

Heavy quarks in a thermal bath

“From Particles and Partons to Nuclei and Fields”
Columbia University
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Happy birthday, Al !

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Motivation

- Experimental (" J/ψ suppression", heavy quark energy loss, etc)
- Theory. Controversial results. No good theory for heavy quark "bound states" in a thermal medium.
- Connection with strong coupling calculations (within AdS/CFT)

WORK IN PROGRESS !

Based on

A. Beraudo, JPB, C. Ratti, NPA 806 (2008) 312 [arXiv: 0712.4394]

A. Beraudo, JPB, P. Faccioli and G. Garberoglio (in preparation)

J. Albacete, E. Iancu and JPB (ongoing discussions)

Screening of binding forces in a quark-gluon plasma

Screened potential

$$V(r) = -\frac{\alpha}{r} e^{-r/r_D(T)}$$

Bound state exists for

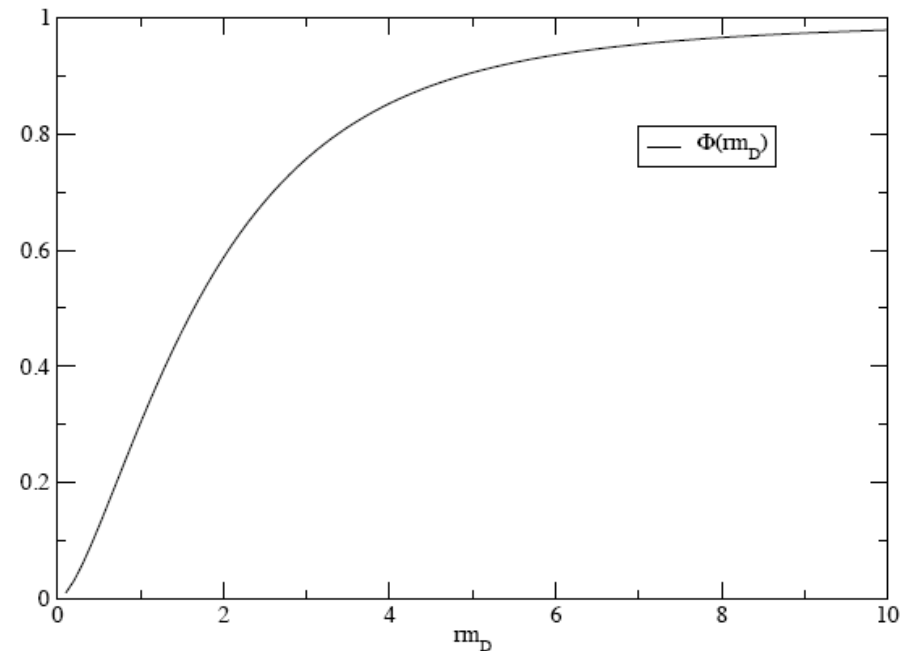
$$r_D(T) > r_D^{\min}$$

that is, for

$$T < T_D$$

Effective potential has imaginary part
(there is more than screening)

$$V(r) = -\frac{g^2 m_D}{4\pi} \left[1 + \frac{e^{-m_D r}}{m_D r} \right] - i \frac{g^2 T}{4\pi} \phi(m_D r)$$



The heavy quark propagator

$$\begin{aligned} S^>(t, r_1; 0, r'_1) &= \frac{1}{Z} \text{Tr} \left\{ e^{-\beta H} \psi(t; r_1) \psi^\dagger(0; r'_1) \right\} \\ &= \frac{1}{Z} \sum_{n,m} e^{-\beta E_n} e^{i(E_n - E_m)t} \langle n | \psi(r_1) | m \rangle \langle \psi^\dagger(r'_1) | n \rangle \end{aligned}$$

note analyticity for $-\beta < \text{Im } t$

For a free particle

$$S^>(t = -i\tau, x) = e^{-M\tau} \left(\frac{M}{2\pi\tau} \right)^{3/2} e^{-\frac{Mx^2}{2\tau}} \quad (x \equiv r_1 - r'_1)$$

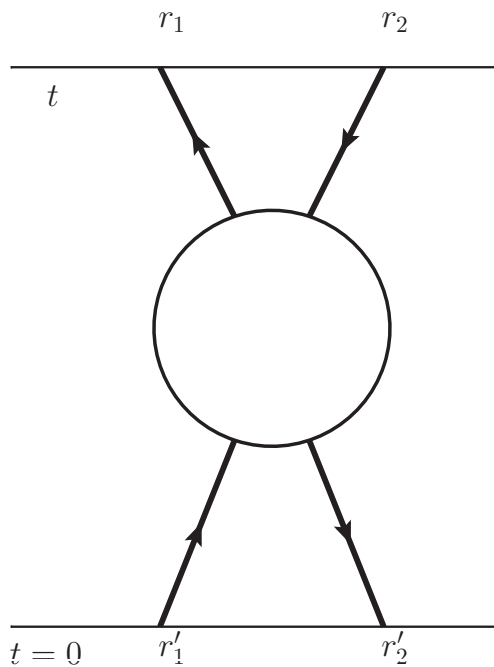
For large M

$$S_0^>(t = -i\tau, x) \sim e^{-M\tau} \delta(x)$$

The heavy quark correlator

$$J_Q^\dagger(t; r_1, r_2) \equiv \psi^\dagger(t, r_1) \chi^\dagger(t, r_2)$$

$$\begin{aligned} G^>(t, r_1; t, r_2 | 0, r'_1; 0, r'_2) &= \frac{1}{Z} \text{Tr} \left\{ e^{-\beta H} J_Q(t; r_1, r_2) J_Q^\dagger(0; r'_1, r'_2) \right\} \\ &= \frac{1}{Z} \sum_{n,m} e^{-\beta E_n} e^{i(E_n - E_m)t} \langle n | J_Q | m \rangle \langle m | J_Q^\dagger | n \rangle \end{aligned}$$



The heavy quarks are treated as “test particles”. They are NOT part of the thermal bath.

A simple model

HQ's in abelian plasma

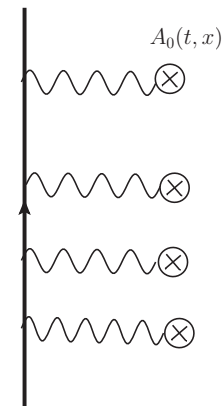
$$(M \rightarrow \infty)$$

Hamiltonian of the form

$$H = g \int dx A_0(x) \left(-\psi^\dagger(x)\psi(x) + \chi^\dagger(x)\chi(x) \right) + H_{Cb} + H_f$$

Propagator in background $A_0(t, x)$

$$S_A^>(t, x) = S_0^>(t, x) \exp\left(ig \int_0^t dt' A_0(x, t') \right)$$



$Q\bar{Q}$ pair in $A_0(t, x)$ background

$$G_A(t; r_1, r_2; r'_1, r'_2) = \delta(r_1 - r'_1)\delta(r_2 - r'_2) \times \\ \times \exp\left(ig \int_0^t dt' A_0(r_1, t')\right) \exp\left(-ig \int_0^t dt' A_0(r_2, t')\right)$$

Exact representation of correlator

$$G^>(t; r_1, r_2; r'_1, r'_2) = \int [\mathcal{D}A] G_A(t; r_1, r_2; r'_1, r'_2) e^{iS[A]} \\ \equiv \delta(r_1 - r'_1)\delta(r_2 - r'_2)\bar{G}(t, r_1 - r_2)$$

$S[A]$ is the hard thermal loop effective action

The diagram shows a wavy line with a black dot (vertex) on it, followed by an equals sign. To the right of the equals sign are three terms separated by plus signs. The first term is a simple wavy line. The second term is a wavy line connected to a circular loop with an arrow indicating a clockwise direction. The third term is an ellipsis (...).

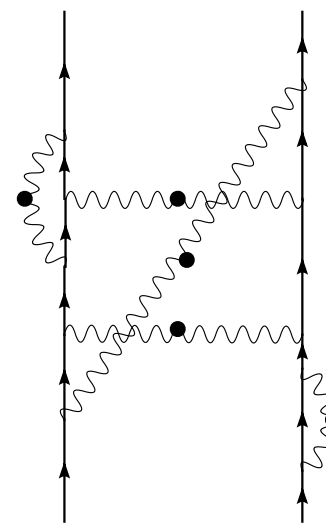
The HTL effective action is quadratic in A

$$S_C^{HTL}[A] = \frac{1}{2} \int_C d^4x \int_C d^4y A^\mu(x) (D^{-1})_{\mu\nu}^{HTL}(x-y) A^\nu(y).$$

and the path integral can be done exactly

$$\bar{G}(t, r_1 - r_2) = \exp \left[-\frac{i}{2} \int_C d^4x \int_C d^4y J^\mu(x) D_{\mu\nu}^{HTL}(x-y) J^\nu(y) \right]$$

$$J^\mu(z) = \delta^{\mu 0} \theta(z^0) \theta(t - z^0) [-g\delta(z - r_1) + g\delta(z - r_2)]$$



Large time behaviour ($t m_D \gg 1$)

$$\overline{G}(t, r_1 - r_2) \underset{t \rightarrow \infty}{\sim} \exp[-iV_{\text{eff}}(r_1 - r_2)t]$$

V_{eff} has real and imaginary part (*)

$$\begin{aligned} V_{\text{eff}}(r_1 - r_2) &\equiv g^2 \int \frac{dq}{(2\pi)^3} (1 - e^{iq \cdot (r_1 - r_2)}) D_{00}(\omega = 0, q) \\ &= g^2 \int \frac{dq}{(2\pi)^3} (1 - e^{iq \cdot (r_1 - r_2)}) \left[\frac{1}{q^2 + m_D^2} - i \frac{\pi m_D^2 T}{|q|(q^2 + m_D^2)^2} \right] \\ &= -\frac{g^2}{4\pi} \left[m_D + \frac{e^{-m_D r}}{r} \right] - i \frac{g^2 T}{4\pi} \phi(m_D r) \end{aligned}$$

(*first observed by M. Laine et al hep-ph/0611300)

Screened potential

$$\begin{aligned} V_{\text{eff}}(r) &= -\alpha m_D - \frac{\alpha}{r} e^{-m_D r} \\ &\underset{r \rightarrow 0}{\sim} -\alpha m_D - \frac{\alpha}{r} + \alpha m_D = -\frac{\alpha}{r} \\ &\underset{r \rightarrow \infty}{\sim} -\alpha m_D \end{aligned}$$

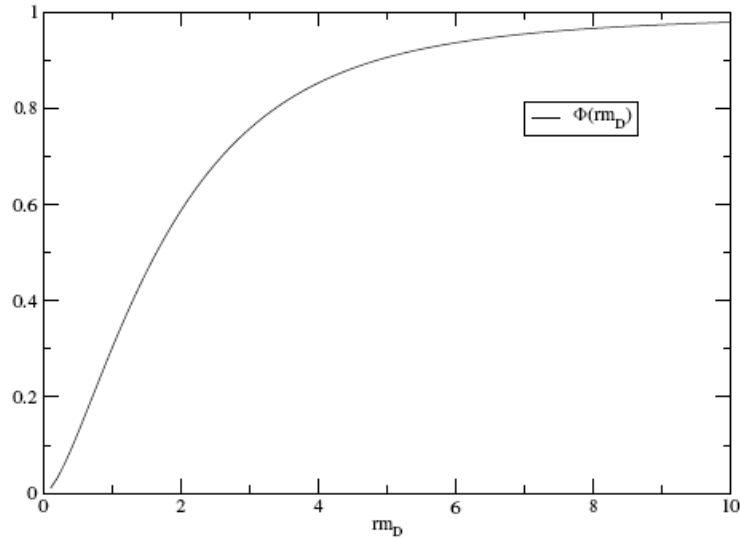
Note: $-\alpha m_D = 2\delta M$

Real part of potential is heavy quark free energy

$$\Delta F_{Q\bar{Q}} = \text{Re} V_{\text{eff}}(r)$$

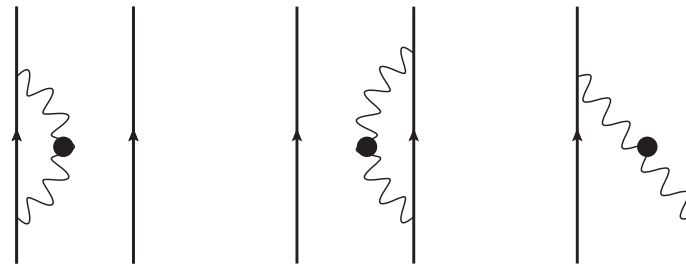
$$S_{Q\bar{Q}} = -\frac{dF}{dT} = \alpha [1 - e^{-m_D r}] \frac{dm_D}{dT}$$

The imaginary part of the effective potential



At large distance the imaginary part is twice the damping rate of the heavy quark

At short distance, interference produces cancellation: a small dipole does not "see" the electric field fluctuations.



Finite mass case

Exact path integral for the propagator

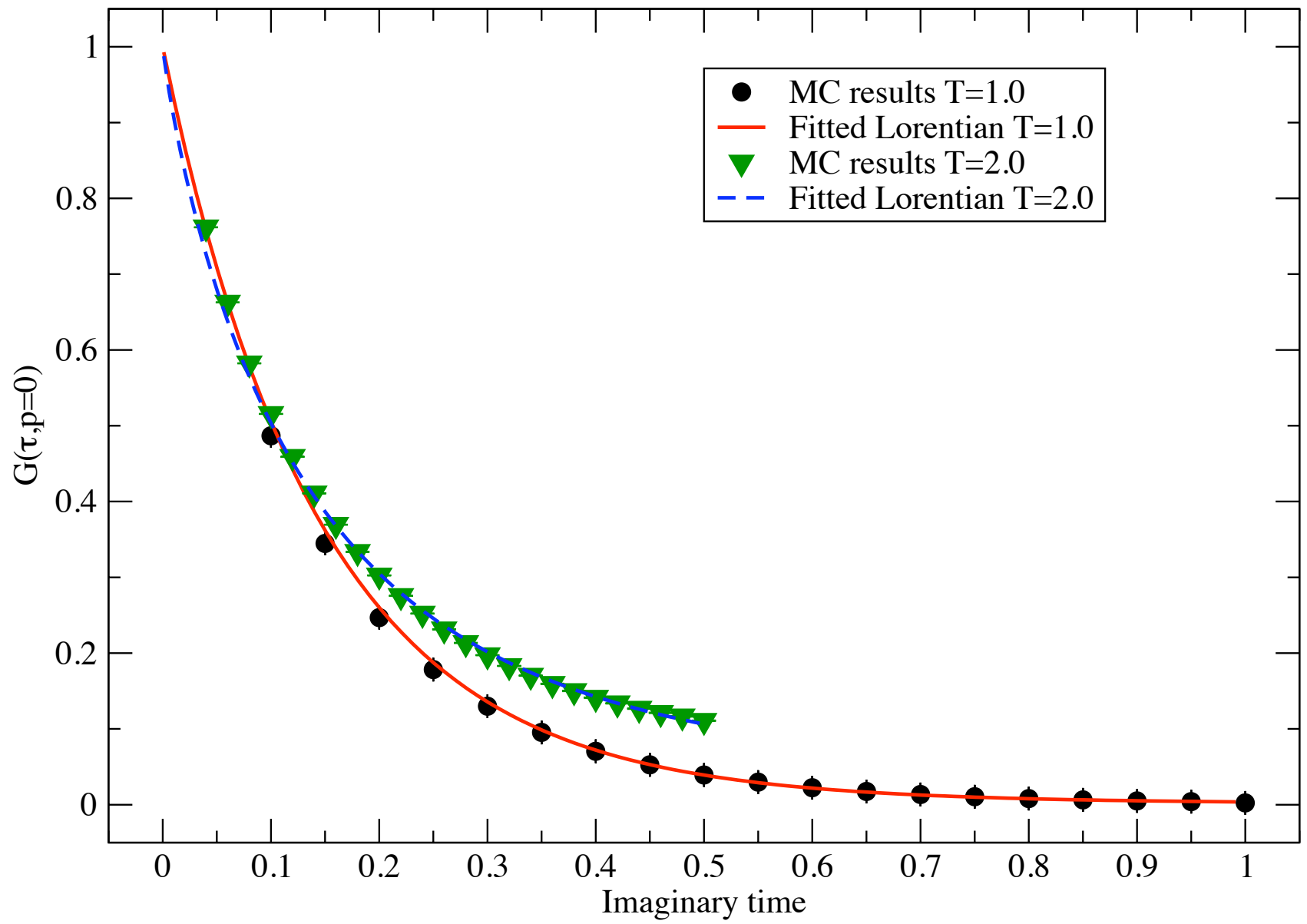
Heavy quark in a background field A

$$\langle r_1 \tau | r'_1 0 \rangle = \int_{x(0)=r'_1}^{x(\tau)=r_1} [\mathcal{D}x(\tau')] \exp \left[- \int_0^\tau d\tau' \left(\frac{1}{2} M \dot{x}^2 + g A_0(\tau', x) \right) \right]$$

Averaging over A yields

$$G^>(-i\tau, r_1 | 0, r'_1) = \int_{z(0)=r'_1}^{z(\tau)=r_1} [\mathcal{D}z] \exp \left[- \int_0^\tau d\tau' \frac{1}{2} M \dot{z}^2 \right] \times \\ \times \exp \left[\frac{g^2}{2} \int_0^\tau d\tau' \int_0^\tau d\tau'' \Delta(\tau' - \tau'', z(\tau') - z(\tau'')) \right]$$

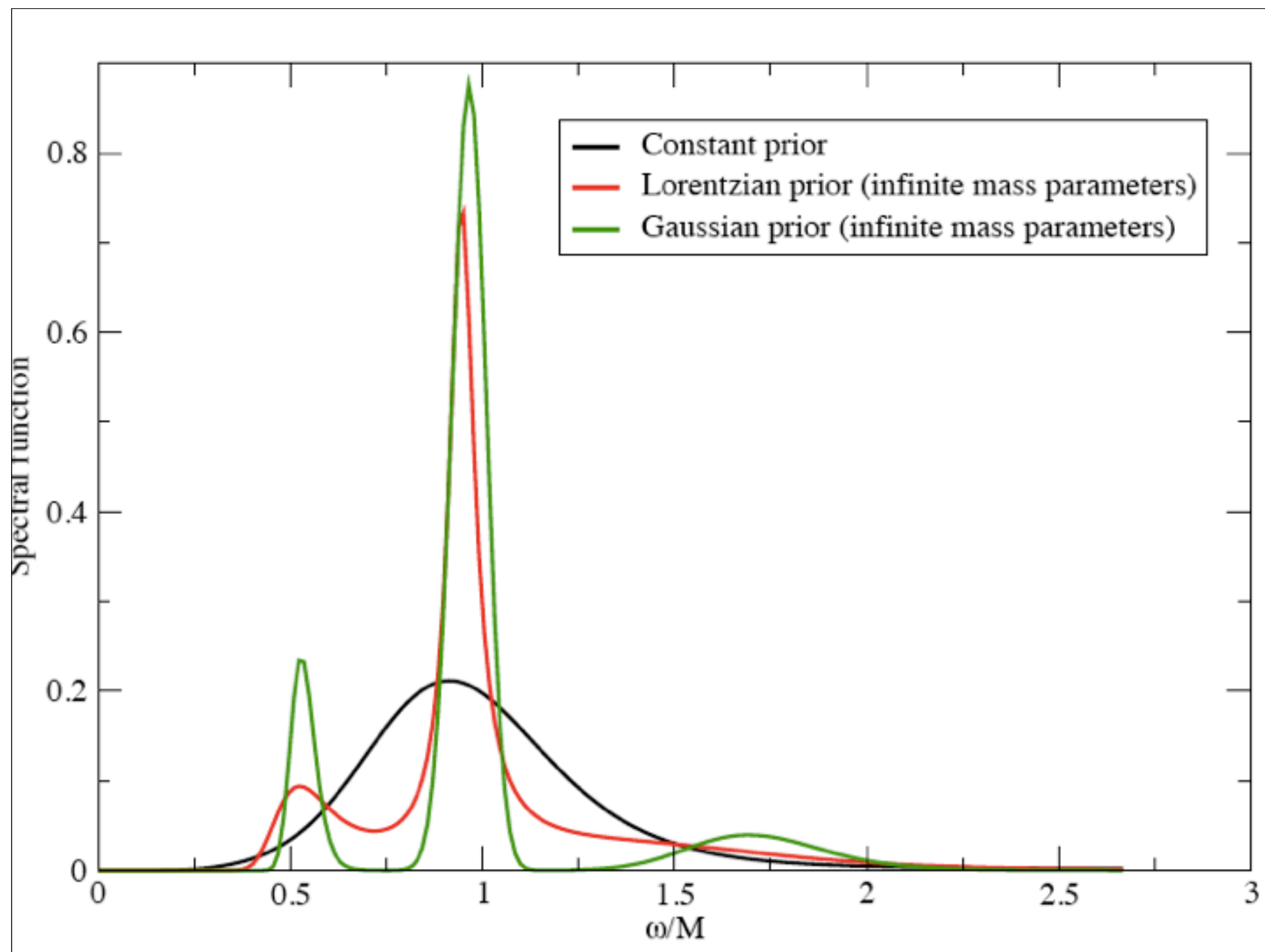
which can be evaluated by Monte Carlo



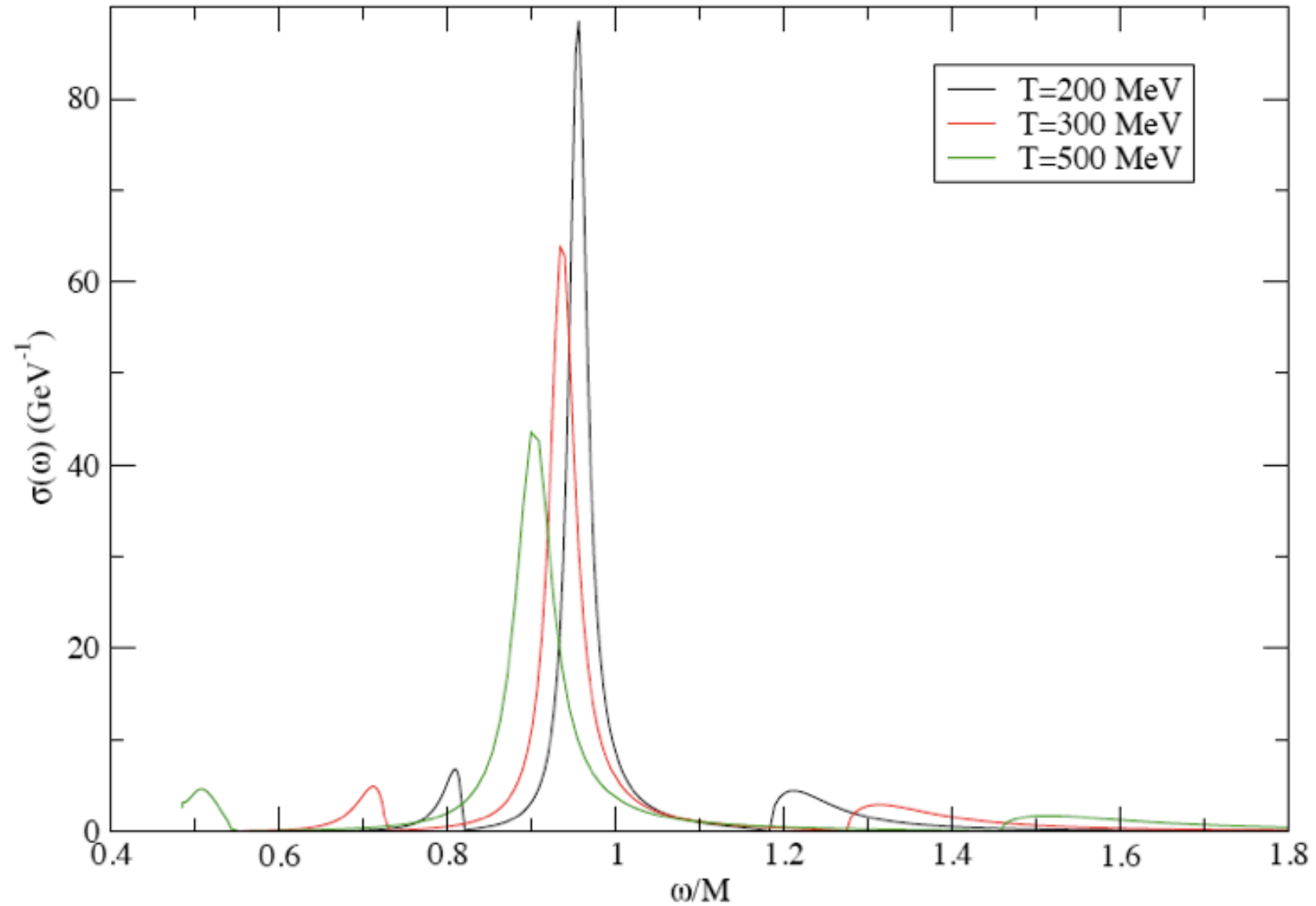
Recovering the spectral function from the (numerical) correlator is a difficult problem

$$G^>(t = -i\tau) = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} e^{-\omega\tau} \sigma(\omega)$$

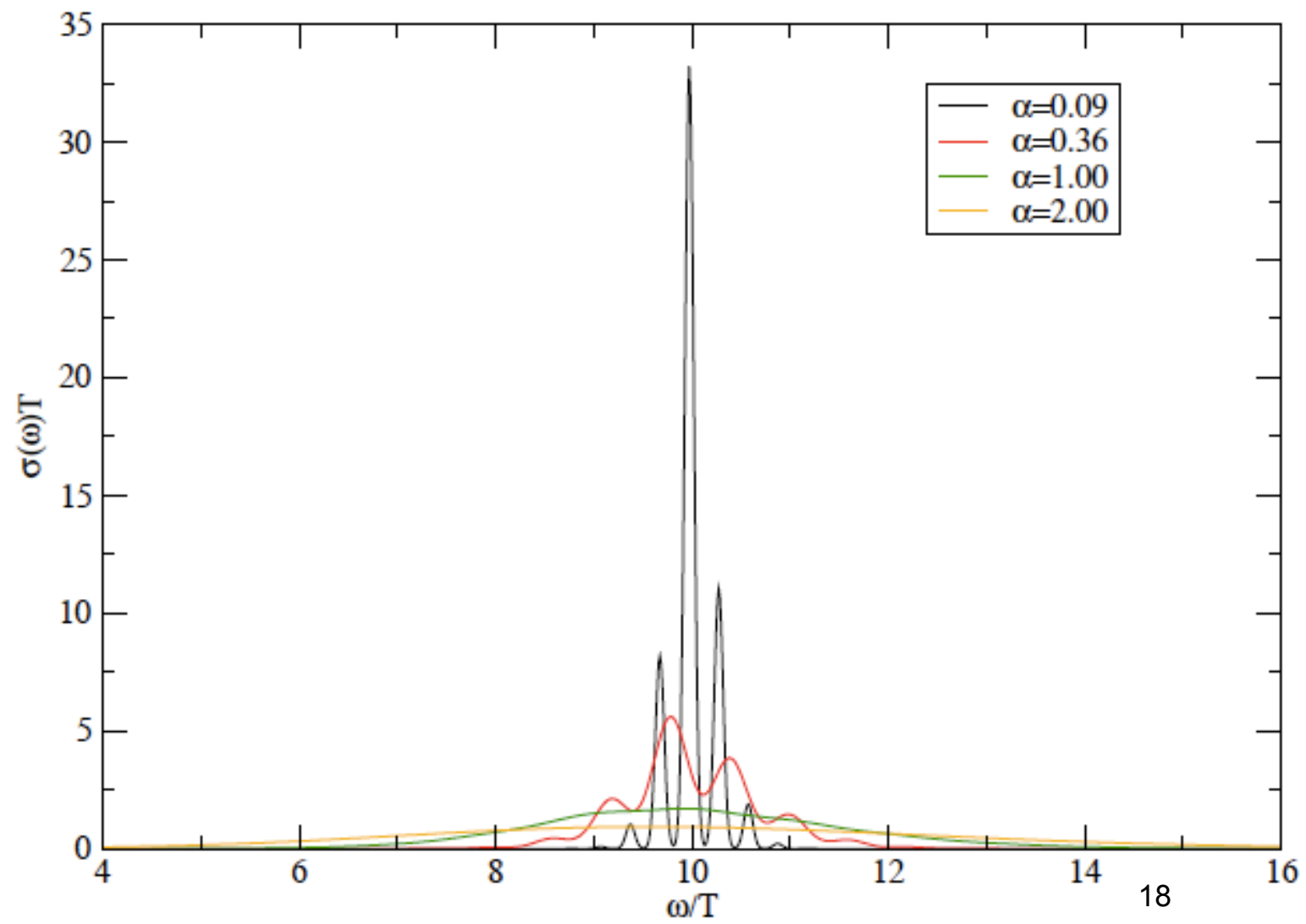
We are using the Maximum Entropy Method....
But results still sensitive to the choice of the
prior - see plot - PRELIMINARY



One loop spectral function



$$E/T = \alpha^{0.5} \quad \gamma/T = \alpha/2$$



CONCLUSIONS

- A simple model was constructed, which takes into account exactly all essential many-body effects in the interaction of a heavy quark pair with a thermal bath
- The model leads to a simple path integral that can be calculated (numerically for finite mass using Euclidean Monte Carlo techniques)
- To reconstruct (numerically) the real time information one has to face similar difficulties as in usual lattice gauge theory, but in a much simpler context.
- Transition from weak to strong coupling can be studied. Detailed comparison with predictions from AdS/CFT calculations should be possible.
- Work in progress on the two particle case to understand bound state formation/dissolution in a thermal bath.