

# Particle Physics

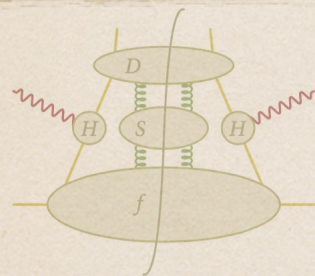


Figure 8.11: Factorisation in SIDIS: the bull diagram. All IR divergences are absorbed in the soft factor S, that hence only interacts with the TMD and FF. Note that there is no real radiation coming from the hard process.



Figure 5.8: All types of first order corrections to the DIS process. Real corrections are on the upper line; virtual on the lower line.

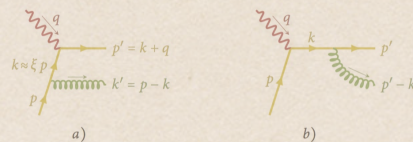


Figure 5.9: a) Initial state gluon radiation. b) Final state gluon radiation.

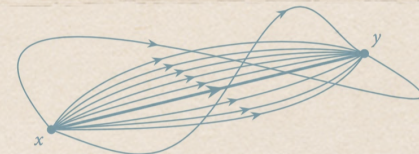
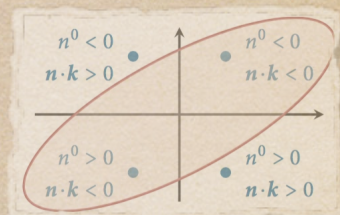


Figure 2.1: As a parallel transporter transforms in function of its path endpoints only, all paths shown will give rise to equivalent  $U_{(y,x)}$ 's, shifting a field at x to a field at y.



## II. Conservation Laws

Minkowski loop integrals are the same as the Euclidian ones, up to a possible sign difference:

$$\int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2 - \Delta)^n} = i \frac{(-1)^n}{(4\pi)^{\frac{d}{2}}} \frac{\Gamma(-\frac{d}{2} + n)}{\Gamma(n)} \Delta^{\frac{d}{2} - n} \quad (d \geq 2n)$$

$$\left( \begin{matrix} d \geq 2n \\ d \text{ even} \end{matrix} \right) = i \frac{\Delta^{\frac{d}{2} - n}}{(4\pi)^{\frac{d}{2}}} \frac{(-1)^n}{(n-1)! (\frac{d}{2} - n)!} \left( \frac{1}{\epsilon} - \gamma_E + \sum_{j=1}^{\frac{d}{2} - n} \frac{1}{j} + \ln 4\pi - \ln \Delta \right),$$

$$\int \frac{d^d k}{(2\pi)^d} \frac{k^2}{(k^2 - \Delta)^n} = i \frac{(-1)^{n+1}}{(4\pi)^{\frac{d}{2}}} \frac{\omega \Gamma(n - \frac{\omega}{2} - 1)}{2 \Gamma(n)} \Delta^{\frac{\omega}{2} + 1 - n}, \quad (B.25b)$$

$$\left( \begin{matrix} d \geq 2n - 2 \\ d \text{ even} \end{matrix} \right) = i \frac{\Delta^{\frac{d}{2} + 1 - n}}{(4\pi)^{\frac{d}{2}}} \frac{\omega}{2} \frac{(-1)^{\frac{d}{2}}}{(n-1)! (\frac{d}{2} + 1 - n)!} \left( \frac{1}{\epsilon} - \gamma_E + \sum_{j=1}^{\frac{d}{2} + 1 - n} \frac{1}{j} + \ln 4\pi - \ln \Delta \right)$$

$$\int \frac{d^d k}{(2\pi)^d} \frac{k^4}{(k^2 - \Delta)^n} = i \frac{(-1)^n}{(4\pi)^{\frac{d}{2}}} \frac{\omega(\omega+2) \Gamma(n - \frac{\omega}{2} - 2)}{4 \Gamma(n)} \Delta^{\frac{\omega}{2} + 2 - n}, \quad (B.25c)$$

$$\left( \begin{matrix} d \geq 2n - 4 \\ d \text{ even} \end{matrix} \right) = i \frac{\Delta^{\frac{d}{2} + 2 - n}}{(4\pi)^{\frac{d}{2}}} \frac{\omega(\omega+2)}{4} \frac{(-1)^{\frac{d}{2}}}{(n-1)! (\frac{d}{2} + 2 - n)!} \left( \frac{1}{\epsilon} - \gamma_E + \sum_{j=1}^{\frac{d}{2} + 2 - n} \frac{1}{j} + \ln 4\pi - \ln \Delta \right)$$

We list some other common Minkowskian integrals:

$$\int \frac{d^d k}{(2\pi)^d} \ln(k^2 - a) = -\frac{i}{(4\pi)^{\frac{d}{2}}} \Gamma\left(-\frac{\omega}{2}\right) a^{\frac{\omega}{2}}, \quad (B.26a)$$

$$\int \frac{d^d k}{(2\pi)^d} e^{ak^2 - ib \cdot k} = \frac{i}{(4\pi)^{\frac{d}{2}}} a^{-\frac{\omega}{2}} e^{\frac{b^2}{4a}}, \quad (B.26b)$$

$$\int \frac{d^d k}{(2\pi)^d} \frac{1}{(-k^2)^\alpha} e^{-ib \cdot k} = \frac{i}{4^\alpha \pi^{\frac{d}{2}}} \frac{\Gamma(\frac{\omega}{2} - \alpha)}{\Gamma(\alpha)} \frac{1}{(-b^2)^{\frac{\omega}{2} - \alpha}}. \quad (B.26c)$$

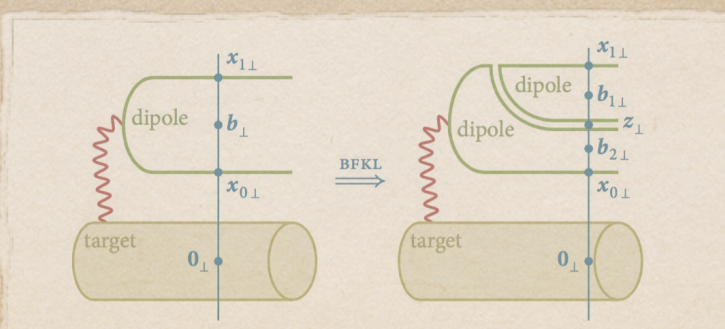
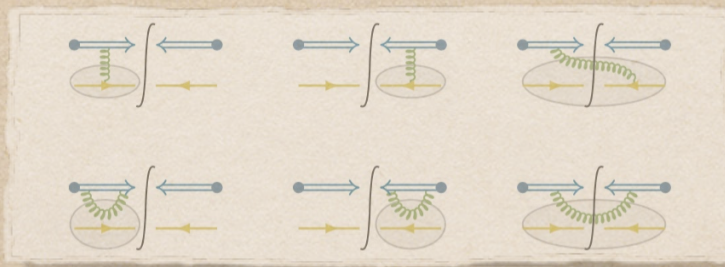


Figure 9.6: In the dipole picture, the BFKL evolution is an evolution in dipoles, i.e. new dipoles are created during the evolution. A gluon that is radiated from the dipole can be represented as two fundamental lines (see Equation 10.13). This essentially splits the dipole in two at the point  $z_\perp$ , as is illustrated in the second diagram.

$$\oint_C dx \cdot A = \int_\Sigma d\sigma \cdot (\partial \wedge A)$$



$$\text{tr}(t^a t^x t^b t^x) = -\frac{1}{4N_c} \delta^{ab},$$

$$\text{tr}(t^b t^x t^y) f^{axy} = -i \frac{N_c}{4} \delta^{ab},$$

$$\text{tr}(t^y t^z) f^{axy} f^{bzx} = -\frac{N_c}{2} \delta^{ab},$$

$$f^{xay} f^{ycz} f^{zbw} f^{wcx} = \frac{N_c^2}{2} \delta^{ab},$$

$$f^{avw} f^{xbv} f^{yvw} f^{zvx} = \frac{N_c^2}{2} \delta^{ab},$$

$$f^{awv} f^{bzw} f^{xzy} f^{yvx} = N_c^2 \delta^{ab},$$

$$f^{xay} f^{ycz} f^{zbw} f^{wcx} = \frac{N_c^2}{2} \delta^{ab},$$

$$f^{vaw} f^{wbz} f^{xzy} f^{yvx} = N_c^2 \delta^{ab},$$

and similarly for the seven remaining diagrams.



# Goals

- ¿ Energy momentum?
- ¿ Conservation Laws ?
  - ¿ Decay ?
  - ¿ Flavour ?



→ **feel free to interrupt  
& ask questions**



# Conservation Laws

- Conservation laws are an important concept in quantum physics. They tell us that a certain quantity is **conserved**
- This means that its values before and after an interaction have to be the same
- For example EM charge is conserved. Not for each particle separately, but the sum of all particles' charges before an interaction has to equal the sum of all particles' charges after that interaction



# Conservation Laws

- Conservation laws help us determine if a certain process can take place or not
- Example:  $p + p \rightarrow n + \pi^+ + \pi^- + p$  is forbidden (initial charge is 2, final charge is 1)
- Certain laws are **exact**: it is believed that they always hold. Others are **approximately exact** (they hold in say 99% of cases), or are valid under certain **conditions** only (for example only when the process is without the weak force)



# Conservation Laws

- Conservation of 4-momentum
- Conservation of charges (EM, weak, colour)
- Conservation of baryon number
- Conservation of lepton number (total & individual)
- Conservation of flavour
- .....



# 4-momentum

- Is the relativistic equivalent of classical momentum
- Relativity combines space and time, and uses 4-vectors instead of 3-vectors
- 4-momentum is a combination of energy and 3-momentum:

$$\mathbf{p} = (E, \vec{p})$$

- The square of a 4-vector is the square of the first component minus the square of the other components:

$$p^2 = E^2 - \vec{p} \cdot \vec{p}$$



# 4-momentum

- In the case of 4-momentum, its square equals the mass squared of the particle under consideration:

$$p^2 = m^2$$

- Combining these two formulas gives

$$E^2 = m^2 + \vec{p} \cdot \vec{p}$$

- We have used  $c = 1$  (speed of light) as this simplifies calculations quite a lot. The correct value can be reintroduced easily:

$$E^2 = m^2 c^4 + \vec{p} \cdot \vec{p} c^2$$

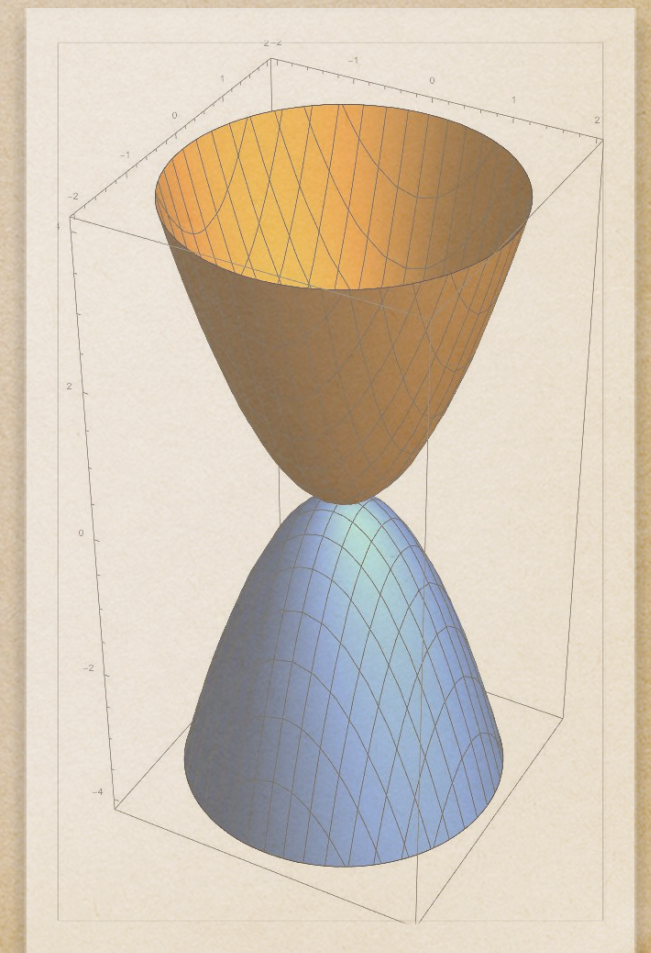
- For a particle at rest, this reduces to the famous formula

$$E = mc^2$$



# 4-momentum

- In particle physics, masses and momenta are always given in scaled energy units ( $\text{MeV}/c^2$  and  $\text{MeV}/c$ ), so we can safely remove all  $c$ 's from the equations
- The Heisenberg uncertainty principle tells us that for a small amount of spacetime, the mass-energy relation  $p^2=m^2$  can be violated. The particle is then called **off-shell**, or **virtual**





# Interludium

- Gamma factor for fast-moving particle:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{E}{mc^2}$$

- Time dilation and Lorentz contraction:

$$t_{\text{LAB}} = \gamma t_{\text{COM}} \quad L_{\text{LAB}} = 1/\gamma L_{\text{COM}}$$

- Cosmic muons:  $E=6\text{GeV}$ ,  $m=105\text{MeV}/c^2 \Rightarrow \gamma=57$   
 $\tau = 2.2\mu\text{s} \rightarrow$  time dilation to the rescue!



# 4-momentum

- 4-momentum is **always conserved**
- Important for decays (where one particle transforms into several). In rest frame of initial particle: mass of initial particle equals sum of energies of final particles. This gives the following condition:

$$m_{\text{initial}} \geq \sum m_{\text{final}}$$



# Charge Conservation

- All charges are always 100% conserved!
- Conservation of **electromagnetic charge** is extremely important and an **easy check** of process validity
- Conservation of **colour charge** is automatically satisfied as long as quarks are combined correctly
- Conservation of weak charge is more complex to check, use rules of weak interaction



# Weak interaction?

- Weak interaction changes **flavour**
- This means between one family:  
$$e^- \leftrightarrow \nu_e \quad u \leftrightarrow d \quad \text{etc}$$
- Charge is not the same  $\Rightarrow$  need other particles to correct this (see lecture on Feynman diagrams)
- Quarks can show **mixing** between families:  
$$u \leftrightarrow s \quad \text{or} \quad u \leftrightarrow b$$

but probability is rather small  
No mixing between leptons!



# Weak interaction?

- No other interaction can change flavour, so if we see a flavour change in a certain process, we know that this process is governed by the weak force
- We say that all interactions except the weak conserve flavour

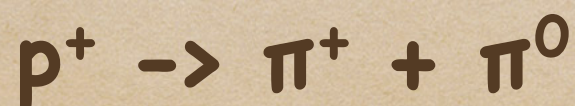


# Baryon number

- The baryon number of a system is the total number of baryons minus the total number of anti-baryons.
- Baryon number is **always conserved**
- Examples:



$$B: 1 = 1 + 0 + 0 \quad \text{OK}$$



$$B: 1 = 0 + 0 \quad \text{NOK}$$

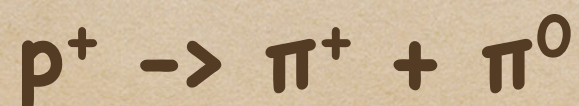


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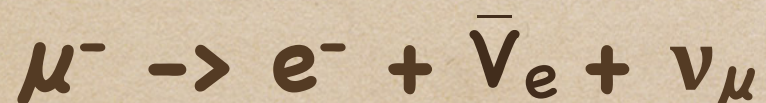
$$B: 1 = 0 + 0 \quad \text{NOK}$$

**No such rule  
for mesons !**



# Lepton number

- The lepton number of a system is the total number of leptons minus the total number of anti-leptons. Similar to baryon number, lepton number is always conserved, but **even per lepton family!**
- Examples:



$$L_e: 0 = 1 - 1 + 0 \quad \text{OK}$$

$$L_\mu: 1 = 0 + 0 + 1 \quad \text{OK}$$

$$L: 1 = 1 - 1 + 1 \quad \text{OK}$$



$$0 \neq 1 - 1 + 1 \quad \text{NOK}$$

$$1 \neq 0 + 0 + 0 \quad \text{NOK}$$

$$1 = 1 - 1 + 1 \quad \text{OK}$$



# Summary

- In decays, the sum of masses of final products cannot be larger than the initial mass
- EM charge is conserved
- Baryon number is conserved
- Lepton number is conserved (total & individual)
- Flavour is conserved unless the process is weak
- Antiparticles have opposite numbers and charges



# Exercises

- Determine if the following processes are possible, and if yes, with which interaction:

$$p + \bar{p} \rightarrow \pi^+ + \pi^- + \pi^0 + \pi^+ + \pi^-$$

$$p + K^- \rightarrow \Sigma^+ + \pi^- + \pi^+ + \pi^- + \pi^0$$

$$p \rightarrow \Lambda^0 + \bar{\Sigma}^0 + \pi^+$$

$$\bar{\nu}_\mu + p \rightarrow \mu^+ + n$$

$$\bar{\nu}_e + p \rightarrow e^+ + \Lambda^0 + K^0$$

$$\Sigma^0 \rightarrow \Lambda^0 + \gamma$$