## Hadronic and New Physics contributions to $\mathrm{b} \rightarrow \mathrm{s}$ transitions

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XIII Meeting on B Physics: Synergy between LHC \& SUPERKEKB in the Quest for New Physics

Radiative and (semi)leptonic rare $B$ decays are highly sensitive probes for new physics

Inclusive decays $B \rightarrow X_{s} \gamma$ and $B \rightarrow X_{s} \ell^{+} \ell^{-}$

- Precise theory calculations
- Heavy mass expansion
- Theoretical description of power corrections available $\rightarrow$ they can be calculated or estimated within the theoretical approach
- Full exploitation possible with Belle-II (complete angular analysis)


## Exclusive decays

- Leptonic: $B_{s} \rightarrow \mu^{+} \mu^{-}$
$\rightarrow$ theory errors under control (decay constant with rather good precision)
- Semileptonic: $B \rightarrow K^{*} \mu^{+} \mu^{-}, B \rightarrow K \mu^{+} \mu^{-}$and $B_{s} \rightarrow \phi \mu^{+} \mu^{-}$
$\rightarrow$ many experimentally accessible observables
$\rightarrow$ issue of hadronic uncertainties in exclusive modes
no theoretical description of power corrections existing within the theoretical framework of QCD factorisation and SCET


## Theoretical framework

## Effective field theory

$$
\mathcal{H}_{\mathrm{eff}}=-\frac{4 G_{F}}{\sqrt{2}} V_{t b} V_{t s}^{*}\left(\sum_{i=1 \cdots 10, S, P}\left(C_{i}(\mu) \mathcal{O}_{i}(\mu)+C_{i}^{\prime}(\mu) \mathcal{O}_{i}^{\prime}(\mu)\right)\right)
$$

Operator set for $b \rightarrow s$ transitions:


+ the chirality flipped counter-parts of the above operators, $\mathcal{O}_{i}^{\prime}$


## Wilson coefficients:

The Wilson coefficients are calculated perturbatively and are process independent.
SM contributions known to NNLL (Bobeth, Missiak, Urban '99; Misiak, Steinhauser '04, Gorbahn, Haisch '04; Gorbahn, Haisch, Misiak '05; Czakon, Haisch, Misiak '06,...)

$$
C_{7}=-0.294 \quad C_{9}=4.20 \quad C_{10}=-4.01
$$

## Rare decays

Many observables, with different sensitivities to different Wilson coefficients.

| decay | obs | $C_{7}^{(\prime)}$ | $C_{9}^{(\prime)}$ | $C_{10}^{(\prime)}$ |
| :---: | :---: | :---: | :---: | :---: |
| $B \rightarrow X_{s} \gamma$ | BR | X |  |  |
| $B \rightarrow K^{*} \gamma$ | $\mathrm{BR}, \mathrm{A}_{\mathrm{I}}$ | X |  |  |
| $B \rightarrow X_{s} \ell^{+} \ell^{-}$ | $\mathrm{dBR} / \mathrm{dq}^{2}, A_{\mathrm{FB}}$ | X | X | X |
| $B \rightarrow K \ell^{+} \ell^{-}$ | $\mathrm{dBR} / \mathrm{d} q^{2}$ | X | X | X |
| $B \rightarrow K^{*} \ell^{+} \ell^{-}$ | $\mathrm{dBR} / \mathrm{d} q^{2}$, angular obs. | X | X | X |
| $B_{s} \rightarrow \phi \ell^{+} \ell^{-}$ | $\mathrm{dBR} / \mathrm{dq}^{2}$, angular obs. | X | X | X |
| $B_{s} \rightarrow \mu^{+} \mu^{-}$ | BR |  |  | X |

The only reason $C_{9}$ is the main player to explain the anomalies is that $C_{7}$ and $C_{10}$ are severely constrained!

$$
\delta\left\langle P_{5}^{\prime}\right\rangle_{[4.3,8.68]} \simeq-0.52 \delta C_{7} \quad-0.03 \delta C_{8} \quad-0.08 \delta C_{9} \quad-0.03 \delta C_{10}
$$

## $B \rightarrow K^{*} \mu^{+} \mu^{-}$

## Angular distributions

The full angular distribution of the decay $\bar{B}^{0} \rightarrow \bar{K}^{* 0} \ell^{+} \ell^{-}\left(\bar{K}^{* 0} \rightarrow K^{-} \pi^{+}\right)$is completely described by four independent kinematic variables: $q^{2}$ (dilepton invariant mass squared), $\theta_{\ell}, \theta_{K^{*}}, \phi$


## Differential decay distribution:


$J\left(q^{2}, \theta_{\ell}, \theta_{K^{*}}, \phi\right)=\sum_{i} J_{i}\left(q^{2}\right) f_{i}\left(\theta_{\ell}, \theta_{K^{*}}, \phi\right)$

## angular coefficients $J_{1-9}$

functions of the spin amplitudes $A_{0}, A_{\|}, A_{\perp}, A_{t}$, and $A_{S}$
Spin amplitudes: functions of Wilson coefficients and form factors
Main operators:

$$
\begin{array}{ll}
\mathcal{O}_{9}=\frac{e^{2}}{(4 \pi)^{2}}\left(\bar{s} \gamma^{\mu} b_{L}\right)\left(\bar{\ell} \gamma_{\mu} \ell\right), & \mathcal{O}_{10}=\frac{e^{2}}{(4 \pi)^{2}}\left(\bar{s} \gamma^{\mu} b_{L}\right)\left(\bar{\ell} \gamma_{\mu} \gamma_{5} \ell\right) \\
\mathcal{O}_{S}=\frac{e^{2}}{16 \pi^{2}}\left(\bar{s}_{L}^{\alpha} b_{R}^{\alpha}\right)(\bar{\ell} \ell), & \mathcal{O}_{P}=\frac{e^{2}}{16 \pi^{2}}\left(\bar{s}_{L}^{\alpha} b_{R}^{\alpha}\right)\left(\bar{\ell}_{5} \ell\right)
\end{array}
$$

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Differential decay distribution:

$$
\frac{d^{4} \Gamma}{d q^{2} d \cos \theta_{\ell} d \cos \theta_{K^{*}} d \phi}=\frac{9}{32 \pi} J\left(q^{2}, \theta_{\ell}, \theta_{K^{*}}, \phi\right)
$$

$$
J\left(q^{2}, \theta_{\ell}, \theta_{K^{*}}, \phi\right)=\sum_{i} J_{i}\left(q^{2}\right) f_{i}\left(\theta_{\ell}, \theta_{K^{*}}, \phi\right)
$$

$\downarrow$ angular coefficients $J_{1-9}$
$\searrow$ functions of the spin amplitudes $A_{0}, A_{\|}, A_{\perp}, A_{t}$, and $A_{S}$
Spin amplitudes: functions of Wilson coefficients and form factors
Main operators:

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\mathcal{O}_{9}=\frac{e^{2}}{(4 \pi)^{2}}\left(\bar{s} \gamma^{\mu} b_{L}\right)\left(\bar{\ell} \gamma_{\mu} \ell\right), & \mathcal{O}_{10}=\frac{e^{2}}{(4 \pi)^{2}}\left(\bar{s} \gamma^{\mu} b_{L}\right)\left(\bar{\ell} \gamma_{\mu} \gamma_{5} \ell\right) \\
\mathcal{O}_{S}=\frac{e^{2}}{16 \pi^{2}}\left(\bar{s}_{L}^{\alpha} b_{R}^{\alpha}\right)(\bar{\ell} \ell), & \mathcal{O}_{P}=\frac{e^{2}}{16 \pi^{2}}\left(\bar{s}_{L}^{\alpha} b_{R}^{\alpha}\right)\left(\bar{\ell} \gamma_{5} \ell\right)
\end{array}
$$

$\qquad$

$\qquad$

## $B \rightarrow K^{*} \mu^{+} \mu^{-}-$Angular observables

Optimised observables: form factor uncertainties cancel at leading order

$$
\begin{aligned}
\left\langle P_{1}\right\rangle_{\text {bin }}=\frac{1}{2} \frac{\int_{\text {bin }} d q^{2}\left[J_{3}+\bar{J}_{3}\right]}{\int_{\text {bin }} d q^{2}\left[J_{2 s}+\bar{J}_{2 s}\right]} & \left\langle P_{2}\right\rangle_{\text {bin }}=\frac{1}{8} \frac{\int_{\text {bin }} d q^{2}\left[J_{6 s}+\bar{J}_{6 s}\right]}{\int_{\text {bin }} d q^{2}\left[J_{2 s}+\bar{J}_{2 s}\right]} \\
\left\langle P_{4}^{\prime}\right\rangle_{\text {bin }}=\frac{1}{\mathcal{N}_{\text {bin }}^{\prime}} \int_{\text {bin }} d q^{2}\left[J_{4}+\bar{J}_{4}\right] & \left\langle P_{5}^{\prime}\right\rangle_{\text {bin }}=\frac{1}{2 \mathcal{N}_{\text {bin }}^{\prime}} \int_{\text {bin }} d q^{2}\left[J_{5}+\bar{J}_{5}\right] \\
\left\langle P_{6}^{\prime}\right\rangle_{\text {bin }}=\frac{-1}{2 \mathcal{N}_{\text {bin }}^{\prime}} \int_{\text {bin }} d q^{2}\left[J_{7}+\bar{J}_{7}\right] & \left\langle P_{8}^{\prime}\right\rangle_{\text {bin }}=\frac{-1}{\mathcal{N}_{\text {bin }}^{\prime}} \int_{\text {bin }} d q^{2}\left[J_{8}+\bar{J}_{8}\right]
\end{aligned}
$$

with

$$
\mathcal{N}_{\text {bin }}^{\prime}=\sqrt{-\int_{\text {bin }} d q^{2}\left[J_{2 s}+\bar{J}_{2 s}\right] \int_{\text {bin }} d q^{2}\left[J_{2 c}+\bar{J}_{2 c}\right]}
$$

+CP violating clean observables and other combinations

$$
\begin{aligned}
& \text { U. Egede et al., JHEP } 0811 \text { (2008) 032, JHEP } 1010 \text { (2010) } 056 \\
& \text { J. Matias et al., JHEP } 1204 \text { (2012) } 104 \\
& \text { S. Descotes-Genon et al., JHEP } 1305 \text { (2013) } 137
\end{aligned}
$$

Or alternatively:

$$
S_{i}=\frac{J_{i(s, c)}+\bar{J}_{i(s, c)}}{\frac{d \Gamma}{d q^{2}}+\frac{d \bar{\Gamma}}{d q^{2}}}, \quad P_{4,5,8}^{\prime}=\frac{S_{4,5,8}}{\sqrt{F_{L}\left(1-F_{L}\right)}}
$$

## $B \rightarrow K^{*} \mu^{+} \mu^{-}-P_{5}^{\prime}$ anomaly

Long standing anomaly 2-3 $\sigma$ :

- $2013\left(1 \mathrm{fb}^{-1}\right)$ : disagreement with the SM for $P_{2}$ and $P_{5}^{\prime}$ (PRL 111, 191801 (2013))
- March $2015\left(3 \mathrm{fb}^{-1}\right)$ : confirmation of the deviations (LHCb-CONF-2015-002)
- Dec. 2015: 2 analysis methods, both show the deviations (JHEP 1602, 104 (2016))


LHCb, JHEP 02 (2016) 104; Belle, PRL 118 (2017); ATLAS, ATLAS-CONF-2017-023; CMS, CMS-PAS-BPH-15-008

- Also measured by ATLAS, CMS and Belle


## $B_{s} \rightarrow \phi \mu^{+} \mu^{-}$

$B_{s} \rightarrow \phi \mu^{+} \mu^{-}$branching fraction

- Same theoretical description as $B \rightarrow K^{*} \mu^{+} \mu^{-}$
- Replacement of $B \rightarrow K^{*}$ form factors with the $B_{s} \rightarrow \phi$ ones
- Also consider the $B_{s}-\bar{B}_{s}$ oscillations
- June $2015\left(3 \mathrm{fb}^{-1}\right)$ : the differential branching fraction is found to be $3.2 \sigma$ below the SM predictions in the [1-6] $\mathrm{GeV}^{2}$ bin

$$
\text { JHEP } 1509 \text { (2015) } 179
$$



## Lepton flavour universality tests

Lepton flavour universality in $B^{+} \rightarrow K^{+} \ell^{+} \ell^{-}$

- Theoretical description similar to $B \rightarrow K^{*} \mu^{+} \mu^{-}$, but different since $K$ is scalar
- SM prediction very accurate
- June $2014\left(3 \mathrm{fb}^{-1}\right)$ : measurement of

$$
R_{K}=B R\left(B^{+} \rightarrow K^{+} \mu^{+} \mu^{-}\right) / B R\left(B^{+} \rightarrow K^{+} e^{+} e^{-}\right)
$$

in the $[1-6] \mathrm{GeV}^{2}$ bin: $2.6 \sigma$ tension (Lhcb, PRL 113, 151601(2014))
 BaBar, PRD 86 (2012) 032012; Belle, PRL 103 (2009) 171801

Lepton flavour universality in $B^{0} \rightarrow K^{* 0} \ell^{+} \ell^{-}$

- LHCb measurement (April 2017): JHEP 08 (2017) 055

$$
R_{K^{*}}=B R\left(B^{0} \rightarrow K^{* 0} \mu^{+} \mu^{-}\right) / B R\left(B^{0} \rightarrow K^{* 0} e^{+} e^{-}\right)
$$

- Two $q^{2}$ regions: [0.045-1.1] and [1.1-6.0] $\mathrm{GeV}^{2}$
- 2.2-2.5 $\sigma$ tension in each bin


BaBar, PRD 86 (2012) 032012; Belle, PRL 103 (2009) 171801

If confirmed this would be a groundbreaking discovery!

## Issue of hadronic effects

Effective Hamiltonian for $b \rightarrow s \ell \ell$ transitions

$$
\begin{gathered}
\mathcal{H}_{\mathrm{eff}}=\mathcal{H}_{\mathrm{eff}}^{\mathrm{had}}+\mathcal{H}_{\mathrm{eff}}^{\mathrm{sl}} \\
\mathcal{H}_{\mathrm{eff}}^{\mathrm{sl}}=-\frac{4 G_{F}}{\sqrt{2}} V_{t b} V_{t s}^{*}\left[\sum_{i=7,9,10} C_{i}^{(\prime)} O_{i}^{(\prime)}\right]
\end{gathered}
$$

$\left\langle\bar{K}^{*}\right| \mathcal{H}_{\text {eff }}^{\text {sl }}|\bar{B}\rangle: B \rightarrow K^{*}$ form factors $V, A_{0,1,2}, T_{1,2,3}$
Transversity amplitudes:

$$
\begin{aligned}
A_{\perp}^{L, R} \simeq & N_{\perp}\left\{\left(C_{9}^{+} \mp C_{10}^{+}\right) \frac{V\left(q^{2}\right)}{m_{B}+m_{K^{*}}}+\frac{2 m_{b}}{q^{2}} C_{7}^{+} T_{1}\left(q^{2}\right)\right\} \\
A_{\|}^{L, R} \simeq & N_{\|}\left\{\left(C_{9}^{-} \mp C_{10}^{-}\right) \frac{A_{1}\left(q^{2}\right)}{m_{B}-m_{K^{*}}}+\frac{2 m_{b}}{q^{2}} C_{7}^{-} T_{2}\left(q^{2}\right)\right\} \\
A_{0}^{L, R} \simeq & N_{0}\left\{\left(C_{9}^{-} \mp C_{10}^{-}\right)\left[(\ldots) A_{1}\left(q^{2}\right)+(\ldots) A_{2}\left(q^{2}\right)\right]\right. \\
& \left.+2 m_{b} C_{7}^{-}\left[(\ldots) T_{2}\left(q^{2}\right)+(\ldots) T_{3}\left(q^{2}\right)\right]\right\} \\
A_{S} & =N_{S}\left(C_{S}-C_{S}^{\prime}\right) A_{0}\left(q^{2}\right) \quad\left(C_{i}^{ \pm} \equiv C_{i} \pm C_{i}^{\prime}\right)
\end{aligned}
$$

## Issue of hadronic effects

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$$
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\mathcal{H}_{\mathrm{eff}}=\mathcal{H}_{\mathrm{eff}}^{\mathrm{had}}+\mathcal{H}_{\mathrm{eff}}^{\mathrm{sl}} \\
\mathcal{H}_{\mathrm{eff}}^{\mathrm{had}}=-\frac{4 G_{F}}{\sqrt{2}} V_{t b} V_{t s}^{*}\left[\sum_{i=1 \ldots 6} C_{i} O_{i}+C_{8} O_{8}\right] \\
\mathcal{A}_{\lambda}^{(\mathrm{had})}=-i \frac{e^{2}}{q^{2}} \int d^{4} x e^{-i q \cdot x}\left\langle\ell^{+} \ell^{-}\right| j_{\mu}^{\mathrm{em}, \text { lept }}(x)|0\rangle \\
\times \int d^{4} y e^{i q \cdot y}\left\langle\bar{K}_{\lambda}^{*}\right| T\left\{j^{\mathrm{em}, \text { had }, \mu}(y) \mathcal{H}_{\mathrm{eff}}^{\mathrm{had}}(0)\right\}|\bar{B}\rangle \\
\equiv \\
\frac{e^{2}}{q^{2}} \epsilon_{\mu} L_{V}^{\mu}[\underbrace{\mathrm{LO} \text { in } \mathcal{O}\left(\frac{\Lambda}{m_{b}}, \frac{\Lambda}{E_{K^{*}}}\right)}_{\begin{array}{c}
\text { Non-Fact., QCDf } \\
\text { Beneke et al.: } \\
106067 ; 0412400
\end{array}}+\underbrace{h_{\lambda}\left(q^{2}\right)}_{\text {power corrections }}]
\end{gathered}
$$

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\mathcal{A}_{\lambda}^{(\mathrm{had})}= \\
-i \frac{e^{2}}{q^{2}} \int d^{4} x e^{-i q \cdot x}\left\langle\ell^{+} \ell^{-}\right| j_{\mu}^{\mathrm{em}, l \mathrm{lept}}(x)|0\rangle \\
\times \int d^{4} y e^{i q \cdot y}\left\langle\bar{K}_{\lambda}^{*}\right| T\left\{j^{\mathrm{em}, \text { had }, \mu}(y) \mathcal{H}_{\mathrm{eff}}^{\text {had }}(0)\right\}|\bar{B}\rangle \\
\equiv \frac{e^{2}}{q^{2}} \epsilon_{\mu} L_{V}^{\mu}[\underbrace{\mathrm{LO} \text { in } \mathcal{O}\left(\frac{\Lambda}{m_{b}}, \frac{\Lambda}{E_{K^{*}}}\right.}_{\begin{array}{c}
\text { Non-Fact., } \mathrm{QCDf} \\
\text { Beneke et al.: } \\
106067 ; 0412400
\end{array}})+\underbrace{\text { unknown }}_{\substack{h_{\lambda}\left(q^{2}\right) \\
\text { power corrections } \\
\text { partial calculation: Khodjamirian et al., } \\
1006.4945}}]
\end{gathered}
$$

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\times \int d^{4} y e^{i q \cdot y}\left\langle\bar{K}_{\lambda}^{*}\right| T\left\{j^{\mathrm{em}, \text { had }, \mu}(y) \mathcal{H}_{\mathrm{eff}}^{\mathrm{had}}(0)\right\}|\bar{B}\rangle \\
\equiv \frac{e^{2}}{q^{2}} \epsilon_{\mu} L_{V}^{\mu}[\underbrace{\text { LO in } \mathcal{O}\left(\frac{\Lambda}{m_{b}}, \frac{\Lambda}{E_{K^{*}}}\right)}_{\begin{array}{l}
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106067 ; 0412400
\end{array}}+\underbrace{h_{\lambda}\left(q^{2}\right)}_{\substack{\underbrace{\rightarrow \text { unknown }}_{\text {power corrections }} \begin{array}{l}
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100064945
\end{array}}}]
\end{gathered}
$$

The significance of the anomalies depends on the assumptions made for the unknown power corrections!

## Issue of hadronic effects

Effective Hamiltonian for $b \rightarrow s \ell \ell$ transitions

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\begin{aligned}
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& \equiv \frac{e^{2}}{q^{2}} \epsilon_{\mu} L_{V}^{\mu}[\underbrace{\operatorname{LO} \text { in } \mathcal{O}\left(\frac{\Lambda}{m_{b}}, \frac{\Lambda}{E_{K^{*}}}\right)}_{\text {Non-Fact., QCDf }}+\underbrace{h_{\lambda}\left(q^{2}\right)}_{\text {power corrections }}] \\
& \text { Beneke et al.: } \\
& \text { 106067; } 0412400 \\
& \rightarrow \text { unknown } \\
& \text { partial calculation: Khodjamirian et al., } \\
& 1006.4945
\end{aligned}
$$

The significance of the anomalies depends on the assumptions made for the unknown power corrections!
This does not affect $R_{K}$ and $R_{K}^{*}$ of course, but does affect the combined fits!

# New Physics or hadronic effects? 

## Global fits

Many observables $\rightarrow$ Global fits of the latest LHCb data
Relevant $\mathcal{O}$ perators:

$$
\mathcal{O}_{7}, \mathcal{O}_{8}, \mathcal{O}_{9 \mu, e}^{\left({ }^{\prime}\right)}, \mathcal{O}_{10 \mu, e}^{\left({ }^{\prime}\right)} \quad \text { and } \quad \mathcal{O}_{S-P} \propto\left(\bar{s} P_{R} b\right)\left(\bar{\mu} P_{L} \mu\right)
$$

NP manifests itself in the shifts of the individual coefficients with respect to the SM values:

$$
C_{i}(\mu)=C_{i}^{\mathrm{SM}}(\mu)+\delta C_{i}
$$

$\rightarrow$ Scans over the values of $\delta C_{i}$
$\rightarrow$ Calculation of flavour observables
$\rightarrow$ Comparison with experimental results
$\rightarrow$ Constraints on the Wilson coefficients $C_{i}$
Several groups doing global fits (with similar results):
B. Capdevila, A. Crivellin, S. Descotes-Genon, J. Matias and J. Virto, JHEP 1801 (2018) 093
W. Altmannshofer, P. Stangl and D. M. Straub, Phys. Rev. D 96 (2017) no.5, 055008
G. D'Amico, M. Nardecchia, P. Panci, F. Sannino, A. Strumia, R. Torre and Urbano, JHEP 1709 (2017) 010
G. Hiller and I. Nisandzic, Phys. Rev. D 96 (2017) no.3, 035003
L. S. Geng, B. Grinstein, S. Jager, J. Martin Camalich, X. L. Ren and R. X. Shi, Phys. Rev. D 96 (2017) no.9, 093006
M. Ciuchini, A. M. Coutinho, M. Fedele, E. Franco, A. Paul, L. Silvestrini and M. Valli, Eur. Phys. J. C 77 (2017) no.10, 688
T. Hurth, FM, D. Martinez Santos and S. Neshatpour, Phys. Rev. D 96 (2017) no.9, 095034

## Theoretical uncertainties and correlations

- Monte Carlo analysis
- variation of the "standard" input parameters: masses, scales, CKM, ...
- decay constants taken from the latest lattice results
- use for the $B_{(s)} \rightarrow V$ form factors of the lattice+LCSR combinations from 1503.05534, including correlations
- $B \rightarrow K$ form factors are obtained from the lattice+LCSR combinations (1411.3161, 1503.05534), including all the correlations
- for $B_{s} \rightarrow \phi \mu^{+} \mu^{-}$, mixing effects taken into account
- Parameterisation of uncertainties from power corrections:

$$
A_{k} \rightarrow A_{k}\left(1+a_{k} \exp \left(i \phi_{k}\right)+\frac{q^{2}}{6 \mathrm{GeV}^{2}} b_{k} \exp \left(i \theta_{k}\right)\right)
$$

Leading Order QCDf of non-factorisable piece
$\left|a_{k}\right|$ between 10 to $60 \%, b_{k} \sim 2.5 a_{k}$
Low recoil: $b_{k}=0$

$$
\Rightarrow \text { Computation of a (theory }+\exp \text { ) correlation matrix }
$$

## Global fits

Global fits of the observables obtained by minimisation of

$$
\chi^{2}=\left(\vec{O}^{\text {th }}-\vec{O}^{\exp }\right) \cdot\left(\Sigma_{\text {th }}+\Sigma_{\exp }\right)^{-1} \cdot\left(\vec{O}^{\text {th }}-\vec{O}^{\exp }\right)
$$

$\left(\Sigma_{\text {th }}+\Sigma_{\text {exp }}\right)^{-1}$ is the inverse covariance matrix.
More than 100 observables relevant for leptonic and semileptonic decays:

- $\operatorname{BR}\left(B \rightarrow X_{s} \gamma\right)$
- $\operatorname{BR}\left(B \rightarrow K^{0} \mu^{+} \mu^{-}\right)$
- $\operatorname{BR}\left(B \rightarrow X_{d} \gamma\right)$
- $\operatorname{BR}\left(B \rightarrow K^{*} \gamma\right)$
- $\Delta_{0}\left(B \rightarrow K^{*} \gamma\right)$
- $\operatorname{BR}\left(B \rightarrow K^{*+} \mu^{+} \mu^{-}\right)$
- $\mathrm{BR}\left(B \rightarrow K^{+} \mu^{+} \mu^{-}\right)$
- $\operatorname{BR}\left(B \rightarrow K^{*} e^{+} e^{-}\right)$
- $\mathrm{BR}^{\text {low }}\left(B \rightarrow X_{s} \mu^{+} \mu^{-}\right)$
- $R_{K}$
- $\operatorname{BR}^{\text {high }}\left(B \rightarrow X_{s} \mu^{+} \mu^{-}\right)$
- $\mathrm{BR}^{\text {low }}\left(B \rightarrow X_{s} e^{+} e^{-}\right)$
- $\mathrm{BR}^{\mathrm{high}}\left(B \rightarrow X_{s} e^{+} e^{-}\right)$
- $\operatorname{BR}\left(B_{s} \rightarrow \mu^{+} \mu^{-}\right)$
- $\mathrm{BR}\left(B_{d} \rightarrow \mu^{+} \mu^{-}\right)$
- $R_{K^{*}}$
- $B \rightarrow K^{* 0} \mu^{+} \mu^{-}: B R, F_{L}, A_{F B}, S_{3}, S_{4}$, $S_{5}, S_{7}, S_{8}, S_{9}$ in 8 low $q^{2}$ and 4 high $q^{2}$ bins
- $B_{s} \rightarrow \phi \mu^{+} \mu^{-}: \mathrm{BR}, F_{L}, S_{3}, S_{4}, S_{7}$ in 3 low $q^{2}$ and 2 high $q^{2}$ bins

Computations performed using Superlso public program

## NP Fit results: single operator

Best fit values considering all observables besides $R_{K}$ and $R_{K^{*}}$
(under the assumption of $10 \%$ non-factorisable power corrections)

| All $b \rightarrow s$ data except <br> $R_{K^{(*)}}$ |  | $\left(\chi_{\mathrm{SM}}^{2}=98.1\right)$ |  |
| :--- | ---: | :---: | :---: |
|  | b.f. value | $\chi_{\min }^{2}$ | Pull $_{\mathrm{SM}}$ |
| $\delta C_{9}$ | $-1.02 \pm 0.20$ | 79.7 | $4.3 \sigma$ |
| $\delta C_{10}$ | $0.18 \pm 0.25$ | 97.6 | $0.8 \sigma$ |
| $\delta C_{9}^{\mu}$ | $-1.05 \pm 0.19$ | 77.5 | $4.5 \sigma$ |
| $\delta C_{9}^{e}$ | $0.72 \pm 0.58$ | 96.9 | $1.1 \sigma$ |
| $\delta C_{10}^{\mu}$ | $0.27 \pm 0.25$ | 96.8 | $1.1 \sigma$ |
| $\delta C_{10}^{e}$ | $-0.56 \pm 0.50$ | 97.1 | $1.0 \sigma$ |

$\rightarrow C_{9}$ and $C_{9}^{\mu}$ solutions are favoured with SM pulls of 4.3 and $4.5 \sigma$
$\rightarrow C_{10}$-like solutions do not play a role

## NP Fit results: single operator

Best fit values considering all observables besides $R_{K}$ and $R_{K^{*}}$
(under the assumption of $10 \%$ non-factorisable power corrections)

| All $b \rightarrow s$ data except |  |  |  |
| :--- | ---: | :---: | :---: |
| $R_{K(*)}$ | $\left(\chi_{\mathrm{SM}}^{2}=98.1\right)$ |  |  |
|  | b.f. value | $\chi_{\min }^{2}$ | Pull ${ }_{\mathrm{SM}}$ |
| $\delta C_{9}$ | $-1.02 \pm 0.20$ | 79.7 | $4.3 \sigma$ |
| $\delta C_{10}$ | $0.18 \pm 0.25$ | 97.6 | $0.8 \sigma$ |
| $\delta C_{9}^{\mu}$ | $-1.05 \pm 0.19$ | 77.5 | $4.5 \sigma$ |
| $\delta C_{9}^{e}$ | $0.72 \pm 0.58$ | 96.9 | $1.1 \sigma$ |
| $\delta C_{10}^{\mu}$ | $0.27 \pm 0.25$ | 96.8 | $1.1 \sigma$ |
| $\delta C_{10}^{e}$ | $-0.56 \pm 0.50$ | 97.1 | $1.0 \sigma$ |

$\rightarrow C_{9}$ and $C_{9}^{\mu}$ solutions are favoured with SM pulls of 4.3 and $4.5 \sigma$
$\rightarrow C_{10}$-like solutions do not play a role

Best fit values in the one operator fit considering only $R_{K}$ and $R_{K^{*}}$

| Only $R_{K}$ and $R_{K^{*}}$ |  |  | $\left(\chi_{\mathrm{SM}}^{2}=18.7\right)$ |  |
| :--- | ---: | :---: | :---: | :---: |
|  | b.f. value | $\chi_{\min }^{2}$ | Pull ${ }_{\mathrm{SM}}$ |  |
| $\delta C_{9}$ | $-1.99 \pm 5.81$ | 18.6 | $0.3 \sigma$ |  |
| $\delta C_{10}$ | $4.09 \pm 12.23$ | 18.5 | $0.5 \sigma$ |  |
| $\delta C_{9}^{\mu}$ | $-1.47 \pm 0.52$ | 5.3 | $3.7 \sigma$ |  |
| $\delta C_{9}^{e}$ | $1.58 \pm 0.49$ | 3.6 | $3.9 \sigma$ |  |
| $\delta C_{10}^{\mu}$ | $1.38 \pm 0.44$ | 2.8 | $4.0 \sigma$ |  |
| $\delta C_{10}^{e}$ | $-1.44 \pm 0.44$ | 2.3 | $4.1 \sigma$ |  |

$\rightarrow \mathrm{NP}$ in $C_{9}^{e}, C_{9}^{\mu}, C_{10}^{e}$, or $C_{10}^{\mu}$ are favoured by the $R_{K^{(*)}}$ ratios (significance: $3.7-4.1 \sigma$ )

## Fit results for two operators



The two sets are compatible at least at the $2 \sigma$ level.

## Fit results for two operators: dependence on hadronic uncertainties

Fits with different assumptions for the form factor uncertainties:

- correlations ignored (solid line)
- normal form factor errors (filled areas)
- $2 \times$ form factor errors (dashed line)
- $4 \times$ form factor errors (dotted line)


The size of the form factor errors has a crucial role in constraining the allowed region!

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The size of the form factor errors has a crucial role in constraining the allowed region!

Fits assuming different power correction uncertainties:

- $10 \%$ uncertainty (filled areas)
- $60 \%$ uncertainty (solid line)


60\% power correction uncertainty leads to only $17-20 \%$ error at the observable level. Significance of the tension depends on the assumption on the size of the power corrections

## New physics or hadronic effects?

The hadronic contributions (in terms of helicity amplitudes) appear in:

$$
\begin{aligned}
& H_{V}(\lambda)=-i N^{\prime}\left\{C_{9}^{\text {eff }} \tilde{V}_{\lambda}\left(q^{2}\right)+\frac{m_{B}^{2}}{q^{2}}\left[\frac{2 \hat{m}_{b}}{m_{B}} C_{7}^{\text {eff }} \tilde{T}_{\lambda}\left(q^{2}\right)-16 \pi^{2} \mathcal{N}_{\lambda}\left(q^{2}\right)\right]\right\} \\
&\left(N^{\prime}=-\frac{4 G_{F} m_{B}}{\sqrt{2}} \frac{e^{2}}{16 \pi^{2}} V_{t b} V_{t s}^{*}\right) \\
& N_{\lambda}\left(q^{2}\right)=\text { leading nonfact. }+h_{\lambda}
\end{aligned}
$$

Helicity FFs $\tilde{V}_{L / R}, \tilde{T}_{L / R}$ are combinations of the standard FFs $V, A_{0,1,2}, T_{1,2,3}$
The most general parametrisation up to higher order terms in $q^{2}$ of the non-factorisable power corrections $h_{\lambda(=+,-, 0)}\left(q^{2}\right)$ which is compatible with the analyticity structure is:



## New Physics effect:

and similarly for $\lambda=0$ and for $C_{7}$

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$$
\begin{gathered}
\delta H_{V}^{\text {p.c. }}(\lambda= \pm)=i N^{\prime} m_{B}^{2} \frac{16 \pi^{2}}{q^{2}} h_{\lambda}\left(q^{2}\right)=i N^{\prime} m_{B}^{2} 16 \pi^{2}\left(\frac{h_{\lambda}^{(0)}}{q^{2}}+h_{\lambda}^{(1)}+q^{2} h_{\lambda}^{(2)}\right) \\
\delta H_{V}^{\text {PC }}(\lambda=0)=i N^{\prime} m_{B}^{2} \frac{16 \pi^{2}}{\sqrt{q^{2}}}\left(h_{0}^{(0)}+q^{2} h_{0}^{(1)}+q^{4} h_{0}^{(2)}\right)
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\end{gathered}
$$

## New Physics effect:

$$
\delta H_{V}^{C_{9}^{N P}}(\lambda= \pm)=-i N^{\prime} \tilde{V}_{L}\left(q^{2}\right) C_{9}^{N P}=-i N^{\prime}\left(a_{\lambda} C_{9}^{N P}+q^{2} b_{\lambda} C_{9}^{\mathrm{NP}}\right)
$$

and similarly for $\lambda=0$ and for $C_{7}$
$\Rightarrow$ NP effects can be embedded in the hadronic effects.

## Wilks' test

We can do a fit for both (hadronic quantities $h_{+,-, 0}^{(0,1,2)}$ (18 parameters) and Wilson coefficients $C_{i}^{\text {NP }}$ ( 1 to 4 parameters $)$ )

Due to this embedding the two fits can be compared with the Wilks' test

| nr. of parameters | $\left(\right.$ Real $\left.\delta C_{9}\right)$ | $\left(\right.$ Real $\left.\delta^{2} C_{7}, \delta C_{9}\right)$ | $\left(\right.$ Complex $\left.\delta C_{9}\right)$ | $\left(\right.$ Complex $\left.\delta C_{7}, \delta C_{9}\right)$ | $\left(\right.$ Complex $\left._{4}^{18} h_{4,-, 0)}^{(0,1,2)}\right)$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| $0($ plain SM$)$ | $4.1 \sigma$ | $4.0 \sigma$ | $4.2 \sigma$ | $4.1 \sigma$ | $3.1 \sigma$ |
| $1\left(\right.$ Real $\left.\delta C_{9}\right)$ | - | $1.5 \sigma$ | $2.1 \sigma$ | $2.0 \sigma$ | $1.5 \sigma$ |
| $2\left(\right.$ Real $\left.\delta C_{7}, \delta C_{9}\right)$ | - | - | - | $1.9 \sigma$ | $1.4 \sigma$ |
| $2\left(\right.$ Complex $\left.\delta C_{9}\right)$ | - | - | - | $1.4 \sigma$ | $1.1 \sigma$ |
| $4\left(\right.$ Complex $\left.\delta C_{7}, \delta C_{9}\right)$ | - | - | - | - | $0.95 \sigma$ |

$\rightarrow$ Adding $\delta C_{9}$ improves over the SM hypothesis by $4.1 \sigma$
$\rightarrow$ Including in addition $\delta C_{7}$, imaginary parts or hadronic parameters improves the situation only mildly
$\rightarrow$ One cannot rule out the hadronic option

Adding 17 more parameters does not improve the fits significantly
The situation is still inconclusive

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| nr. of parameters | 1 <br> $\left(\right.$ Real $\left.\delta C_{9}\right)$ | $\left(\right.$ Real $\left.\delta C_{7}, \delta C_{9}\right)$ | 2 <br> $\left(\right.$ Complex $\left.\delta C_{9}\right)$ | 4 <br> $\left(\right.$ Complex $\left.\delta C_{7}, \delta C_{9}\right)$ | 18 <br> $\left(\right.$ Complex $\left.h_{+,-, 0}^{(0,1,2)}\right)$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| $0($ plain SM$)$ | $4.1 \sigma$ | $4.0 \sigma$ | $4.2 \sigma$ | $4.1 \sigma$ | $3.1 \sigma$ |
| 1 (Real $\left.\delta C_{9}\right)$ | - | $1.5 \sigma$ | $2.1 \sigma$ | $2.0 \sigma$ | $1.5 \sigma$ |
| $2\left(\right.$ Real $\left.\delta C_{7}, \delta C_{9}\right)$ | - | - | - | $1.9 \sigma$ | $1.4 \sigma$ |
| $2\left(\right.$ Complex $\left.\delta C_{9}\right)$ | - | - | - | $1.1 \sigma \sigma$ | $0.95 \sigma$ |
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## Estimates of hadronic effects

## Various methods for hadronic effects

$$
\frac{e^{2}}{q^{2}} \epsilon_{\mu} L_{V}^{\mu}\left[Y\left(q^{2}\right) \tilde{V}_{\lambda}+\mathrm{LO} \text { in } \mathcal{O}\left(\frac{\Lambda}{m_{b}}, \frac{\Lambda}{E_{K^{*}}}\right)+h_{\lambda}\left(q^{2}\right)\right]
$$

|  | factorisable | non- <br> factorisable | power corrections <br> (soft gluon) | region of <br> calculation | physical region <br> of interest |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Standard | $\checkmark$ | $\checkmark$ | $x$ | $q^{2} \lesssim 7 \mathrm{GeV}^{2}$ | directly |
| Khodjamirian et al. <br> [1006.4945] | $\checkmark$ | $x$ | $\checkmark$ | $q^{2}<1 \mathrm{GeV}^{2}$ | extrapolation by <br> dispersion relation |
| Bobeth et al. <br> $[1707.07305]$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $q^{2}<0 \mathrm{GeV}^{2}$ | extrapolation by <br> analyticity |



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## More realistic analyses

In a New Physics model:

- new vector bosons: $C_{7}, C_{9}, C_{10}$
- new fermions: $C_{7}, C_{8}, C_{9}, C_{10}$
- extended Higgs sector/new scalars: $C_{S}, C_{P}$
e.g. in the MSSM, $2 \mathrm{HDM}, \ldots: C_{7}, C_{8}, C_{9}, C_{10}, C_{S}, C_{P}$

Considering only one or two Wilson coefficients may not give the correct picture!
$C_{S, P}$ are usually assumed to be highly constrained by $\operatorname{BR}\left(B_{s} \rightarrow \mu^{+} \mu^{-}\right)$
$\rightarrow$ not considered in the global fits

Not quite true!

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Not quite true!

## More realistic fits

If $C_{S}$ and $C_{P}$ independent, there exists a degeneracy between $C_{10}$ and $C_{P}$ so that large values for $C_{P}$ are possible



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If $C_{S}$ and $C_{P}$ independent, there exists a degeneracy between $C_{10}$ and $C_{P}$ so that large values for $C_{P}$ are possible



Even if $C_{S}=-C_{P}$, allowing for small variations of $C_{S, P}$ alleviates the constraints from $B_{s} \rightarrow \mu^{+} \mu^{-}$on $C_{10}$


## Effective number of degrees of freedom

A generic set of Wilson coefficients:

$$
\text { complex } C_{7}, C_{8}, C_{9}^{\ell}, C_{10}^{\ell}, C_{S}^{\ell}, C_{P}^{\ell}+\text { primed coefficients }
$$

The available observables are mainly insensitive to the imaginary parts, one can limit the set to
real $C_{7}, C_{8}, C_{9}^{\ell}, C_{10}^{\ell}, C_{S}^{\ell}, C_{P}^{\ell}+$ primed coefficients
corresponding to 20 degrees of freedom
Some of the coefficients may have only weak effects on the observables, and affect the number of dof without affecting the $\chi^{2}$, acting as spurious degrees of freedom.
effective degrees of freedom (e-dof): degrees of freedom minus the parameters $\delta C_{i}$ only weakly affecting the $\chi^{2}$, defined such as

$$
\left|\chi^{2}\left(\delta C_{i}=1\right)-\chi^{2}\left(\delta C_{i}=0\right)\right|<1
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Set: real $C_{7}, C_{8}, C_{9}^{\ell}, C_{10}^{\ell}, C_{S}^{\ell}, C_{P}^{\ell}+$ primed coefficients

```
20 degrees of freedom, 108 observables
```

| Set of WC | Nr. parameters (e-dof) | $\chi_{\text {min }}^{2}$ | PullsMI | Improvement |
| :---: | :---: | :---: | :---: | :---: |
| SM | 0 | 118.8 |  |  |
| $C_{9}^{\mu}$ | 1 | 85.1 | $5.8 \sigma$ | - |
| $C_{9}^{(, \mu)}$ | 2 | 83.9 | $5.6 \sigma$ | $1.1 \sigma$ |
| $C_{7}, C_{8}, C_{9}^{(e, \mu)}, C_{10}(\mu)$ | 6 | 81.2 | $4.8 \sigma$ | $0.5 \sigma$ |
| All non-primed WC | $10(8)$ | 81.0 | $4.1(4.5) \sigma$ | $0.0(0.1) \sigma$ |
| All WC (incl. primed) | $20(16)$ | 70.2 | $3.6(4.1) \sigma$ | $0.9(1.2) \sigma$ |

- No real improvement in the fits when going beyond the $C_{9}^{(e, \mu)}$ case
- Pull with the SM decreases when all WC are varied
- Many parameters are very weakly constrained


## Fit results with more than two operators: All observables

Set: real $C_{7}, C_{8}, C_{9}^{\ell}, C_{10}^{\ell}, C_{S}^{\ell}, C_{P}^{\ell}+$ primed coefficients

20 degrees of freedom, 108 observables

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## Fit results with more than two operators: All observables

| All observables with $\chi_{S M}^{2}=118.8$ <br> $\left(\chi_{\min }^{2}=70.2 ;\right.$ Pull $\left._{\text {SM }}=3.5(4.1) \sigma\right)$ |  |  |  |
| :---: | :---: | :---: | :---: |
| $\delta C_{7}$ |  | $\delta C_{8}$ |  |
| $-0.01 \pm 0.05$ | $0.89 \pm 0.81$ |  |  |
| $\delta C_{7}^{\prime}$ |  | $\delta C_{8}^{\prime}$ |  |
| $0.01 \pm 0.03$ | $-1.70 \pm 0.46$ |  |  |
| $\delta C_{9}^{\mu}$ | $\delta C_{9}^{e}$ | $\delta C_{10}^{\mu}$ | $\delta C_{10}^{e}$ |
| $-1.40 \pm 0.26$ | $-4.02 \pm 5.58$ | $-0.07 \pm 0.28$ | $1.32 \pm 2.02$ |
| $\delta C_{9}^{\prime \mu}$ | $\delta C_{9}^{\prime e}$ | $\delta C_{10}^{\prime \mu}$ | $\delta C_{10}^{\prime e}$ |
| $0.23 \pm 0.65$ | $-1.10 \pm 5.98$ | $-0.16 \pm 0.38$ | $2.70 \pm 2$ |
| $C_{Q_{1}}^{\mu}$ | $C_{Q_{1}}^{e}$ | $C_{Q_{2}}^{\mu}$ | $C_{Q_{2}}^{e}$ |
| $-0.13 \pm 1.86$ | undetermined | $-0.05 \pm 0.58$ | undetermined |
| $C_{Q_{1}}^{\prime \mu}$ | $C_{Q_{1}}^{\prime e}$ | $C_{Q_{2}}^{\prime \mu}$ | $C_{Q_{2}}^{\prime e}$ |
| $0.01 \pm 1.87$ | undetermined | $-0.18 \pm 0.62$ | undetermined |

- No real improvement in the fits when going beyond the $C_{9}^{(e, \mu)}$ case
- Pull with the SM decreases when all WC are varied
- Many parameters are very weakly constrained


# Future prospects 

## How to resolve the issue?

## 1) Future LHCb prospects

Global fits using the angular observables only (NO theoretically clean $R$ ratios)
Considering several luminosities, assuming the current central values



LHCb may be able to establish new physics within the angular observables even in the pessimistic case that there will be no theoretical progress on non-factorisable power corrections!

## How to resolve the issue?

Pull ${ }_{\text {SM }}$ for the fit to $\Delta C_{9}^{\mu}$ based on $R_{K}$ and $R_{K^{*}}$ only for the LHCb upgrade Assuming current central values remain

| $\Delta C_{9}^{\mu}$ | Syst. <br> Pull $_{\text {SM }}$ | Syst./2 <br> Pull | Syst./3 <br> Pull |
| :--- | :---: | :---: | :---: |
| $12 \mathrm{fb}^{-1}$ | $6.1 \sigma(4.3 \sigma)$ | $7.2 \sigma(5.2 \sigma)$ | $7.4 \sigma(5.5 \sigma)$ |
| $50 \mathrm{fb}^{-1}$ | $8.2 \sigma(5.7 \sigma)$ | $11.6 \sigma(8.7 \sigma)$ | $12.9 \sigma(9.9 \sigma)$ |
| $300 \mathrm{fb}^{-1}$ | $9.4 \sigma(6.5 \sigma)$ | $15.6 \sigma(12.3 \sigma)$ | $19.5 \sigma(16.1 \sigma)$ |

(): assuming 50\% correlation between each of the $R_{K}$ and $R_{K^{*}}$ measurements

Only a small part of the $50 \mathrm{fb}^{-1}$ is needed to establish NP in the $R_{K^{(*)}}$ ratios even in the pessimistic case that the systematic errors are not reduced by then at all.

This is independent of the hadronic uncertainties!

## How to resolve the issue?

2) Cross-check with other $R_{\mu / e}$ ratios

- $R_{K}$ and $R_{K^{*}}$ ratios are theoretically very clean
- The tensions cannot be explained by hadronic uncertainties

Cross-checks needed with other ratios:

|  | Predictions assuming $12 \mathrm{fb}^{-1}$ luminosity |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Obs. | $C_{9}^{\mu}$ | $C_{9}^{e}$ | $C_{10}^{\mu}$ | $C_{10}^{e}$ |
| $R_{F_{L}}^{[1.1,6.0]}$ | [0.785, 0.913] | [0.909, 0.933] | [1.005, 1.042] | [1.001, 1.018] |
| $R_{A_{F B}}^{[1.1,6.0]}$ | [6.048, 14.819] | [-0.288, -0.153] | [0.816, 0.928] | [0.974, 1.061] |
| $R_{\left.S_{5}, 6.0\right]}^{[1.1,6.0]}$ | [-0.787, 0.394] | [0.603, 0.697] | [0.881, 1.002] | [1.053, 1.146] |
| $R_{F_{L}}^{[15,19]}$ | [0.999, 0.999] | [0.998, 0.998] | [0.997, 0.998] | [0.998, 0.998] |
| $R_{A_{F B}}^{[15,19]}$ | [0.616, 0.927] | [1.002, 1.061] | [0.860, 0.994] | [1.046, 1.131] |
| $R_{S_{5}}^{[15,19]}$ | [0.615, 0.927] | [1.002, 1.061] | [0.860, 0.994] | [1.046, 1.131] |
| $R_{K^{*}}^{[15,19]}$ | [0.621, 0.803] | [0.577, 0.771] | [0.589, 0.778] | [0.586, 0.770] |
| $R_{K}^{[15,19]}$ | [0.597, 0.802] | [0.590, 0.778] | [0.659, 0.818] | [0.632, 0.805] |
| $R_{\phi}^{[1.1,6.0]}$ | [0.748, 0.852] | [0.620, 0.805] | [0.578, 0.770] | [0.578, 0.764] |
| $R_{\phi}^{[15,19]}$ | [0.623, 0.803] | [0.577, 0.771] | [0.586, 0.776] | [0.583, 0.769] |

A confirmation of the deviations in the ratios would indirectly confirm the NP interpretation of the anomalies in the angular observables!

## How to resolve the issue?

3) Cross-check with inclusive modes

Inclusive decays are theoretically cleaner (see e.g. T. Huber, T. Hurth, E. Lunghi, JHEP 1506 (2015) 176)
At Belle-II, for inclusive $b \rightarrow s \ell \ell$ :

T. Hurth, FM, JHEP 1404 (2014) 097
T. Hurth, FM, S. Neshatpour, JHEP 1412 (2014) 053

Predictions based on our model-independent analysis
black cross: future measurements at Belle-II assuming the best fit solution red cross: SM predictions
$\rightarrow$ Belle-II will check the NP interpretation with theoretically clean modes

## Conclusion

- The full LHCb Run 1 results still show some tensions with the SM predictions
- Significance of the anomalies depends on the assumptions on the power corrections
- Model independent fits point to about $25 \%$ reduction in $C_{9}$, and new physics in muonic $C_{9}^{\mu}$ is preferred
- The tension with the SM including all observables (assuming $10 \%$ non-factorisable power corrections) and all relevant Wilson coefficents is at the $4 \sigma$ level at the moment.
- We can compare the fits for NP and hadronic parameters using the Wilks' test
$\rightarrow$ At the moment adding the hadronic parameters does not improve the fit compared to the new physics fit, but the situation is inconclusive
- The LHCb upgrade will have enough precision to distinguish between NP and power corrections

