1 Particles

1. Can a gluon interact with a photon?

   No, because a gluon has no electromagnetic charge, nor does a photon have a colour charge.

2. Can a gluon interact with a W^-?

   No, for the same reason as above.

3. Can a photon interact with itself? Why (not)?

   No, because a photon itself has no charge. So even though it is the carrier of the electromagnetic force, it cannot interact with itself.

4. What is the only elementary boson that can interact with neutrinos without changing them?

   The Z^0. Only weak bosons can interact with neutrinos because the latter have no EM charge nor colour. But the $W^\pm$ bosons change the neutrinos into their corresponding charged lepton (i.e. electron-neutrino into electron etc). Only the Z^0 interacts with them without changing them, like a photon does to charged particles.

5. Can we have a meson with charge $++$?

   No, because mesons consist of two quarks (one quark and one antiquark), we can hence maximally have a charge of $\frac{1}{3} + \frac{2}{3} = 1$.

6. What is the quark content of:

   Λ^0: (uds)

   D_s^+: $(c\bar{s})$

   Ω^-: (sss)

   Ξ_{cc}^{++}: (ucc)

2 Conservation Laws

For this exercise, we refer to section 2 in the cheat sheet. We need to check four strict conservation laws (conservation of charge, energy, lepton number and baryon number) and one less strict (not satisfied by the weak interaction) conservation law (conservation of flavour).

a) $p + \overline{p} \to 2\pi^+ + 2\pi^- + \pi^0$:

i) *Charge*: is conserved (the charge of an antiproton is -1).

ii) *Energy*: is trivially conserved (in a collision, one can add arbitrary amounts of energy in the initial particles).

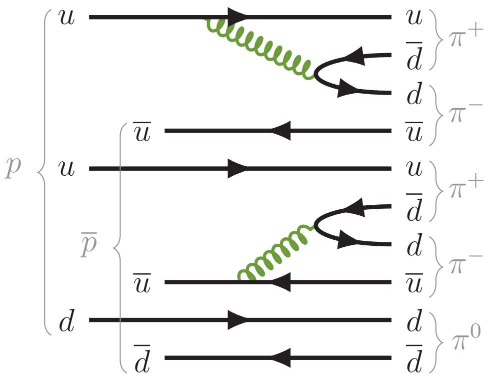
iii) *Lepton number*: is trivially conserved (no leptons).

iv) *Baryon number*: is conserved:

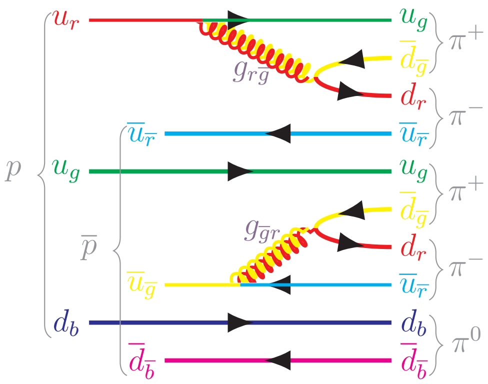
$$\begin{array}{rcccc} & p + \overline{p} \to & 2\pi^+ & + 2\pi^- & + \pi^0 \\ B: & 1 - 1 = & 0 & + 0 & + 0 \end{array}$$

v) *Flavour*: is conserved (both the proton and the pion have strangeness $S = 0$).

So this interaction is allowed, by any interaction (because flavour is conserved). If an interaction can occur via all interactions, it will interact using the strong force (unless there are neutrinos or photons involved) because the strong force has much higher probability. As an illustration, we add the Feynman diagram:



Note that, just for fun, we could add colour charge to our drawing (remember that, to draw your Feynman diagram such that it satisfies colour conservation, it is enough to make sure that quarks only combine per three or per quark-antiquark pair):



b) $p + K^- \to \Sigma^+ + \pi^+ + 2\pi^- + \pi^0$:

i) *Charge*: is conserved.

ii) *Energy*: is trivially conserved (collision).

iii) *Lepton number*: is trivially conserved (no leptons).

iv) *Baryon number*: is conserved:

$$\begin{array}{rl} & p + K^- \rightarrow \Sigma^+ + \pi^+ + 2\pi^- + \pi^0 \\ B: & 1 + 0 \quad = \quad 1 \ + \ 0 \ + \ 0 \quad + 0 \end{array}$$

v) *Flavour*: is conserved (both the kaon and the Σ have strangeness $S = -1$, see the particle table).

This collision is allowed by all forces, thus it will occur using the strong force.

c) $p \rightarrow \Lambda^0 + \overline{\Sigma}^0 + \pi^+$: This interaction is not allowed, because baryon number is not conserved:

$$\begin{array}{rl} & p \rightarrow \Lambda^0 + \overline{\Sigma}^0 + \pi^+ \\ B: & 1 \quad \neq 1 \quad -1 \quad +0 \end{array}$$

d) $\overline{\nu}_\mu + p \rightarrow \mu^+ + n$:

i) *Charge*: is conserved.

ii) *Energy*: is trivially conserved (collision).

iii) *Lepton number*: is conserved:

$$\begin{array}{rl} & \overline{\nu}_\mu + p \rightarrow \mu^+ + n \\ L_\mu: & -1 + 0 \ = \ -1 + 0 \end{array}$$

iv) *Baryon number*: is conserved ($B(p) = 1$ and $B(n) = 1$).

v) *Flavour*: is not conserved, because we change an antimuon-neutrino into an antimuon.

Since flavour is violated, this interaction is only allowed using the weak interaction.

e) $\overline{\nu}_e + p \rightarrow e^+ + \Lambda^0 + K^0$:

i) *Charge*: is conserved.

ii) *Energy*: is trivially conserved (collision).

iii) *Lepton number*: is conserved:

$$\begin{array}{rl} & \overline{\nu}_e + p \rightarrow e^+ + \Lambda^0 + K^0 \\ L_e: & -1 + 0 \ = \ -1 + 0 \ + 0 \end{array}$$

iv) *Baryon number*: is conserved:

$$\begin{array}{rl} & \overline{\nu}_e + p \rightarrow e^+ + \Lambda^0 + K^0 \\ B: & 0 + 1 \ = \ 0 \ \ +1 \ \ +0 \end{array}$$

v) *Flavour*: is not conserved, because we change an antielectron-neutrino into an antielectron (also called positron), but strangeness is conserved:

$$\begin{array}{rl} & \overline{\nu}_e + p \rightarrow e^+ + \Lambda^0 + K^0 \\ S: & 0 + 0 \ = \ 0 \ \ -1 \ \ +1 \end{array}$$

Since flavour is violated, this interaction is only allowed using the weak interaction. But since strangeness is conserved, we will need the strong interaction as well in order to create a $s\overline{s}$ pair for the Λ^0 (uds) and the K^0 $(d\overline{s})$.[1]

f) $\Sigma^0 \rightarrow \Lambda^0 + \gamma$:

i) *Charge*: is conserved.

ii) *Energy*: is conserved (now we need to check energy conservation, because it is a decay, not a collision):

$$m_{\Sigma^0} \ {\scriptstyle (1193\,\mathrm{MeV})} \geq m_{\Lambda^0} \ {\scriptstyle (1116\,\mathrm{MeV})} + m_\gamma \ {\scriptstyle (0\,\mathrm{MeV})}$$

iii) *Lepton number*: is trivially conserved (no leptons).

iv) *Baryon number*: is conserved:

$$\begin{array}{rccc} & \Sigma^0 & \rightarrow & \Lambda^0 + \gamma \\ B: & 1 & = & 1 \ \ + 0 \end{array}$$

v) *Flavour*: is conserved since strangeness is conserved:

$$\begin{array}{rccc} & \Sigma^0 & \rightarrow & \Lambda^0 + \gamma \\ S: & -1 & = & -1 + 0 \end{array}$$

This decay is allowed, but since there is a photon involved, we know it has to occur using the electromagnetic interaction (because the strong force doesn't interact with photons, nor does the weak).

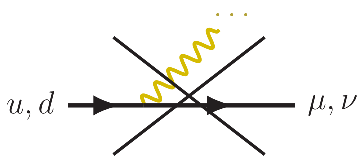
[1] We could as well create the s and the $\overline{s}$ using the weak interaction, but than we would need two W's which both would show mixing. This is highly unprobable, and since it is possible to create the two strange quarks with the strong interaction, the latter is preferred.

3 Feynman Diagrams

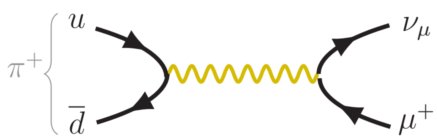
We refer to the particle table for the quark contents of the baryons, and to section 1 of the cheat sheet to know the rules to draw Feynman diagrams.

a) $\pi^+ \to \mu^+ + \nu_\mu$: First we list the particles (and their quark content):

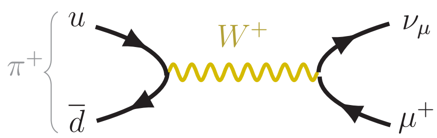
Quarks never couple directly to leptons, in other words it is impossible to transform a quark into a lepton:



Which leaves us with only one solution: we have to annihilate the quarks into a weak boson (we are using the weak interaction, as requested in the task), and then recreate the leptons from it:

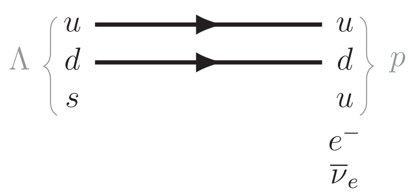


Note the direction of the arrows: for the incoming part the arrows go *from* the particle (u) *to* the *anti*particle ($\overline{d}$); for the outgoing part this is reversed: from the antiparticle (μ^+) to the particle (ν_μ). The only open question is which type of boson we have. It could be a W^+, a W^- or a Z^0. Since we change flavour (from u to d and from μ to ν_μ), it should be a W boson. Then the charge is easily determined from charge conservation: we have a total charge $+1$, thus it should be a W^+. This gives the final diagram:

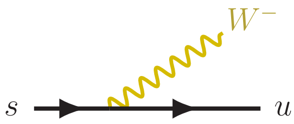


b) $\Lambda \to p + e^- + \overline{\nu}_e$: We again list the particles (and their quark contents) first:

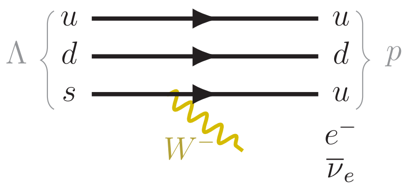
Before drawing any bosons, we connect the particles that are both incoming and outgoing with a straight line:



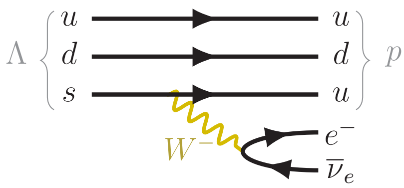
Now we are left with one incoming particle, going to three outgoing particles. This means the incoming particle will emit a boson (possible transforming the incoming particle into another), which in turn will decay into two new particles. The incoming particle is a quark, and we know quarks can only change into quarks (possibly from other families, because there exists some mixing). From the mixing table in the addendum, we see that a s can change into an u by emitting a W:



This gives:

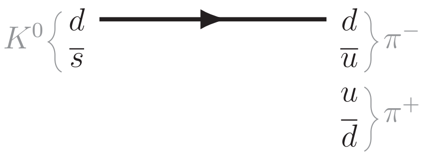


Last, we see that the W^- will decay into the electron and the neutrino (which is correct because a W transforms an electron into a neutrino and vice versa):

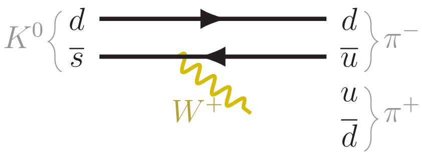


c) $K^0 \to \pi^+ + \pi^-$: Again we list the particles (and their quark contents):

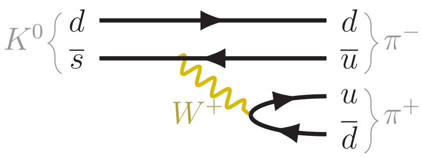
We connect the d, which remains the same during the interaction:



Next, we know the $\overline{s}$ can emit a W^+ and transform, using mixing, into an $\overline{u}$:

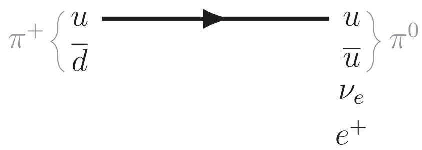


Last, the W^+ will decay into an u and a $\overline{d}$:

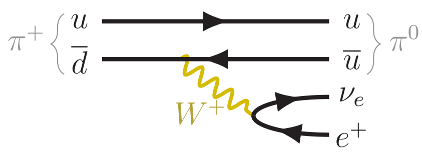


d) $\pi^+ \rightarrow \pi^0 + \pi^-$: Again we start by listing the particles (and their quark contents):

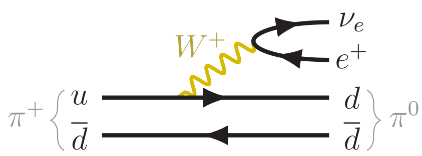
Note that for the quark content of a π^0 we have the choice between $(u\overline{u})$ and $(d\overline{d})$. In this case, both choices give a valid Feynman diagram (as we will show a bit further). If we choose $u\overline{u}$, we can leave the u quark unchanged:



The $\overline{d}$ quark will emit a W^+ (transforming the quark into an $\overline{u}$, no mixing is needed), which in turn will decay into the lepton pair:



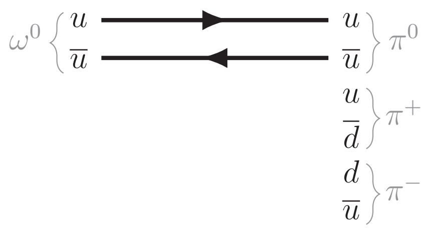
Note that, if we choose to write $(d\overline{d})$ for the π^0, the $\overline{d}$ from the π^+ will remain and the u will radiate a W^+:



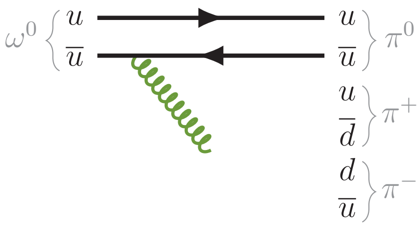
For the next 3 exercises, it is requested to use the strong force, in other words, all bosons we draw will need to be gluons (and no leptons will be involved).

e) $\omega^0 \rightarrow \pi^+ + \pi^- + \pi^0$: The quark contents are:

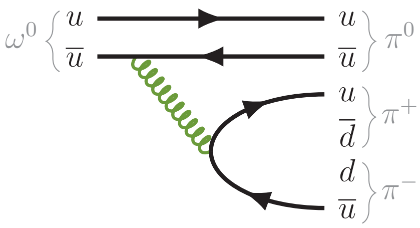
Where we chose to take the quark content of both the ω^0 and the π^0 equal to $(u\overline{u})$. We start by leaving the u and the $\overline{u}$ unchanged:



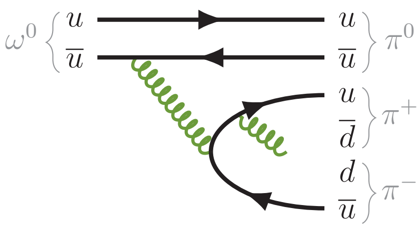
One of these two will radiate a gluon:



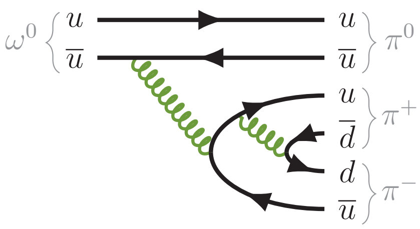
Which will for instance split into an $u\overline{u}$ pair:



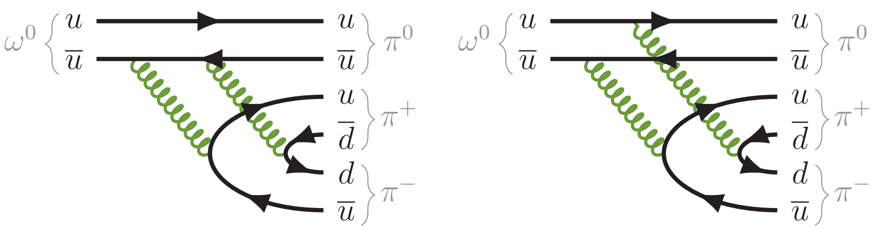
From which one of the two quarks can radiate again a gluon:



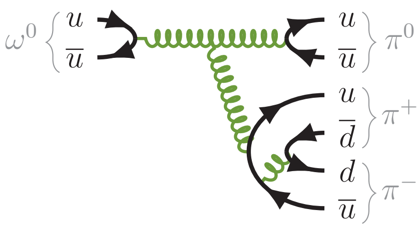
Which will split in a $d\overline{d}$ pair:



Which gives the requested. Note that because every quark can radiate a gluon without changing, there are lots of different diagrams possible:



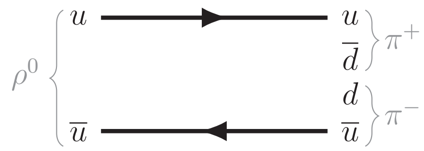
and so on. There is even a totally different set of diagrams, making use of the three-gluon interaction:



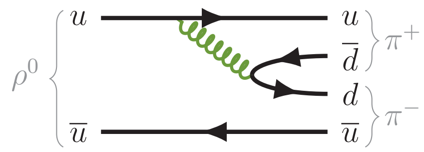
However, because of the fact that we now have six vertices instead of four, this diagram has a lower probability to occur.[2] Note that much more diagrams are possible, because we can choose to use $d\overline{d}$ for the ω^0 or the π^0 (or for both) instead of $u\overline{u}$.

f) $\rho^0 \to \pi^+ + \pi^-$: This goes completely analogous to the former:

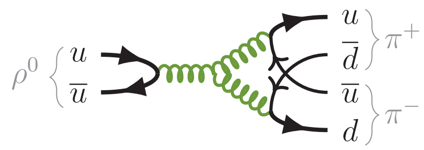
We keep the u and the $\overline{u}$ unchanged:



One of them will radiate a gluon, which will create a quark-antiquark pair:

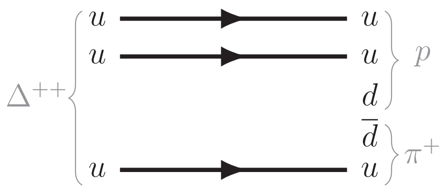


which is the requested. Note that again, there exist several possible diagrams, including a (less probable) three-gluon diagram:

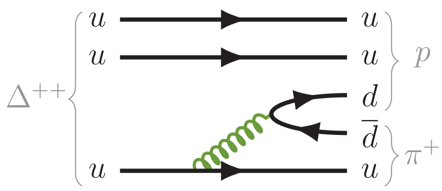


g) $\Delta^{++} \to p + \pi^+$: By now we know the procedure:

We keep all three u quarks unchanged:



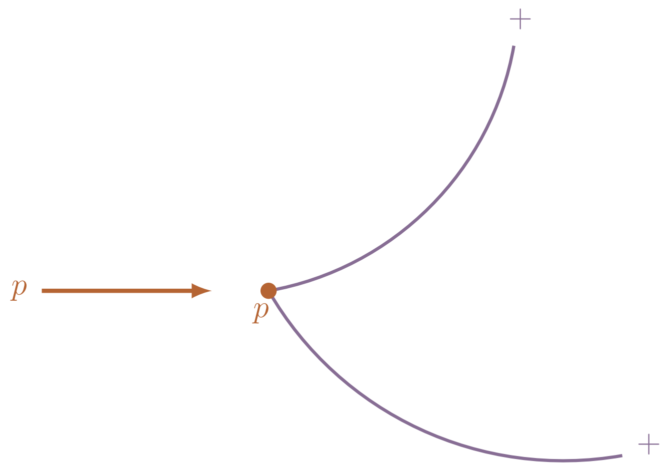
and one of them will radiate a gluon that decays into a $d\overline{d}$ pair:



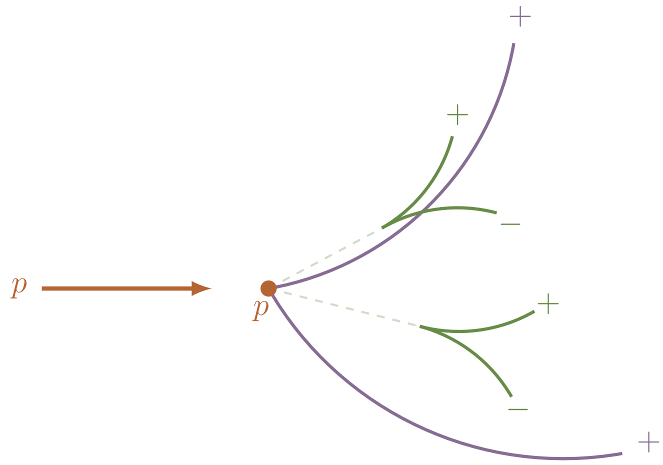
[2]This is based on the idea of perturbative expansion: the more interaction points, the higher the order, the lower the probability. However, this is only true if the coupling at every interaction point is $g < 1$ (which is needed to get a convergent series), and for the strong interaction this is not always true. But we neglect this fact for now.

4 Challenge Exercise

1. We have two outgoing particles that are positive (implying they will have the same direction of curvature):



   Next we have two pairs of opposite charge (and thus opposite curvature) being the decay product of neutral (and thus invisible and following a straight path) particles:



2. To choose possible candidates, we have to satisfy energy conservation. The total energy at hand is:

$$\begin{aligned} s &= (p_1 + p_2)^2 \\ &= 2\,m_p^2 + 2\,m_p E_p = 2m_p(m_p + E_p) \\ E_p &= \sqrt{m_p^2 + |\mathbf{p}|^2} \approx |\mathbf{p}| \\ \Rightarrow \sqrt{s} &\approx 4.9\,\mathrm{GeV} \end{aligned}$$

   Thus the sum of the masses of the outgoing particles (two positive particles and two neutral particles) should be smaller than 4.9 GeV. Furthermore we know that we need to have a total baryon number $B = 2$. The neutral particles decay after a few cm; from this we can make an estimate of their lifetime:

$$x = vt \approx ct$$

   but we shouldn't forget that the lifetime gets Lorentz dilated:

$$x = ct\gamma$$

where we can calculate the Lorentz factor from the incoming energy:

$$\begin{aligned}\gamma &= \frac{E}{m} \\ &\approx \frac{12\,\text{GeV}}{1\,\text{GeV}} = 12\end{aligned}$$

So the lifetime that corresponds to a travelled distance of a few cm's is

$$\begin{aligned}t &= \frac{x}{c\gamma} \\ &\approx 1\,\text{ps} - 50\,\text{ps} \qquad (1\,\text{ps} = 10^{-12}\text{s})\end{aligned}$$

Watch out, because sometimes the symbol used to denote the lifetime is τ (not to be confused with the Minkowskian time $\tau = ct$), and sometimes the lifetime is given as a length (Minkowskian time). In the latter case, we need to have

$$\begin{aligned}\tau &= \frac{cx}{c\gamma} \\ &\approx 0.1\,\text{cm} - 5\,\text{cm}\end{aligned}$$

To know which convention is used in your reference, just look at the units used (seconds or meters). If the lifetime is not given, most of the time the resonance width Γ is given. It is related to the lifetime:

$$t = \frac{\hbar}{\Gamma}$$

So the resonance width needed to get a travelled distance of a few cm's is

$$\begin{aligned}\Gamma &= \frac{\hbar c\gamma}{x} \\ &\approx \frac{197\,\text{MeVfm} \cdot 12}{1\,\text{cm} - 50\,\text{cm}} \\ &\approx 5 \cdot 10^{-4}\,\text{eV} - 5 \cdot 10^{-6}\,\text{eV}\end{aligned}$$

Now we turn to the PDG handbook to investigate possible candidates for the neutral particles.

All neutral mesons built from u and d quarks have a really short lifetime and are thus ruled out. All neutral mesons built from heavy quarks (c, b, t) are ruled out, because there is not enough energy available to create two such mesons and keep enough energy to create the two remaining positive particles (which need to be baryons because of baryon number conservation). The only possible meson candidate is built from a s quark; it is the K_S^0.

We can use an analogous reasoning to find possible neutral baryon candidates (then the two remaining positive particles can be mesons, or a baryon and an antibaryon). We find that only the Λ^0 and the $\overline{\Lambda}^0$ are good baryon candidates.

The (dominant) decay channels for these particles are:

$$\begin{aligned}K_S^0 &\to \pi^+ + \pi^- \\ \Lambda^0 &\to p + \pi^- \\ \overline{\Lambda}^0 &\to \overline{p} + \pi^+\end{aligned}$$

3. The mass of the parent neutral particle X (a K^0_S, Λ^0 or $\overline{\Lambda}^0$) that decays into one of the pairs equals

$$\begin{aligned} m_X^2 &= s \\ &= (p_+ + p_-)^2 \\ &= m_+^2 + m_-^2 + 2\,E_+E_- - 2\,\mathbf{p}_+\cdot\mathbf{p}_- \\ &= m_+^2 + m_-^2 + 2\sqrt{m_+^2 + |\mathbf{p}_+|^2}\sqrt{m_-^2 + |\mathbf{p}_-|^2} - 2\,|\mathbf{p}_+||\mathbf{p}_-|\cos\theta \end{aligned}$$

Knowing the masses of all particles in the three possible decay channels

$$\begin{aligned} m_{\pi^\pm} &= 140\,\text{MeV} \\ m_{K^0_S} &= 498\,\text{MeV} \\ m_p &= 938\,\text{MeV} \\ m_{\Lambda^0} &= 1116\,\text{MeV} \end{aligned}$$

we can check which decay channels agree with the measured momenta.

i) For the first pair, we have $|\mathbf{p}_+| = 680\,\text{MeV}$, $|\mathbf{p}_-| = 270\,\text{MeV}$ and $\theta = 11°$.

I) $K^0_S \to \pi^+ + \pi^-$

$$m_K^2 \stackrel{?}{=} 2\,m_\pi^2 + 2\sqrt{m_\pi^2 + |\mathbf{p}_+|^2}\sqrt{m_\pi^2 + |\mathbf{p}_-|^2} - 2\,|\mathbf{p}_+||\mathbf{p}_-|\cos\theta$$
$$(498\,\text{MeV})^2 \neq (318\,\text{MeV})^2$$

II) $\Lambda^0 \to p + \pi^-$

$$m_\Lambda^2 \stackrel{?}{=} m_p + m_\pi + 2\sqrt{m_p^2 + |\mathbf{p}_+|^2}\sqrt{m_\pi^2 + |\mathbf{p}_-|^2} - 2\,|\mathbf{p}_+||\mathbf{p}_-|\cos\theta$$
$$(1116\,\text{MeV})^2 \stackrel{!}{\approx} (1115\,\text{MeV})^2 \qquad \text{OK}$$

III) $\overline{\Lambda}^0 \to \overline{p} + \pi^+$

$$m_\Lambda^2 \stackrel{?}{=} m_\pi + m_p + 2\sqrt{m_\pi^2 + |\mathbf{p}_+|^2}\sqrt{m_p^2 + |\mathbf{p}_-|^2} - 2\,|\mathbf{p}_+||\mathbf{p}_-|\cos\theta$$
$$(1116\,\text{MeV})^2 \neq (1376\,\text{MeV})^2$$

ii) For the second pair, we have $|\mathbf{p}_+| = 250\,\text{MeV}$, $|\mathbf{p}_-| = 2160\,\text{MeV}$ and $\theta = 16°$.

I) $K^0_S \to \pi^+ + \pi^-$

$$m_K^2 \stackrel{?}{=} 2\,m_\pi^2 + 2\sqrt{m_\pi^2 + |\mathbf{p}_+|^2}\sqrt{m_\pi^2 + |\mathbf{p}_-|^2} - 2\,|\mathbf{p}_+||\mathbf{p}_-|\cos\theta$$
$$(498\,\text{MeV})^2 \stackrel{!}{\approx} (491\,\text{MeV})^2 \qquad \text{OK}$$

II) $\Lambda^0 \to p + \pi^-$

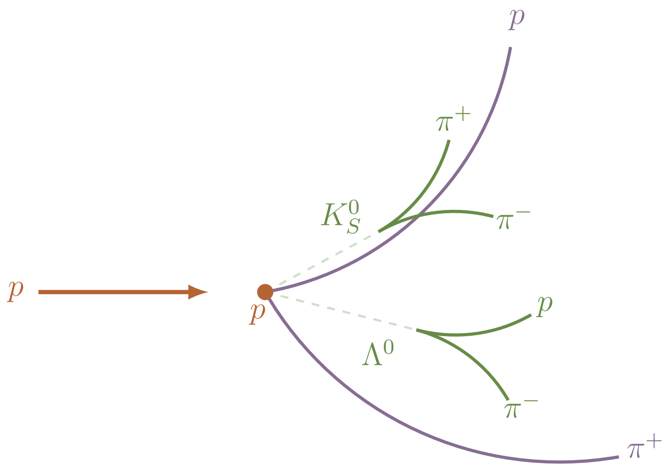
$$m_\Lambda^2 \stackrel{?}{=} m_p + m_\pi + 2\sqrt{m_p^2 + |\mathbf{p}_+|^2}\sqrt{m_\pi^2 + |\mathbf{p}_-|^2} - 2\,|\mathbf{p}_+||\mathbf{p}_-|\cos\theta$$
$$(1116\,\text{MeV})^2 \neq (2016\,\text{MeV})^2$$

III) $\overline{\Lambda}^0 \to \overline{p} + \pi^+$

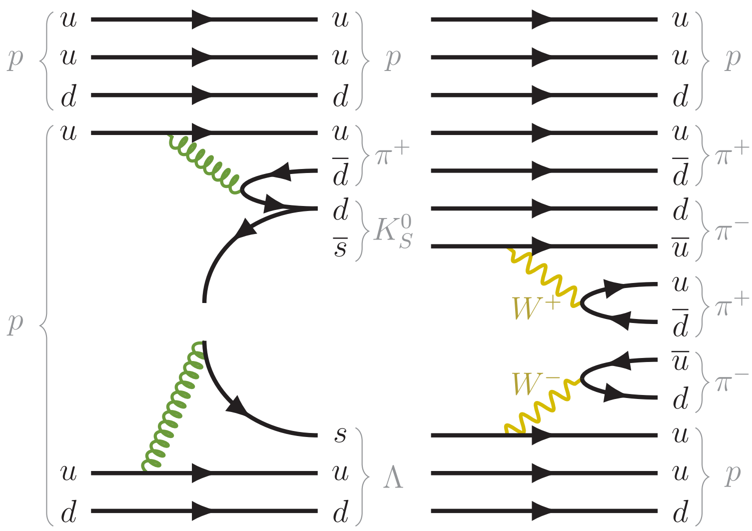
$$m_\Lambda^2 \stackrel{?}{=} m_\pi + m_p + 2\sqrt{m_\pi^2 + |\mathbf{p}_+|^2}\sqrt{m_p^2 + |\mathbf{p}_-|^2} - 2\,|\mathbf{p}_+||\mathbf{p}_-|\cos\theta$$
$$(1116\,\text{MeV})^2 \stackrel{\pm!}{\approx} (1100\,\text{MeV})^2 \qquad \text{not good enough}$$

Thus, only by comparing the calculated dynamics with experiment, we can state (within 95% confidence) that the first particle pair will be a Λ^0 decaying into a proton and a π^-, and that the second particle pair will be a K_S^0 decaying into a π^+ and a π^-.

4. The two remaining (positive) particles will be a baryon and a meson (because of baryon number conservation), both (quasi-) stable (in other words, having a long lifetime, because they don't decay inside the detector). The most obvious candidates are a proton and a π^+. Finally we can complete our sketch of the reaction:



and draw the corresponding Feynman diagram:[3]



email: frederik.van.der.veken@cern.ch

[3]Note that the K_S^0 is a mixture of a K^0 and a $\overline{K}^0$, thus its quark content is $(d\bar{s} \oplus s\bar{d})$, meaning that we can choose between $(d\bar{s})$ and $(s\bar{d})$.