AlFest

From Particles and Partons to Nuclei and Fields

October 23–25, 2009 Columbia University

On Anomalous Quark Triangles

Arkady Vainshtein

William I. Fine Theoretical Physics Institute
University of Minnesota

The Axial Anomaly at Finite Temperature

Itoyama and Mueller, Nucl. Phys. B **218**, 349 (1983)

The Role of the Axial Anomaly in Measuring Spin Dependent Parton Distributions Carlitz, Collins and Mueller, Phys. Lett. B 214, 229 (1988)

Landau Levels and the Partonic Interpretation of the Axial Anomaly

Mueller, Phys. Lett. B **234**, 517 (1990)

Quark triangles show up in the muon anomalous magnetic moment

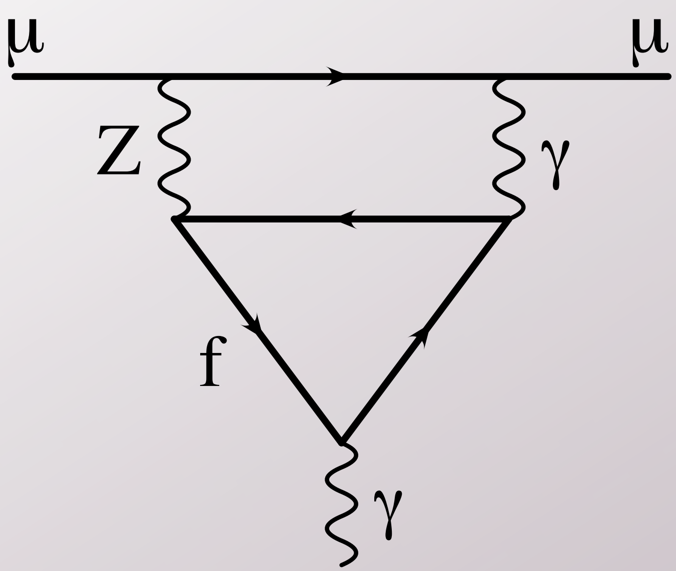
Refinements in electroweak contributions to the muon anomalous magnetic moment
Czarnecki, Marciano and AV, Phys. Rev. D 67, 073006 (2003)

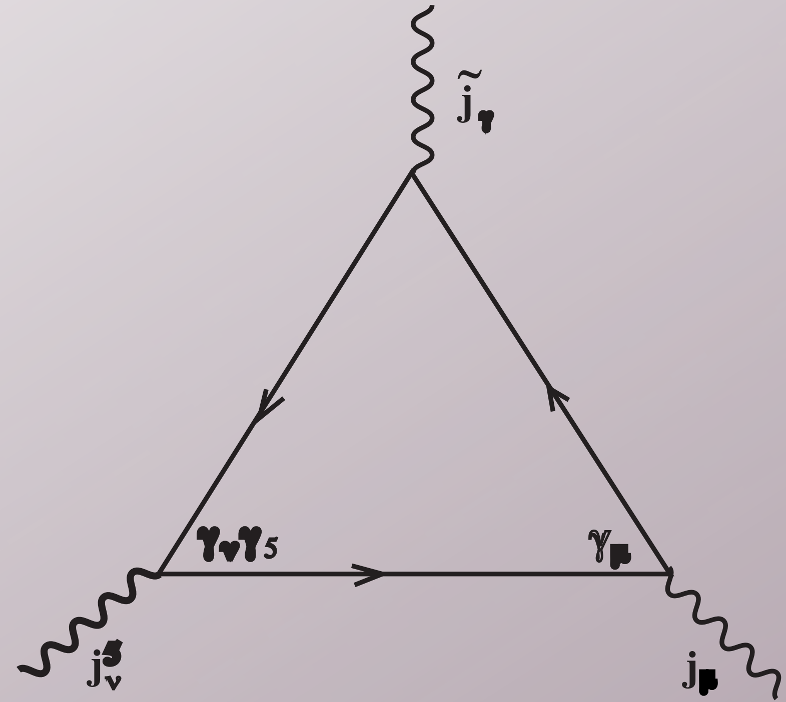
Perturbative and nonperturbative renormalization of anomalous quark triangles
AV, Phys. Lett. B 569, 187 (2003)

Triangle anomaly and the muon g-2 Czarnecki, Marciano and AV, Acta Phys. Polon. B 34, 5669 (2003)

Hadronic light-by-light scattering contribution to the muon anomalous magnetic moment revisited Melnikov and AV, Phys. Rev. D **70**, 113006 (2004)

Perturbative calculations





$$j_\mu = \bar{q}\, V \gamma_\mu q, \qquad j^5_\nu = \bar{q}\, A \gamma_\nu \gamma_5 q \qquad \tilde{j}_\gamma = \bar{q}\, \widetilde{V} \gamma_\mu q$$

$$T_{\mu\gamma\nu} = -\int \mathrm{d}^4x \mathrm{d}^4y \, \mathrm{e}^{iqx - iky} \, \langle 0 | \, T\{ j_\mu(x) \, \tilde{j}_\gamma(y) \, j^5_\nu(0) \} | 0 \rangle$$

Soft photon

$$T_{\mu\nu} = T_{\mu\gamma\nu} e^\gamma(k) = i \int \mathrm{d}^4x \, \mathrm{e}^{iqx} \, \langle 0 | \, T\{ j_\mu(x) \, j^5_\nu(0) \} | \, \gamma(k) \rangle$$

A general form of $T_{\mu\nu}$ contains two Lorentz structures

$$T_{\mu\nu} = -\frac{i}{4\pi^2}\left[w_T(q^2)\left(-q^2\tilde{f}_{\mu\nu} + q_\mu q^\sigma \tilde{f}_{\sigma\nu} - q_\nu q^\sigma \tilde{f}_{\sigma\mu}\right) + w_L(q^2)\, q_\nu q^\sigma \tilde{f}_{\sigma\mu}\right]$$

$$\tilde{f}_{\mu\nu} = \frac{1}{2}\,\epsilon_{\mu\nu\gamma\delta} f^{\gamma\delta}\,, \qquad f_{\mu\nu} = k_\mu e_\nu - k_\nu e_\mu$$

Rosenberg '63, Bell-Jackiw '69, Adler '69 calculated 1 loop

$$w_L^{1-\mathrm{loop}} = 2\, w_T^{1-\mathrm{loop}} = 2N_c \,\mathrm{Tr}\, A\, V\, \widetilde{V} \int_0^1 \frac{\mathrm{d}\alpha\, \alpha(1-\alpha)}{\alpha(1-\alpha)Q^2 + m^2} \qquad Q^2 = -q^2$$

In the chiral limit $m = 0$

$$w_L^{1-\mathrm{loop}}[m=0] = 2\, w_T^{1-\mathrm{loop}}[m=0] = \frac{2N_c\, \mathrm{Tr}\,(A\, V\, \widetilde{V})}{Q^2}$$

Nonvanishing longitudinal part represents ABJ anomaly

$$q^\nu T_{\mu\nu} = \frac{i}{4\pi^2}\, Q^2 w_L\, q^\sigma \tilde{f}_{\sigma\mu} = \frac{i}{2\pi^2}\, N_c\, \mathrm{Tr}\,(A\, V\, \widetilde{V})\, q^\sigma \tilde{f}_{\sigma\mu}$$

Nonrenormalization, Adler-Bardeen theorem, implies that w_L stays intact when gluon interaction is switched on.

Nonrenormalization theorem for the transversal part

The relation

$$w_L[m=0] = 2\, w_T[m=0]$$

valid at the one-loop level gets no perturbative corrections. At $m=0$ the diagrams are symmetric under permutation $\mu \leftrightarrow \nu$, indices of the vector and axial currents.

For the symmetry to hold it is important that the part $q^2 \tilde{f}_{\mu\nu}$ produces just a constant in q term in $T_{\mu\nu}$. The singular part is symmetric and the constant term is fixed by the conservation of the vector current. (Independence on q is an alternative derivation of the AB theorem).

If Pauli-Vilars regularization is used to provide the vector current conservation then the antisymmetric part comes just from regulators.

Thus, the crossing symmetry relates the transversal and longitudinal parts and the AB theorem on the absence of perturbative corrections works for both.

For a general kinematics the relation was found in
‘2004 Knecht, Peris, Perrottet and E. de Rafael and checked in
‘2006 Jegerlehner and Tarasov .

What about nonperturbative corrections?
None in the longitudinal part (‘t Hooft consistency condition) should present in the transversal part -- there is no massless spin one states.

Nonperturbative effects and OPE

$$\hat{T}_{\mu\nu} \equiv i \int \mathrm{d}^4x \, \mathrm{e}^{iqx} \, T\{j_\mu(x) \, j^5_\nu(0)\} = \sum_i c^i_{\mu\nu\gamma_1\ldots\gamma_i}(q) \, \mathcal{O}_i^{\gamma_1\cdots\gamma_i}$$

$$T_{\mu\nu} = \langle 0| \, \hat{T}_{\mu\nu} \, |\gamma(k)\rangle = \sum_i c^i_{\mu\nu\alpha_1\ldots\alpha_i}(q) \, \langle 0| \, \mathcal{O}_i^{\alpha_1\ldots\alpha_i} \, |\gamma(k)\rangle$$

$$\langle 0| \, \mathcal{O}_i^{\alpha\beta} \, |\gamma(k)\rangle = -\frac{i}{4\pi^2} \, \kappa_i \, \tilde{f}^{\alpha\beta}$$

$$\hat{T}_{\mu\nu} = \sum_i \left\{ c^i_T(q^2) \left(-q^2 \mathcal{O}^i_{\mu\nu} + q_\mu q^\sigma \mathcal{O}^i_{\sigma\nu} - q_\nu q^\sigma \mathcal{O}^i_{\sigma\mu} \right) + c^i_L(q^2) \, q_\nu q^\sigma \mathcal{O}^i_{\sigma\mu} \right\}$$

$$w_{T,L}(q^2) = \sum c^i_{T,L}(q^2) \, \kappa_i$$

The leading d=2 operator

$$\mathcal{O}_F^{\alpha\beta} = \frac{1}{4\pi^2} \, \tilde{F}^{\alpha\beta} = \frac{1}{4\pi^2} \, \epsilon^{\alpha\beta\rho\delta} \partial_\rho A_\delta$$

$$c^F_L[\text{1-loop}] = 2 c^F_T[\text{1-loop}] = \frac{2 \, N_c}{Q^2} \, \mathrm{Tr} \, A \, V \widetilde{V} \left[1 + \mathcal{O}\left(\frac{m^2}{Q^2} \right) \right]$$

The next d=2 operator

$$\mathcal{O}_f^{\alpha\beta} = -i\,\bar{q}_f\,\sigma^{\alpha\beta}\gamma_5\,q^f \equiv \frac{1}{2}\,\epsilon^{\alpha\beta\gamma\delta}\bar{q}_f\,\sigma_{\gamma\delta}\,q^f$$

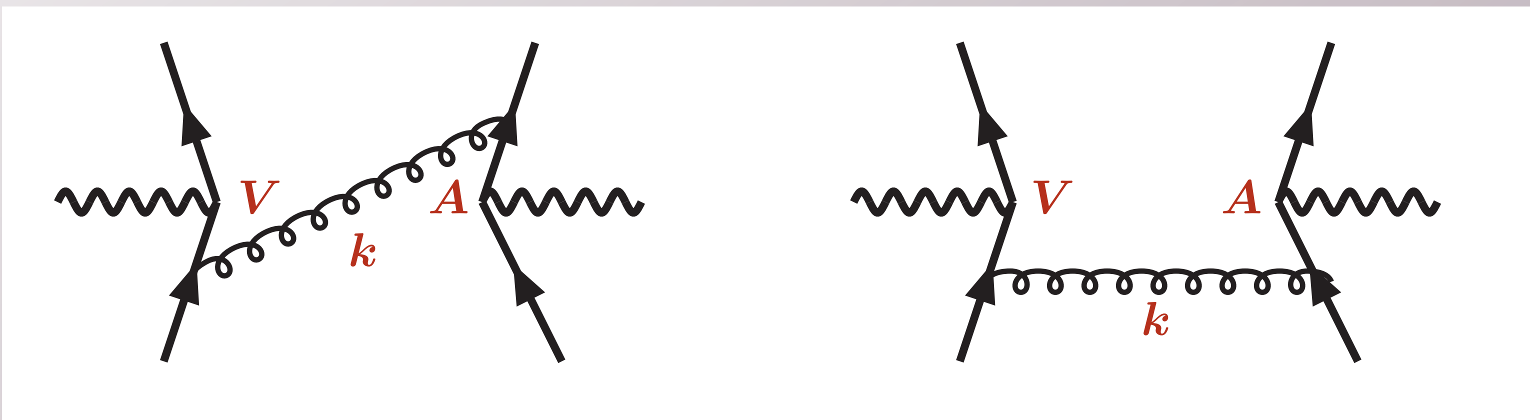
$$c_L^f = 2c_T^f = \frac{4\,A_f V_f\,m_f}{Q^4}$$

$$\Delta^{(d=3)} w_L = 2\,\Delta^{(d=3)} w_T = \frac{4}{Q^4}\sum_f A_f V_f m_f \kappa_f$$

$$\kappa_f = -4\pi^2 \widetilde{V}_f\,\langle \bar{q}q \rangle_0\,\chi$$

Quark condensate magnetic susceptibility χ Ioffe, Smilga

In the chiral limit the difference between longitudinal and transversal parts shows up at the level of d=6 four-fermion operators.



$$-\frac{8\pi\alpha_s Q_q}{k^6}\,\bar{q}\,t^a(\gamma_\alpha\hat{k}\gamma_\mu-\gamma_\mu\hat{k}\gamma_\alpha)q\otimes\bar{q}\,t^a(\gamma_\nu\hat{k}\gamma^\alpha-\gamma^\alpha\hat{k}\gamma_\nu)\gamma_5 q$$

Here the momentum q is substituted by k .

The model

$$w_{\mathrm{T}}[u,d]=\frac{1}{m_{a_1}^2-m_\rho^2}\left(\frac{m_{a_1}^2-m_\pi^2}{K^2+m_\rho^2}-\frac{m_\rho^2-m_\pi^2}{K^2+m_{a_1}^2}\right)$$

Applications

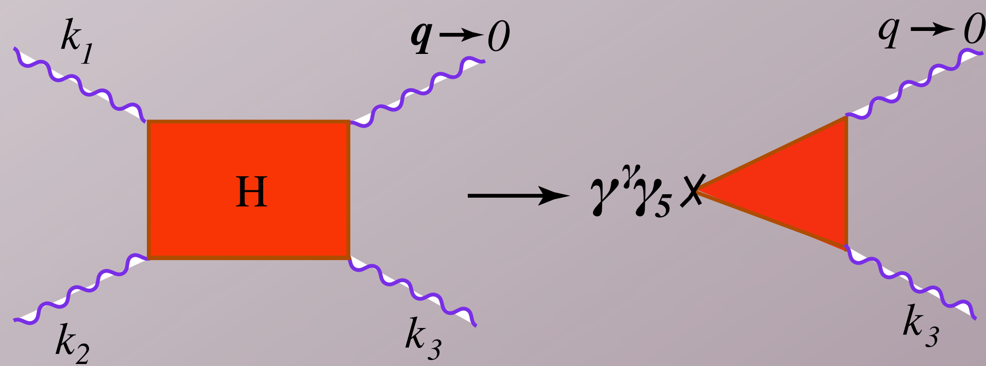
- Hadrons in the electroweak corrections to g-2

$$a_\mu^{\mathrm{EW}} = 154(1)(2) \times 10^{-11}$$

- Magnetic susceptibility of quark condensate

$$\chi = -\frac{N_c}{4\pi^2 F_\pi^2} = -\frac{1}{(335\ \mathrm{MeV})^2}$$

- Hadronic light-by-light in the muon g-2



$$a^{\mathrm{HLbL}} = (105 \pm 26) \times 10^{-11}$$

Many more happy years in physics and life, Al !