UNCERTAINTIES IN THE TAGS MEASUREMENTS

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And the TAGS collaboration
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« Workshop NACRE »
Cadarache June 2018

Subatech

IAEA

Outline

- Introduction: the recent experiments & their setup
- The TAGS technique
- Data Analysis
- Contaminants
- Detector Response
- Study of impact of sources of systematic uncertainties in the case of ^{92}Rb (Z. Issoufou 's PhD), and ^{87,88}Br, ^{94}Rb (Rocinante detector)
- Study of impact of sources of systematic uncertainties with the DTAS detector (V. Guadilla's (2017) and L. Le Meur's (2018) PhDs)

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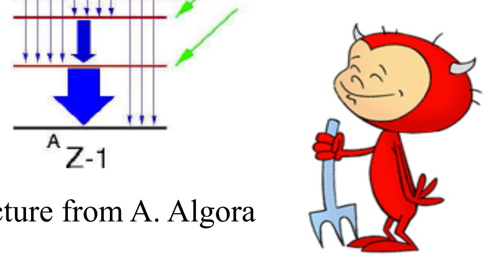
TAGS Solution to Pandemonium Effect

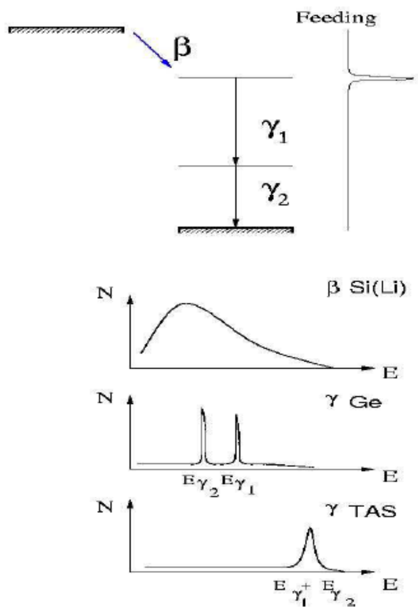
Pandemonium effect** :

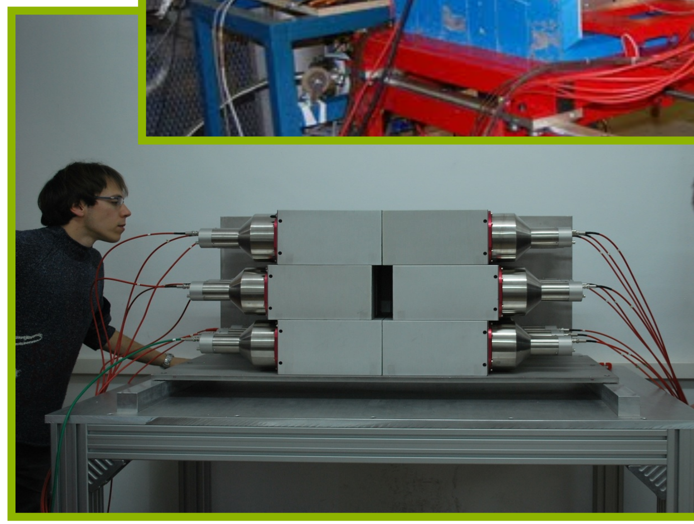
Due to the use of Ge detectors to measure the decay schemes: lower efficiency at higher energy → underestimate of β branches towards high energy excited states: overestimate of the high energy part of the FP β spectra

⇒ Solution is Total Absorption γ-ray Spectroscopy (TAGS)
Big cristal, 4π => A TAGS is a calorimeter !

Picture from A. Algora







2 TAGS arrays developed by the Valencia team (Spain, B. Rubio, J.L. Tain, A. Algora et al.): Rocinante (12 BaF2) & DTAS (18 NaI)

** J.C.Hardy et al., Phys. Lett. B, 71, 307 (1977)

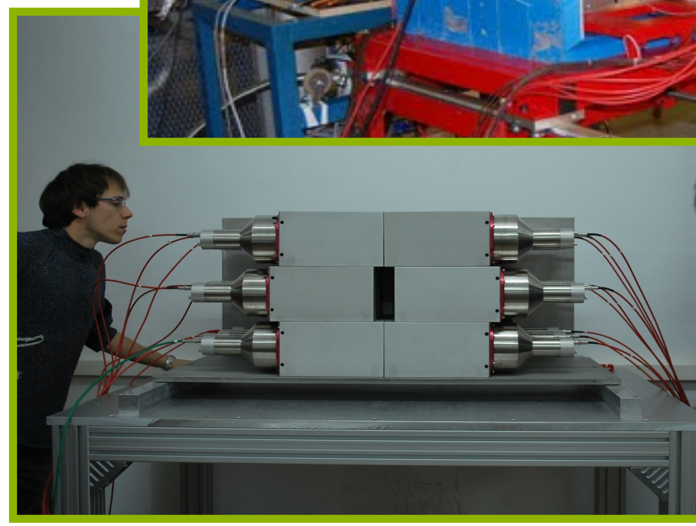
TAGS Experimental Campaigns

- **Decay Total Absorption Spectrometer (DTAS) for FAIR (IFIC Valencia): used in Jyväskylä in Feb. 2014 for our reactor antineutrino proposal:** 18 modules 15x15x25 cm^3 NaI(Tl) + 5” PMT
  - ❑ 12 nuclei for antineutrinos measured & 11 for decay heat
  - ❑ See V. Guadilla’s talk at JEFF/CHANDA meeting in Nov. 2017
- **BAF_2 TAGS (Surrey-Valencia): used for the 2009 measurement at IGISOL-JYFLTRAP**: ^{86}Br, ^{87}Br, ^{88}Br, ^{91}Rb, **^{92}Rb, ^{93}Rb,** ^{94}Rb
  - ❑ ^{92,93}Rb results already shown at last meetings, see **A. Zakari-Issoufou et al., PRL 115, 102503 (2015)**
  - ❑ ^{87}Br, ^{88}Br, ^{94}Rb **E. Valencia et al. PRC 95 024320 (2017)**
  - ❑ ^{86}Br, ^{91}Rb **S. Rice et al. PRC 96 014320 (2017)**

Antineutrino Proposal in Jyväskylä: Subatech-IFIC collaboration

V.Guadilla et al.,, Nucl. Inst. and Meth. B, in press. Online (2015) : http://www.sciencedirect.com/science/article/pii/S0168583X15012628

Pure beams required: Use of the double Penning trap from JYFL

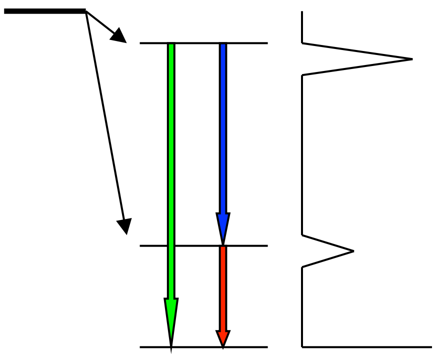


2 TAGS arrays developed by the Valencia team (Spain, B. Rubio, J.L. Tain, A. Algora et al.):

TAGS Technique

⇒ **Solution is Total Absorption γ-ray Spectroscopy (TAGS)**
Big cristal, 4π => A TAS is a calorimeter !

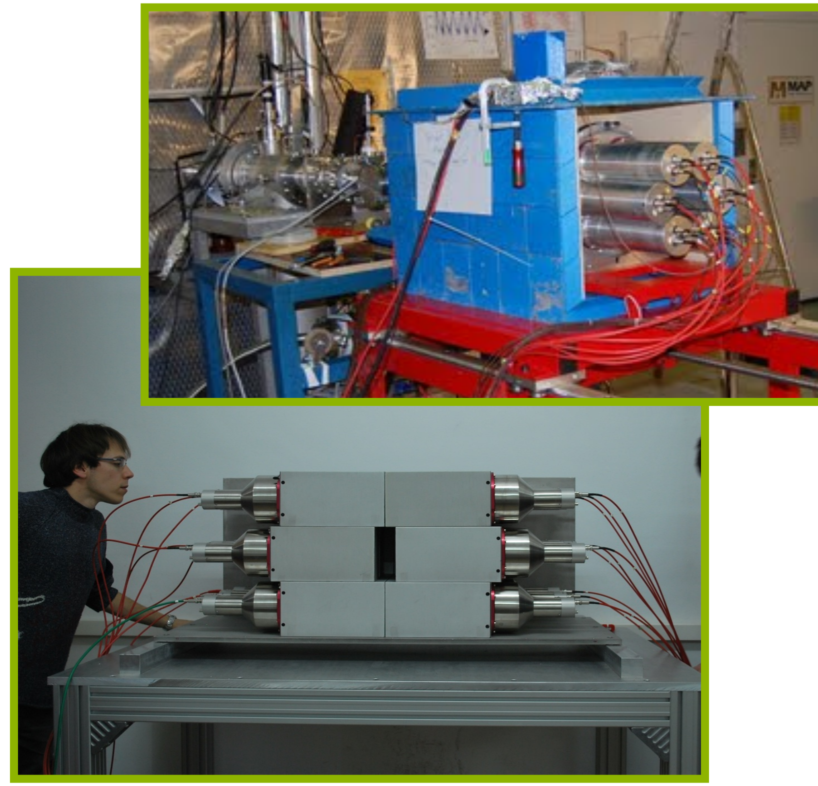
Observable: β-intensity => β-strength:



$$I_i = \frac{f_i}{\sum_k f_k}$$

$$S_i = \frac{I_i}{f(Q_\beta - E_i)T_{1/2}} \quad \left[s^{-1}\right]$$

- **Spectrum must be clean**
- **Response must be accurately known**
- **Solution of inverse problem must be stable**

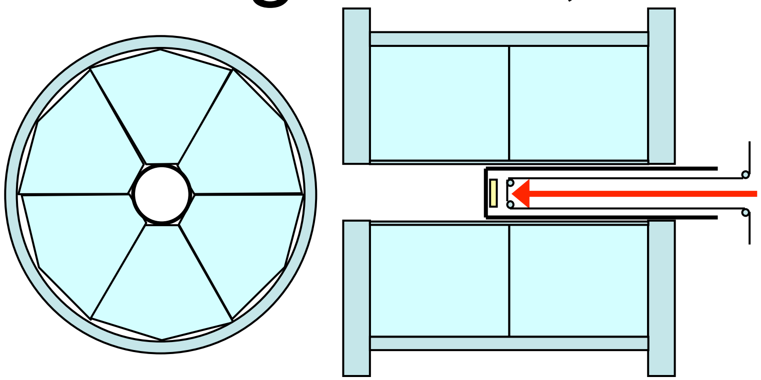


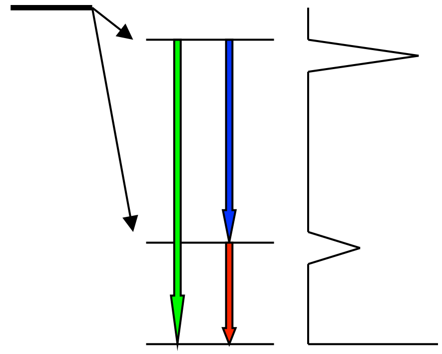
2 TAGS arrays developed by the Valencia team (Spain, B. Rubio, J.L. Tain, A. Algora et al.): Rocinante (12 BaF2) & DTAS (18 NaI)

NIM A430 (1999) 333 NIM A571 (2007) 719
NIM A430 (1999) 488 NIM A571 (2007) 728

Total Absorption Spectroscopy (TAS)

Big cristal, 4π => A TAS is a calorimeter !





Observable:

β-intensity => β-strength:
An ideal TAS would give directly the β-intensity I_β which is linked with the β-strength S_β:

$$S_i = \frac{I_i}{f(Q_\beta - E_i)T_{1/2}} \quad [s^{-1}]$$

$$I_i = \frac{f_i}{\sum_k f_k}$$

Statement of the problem:

Relation between TAS data and the β-intensity distribution:

$$d_i = \sum_j R_{ij} f_j$$

$$\mathbf{R_j} = \sum_{k=0}^{j-1} b_{jk} \mathbf{g_{jk}} \otimes \mathbf{R_k}$$

Deconvolution (Inverse problem) algorithms

Monte Carlo simulations
+
Nuclear statistical model

- Spectrum must be clean
- Response must be accurately known
- Solution of inverse problem must be stable

NIM A430 (1999) 333
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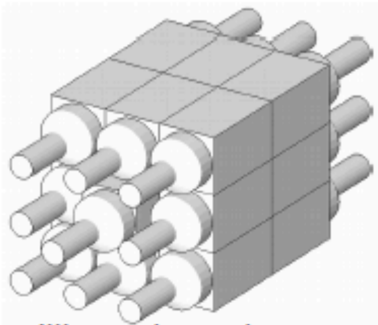
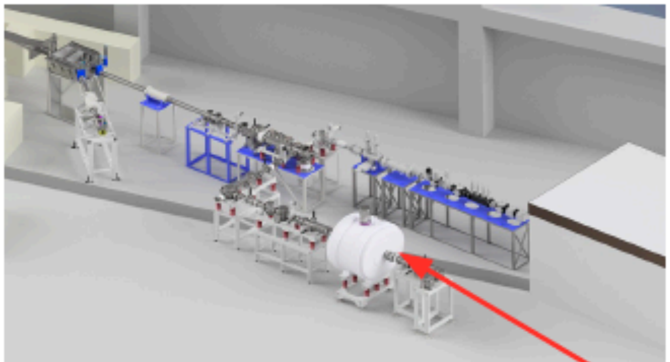
Experimental setup at Jyväskylä (^{142}Cs, ^{99}Y, ^{138}I, ^{96,96m}Y)



V. Guadilla et al., Nucl. Instrum. Methods B 376 (2016), p. 334



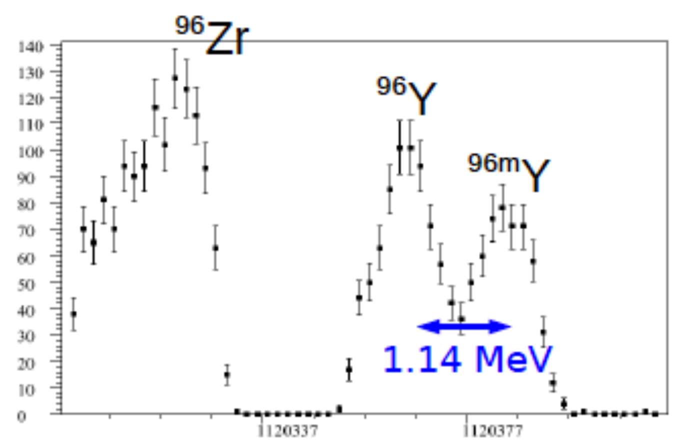
- DTAS = 18 crystals of NaI(Tl)
  - ~90% efficiency for a 1 MeV gamma
  - ΔE/E ~ 5% at 1.3 MeV
- β detector = plastic detector
  - In coincidence with γ → suppression of the background
  - 30% detection efficiency
- HPGe detector
  - Allow identification of possible contaminants coming from the decay chain

Why Jyväskylä IGISOL-4 facility ?

→ Because of the JYFLTRAP, a double Penning Trap

→ Mass resolution of $\delta m/m \sim 10^{-6}$

→ A very pure beam is needed



L. Le Meur's PhD

Data analysis

- Aim of TAS analysis = β feeding → Solve the Inverse Problem

$$d_i = \sum_j R_{ij} \cdot f_j$$

→ Requires clean spectrum

➡ Solved by an iterative procedure based on the Bayes Theorem

J.L. Tain, D. Cano-Ott, Nucl. Inst. and Meth. in Phys. Res. A 571 (2007) 728

d_i : Experimental data

f_j : β feeding

R_{ij} : Detector response matrix

→ Response matrix calculation (R_{ij})

probability that feeding at a level *j* gives counts in data channel *i* of the data spectrum

→ Recursive convolution :

$$r_j = \sum_{k=0}^{j-1} b_{jk} g_{jk} * r_k$$

$$R_j^{\beta^-} = b^- * r_j$$

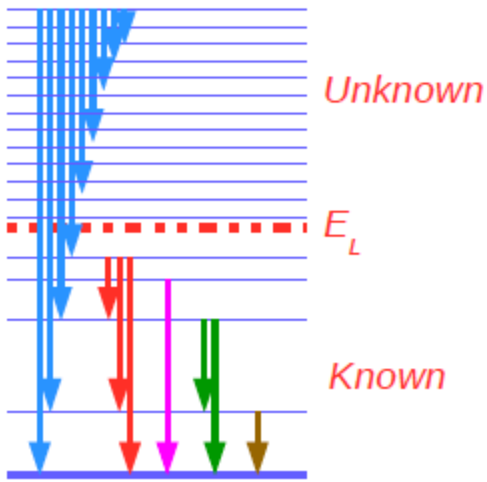
- **r_k** : response for *k* level
- **b_{jk}** : branching ratio for *j* → *k* transition
- **g_{jk}** : γ-response for *j* → *k* transition
- **b^-** : response to the e^- emission in β-decay to level *j*

➜ **b_{jk}** calculation separates in two part :

- *Known (discrete) / Unknown (continue)*

➜ Energy E_L depends on the knowledge of the daughter

➜ Models dependent (Strengths E1/E2/M1, levels density)



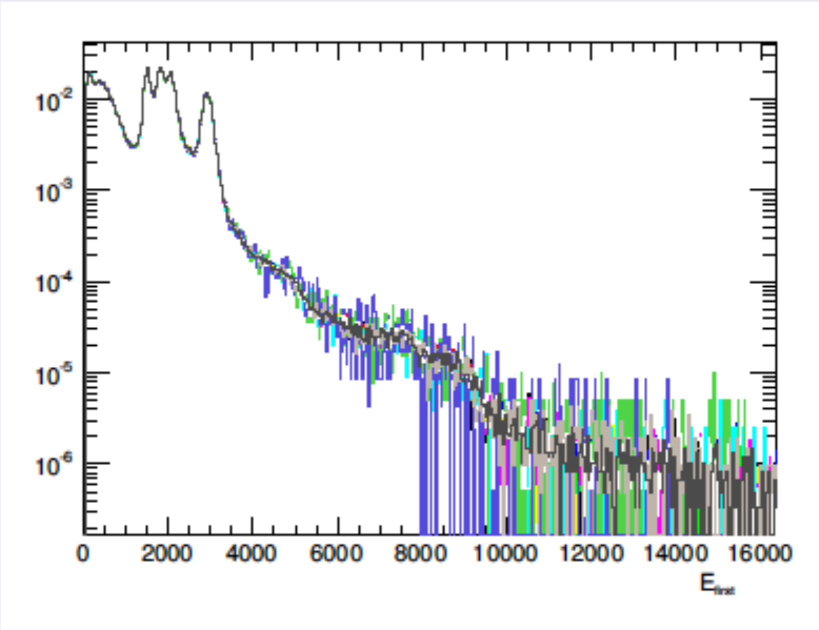
➜ γ-response **g_{jk}** and β-response **b^-** are simulated (Geant4)

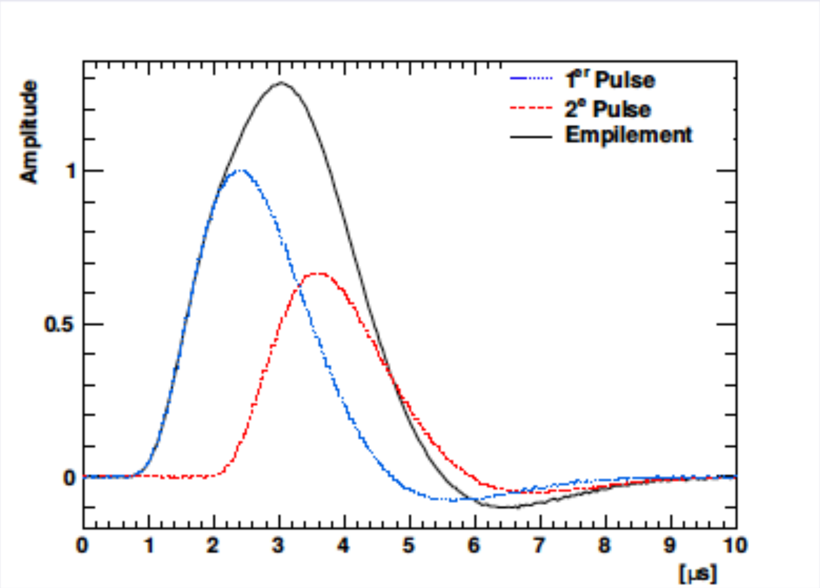
L. Le Meur's PhD

Cleaning from contaminants

Background

- Spectra are dominated by α peaks
- Spectra recorded in between sources
- Normalisation by acquisition time





Pile-Up

- False events stored in data
- constrained random coincidences
- 1^{st} order normalisation

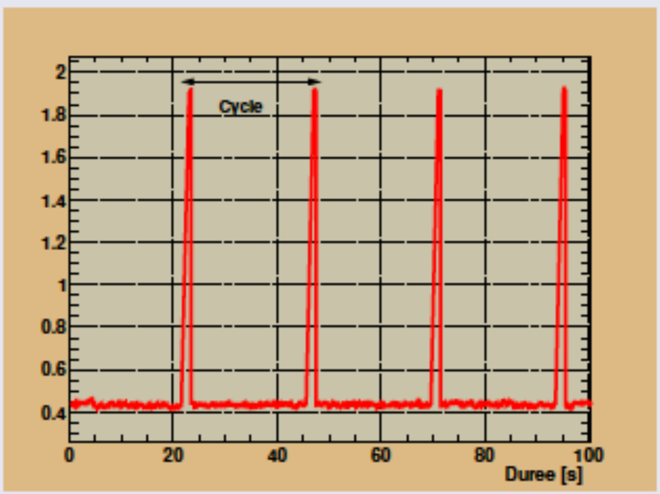
Zakari-Issoufou's PhD

Cleaning from contaminants

Contamination from daughters should be well estimated and removed

$^{92}\mathrm{Rb}(4.48s) \rightarrow {}^{92}\mathrm{Sr}(2.6h) \rightarrow {}^{92}\mathrm{Y}$
$^{93}\mathrm{Rb}(5.84s) \rightarrow {}^{93}\mathrm{Sr}(7.4mn) \rightarrow {}^{93}\mathrm{Y}$
$^{93}\mathrm{Rb}(1.39\%) \rightarrow {}^{92}\mathrm{Sr}(2.6h) \rightarrow {}^{92}\mathrm{Y}$

- TAS spectra from daughter is calculated $\Rightarrow \vec{d} = R\vec{f}$
- Integral dictated by Bateman



- neutron emission is simulated
- $N_{\beta n\gamma} = P_n N_d \varepsilon_{\beta n\gamma}$

Zakari-Issoufou's PhD

Data analysis

- Aim of TAS analysis = β feeding → Solve the Inverse Problem

$$d_i = \sum_j R_{ij} . f_j$$

→ Requires clean spectrum

➡ Solved by an iterative procedure based on the Bayes Theorem

J.L. Tain, D. Cano-Ott, Nucl. Inst. and Meth. in Phys. Res. A 571 (2007) 728

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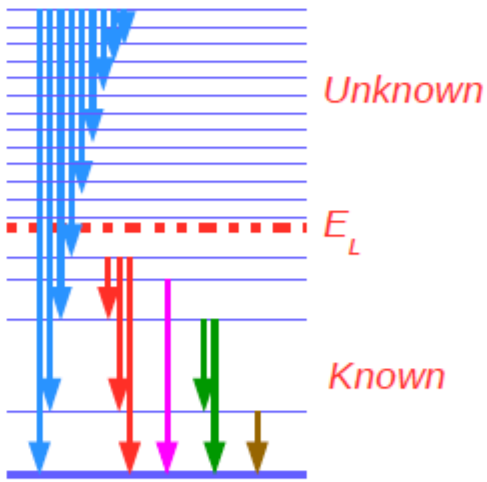

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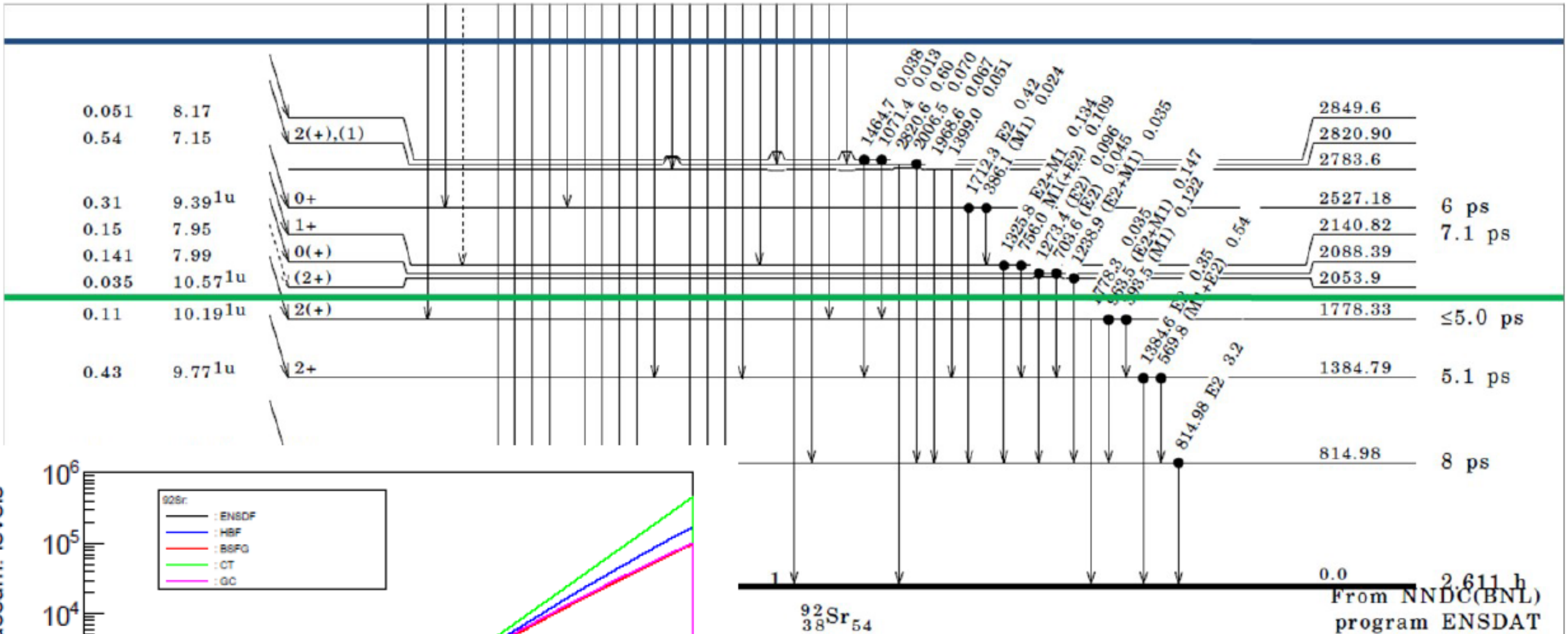
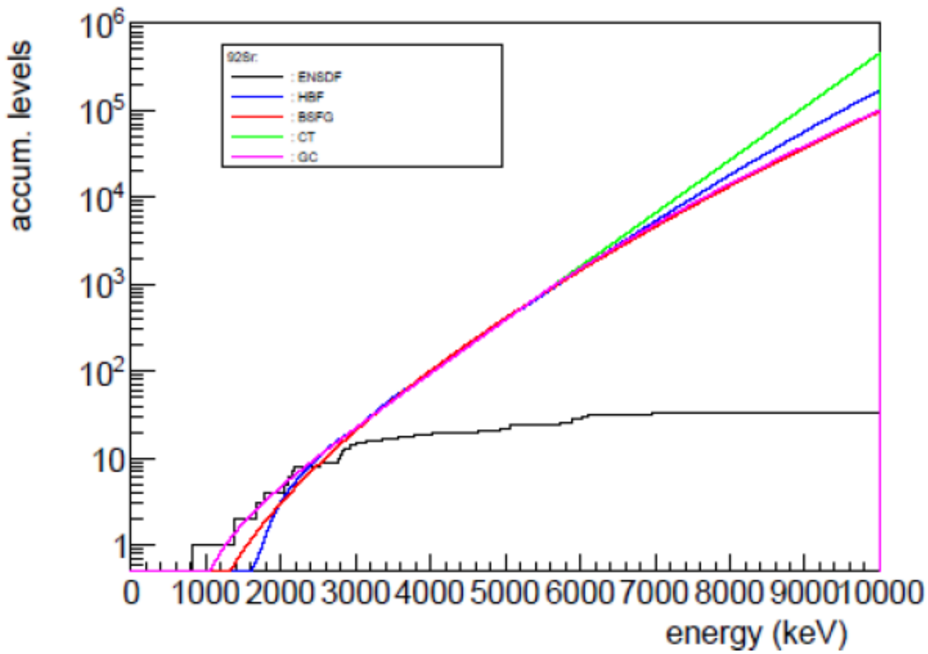
L. Le Meur's PhD





- **Level densities information of daughter nucleus**
- **J^{π} information, I_{β} of the known part**
- **I_{γ} and resonance parameters**

Zakari-Issoufou's PhD

γ strength

$$T_{XL}(E_\gamma) = 2\pi E_\gamma^{2L+1} f_{XL}(E_\gamma)$$

$$f_{XL}(E_\gamma) = \frac{26 \times 10^{-8}}{2L+1} \sigma_0 \Gamma E_\gamma^{3-2L} \frac{\Gamma_0}{(E_\gamma^2 - E_0^2)^2 + E_\gamma^2 \Gamma_0^2}$$

T_{XL} : Transmission coefficient

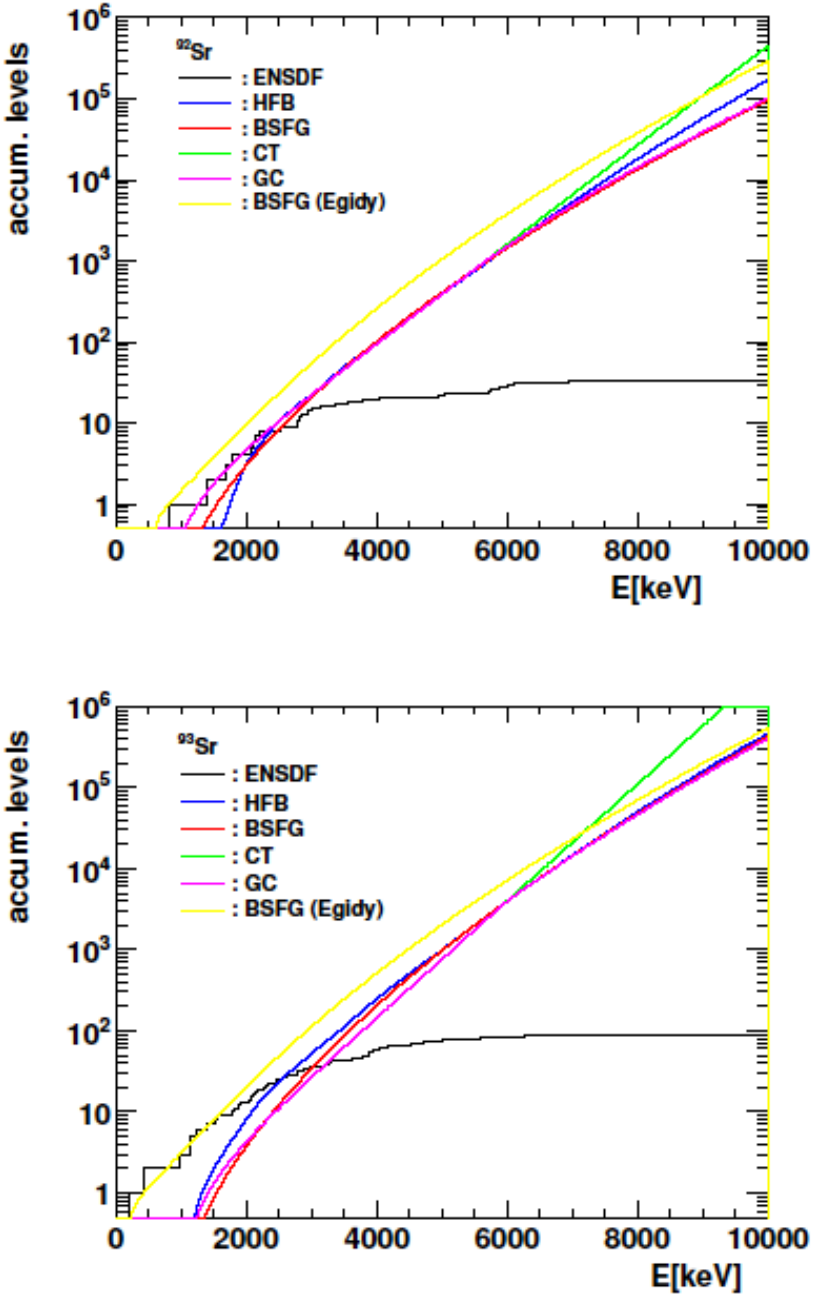
f_{XL} : Strength function, $XL =$ E1,M1,E2

E_γ : Transition energy

Resonance parameters σ_0, Γ_0, E_0 are obtained from RIPL
https://www-nds.iaea.org/RIPL-3

Zakari-Issoufou's PhD

Level densities

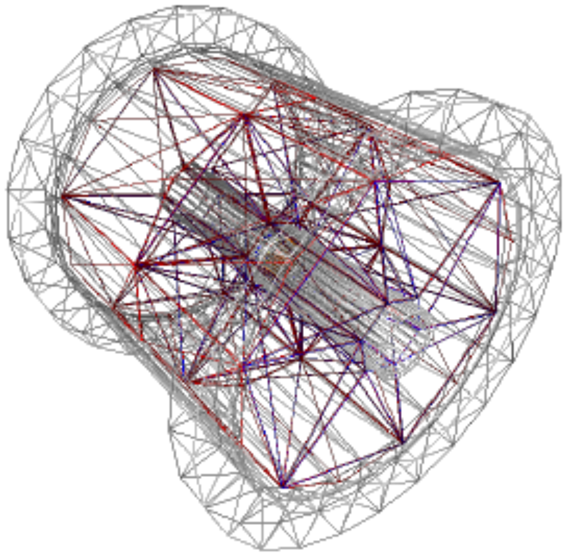


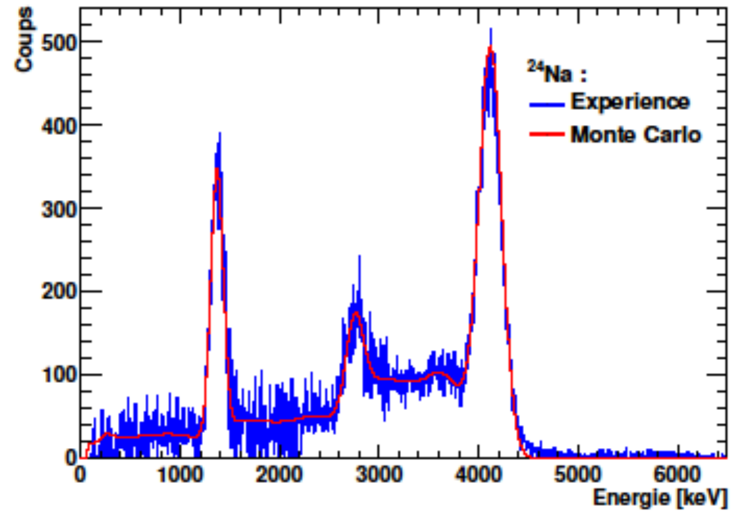
Egidy et al. PRC 72 (2005), Dilg et al. NPA 212 (1973), Gilbert et al. Canadian Journal of Physics 43 (1965)

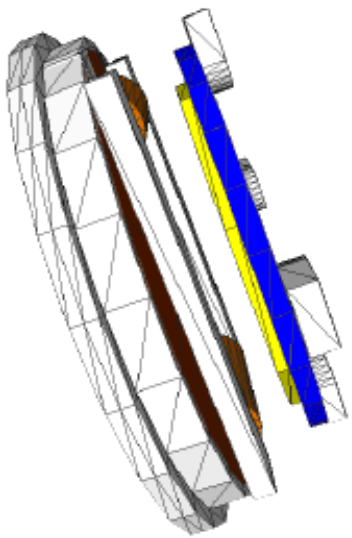
- Semi-empiric models for levels densities:
  - Back Shifted Fermi Gas
  - Constant Temperature
  - Gilbert Cameron
- Parameters obtained from fits
  - experimental data for known
  - theoretical calculation for unknown (HFB)
- Chose the one that is more in agreement with experimental data in the lower energy part

A.-A. Zakari-Issoufou's PhD

E. Valencia, S. Rice Phd work



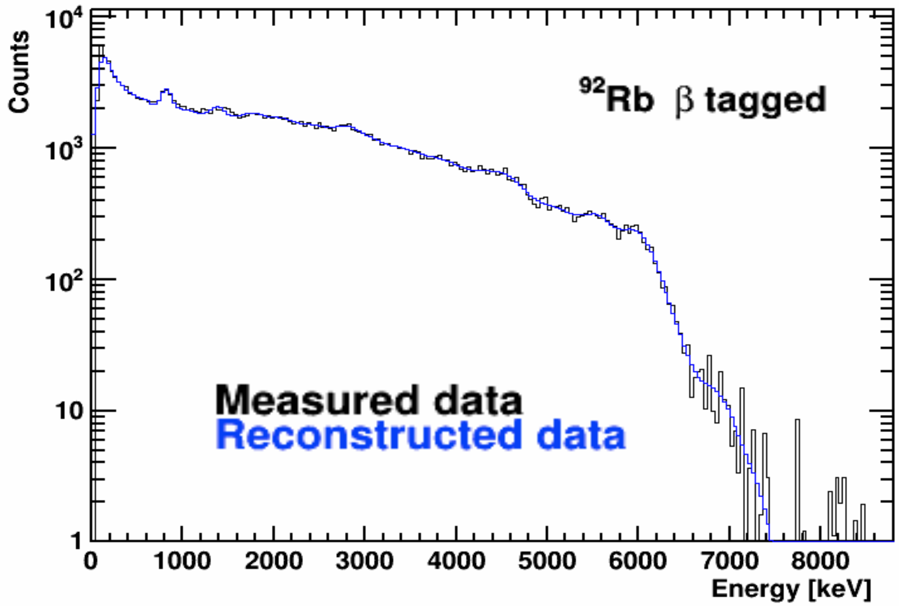




- Precise description of the TAS geometry
- GEANT4 simulation of β and γ
- Compare simulated spectra to experiment for validation

Zakari-Issoufou's PhD

The case of ^{92}Rb



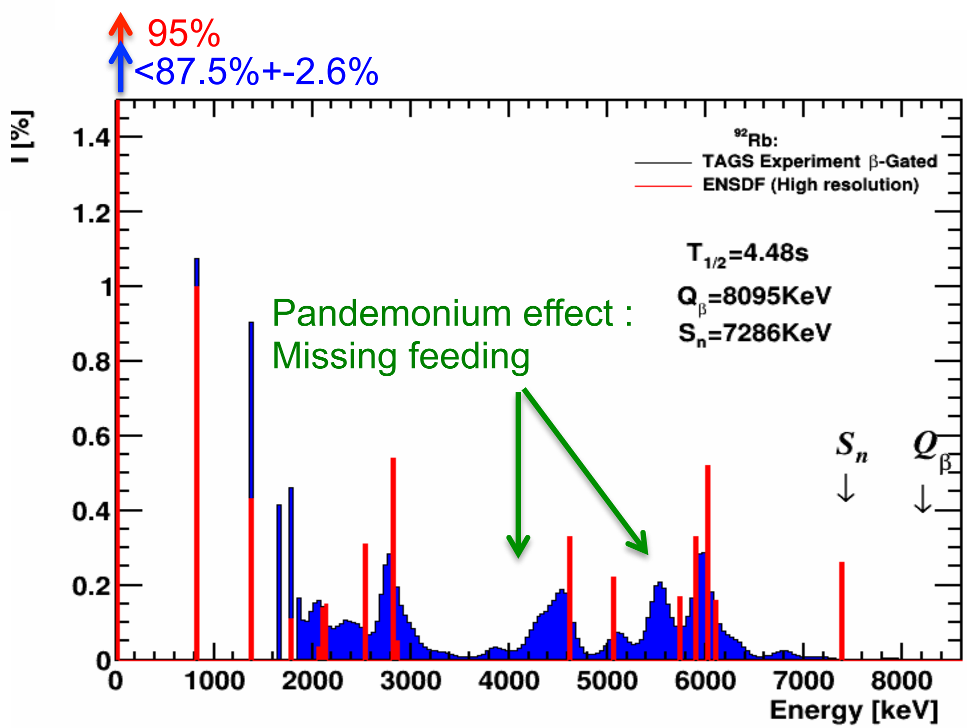
Calculation of level energy feeding through the resolution of the inverse problem

$$d_i=\Sigma R_{ij}*f_j$$

Clean data

Response matrix

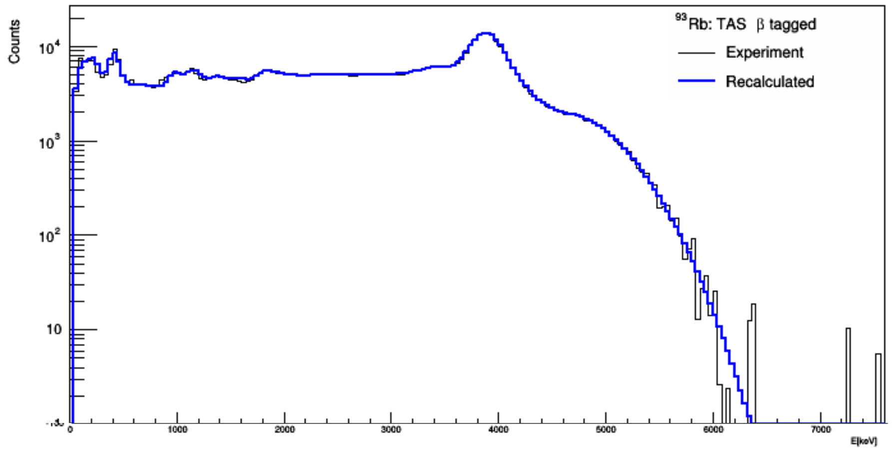
Level feeding



PhD Thesis work:
Zakari Issoufou (Subatech, Nantes)
A.-A. Zakari-Issoufou et al.
Phys. Rev. Lett. 115, 102503 (2015)

Subatech

The case of ^{93}Rb



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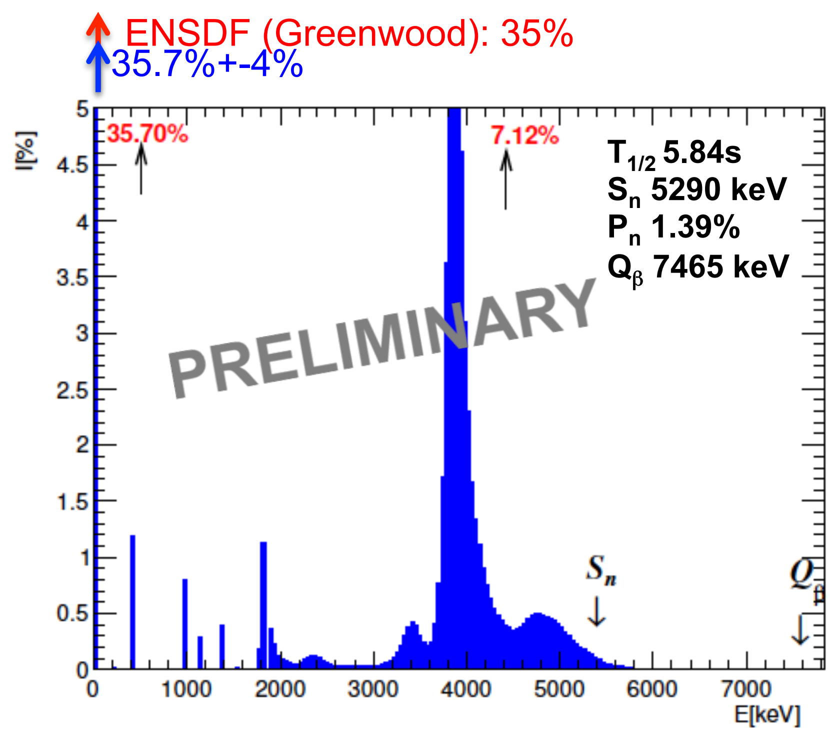

PhD Thesis work:
Zakari Issoufou (Subatech, Nantes)

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Impact of sources of systematic uncertainties

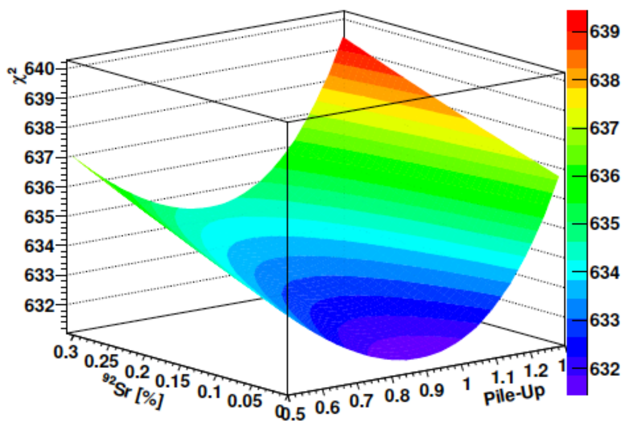


Figure 3.33 – *Courbe des valeurs de χ^2 obtenues lors de la résolution du problème inverse pour différents paramètres de normalisation des empilements et différents taux de contamination du ^{92}Sr. On trouve le minimum pour un facteur de 0.90 de la valeur que nous avions estimée par le calcul pour les empilements.*

A.-A. Zakari-Issoufou's PhD

Impact de la normalisation des empilements et de la contamination du noyau fils

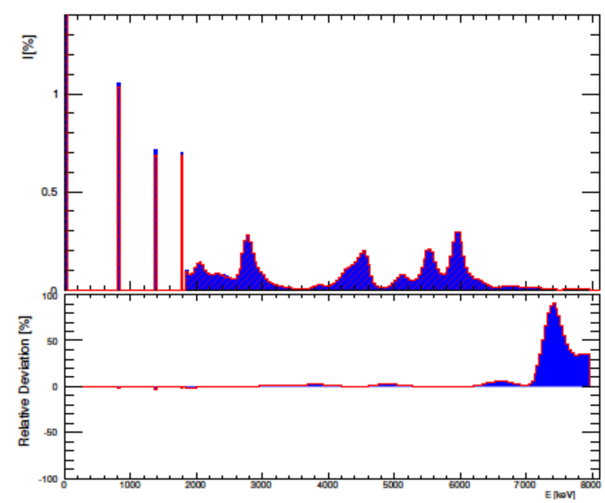


Figure 3.34 – *Comparaison de la distribution d'alimentation bêta obtenue (rouge) sous les conditions de normalisation des contaminants qui minimisent la valeur de χ^2 entre le spectre reconstruit et les données expérimentales et le résultat que nous avions adopté (bleu). En bas l'écart relatif entre les deux distributions, avec des variations plus prononcées en fin de spectre où la contribution des empilements est relativement plus importante.*

Impact des paramètres d’étalonnage

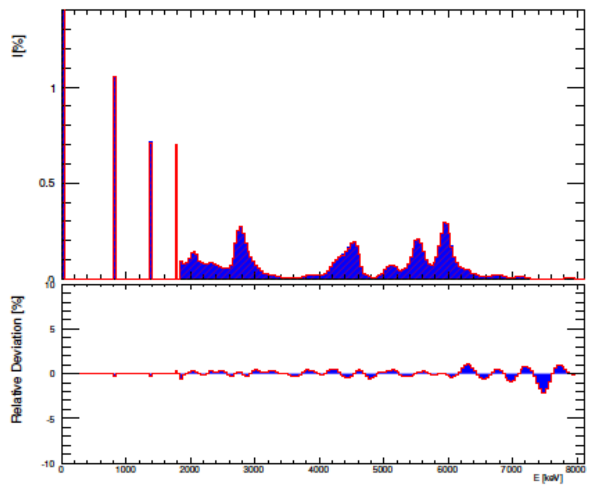


Figure 3.35 – *Comparaison des alimentations bêtas obtenues (rouge) en modifiant la pente de la relation d’étalonnage en énergie de 1% avec le résultat adopté (bleu). Les écart relatifs sont compatibles avec les incertitudes statistiques.*

A.-A. Zakari-Issoufou’s PhD

Impact de l’étalonnage

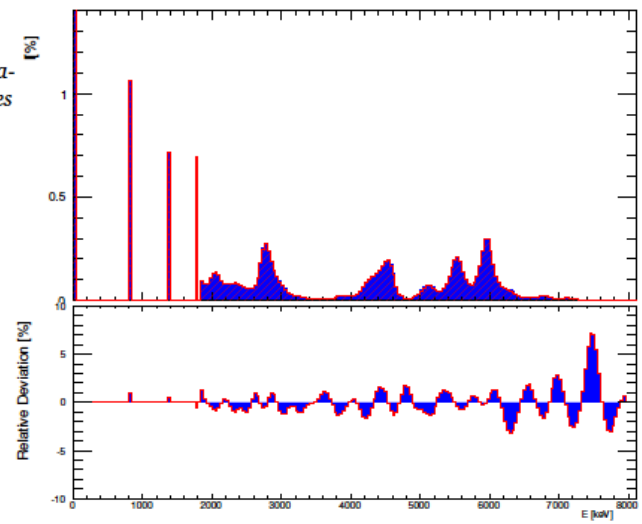
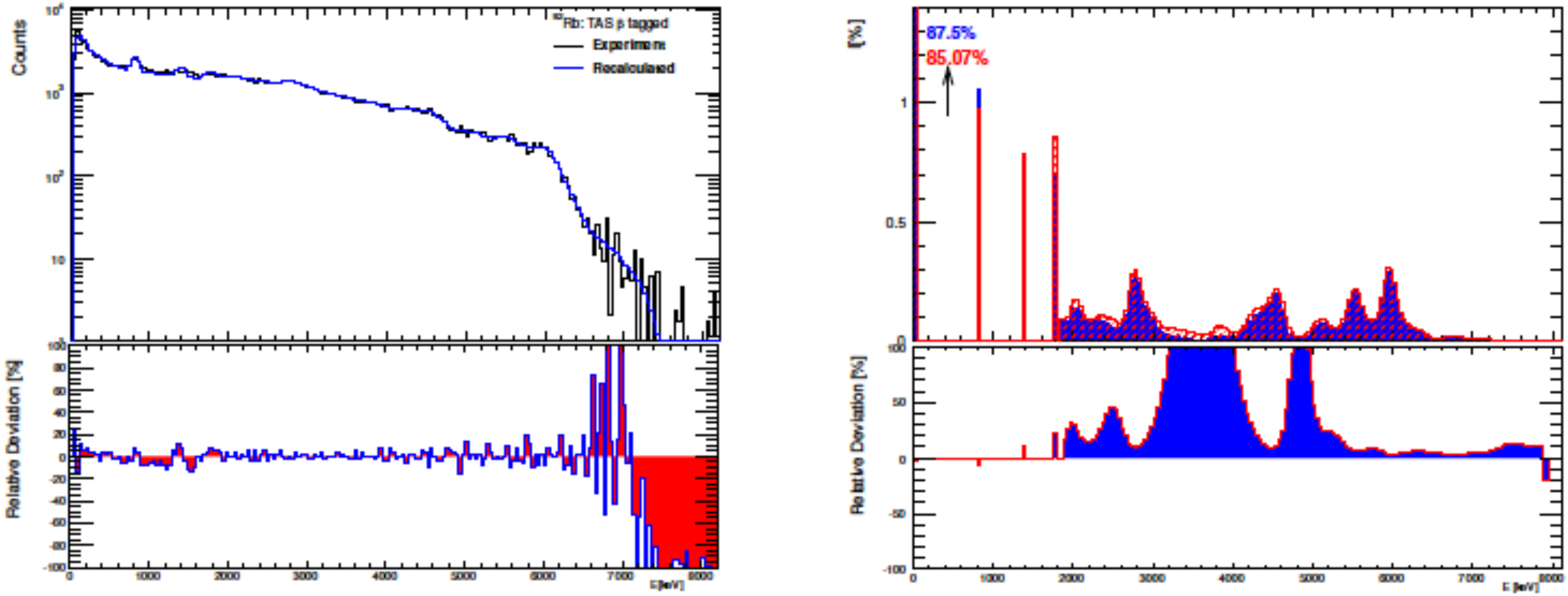


Figure 3.36 – *Comparaison des alimentations bêtas obtenues (rouge) en prenant les bornes supérieures des paramètres de la relation d’étalonnage en résolution avec le résultat adopté (bleu). L’impact est plus prononcé que celui de l’étalonnage en énergie.*

Impact de l'épaisseur du Si



(a) Reconstruction des données

(b) Comparaison des alimentations

FIGURE 3.37 – *En (a) superposition entre les données expérimentales et la reconstruction à partir des alimentations bêtas obtenues lorsque l'épaisseur du silicium est augmenté de 10%. En (b) cette alimentation (rouge) est comparée avec l'alimentation bêta de référence (bleu).*

A.-A. Zakari-Issoufou's PhD

Impact des densités de niveau

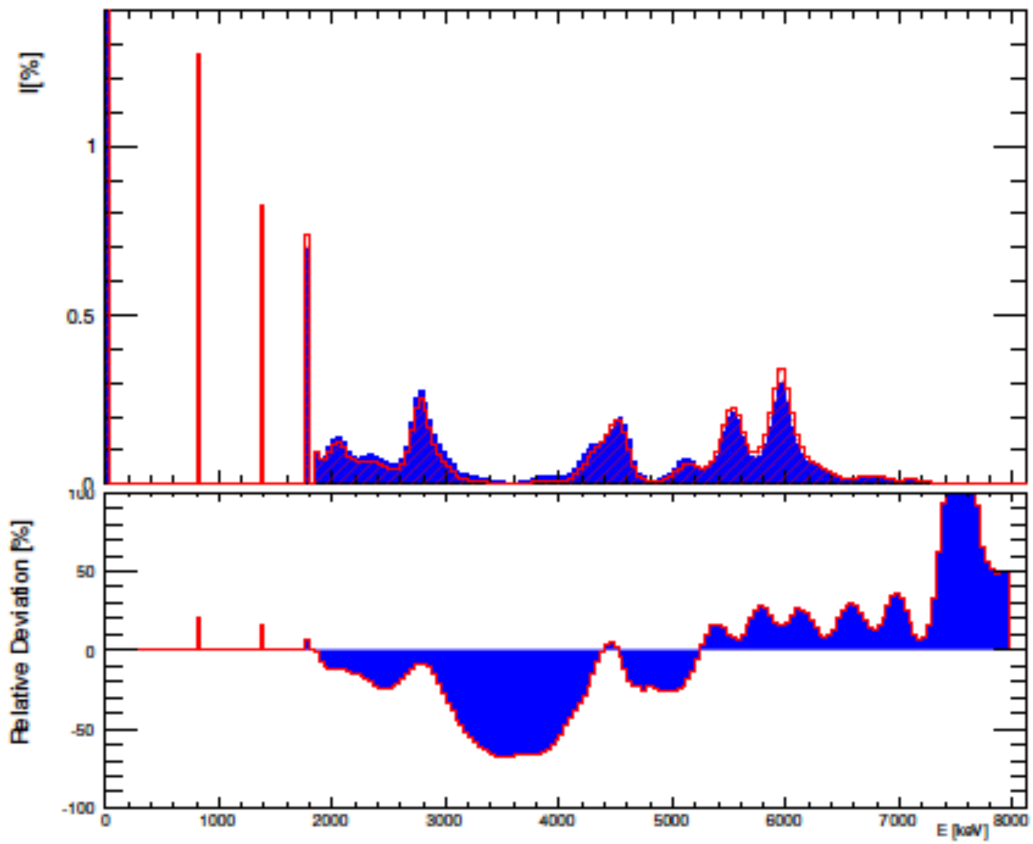


FIGURE 3.38 – *Comparaison des alimentations bêta obtenue pour le* ^{92}Rb, *en utilisant des densités de niveaux en moyenne 2 fois plus élevées que celle utilisées dans l'analyse de référence dont les alimentations sont représentées en bleu.*

A.-A. Zakari-Issoufou's PhD

Tableau récapitulatif

Pour l'instant : Somme quadratique des incertitudes, malgré les corrélations entre paramètres

E_{level}	I_β	Incertitudes[%]						Total[%]
		Statistique	Soustraction	E_{Cal}	R_{Cal}	Δ_{Si}	Densité	
0	87.5	0.23	0.16	0.02	0.05	2.77	0.57	2.84
814.97998	1.05836	0.43	0.80	0.21	1.87	7.40	13.70	15.71
1384.79004	0.713029	0.40	3.47	0.24	1.44	10.81	8.04	14.00
1778.32996	0.69934	0.37	0.71	0.67	0.15	23.25	−0.98	23.29
1860	0.0956618	0.41	0.76	0.88	3.25	0.04	−2.33	4.19
1900	0.0765636	0.41	1.22	0.03	1.37	16.95	−19.52	25.92
1940	0.082986	0.41	1.35	0.38	0.50	25.45	−23.74	34.83
1980	0.110346	0.40	1.41	0.63	0.23	32.55	−20.89	38.71
2020	0.133679	0.41	1.48	0.71	0.57	29.90	−19.18	35.57
2060	0.14103	0.41	1.65	0.68	0.64	23.69	−18.09	29.87
2100	0.126544	0.41	1.89	0.50	0.25	18.44	−18.32	26.07
2140	0.0972783	0.41	2.21	0.19	0.67	15.82	−20.34	25.88
2180	0.082471	0.41	2.35	0.02	1.23	13.80	−22.30	26.36
2220	0.0768429	0.41	2.37	0.05	1.08	13.80	−24.53	28.27
2260	0.0793313	0.41	2.31	0.37	0.13	16.83	−25.22	30.41
2300	0.0831362	0.41	2.25	0.60	0.54	20.13	−26.21	33.14
2340	0.0859162	0.41	2.15	0.69	0.74	26.55	−26.73	37.75
2380	0.0801968	0.41	2.11	0.55	0.19	33.03	−28.13	43.44
2420	0.0729589	0.41	2.06	0.46	0.06	39.35	−29.77	49.39
2460	0.0670209	0.41	1.97	0.60	0.28	45.28	−30.58	54.68
2500	0.060217	0.41	1.85	0.67	0.56	45.85	−30.35	55.03
2540	0.0556243	0.41	1.72	0.48	0.05	42.96	−29.86	52.35

A.-A. Zakari-Issoufou's PhD

Impact of several sources of systematic uncertainty on the shape of the intensity distribution.

In total the blue areas are 14 solutions for ^{87}Br, 13 for ^{88}Br, and 15 for ^{94}Rb.

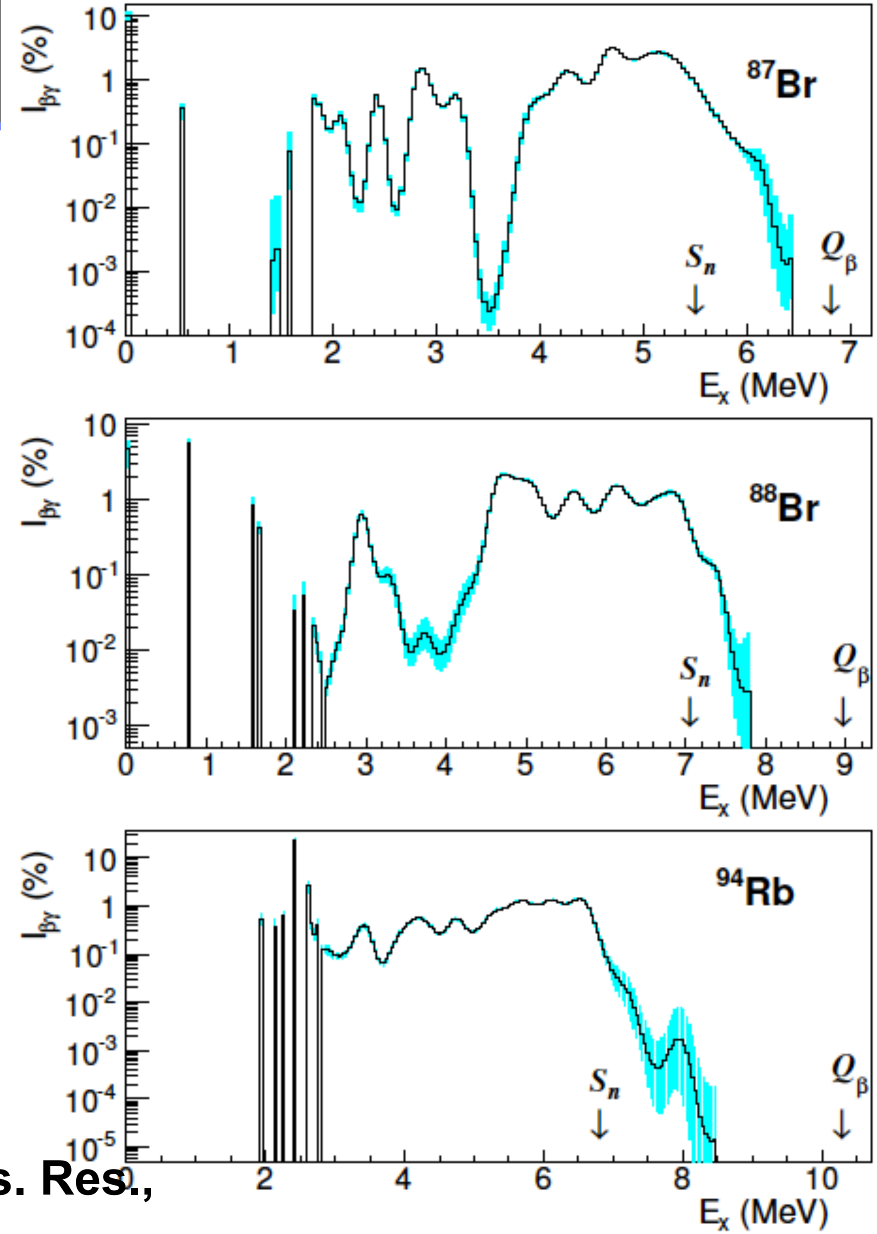
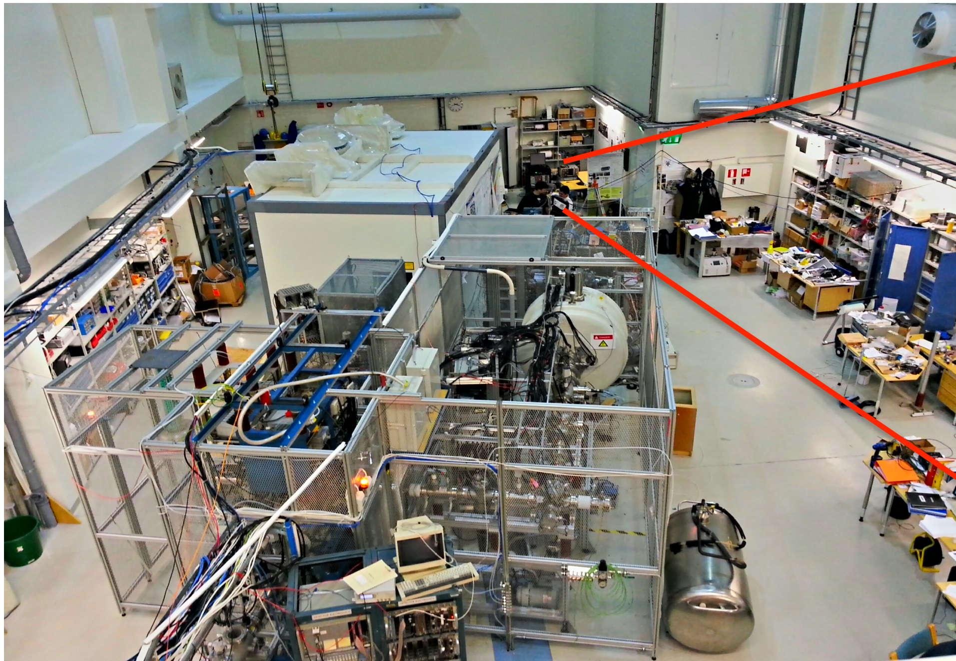
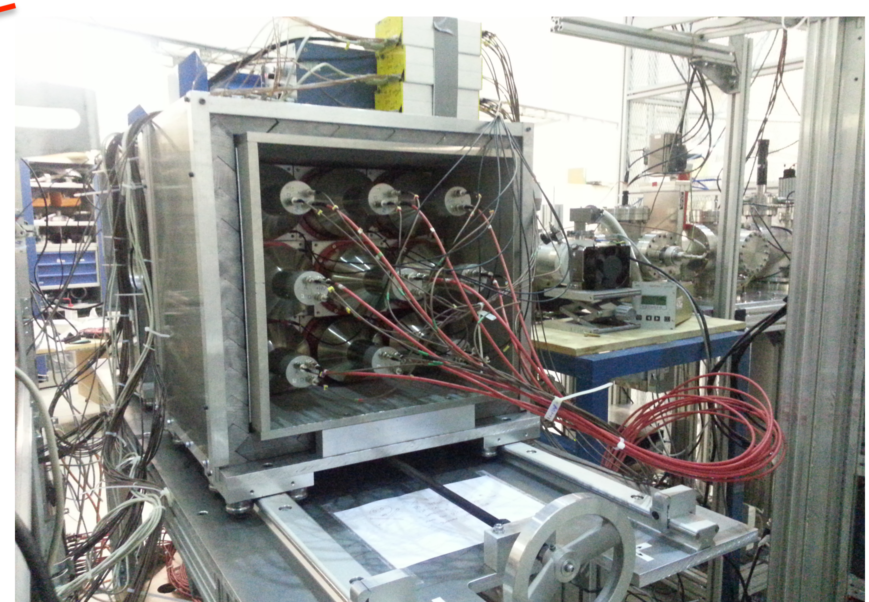


FIG. 4. (Color online) Beta intensity distributions from TAGS. The thin black line is the adopted solution, the light blue filled region indicates the spread of solutions due to the systematic effects investigated. See text for details.

J. Agramunt et al., Nucl. Instrum. Methods Phys. Res., Sect. A 807, 69 (2016).
E. Valencia et al. PRC 95 024320 (2017)

Experiment @ Jyväskylä in 2014





Reactor antineutrino proposal:

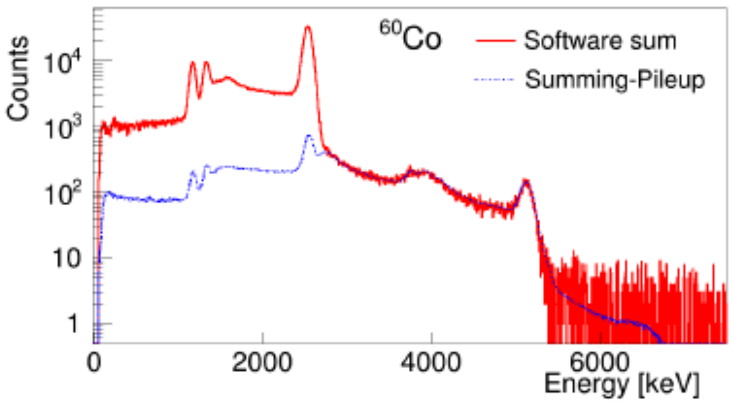
- 12 nuclei for antineutrinos measured & 11 for decay heat
- First use of new DTAS (17+1 NaI) developed by IFIC (Valencia)
- Successful use of the new IGISOL-4 facility
- First use of precision trap with IGISOL-4

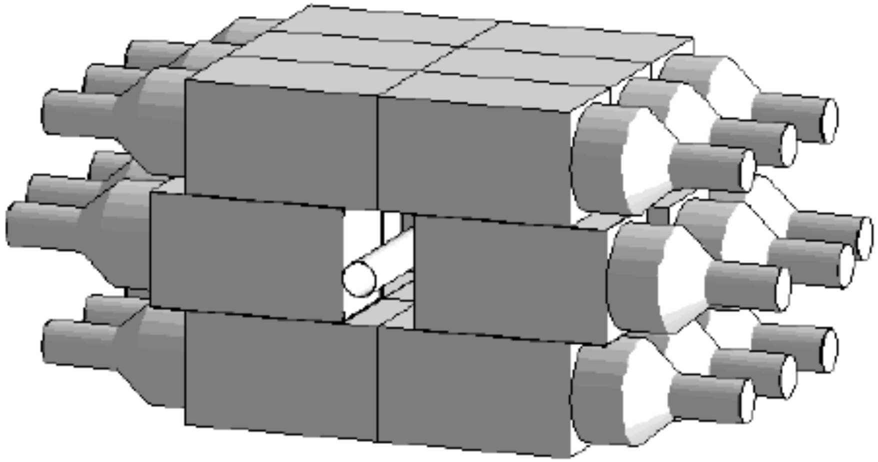
V.Guadilla et al.,, Nucl. Inst. and Meth. B, in press. Online (2015) : http://www.sciencedirect.com/science/article/pii/S0168583X15012628

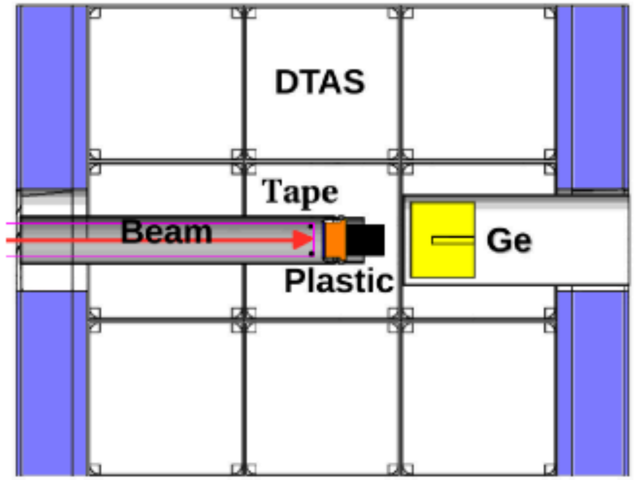
MC characterization: the path to R_{ij}

How?

- Geant4 simulation code
- Event generator for the sources
- Geometry of the set-up
- Non-proportionality NaI(TI)







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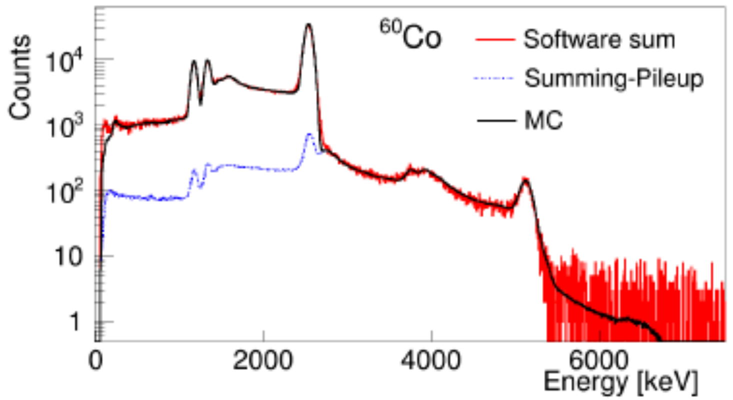
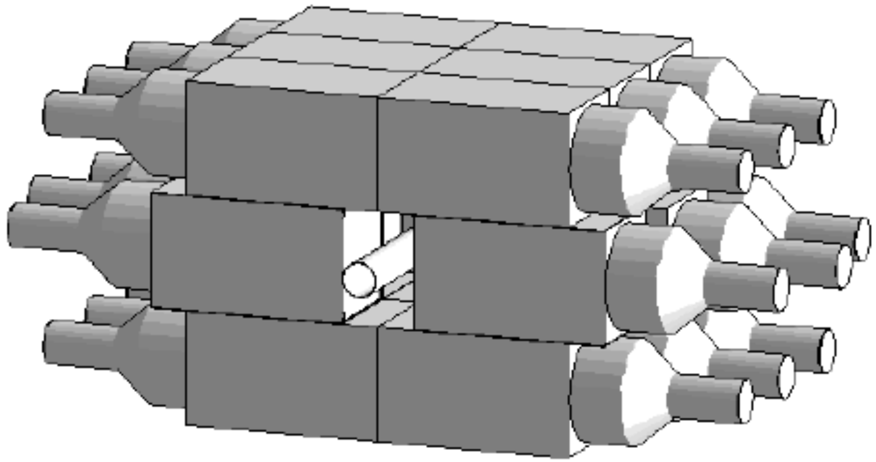
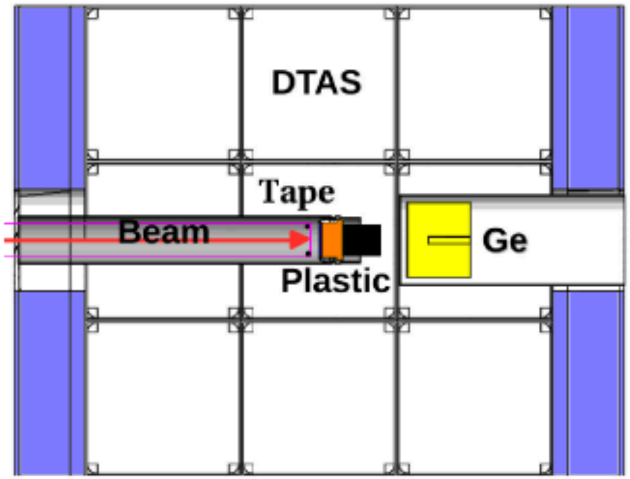


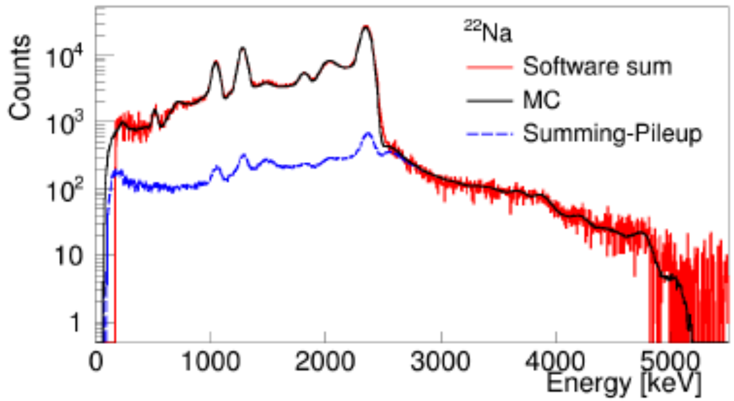




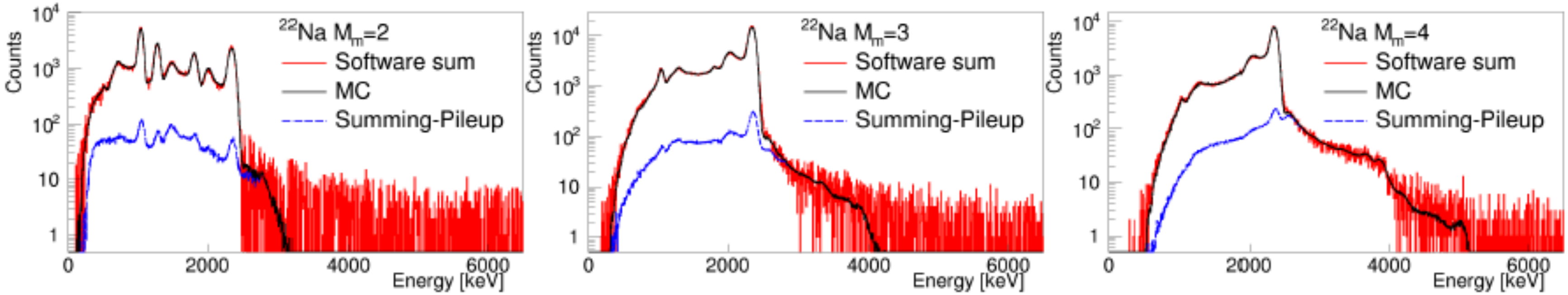
MC characterization: the path to R_{ij}

How?

- Geant4 simulation code
- Event generator for the sources
- Geometry of the set-up
- Non-proportionality NaI(Tl)



Segmentation can be useful to study different multiplicities:



M_m: number of modules with energy deposited in the event

MC characterization: the path to R_{ij}

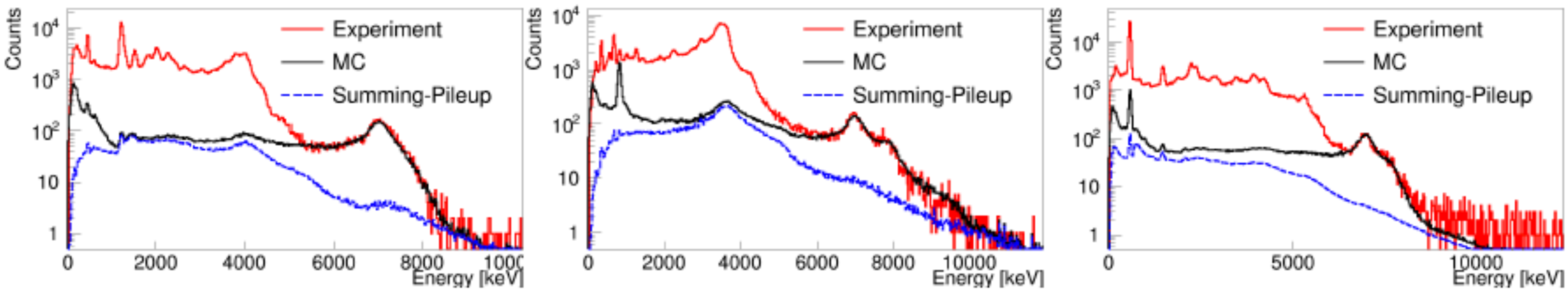
How?

- Geant4 simulation code
- Event generator for the sources
- Geometry of the set-up
- Non-proportionality NaI(Tl)

$LaBr_3$:Ce J.L. Tain et al., NIMA 774 (2015) 17

BaF_2 E. Valencia et al., PRC 95 (2017) 024320

NaI(Tl) → Response of DTAS to neutrons (inelastic, capture...):



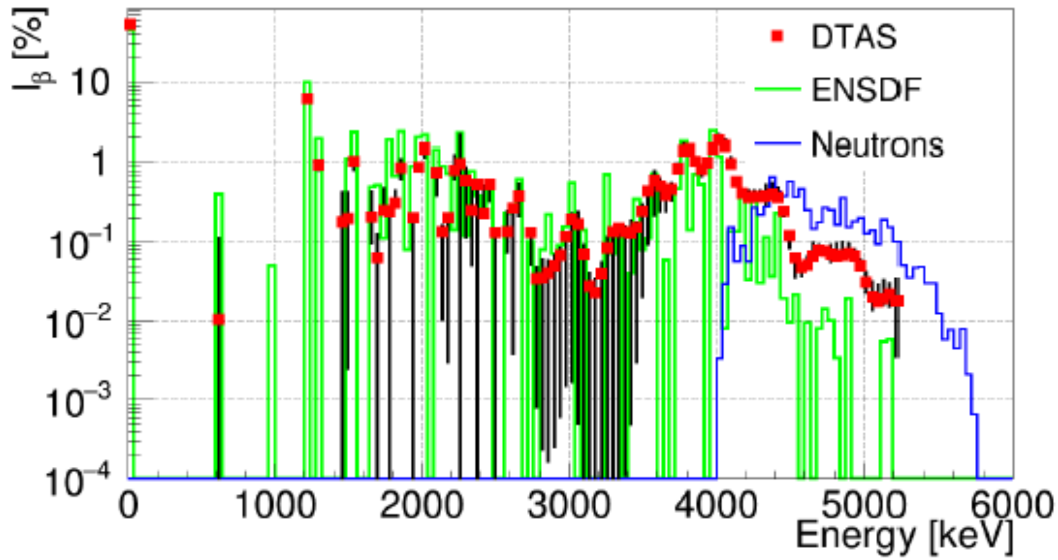
^{137}I ^{95}Rb ^{138}I

V. Guadilla et al. submitted to NIMA (2018), arXiv:

^{137}I: errors

Following E. Valencia et al., PRC 95 (2017) 024320: envelope of solutions compatible with data

- Daughter normalization
- β-n normalization
- Summing-pileup normalization
- Known level scheme
- Deconvolution algorithm
- β detector efficiency
- β-gated **vs.** singles **vs.** neutron veto



- Branching ratio variations: parameters of level density, γ-strength function, spin-parity values of known level scheme, better reproduction of known γ-intensities etc.

^{137}I: errors

Following E. Valencia et al., PRC 95 (2017) 024320: envelope of solutions compatible with data

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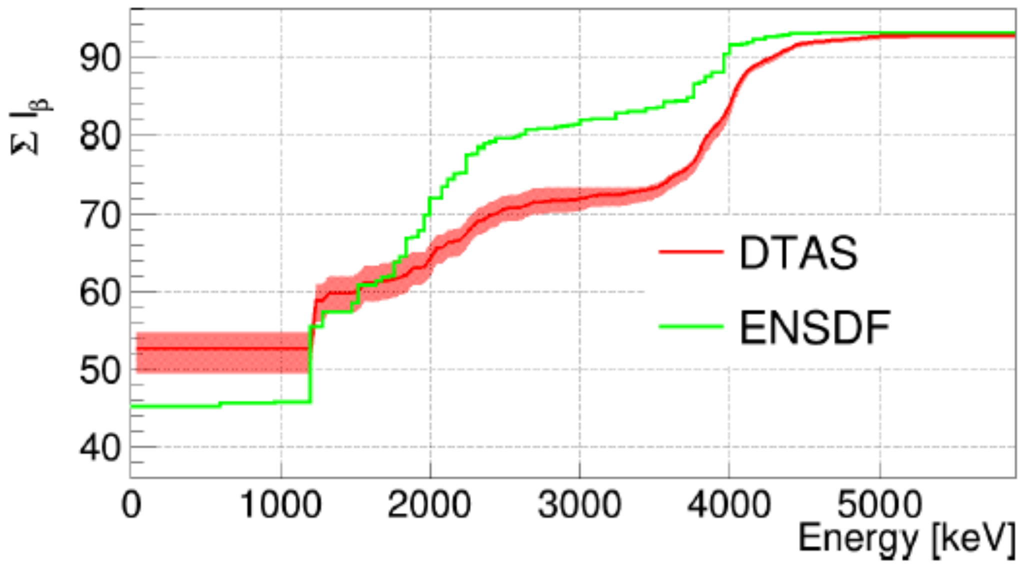


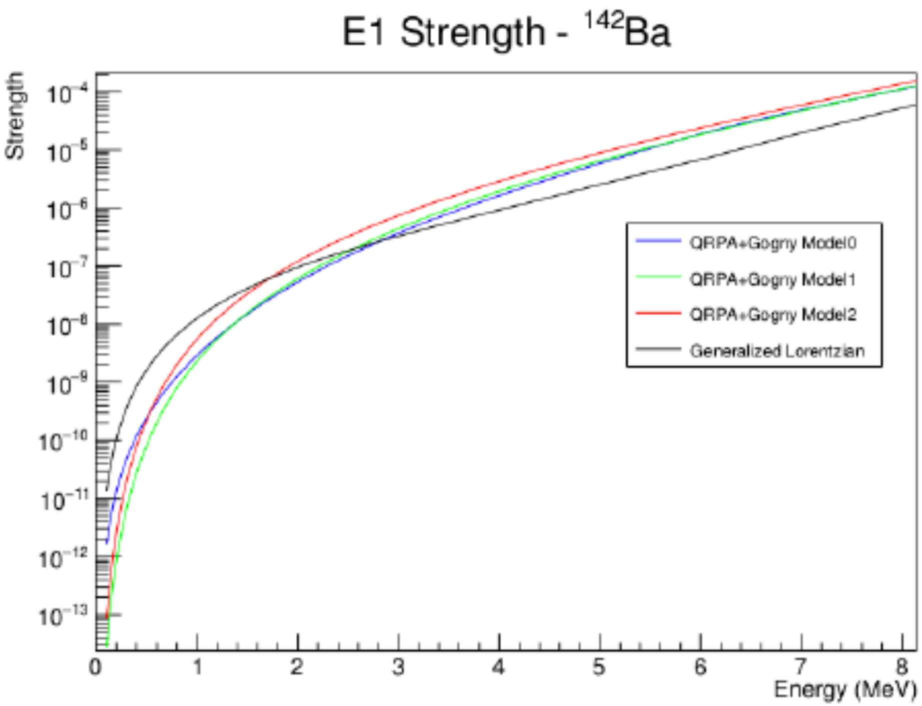
- Branching ratio variations: parameters of level density, γ-strength function, spin-parity values of known level scheme, better reproduction of known γ-intensities etc.

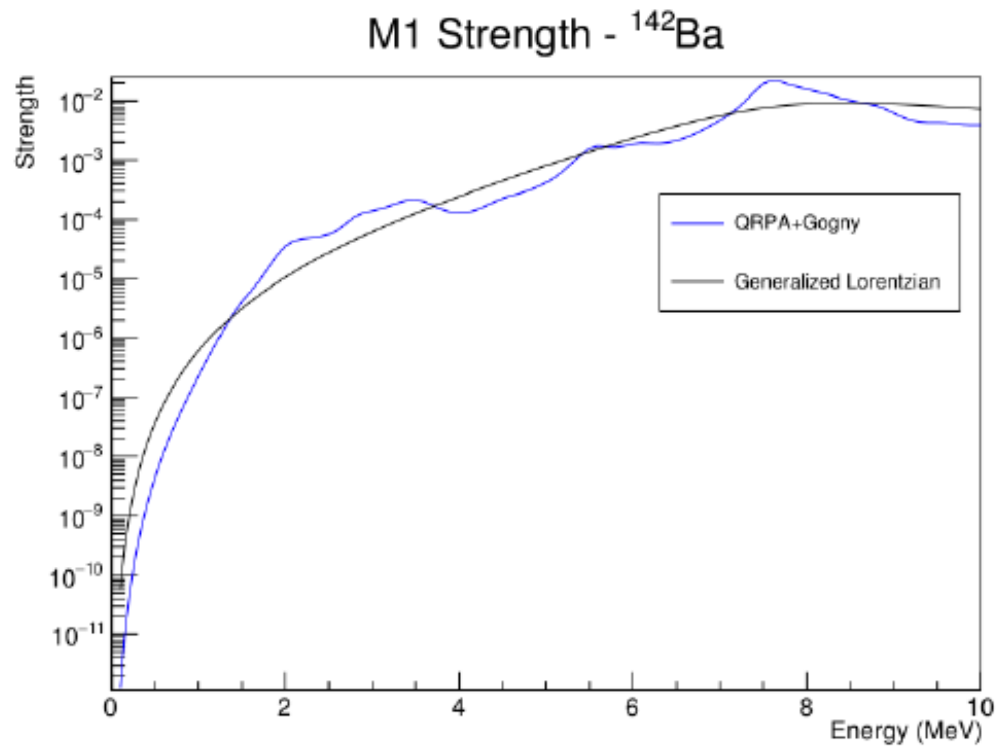
L. Le Meur's PhD thesis

Input parameters for branching ratio matrix :

- γ-strength (E1, M1, E2)…
- … and parametrization (E_R,Γ_R,σ_R)
- Level densities and parameter *a*
- Nucleus deformation β
- Limit between *Known* and *Unknown* part (here = 1880 keV)
    - Depends on the knowledge of the daughter nucleus

Impact of input models for the γ-strength (generalized Lorentzian, QRPA-Gogny v0, 1 and 2 from S. Péru-Désenfants and M. Martini)





Outlooks

- **Study the propagation of uncertainties with correlations**
- **Extract correlation/covariance matrices**
- **Include the TAGS data and their uncertainties in the evaluated databases…**
- **We regularly include the new TAGS data in summation calculations to compare with integral data and extract new priority lists (decay heat, antineutrinos, Beff)**

=> cf. Our proposal to the next European Nuclear Data project + next NACRE 2019-2021

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TOTAL ABSORPTION SPECTROSCOPY FOR NUCLEAR STRUCTURE, NUCLEAR ASTROPHYSICS, NEUTRINO AND REACTOR PHYSICS

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PAC ALTO Jan. 24 2014

Laboratoire de physique subatomique et des technologies associées

ECOLE DES MINES DE NANTES

Bayes theorem

$$P(f_j|d_i) = \frac{P(d_i|f_j)P(f_j)}{\sum_{j=1}^{m} P(d_i|f_j)P(f_j)}$$

Where :

$P(f_j)$ $(\equiv f_j/\sum_{j=1}^{m} f_j)$ is the a priori probability on the feedings f_j

$P(d_i|f_j)$ is the condition probability that d_i is due to the parameter f_j and is equivalent to R_{ij}

$P(f_j|d_i)$ is the a posteriori conditional probability that parameter f_j was the cause of d_i

$$\hat{f}_j = \frac{1}{\sum_{i=1}^{n} R_{ij}} \sum_{i=1}^{n} P(f_j|d_i)\hat{d}_i, \quad j = 1, \ldots, m$$

→ Relation between expected feedings and data

Include the latter formula into the first gives an iterative algorithm

$$f_j^{(s+1)} = \frac{1}{\sum_{i=1}^{n} R_{ij}} \sum_{i=1}^{n} \frac{R_{ij} f_j^{(s)} d_i}{\sum_{k=1}^{m} R_{ik} f_k^{(s)}}, \quad j = 1, \ldots, m.$$