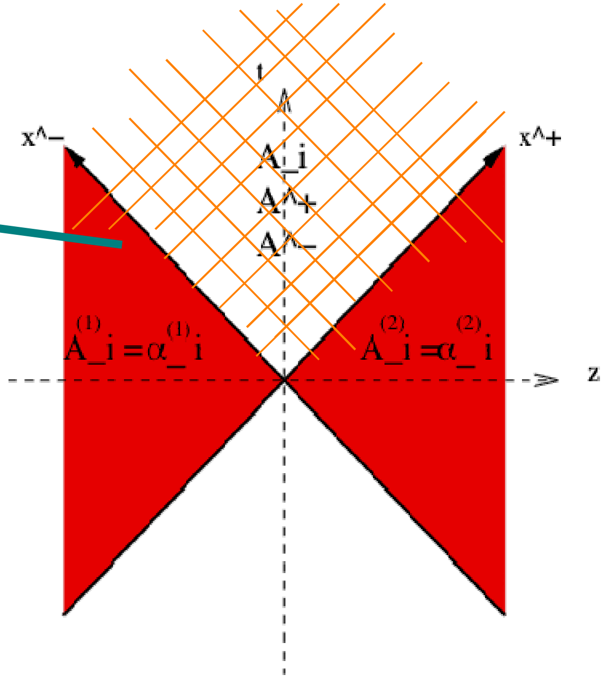
# The Glasma at LO:Yang-Mills eqns. for two nuclei

(=O($1/g^2$) and all orders in $(g\rho)^n$ )

$$D_\mu F^{\mu\nu,a} = \delta^{\nu+}\rho_1^a(x_\perp)\delta(x^-) + \delta^{\nu-}\rho_2^a(x_\perp)\delta(x^+)$$

**Glasma initial conditions from matching classical CGC wave-fns on light cone**

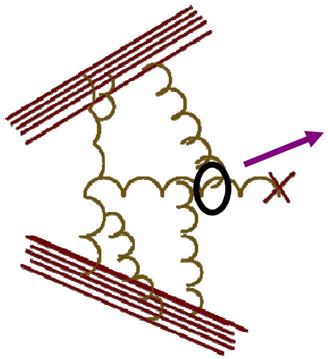
Kovner,McLerran,Weigert




Sources become *time dependent*
after collision:
field theory formalism--
particle production in strong external fields
(e.g., Schwinger mechanism of e+e- production in strong QED fields).

# Numerical Simulations of classical Glasma fields

Krasnitz, Nara, RV
Lappi



All such diagrams
of order O(1/g)

**LO Glasma fields are boost invariant**

$$E_p \frac{d\langle n\rangle_{LO}}{d^3p} = \frac{1}{16\pi^3} \lim_{x^0,y^0\to\infty} \int d^3x\, d^3y\, e^{ip\cdot(x-y)} (\partial_{x^0} - iE_p)(\partial_{y^0} + iE_p) \times \sum_{\text{phys.}\Lambda} \varepsilon_\mu^\lambda(p) \varepsilon^{\star\lambda}_\nu(p) A_a^\mu(x) A_c^\nu(y)$$

$$\frac{1}{\pi R^2}\frac{dN}{dy} = c_N \frac{C_F Q_S^2}{2\pi^2 \alpha_S}$$

**with "gluon liberation coefficient"**

(A.H. Mueller, hep-ph/9906322; Krasnitz, RV, hep-ph/0007108)

$c_N \sim 1.1$ Lappi, arXiv:0711.3039

$$\varepsilon \approx 20 - 40\,\mathrm{GeV/fm^3} \text{ at } \tau \sim 0.3 \text{ fm}$$

**for** $Q_S^A \approx 1 - 1.2\,\mathrm{GeV}$

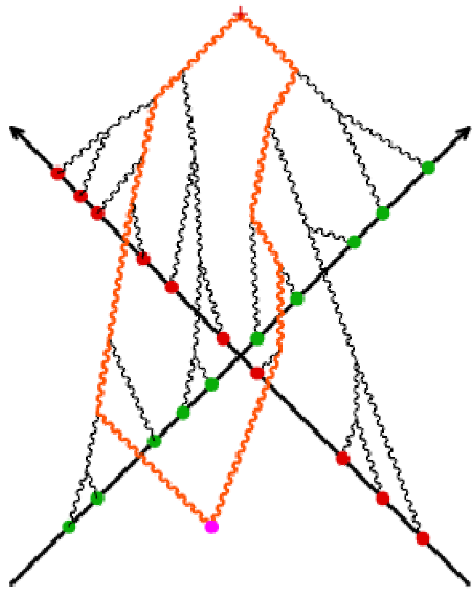
from extrapolating DIS data to RHIC energies

# Multiplicity to NLO (=O(1) in g and all orders in $(g\rho)^n$ )

**Schwinger-Keldysh formalism**

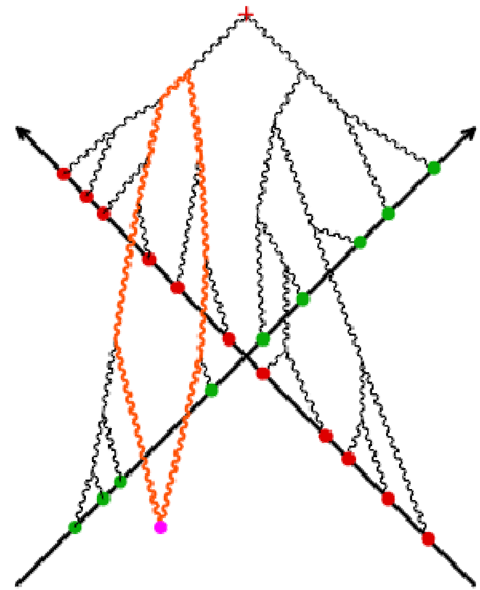

Gelis, RV (2006)

$$\langle n \rangle_{\rm NLO} =$$



Gluon pair production

**+**



One loop contribution to classical field

***Initial value problem*** **with retarded boundary conditions**
**- can be solved on a lattice in real time**

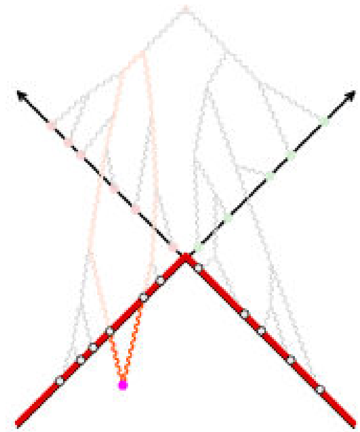
(*a la* Gelis,Kajantie,Lappi for Fermion pair production)

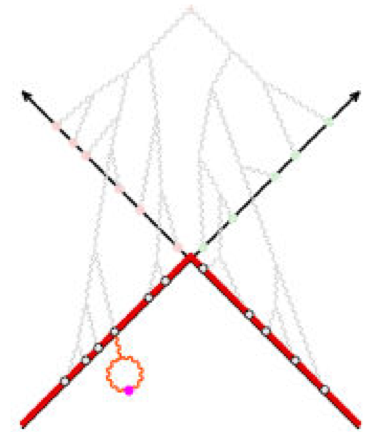
# RG evolution for 2 nuclei

Gelis,Lappi,RV (2008)

**(talk by F. Gelis)**

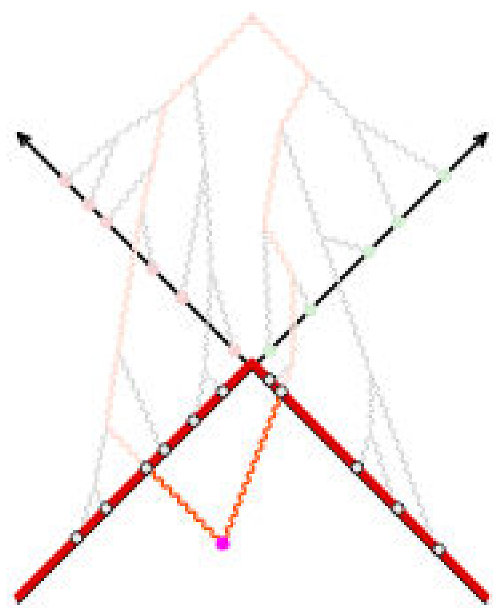
**Log divergent contributions crossing nucleus 1 or 2:**





**Contributions across both nuclei are finite-no log divergences**

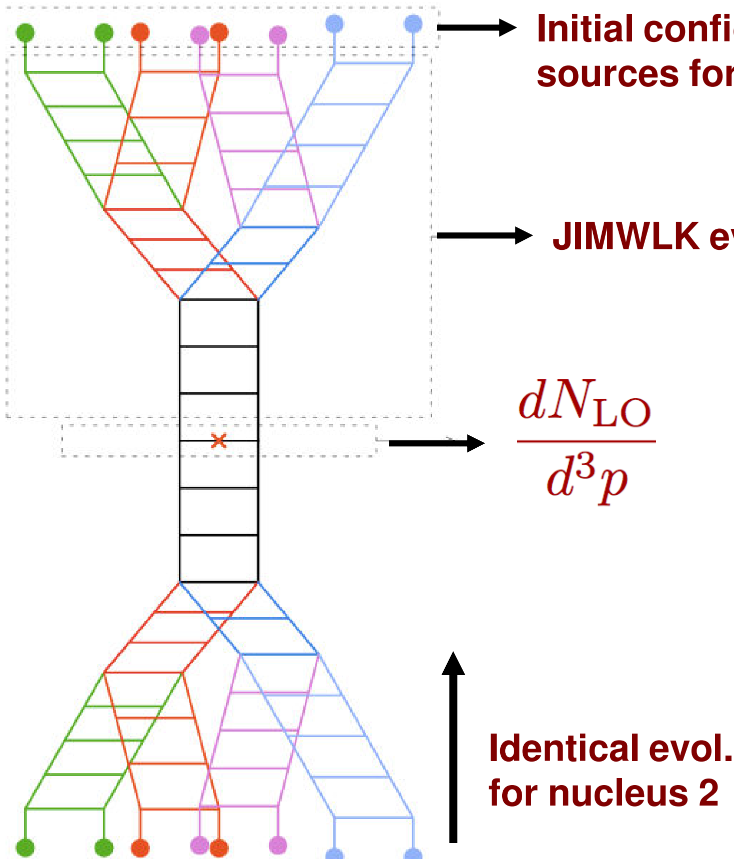
**=> factorization**



# High energy factorization for inclusive multi-gluon production in A+A collisions

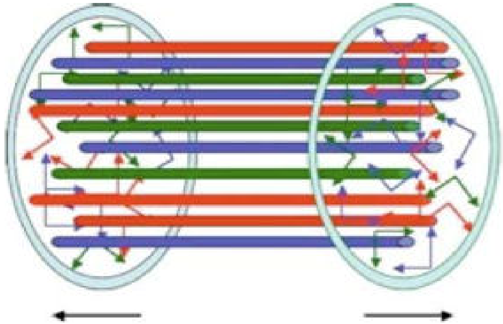
Gelis,Lappi,RV
arXiv:0804.2630 [hep-ph];
arXiv:0807.1306 [hep-ph]
arXiv:0810.4829 [hep-ph]

## Multiplicity distribution



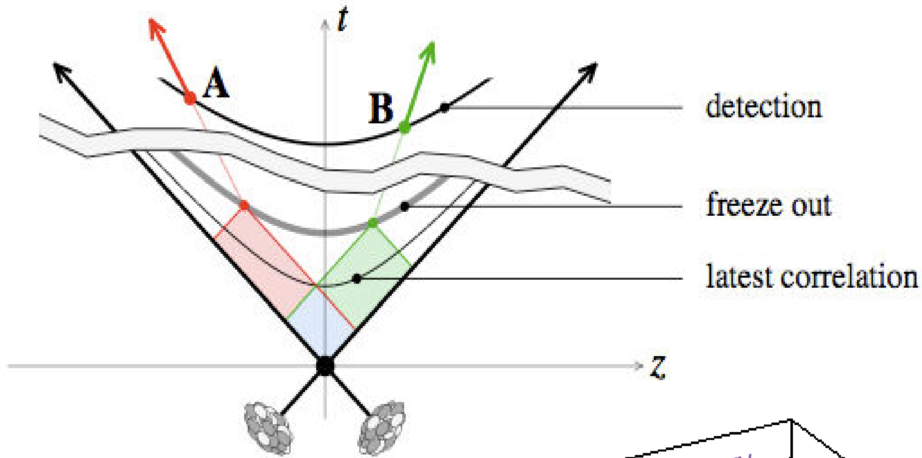

## Basis of Glasma flux tube picture

Dumitru, Gelis ,McLerran, RV, arXiv:0804.3858[hep-ph]



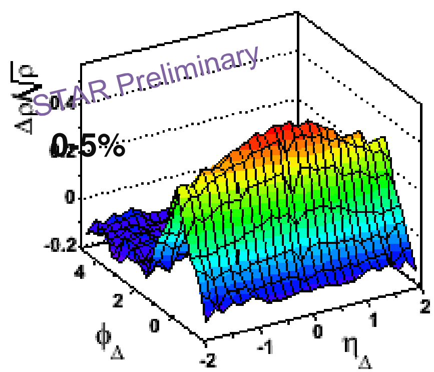
(McLerran talk)

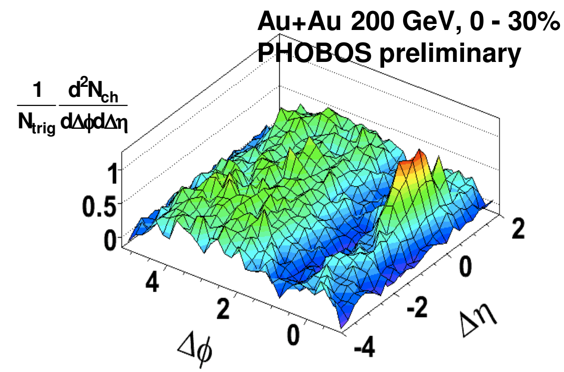
# Long range A+A rapidity correlations



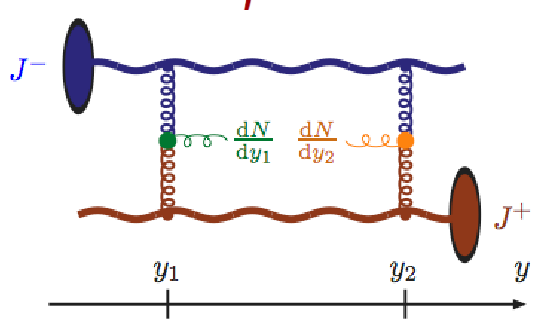

$$\tau \leq \tau_{\text{freeze-out}} \exp\left(-\frac{1}{2}|y_A - y_B|\right)$$

**τ < 1 fm for Δy > 4**



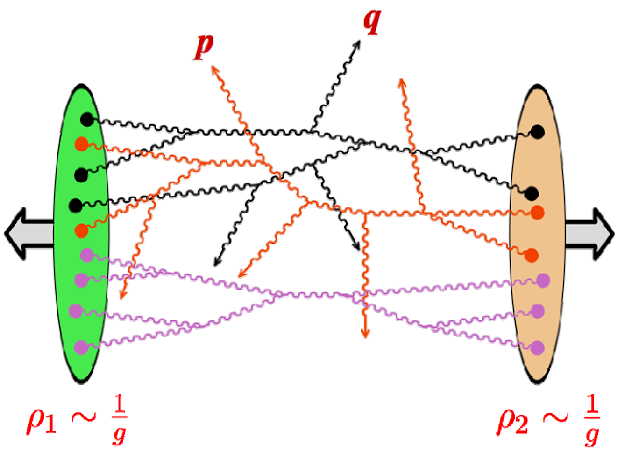

$$\langle \mathcal{O} \rangle_{\text{LLog}} = \int [d\Omega_1(y, x_\perp)]\, [d\Omega_2(y, x_\perp)]\, W[\Omega_1(y, x_\perp)] W[\Omega_2(y, x_\perp)] \mathcal{O}_{\text{LO}}$$



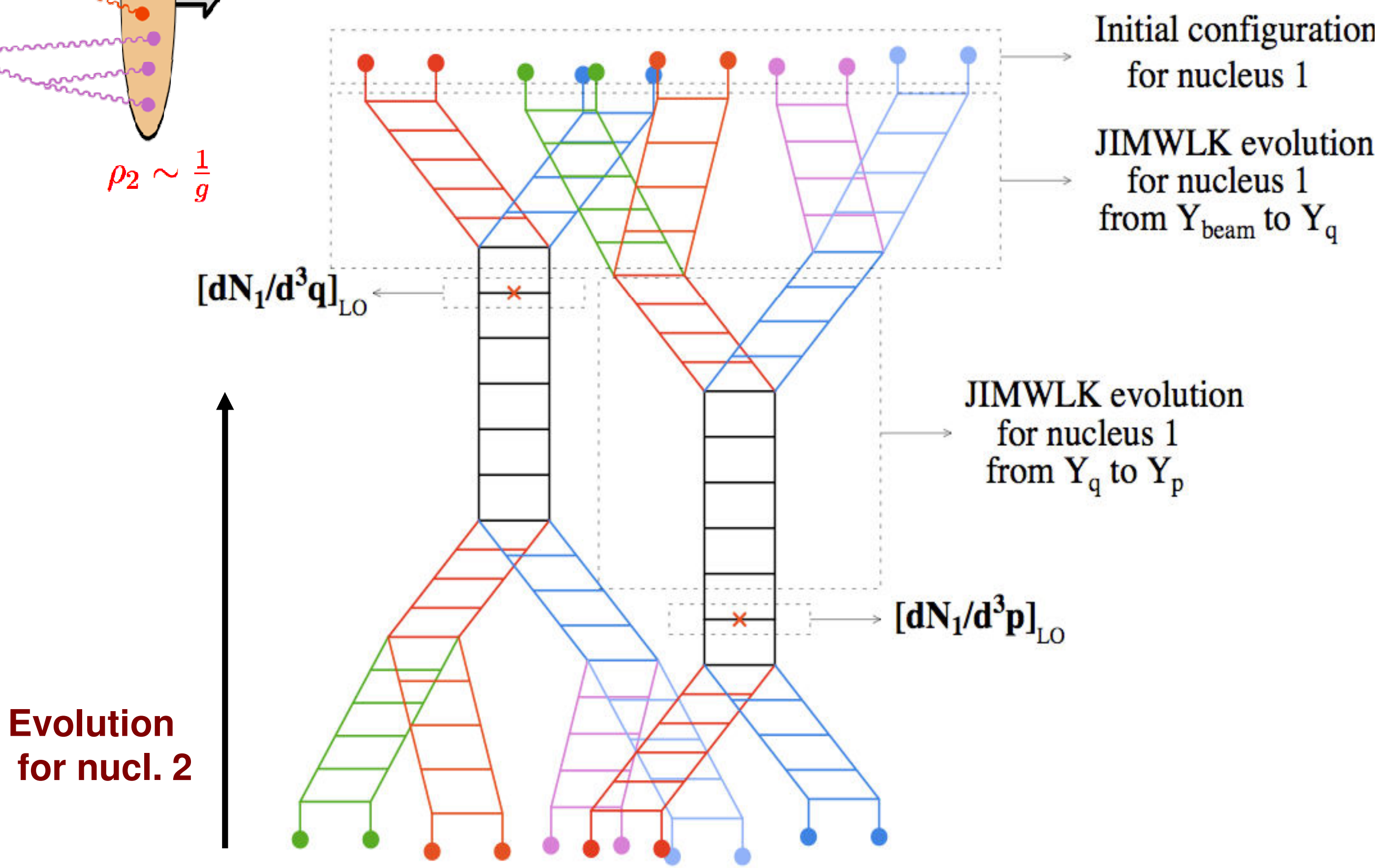

**Master formula encodes information about multi-particle correlations at all rapidities**

Blaizot,Iancu,Weigert

# Two particle inclusive distribution




Gelis,Lappi,RV



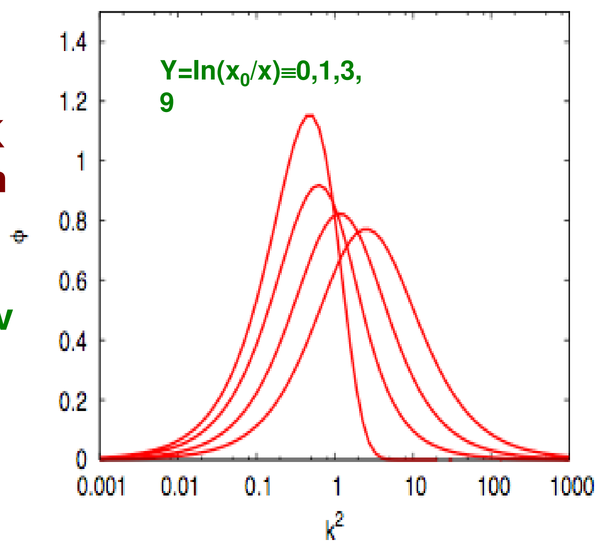
# Two particle inclusive distribution:JIMWLK-> BK

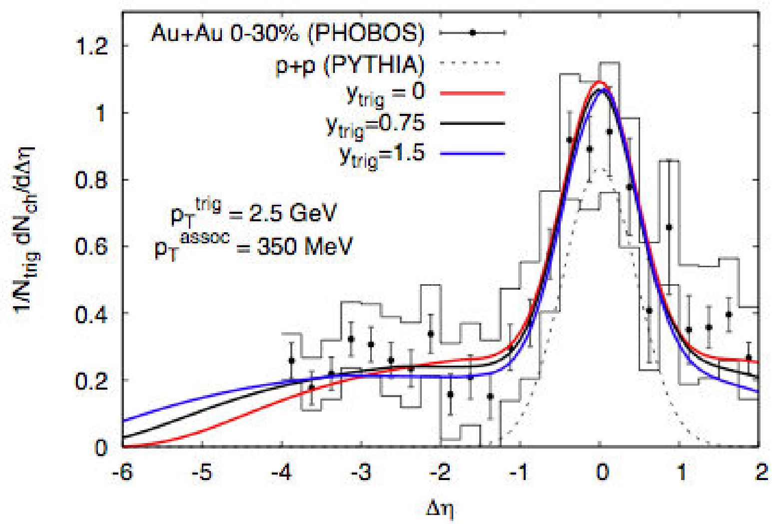
Dusling,Gelis,Lappi,RV

$$\frac{dN_2}{dy_p dy_q d^2q d^2q}\Big|_{\rm corr.} \propto \int d^2k_\perp \Big( \phi^2_{A1}(y_p,|k_\perp|)\phi_{A2}(y_p,|p_\perp - k_\perp|)\Big[\phi_{A2}(y_q,|q_\perp + k_\perp|) + \phi_{A_2}(y_q,|q_\perp - k_\perp|)\Big] + \phi^2_{A2}(y_q,k_\perp)\phi_{A1}(y_p,p_\perp - k_\perp)\Big[\phi_{A1}(y_q,|q_\perp + k_\perp|) + \phi_{A1}(y_q,|q_\perp - k_\perp|)\Big]\Big)$$

**unintegrated gluon dist. from NLL BK RG evolution**

Albacete,Kovchegov

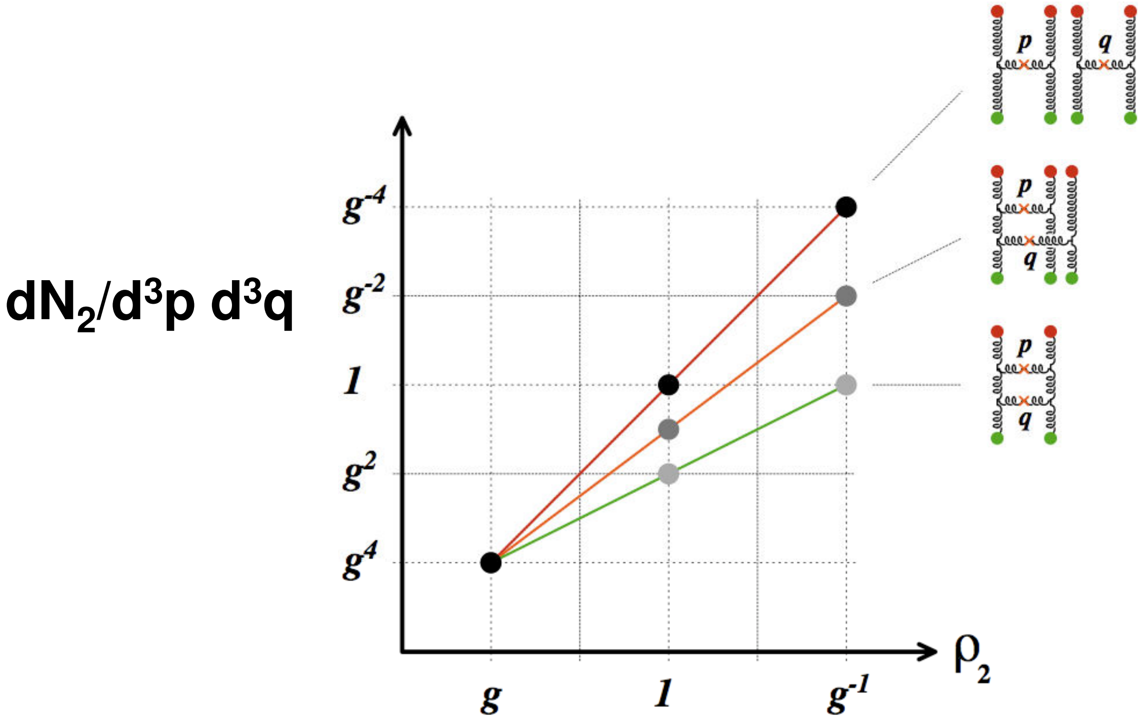



Dusling et al.

# A+A collisions are simpler for n>1 correlations

Gelis,Lappi,RV




**AA:** $\rho_1 \sim \frac{1}{g}\,;\ \rho_2 \sim \frac{1}{g}$ **pA:** $\rho_1 \sim \frac{1}{g}\,;\ \rho_2 \sim g$

**More diagrams even at LO in pA relative to AA**
**At NLO: AA has only "pomeron merging" contributions**
**pA has both merging + splitting contributions**

pLoops: Jalilian-Marian, Kovchegov; Iancu, Triantafyllopolous;
Mueller, Shoshi,Wong; Kovner,Lublinsky,...

# *Time line after a heavy ion collision…*

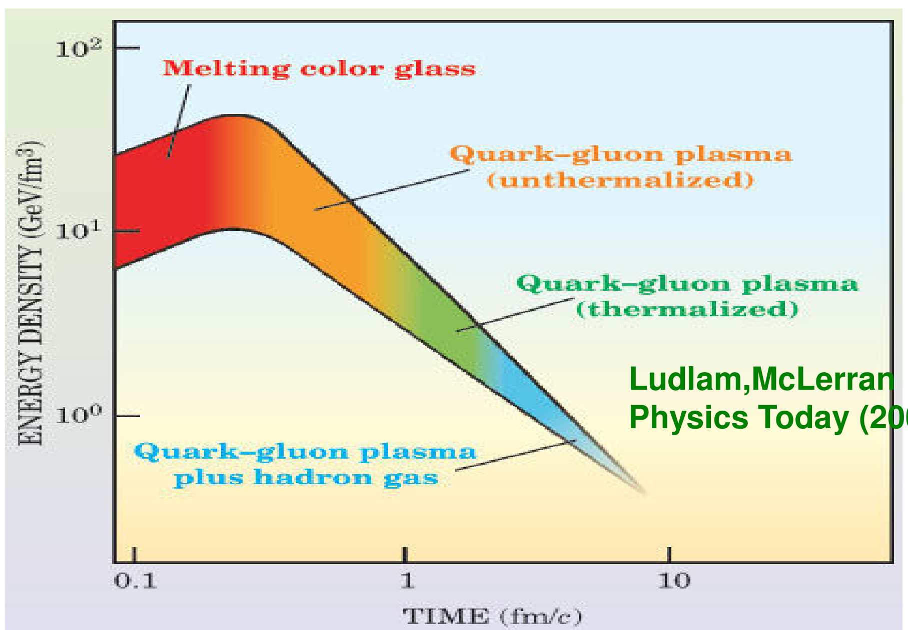
**Classical field** | **Classical field / Particle** | **Particle** → $\tau$

$$f \sim \frac{1}{\alpha_S}$$

$$1 \leq f < \frac{1}{\alpha_S}$$

$$f < 1$$

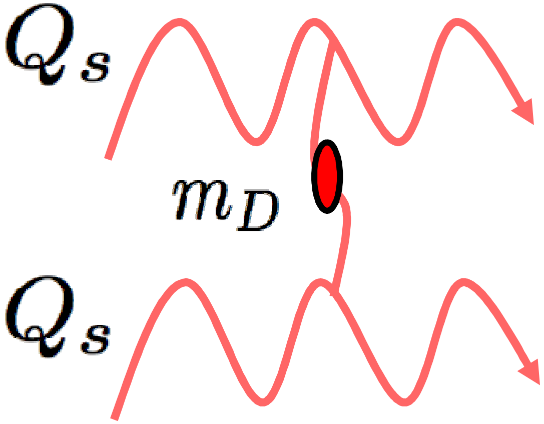
Mueller,Son
Gelis,Jeon,RV




Ludlam,McLerran
Physics Today (2003)

# The “bottom up” scenario

Baier, Mueller, Schiff, Son




Scale for scattering of produced gluons (for t > 1/Q_s) set by

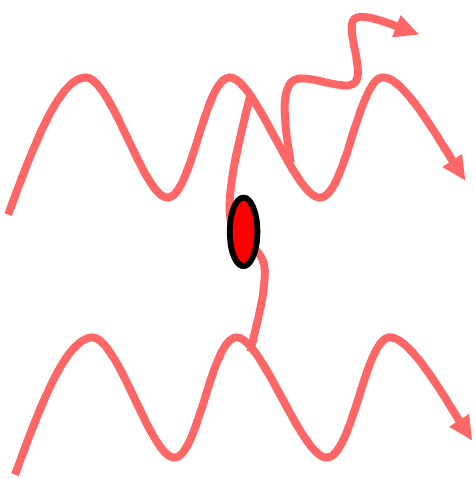
$$m_D^2 \propto g^2 \int_{\vec{p}} \frac{f_{\mathrm{hard}}}{p}$$

**Multiple collisions:** $p_z^2 = N_{\mathrm{coll.}}\, m_D^2$

$$=> p_z \sim \frac{Q_s}{(Q_s \tau)^{1/3}}$$

Occupation # $f = \frac{1}{p_z Q_s^2} \frac{Q_s^3}{\alpha_S(Q_s \tau)}$

$$f < 1 \text{ for } \tau > \frac{1}{\alpha_S^{3/2}} \frac{1}{Q_s}$$



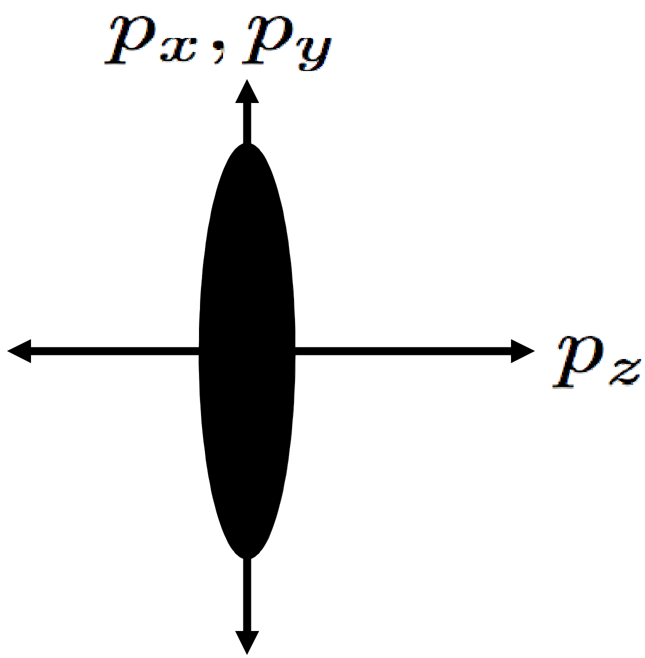
**Radiation of soft gluons important**

for $f \geq \frac{1}{\alpha_S^{5/2}} \frac{1}{Q_s}$

**Thermalization for:** $\tau_{\text{therm.}} \sim \frac{1}{\alpha_S^{13/5}} \frac{1}{Q_s}$ **and** $T_i \sim \alpha_S{}^{2/5} Q_s$

# Weibel instabilities…

Mrowczynski
Arnold,Lenaghan,Moore,Yaffe;
Rebhan, Romatschke, Strickland; …




Anisotropic momentum distributions of hard modes cause $m_D^2 < 0$

-**exponential growth of soft field modes**

$$\left[(-\omega^2 + q^2) g^{\mu\nu} - Q^\mu Q^\nu + \Pi^{\mu\nu}(\omega, q)\right] A_\nu = 0$$

**Changes sign for anisotropic distributions**

## Effective potential interpretation:

$$V[A(\vec{n}\cdot\vec{x})] = \int d^3x \left[\frac{1}{4}F^a_{ij}F^a_{ij} + \frac{1}{2}A^a_i \Pi_{ij}(0,\vec{n}) A^a_j\right]$$

**-ve eigenvalue => potential unbounded from below**

Large magnetic fields can cause O(1) change in hard particle trajectories on short time scales - $1/\gamma \sim 1/m_D$

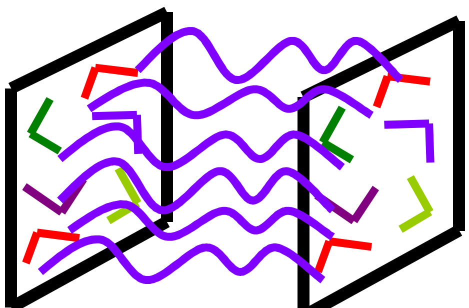
**- possible mechanism for isotropization of hard modes**

# From Glasma to Plasma

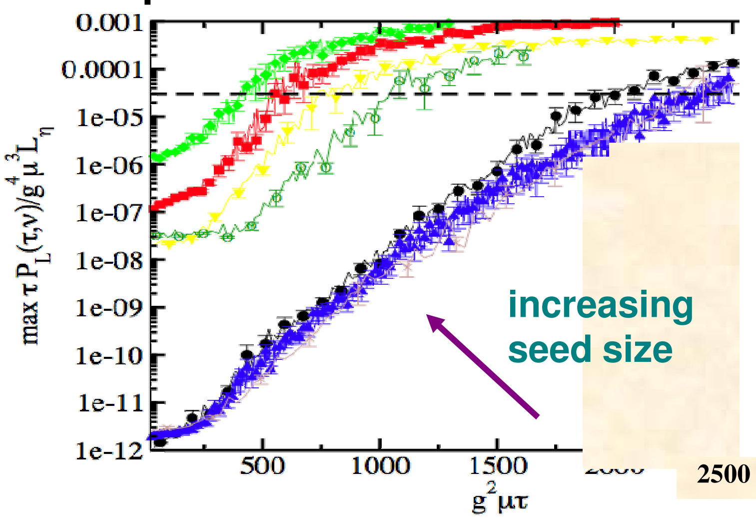
- **NLO factorization formula:**

$$\frac{dN_{\mathrm{LO+NLO}}}{dY\,d^2p_\perp} = \int [D\rho_1][D\rho_2] Z_{\mathrm{Y_{beam}-Y}}[\rho_1] Z_{\mathrm{Y_{beam}+Y}}[\rho_2]$$
$$\times \int [Da(u)] \tilde{G}[a] \frac{dN_{\mathrm{LO}}[A_{\mathrm{cl.}} + a(u)]}{dY\,d^2p_\perp}\Bigg|_{\rho_1,\rho_2}$$

**"Holy Grail" spectrum of small fluctuations.**



**Quant. Fluct. grow exponentially after collision**




Romatschke,RV
Fukushima,Gelis,McLerran
Gelis,Lappi,RV

- **With spectrum, can compute $T_{\mu\nu}$ - and match to hydro/kinetic theory--many subtle issues here…**

**Happy Birthday Al !**
**I wish you many more summers in Paris…**
**and equally many years of rich and**
**exciting physics insights!**