FEMTOLENSING REVISITED

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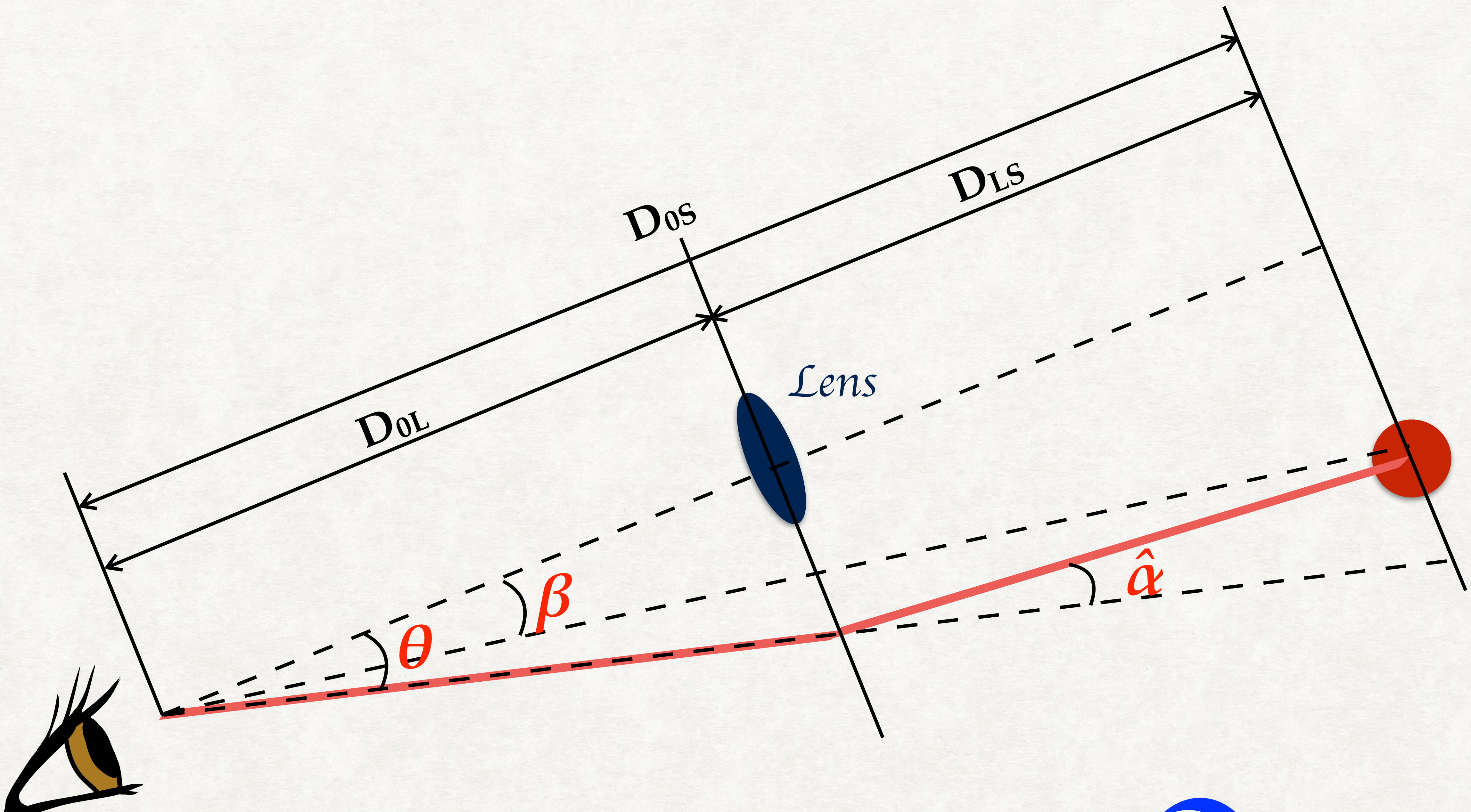
CERN

OUTLINE

- Femtolensing: how does it work?
- Motivation: the dark matter
- Femtolensing of ultra-compact minihalos — (future) data analysis
- Going beyond: speculations about FRB's
- Conclusions and outlook

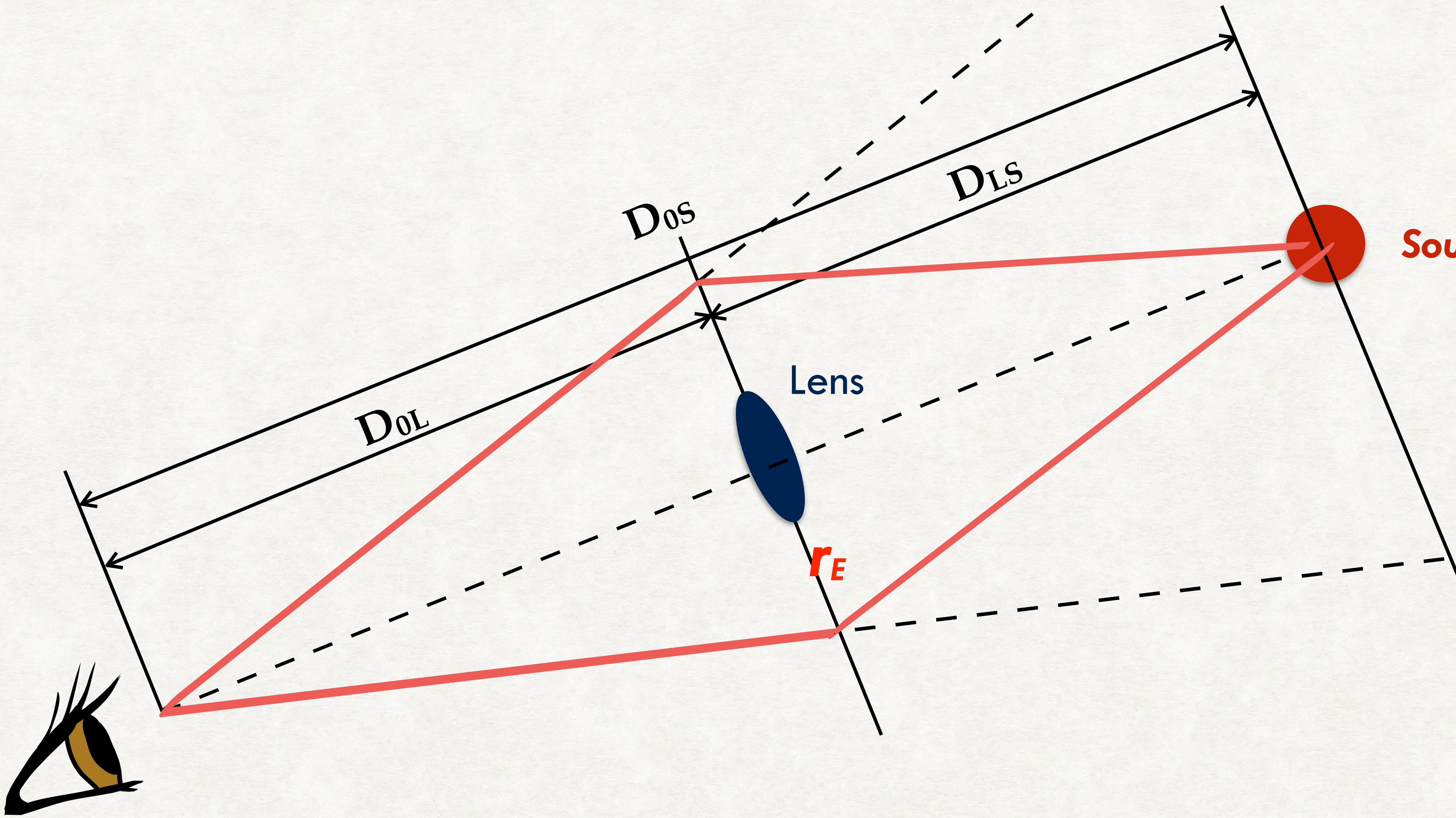
WHAT IS LENSING?



Lens equation: $$\theta = \beta - \hat{\alpha}(\theta) \times \frac{D_s}{D_{LS}} \equiv \beta - \alpha(\theta)$$

reduced deflection angle

THE EINSTEIN RADIUS



$$r_e = \sqrt{\frac{4GM}{c^2}\frac{D_L D_{LS}}{D_S}}$$

Characteristic distance from the (point-like) lens to the ray of light

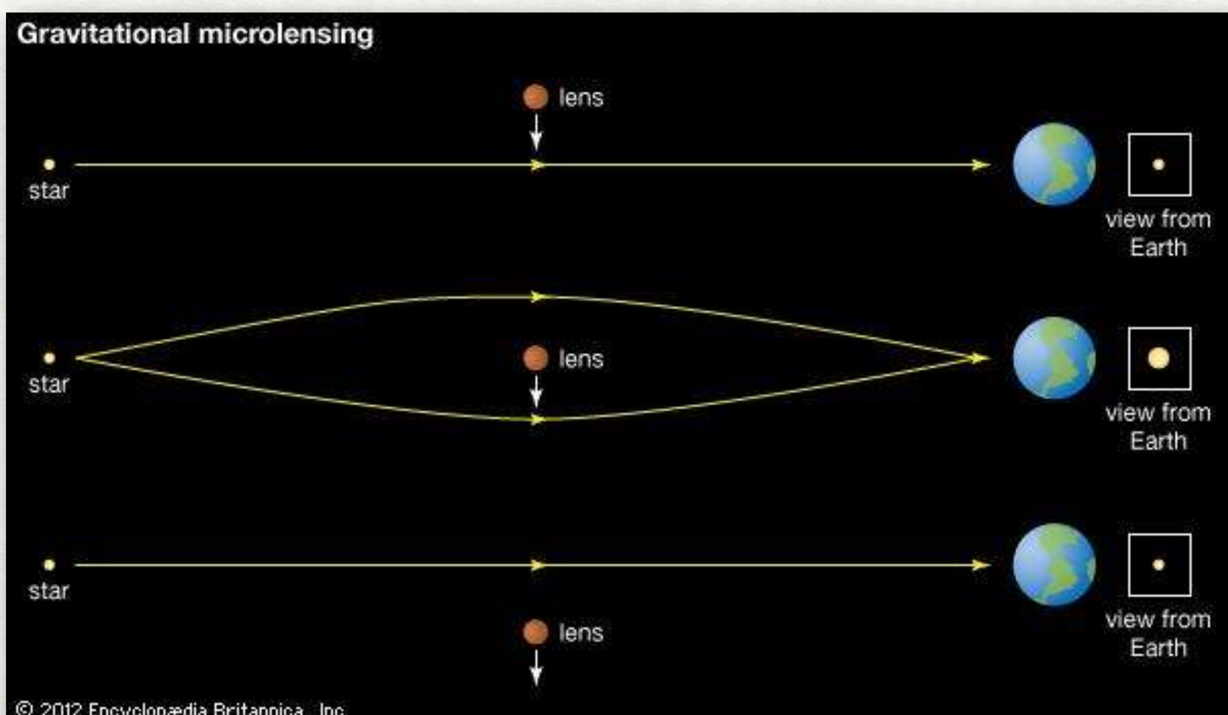
BELOW THE TWO-IMAGES RESOLUTION SCALE

MICRO-, PICO-, FEMTO-LENSING

WE SHOULD NOT NECESSARILY RESOLVE BOTH IMAGES TO LENS HEAVY OBJECTS:

Magnification of (unresolved) object: function of time

$$\mu = \frac{(D_L\theta(t))^4}{(D_L\theta(t))^4 - r_e^4}$$



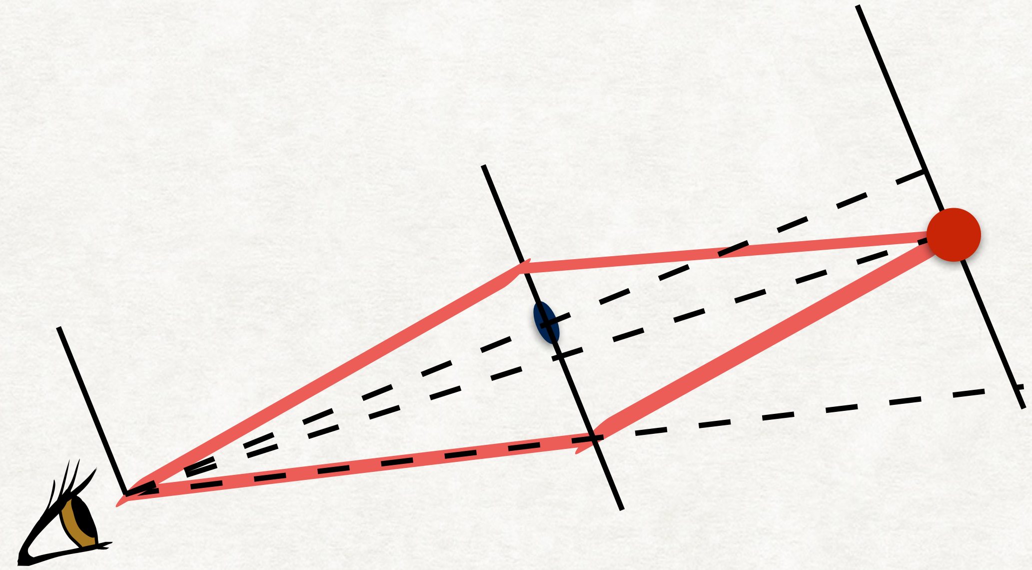
Detect magnification as a function of time. Typical scale of the Einstein angle — micro- arcsecond

Can we go below the micro-arcsec?

- pico — magnification from different points, separation ~1 AU
- femtolensing — interference picture of the two rays

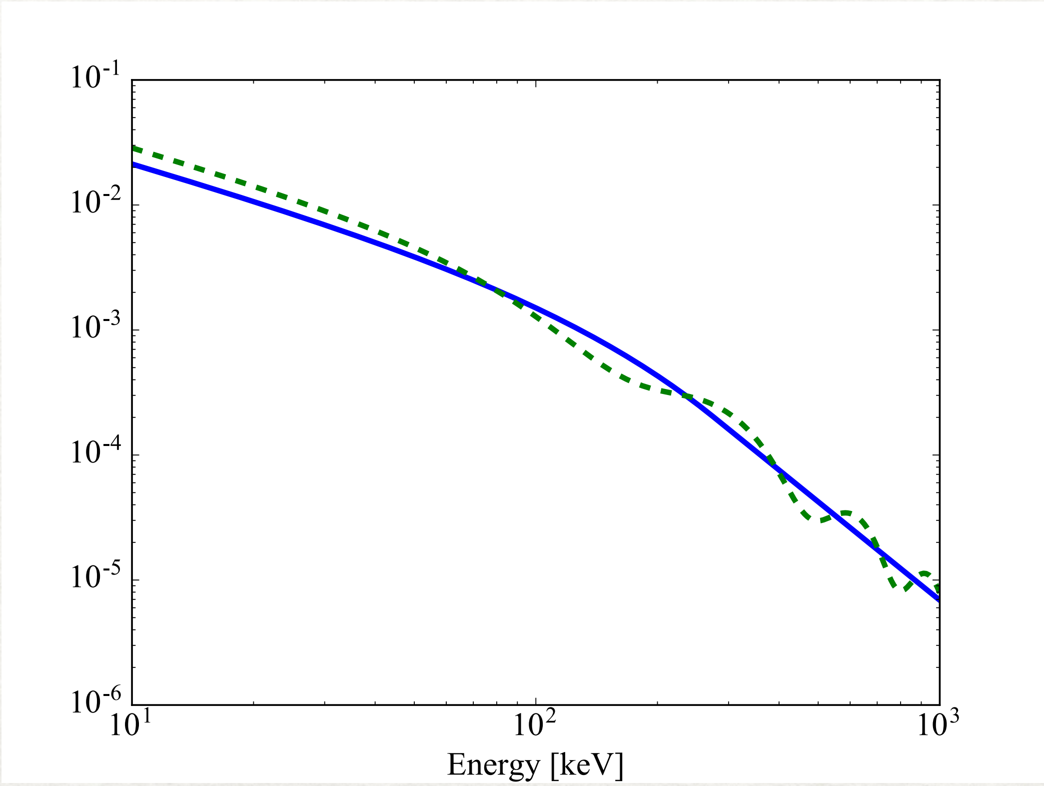
FEMTOLENSING: THE INTERFERENCE PICTURE

The phase shift of each ray:



$$\Delta\phi = \omega\Delta t \qquad \Delta t \sim \frac{4GM}{c^3}$$

Do observe the modulation in the frequency space, this must be ~1.



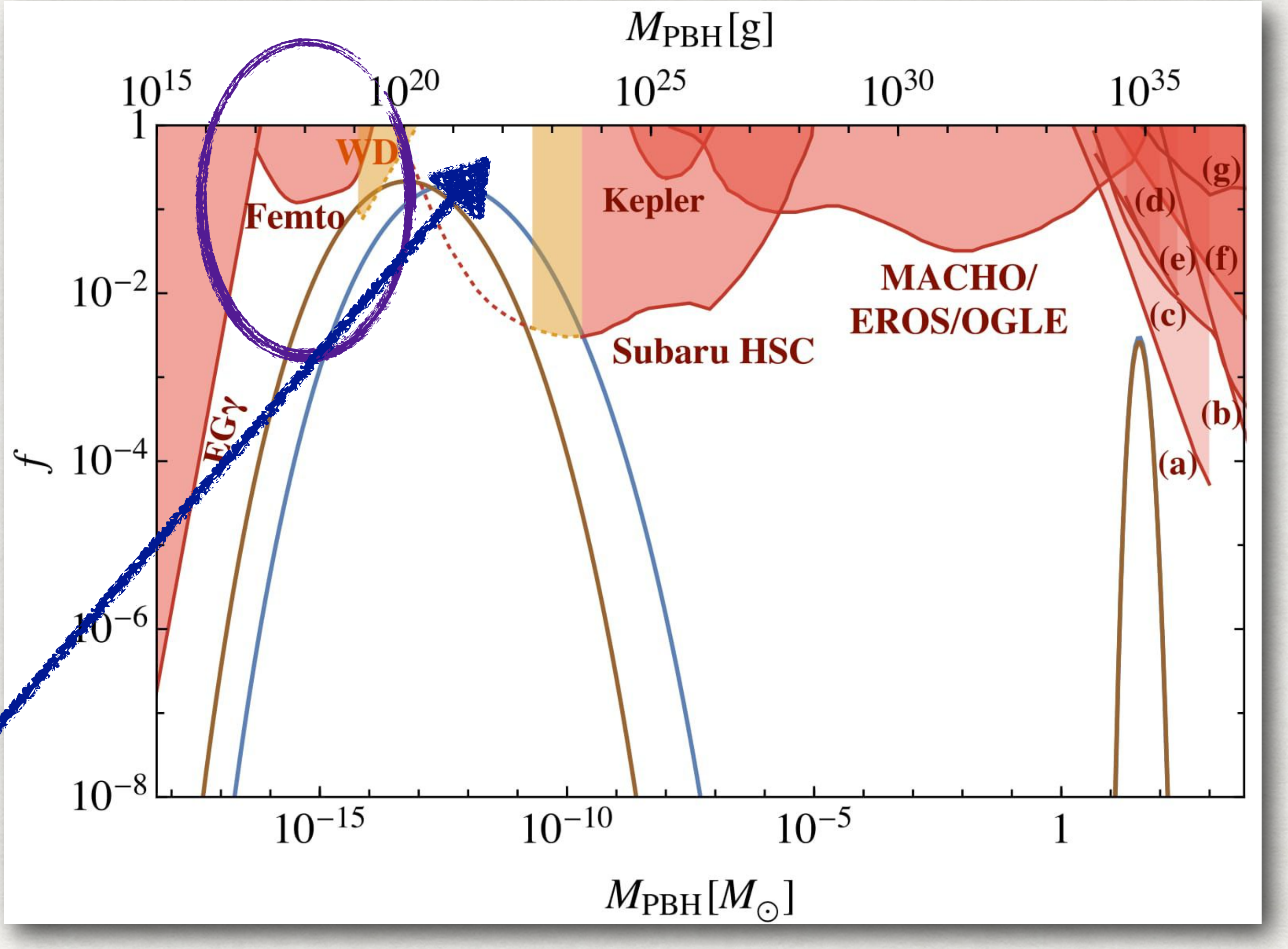
Scale estimations for GRB:

$$E \sim 10 \text{ keV} \Rightarrow \Delta t \sim 10^{-19} \text{ sec} \Rightarrow M \sim 3 \times 10^{-15} \ M_{\odot}$$

with the GRB we probe the BHs with masses of order $10^{-14}...10^{-17}$ solar masses

FEMTOLENSING: OBSERVATIONS UNTIL NOW

Nominal femtolensing constraints



Origin of the bound: analysis of *Barnacka, Glicenstein and Modesrki (2012)* based on 37 Fermi events with known z.

Can we rely on all of them?

Note that this region, although nominally "excluded" by HSC Subaru is kept blank. Here $\lambda > R_s$. Why does this matter. What are the implications on femtolensing?

Are the sources really pointlike projected on the lens plain?

IS THE STORY CORRECT?

GEOMETRIC OPTIC APPROXIMATION

FULL EXPRESSION FOR THE WAVEFUNCTION MAGNIFICATION (FRESNEL INTEGRAL):

$$F(\omega) = \frac{(-i\omega)}{2\pi}\frac{4GM}{c^3}\int d^2\vec{\theta}e^{i\omega\Delta t(\vec{\theta},\vec{\beta})}$$

geometric optics = saddle point approximation

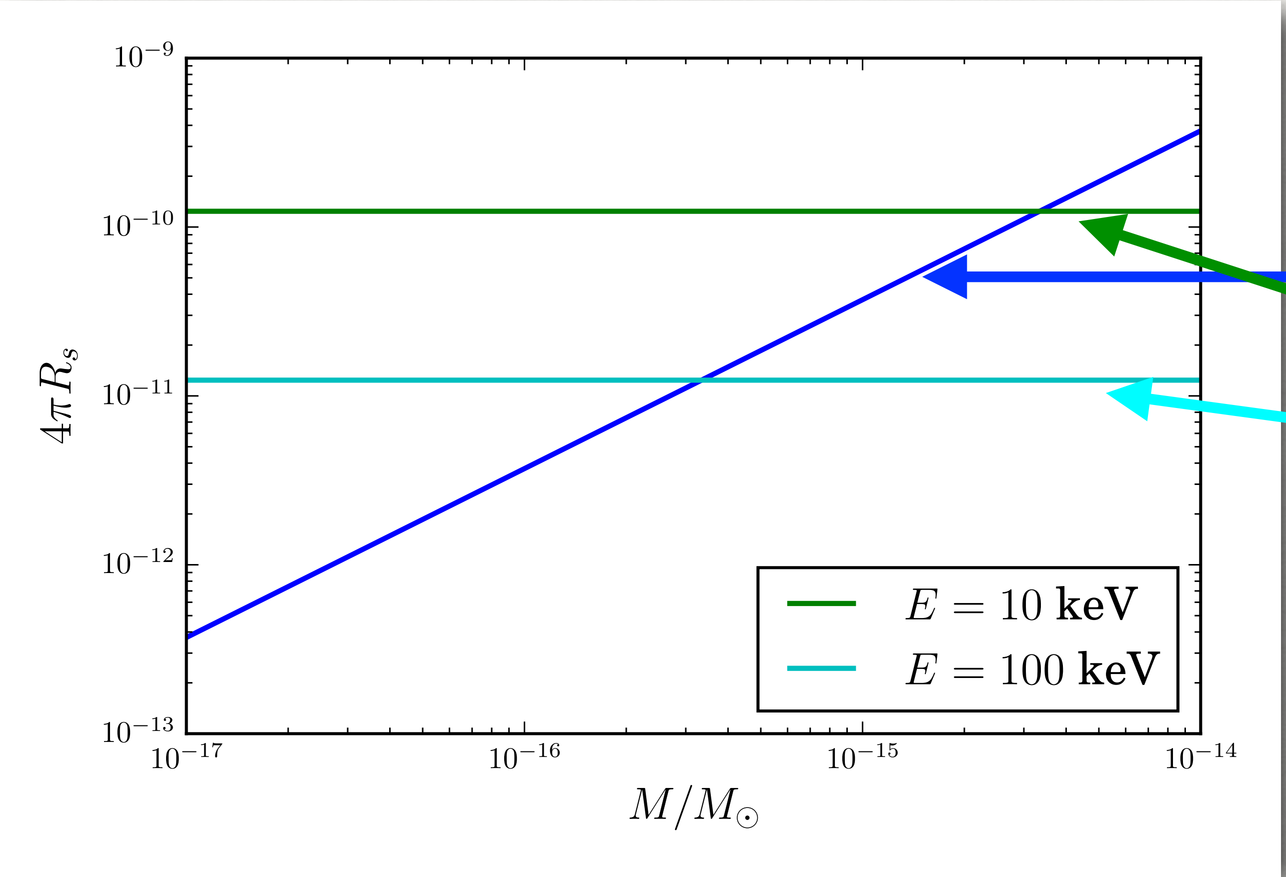
$$\partial_{\theta_i}\Delta t = 0 \quad \Rightarrow \quad \beta^i = \theta^i - \alpha^i(\theta^j)$$

Saddle point approximation is exactly equivalent to the lens equation

VALIDITY OF THE SADDLE POINT APPROXIMATION

Basic criterion: $$\omega \Delta t \gg 1$$ with $$\Delta t \sim \frac{4GM}{c^3} \sim \frac{2R_s}{c}$$

$$\boxed{4\pi R_s \gg \lambda}$$



Is it relevant for the femtolensing?

Schwarzschild radius

typical wavelengths

In the bulk of the femtolensing range wave optics effects cannot be neglected

WHY FEMTOLENSING? DM MINICLUSTERS

AXIONS AS THE BENCHMARK POINT

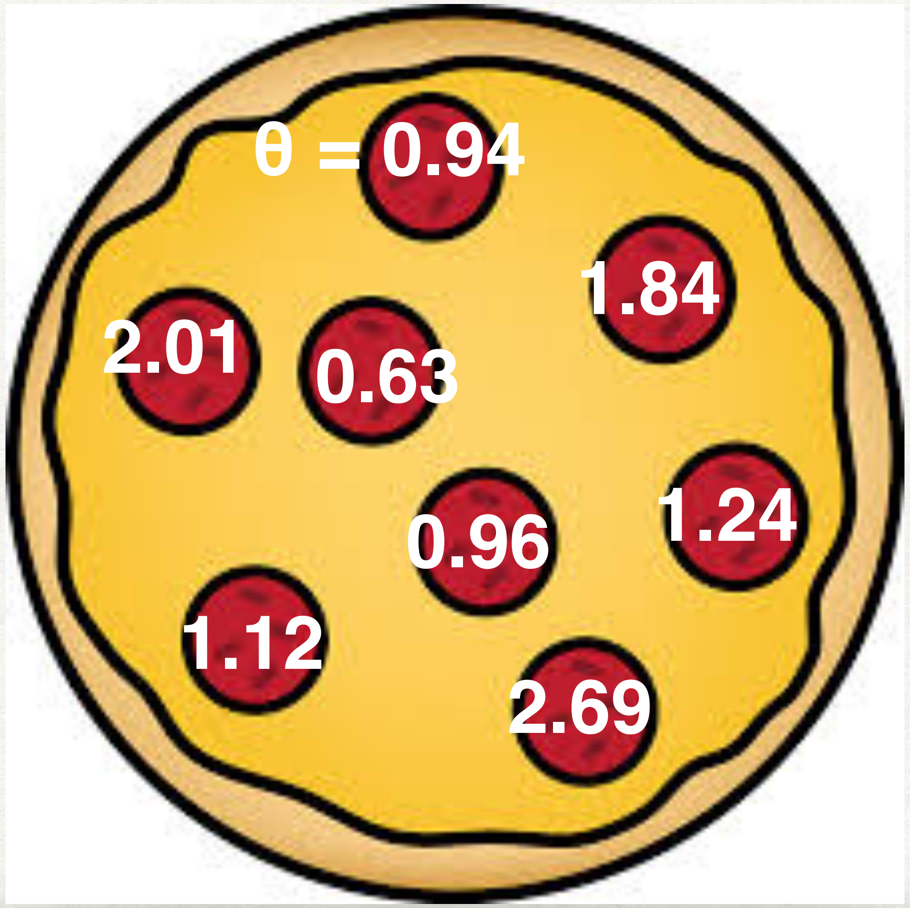
Assume PQ symmetry breaking after the inflation:

During the PQ PT: $m_a = 0$

θ can be anywhere in the range between -π and π. These values vary spatially in the Early Universe.

After the axions get their masses from the QCD instantons, the energy density is

$$\rho_a \sim \Lambda_{QCD}^4 \theta^2$$



This variance in initial values of θ leads to tiny density density fluctuations in the radiation dominated Universe, but $\mathcal{O}(1)$ in axion energy density

AXIONS AS A BENCHMARK

At matter-radiation equality the density perturbations are $\mathcal{O}(1)$ and they start collapsing into mini-clusters.

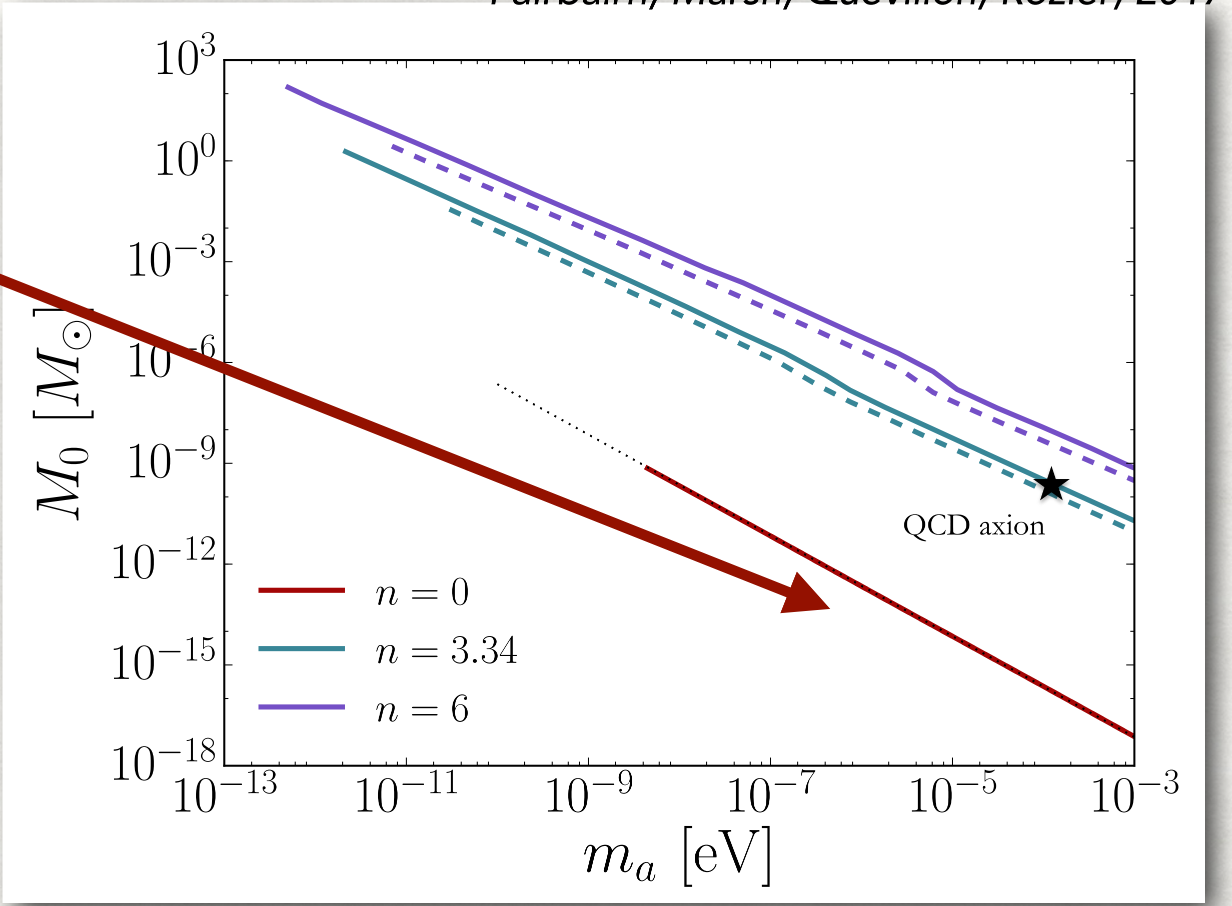
Dark matter contained at one Hubble patch when the axion mass ~ Hubble rate

$$M_0 = \bar{\rho}_a \frac{4}{3}\pi \left(\frac{\pi}{\mathcal{H}(t_0)}\right)^3$$

For temperature-independent axion mass the expected masses are much smaller:

$$M_0(m_a, n=0) \approx 2.3 \times 10^{-7} M_\odot \left(\frac{m_a}{10^{-10}\ \mathrm{eV}}\right)^{-3/2} \left(\frac{\Omega_c h^2}{0.12}\right) \left(\frac{\Omega_m}{0.32}\right)^{-3/4} \left(\frac{1+z_{\rm eq}}{3403}\right)^{3/4},$$

Fairbairn, Marsh, Quevillon, Rozier; 2017



FEMTOLENSING– PREFERRED MASSES ARE POSSIBLE EVEN WITH QCD AXIONS, AND CLEARLY FAVORED FOR THE AXIONS, THAT GET THEIR MASSES FROM NON-QCD SOURCES

PROFILES AND CLUSTER SIZES

- If the structure formation is hierarchical, we expect to get a "standard" NFW profile on a much smaller scaler than the "host" profile. It is a very shallow profile and unlikely can be reasonably femto-lensed

- Self -similar infall can potentially trigger much steeper profile (confirmed by N-body simulations of *Zurek et.al*, *Vogelsberger et. al.*, disagreed on by *Delos et. al.*)

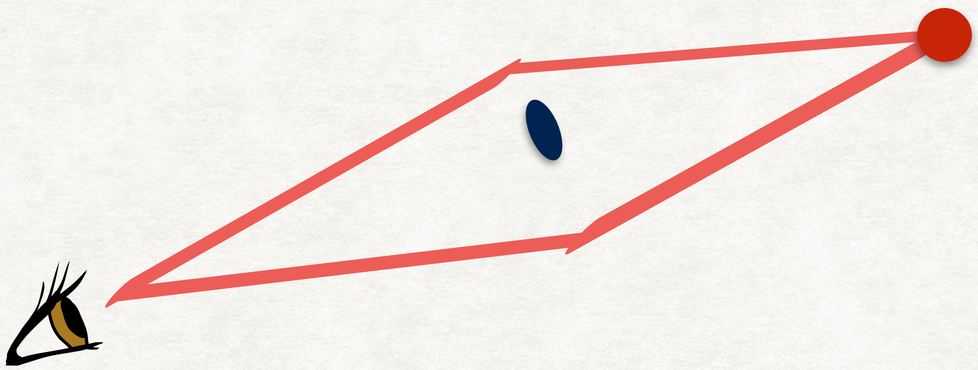
$$\rho(r) = \rho_s \left(\frac{r_s}{r}\right)^{9/4}$$

Radial extend of the self-similar infall profiles (from *Zurek et. al*)

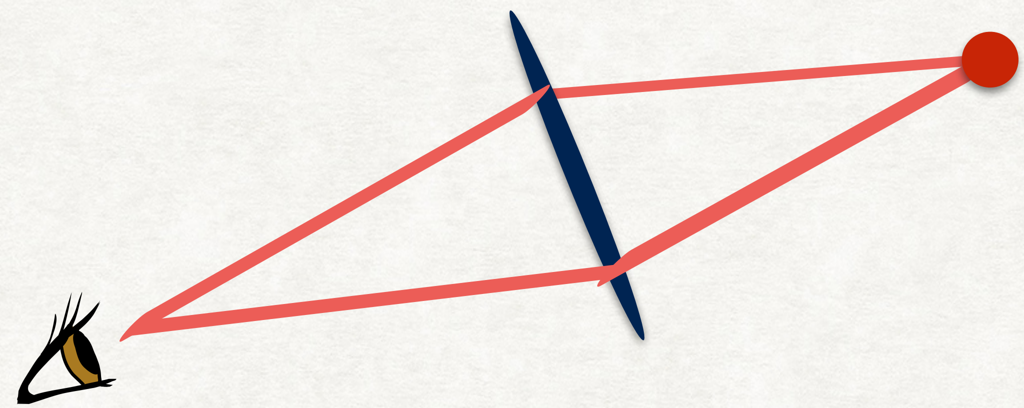
$$R_{ScaM} = 4 \times 10^{16} \frac{1}{\delta\left((\delta+1)\Omega_\phi\right)^{1/3}} \left(\frac{M_{ScaM}}{1 M_\odot}\right)^{1/3} \text{ cm.}$$

It almost always at least 1 order of magnitude bigger than the Einstein radius of these objects

WHERE DOES THE RAY TRAVEL IN THE EXTENDED PROFILE?



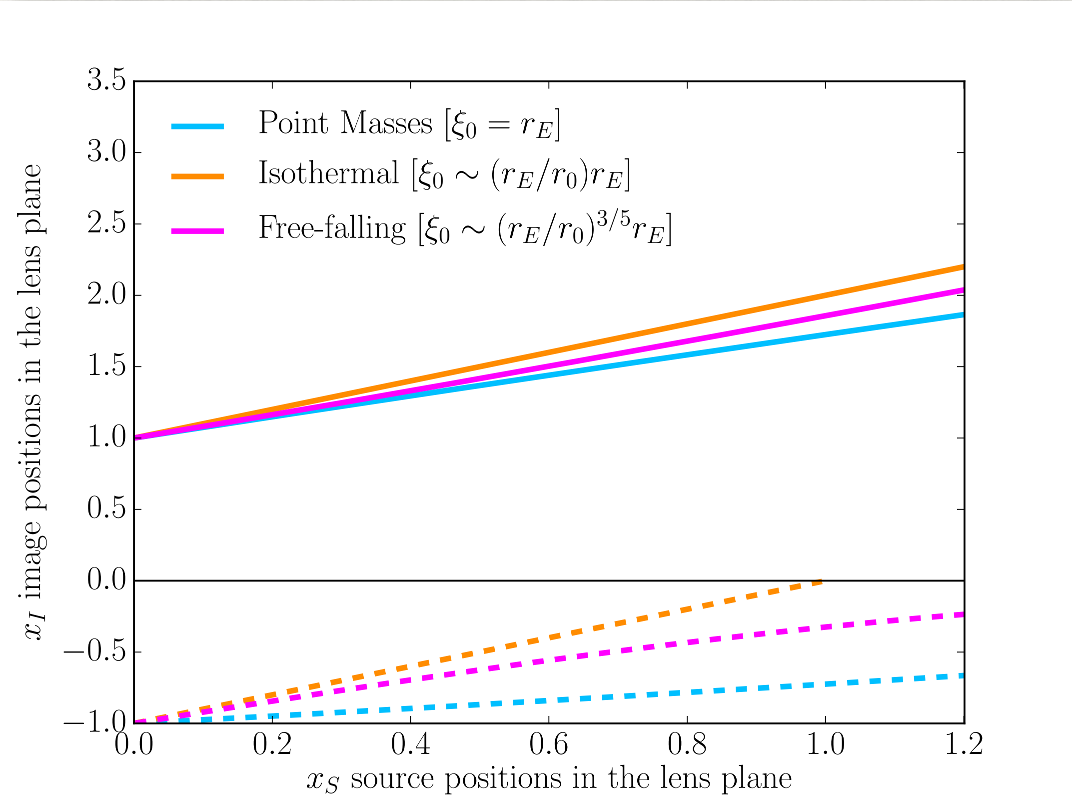
$R_0 < R_E$ ☞ no practical difference from the BH scenario

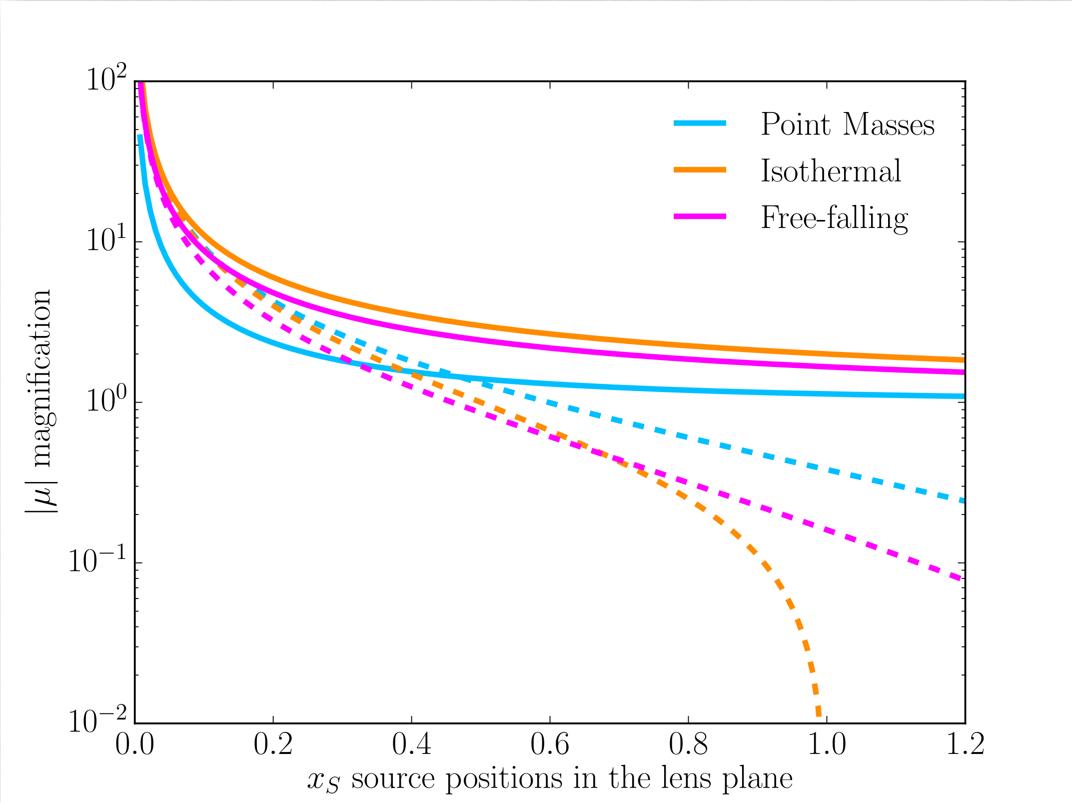


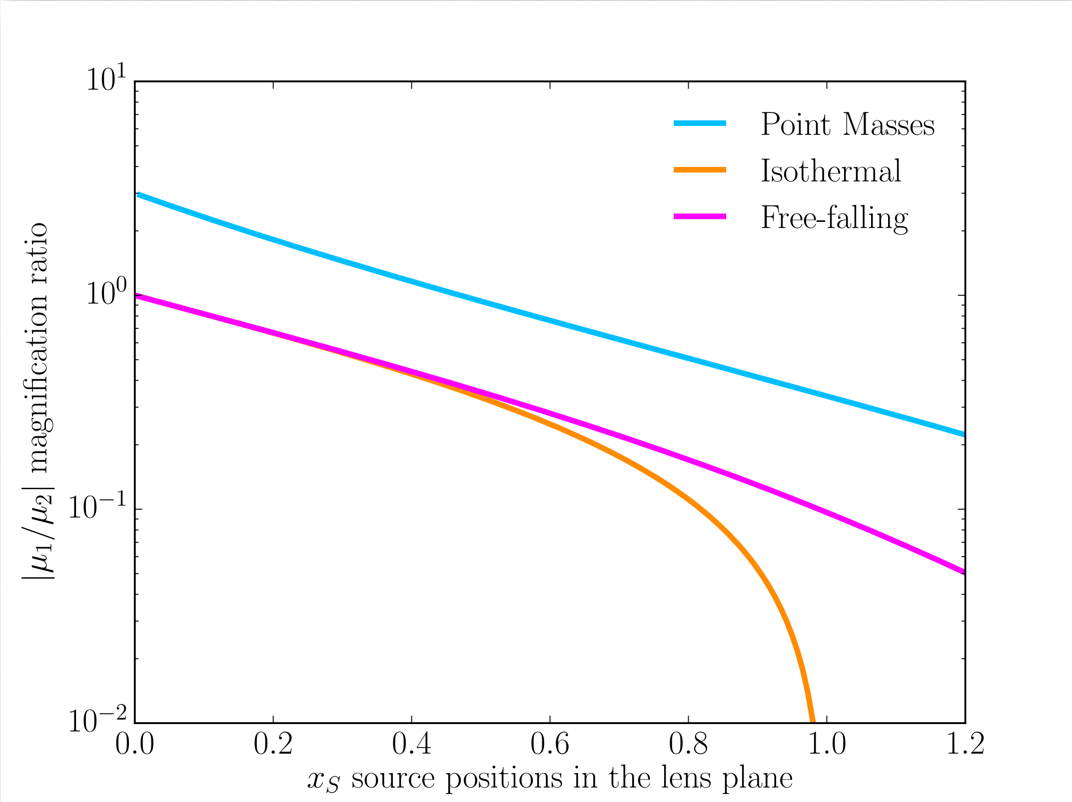
The distance scale os different, because only the mass that is “inside” contributes

- BH: $\xi_0 = R_E$
- Self similar infall (9/4): $(R_E/R_0)^{3/5}\, R_E$
- Isothermal profile (for comparison only, $\boldsymbol{\rho} \sim r^{-2}$): $(R_E/R_0)\, R_E$

IMAGES AND MAGNIFICATIONS

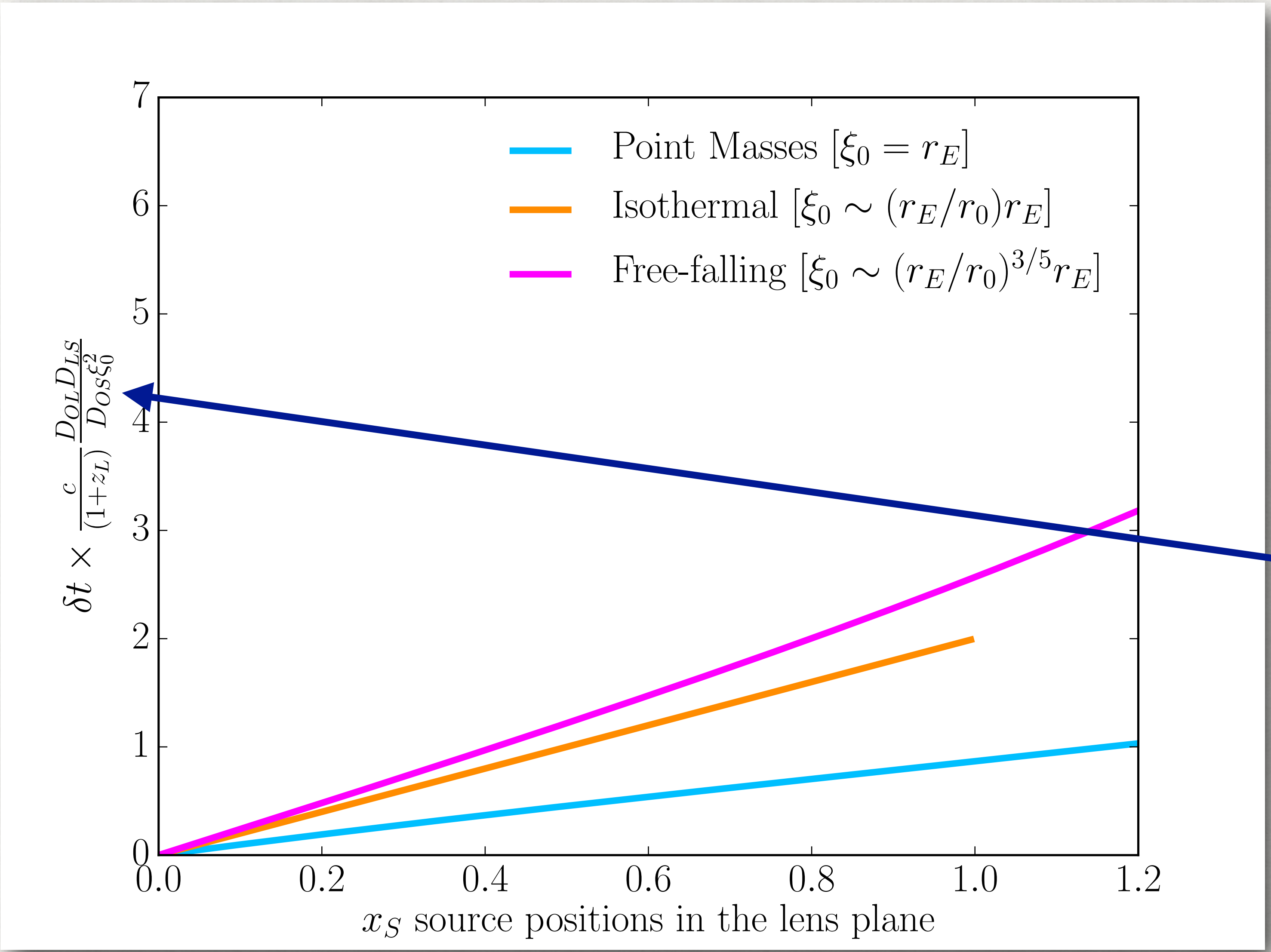






Profiles that fall faster than -2 behave very similar to the BH, isothermal is different as it just preserves one image behind the characteristic scale ξ_0

TIME DELAYS



$$\delta t \sim \frac{D_S}{D_L D_{LS}} \frac{\xi_0^2}{c}$$

Rescaled with ξ_0! Physically (w/o rescaling) the time delay will always be smaller for more dilute profiles.

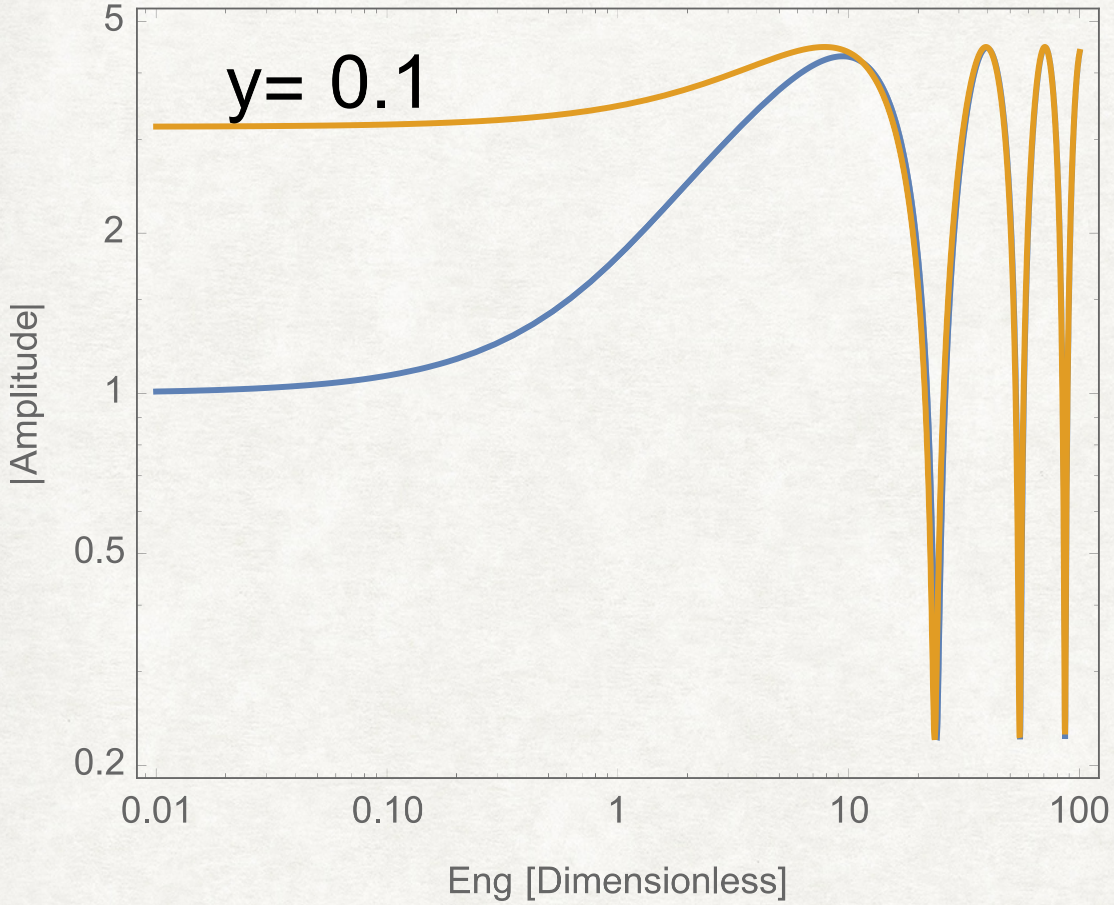
Measured amplitude:

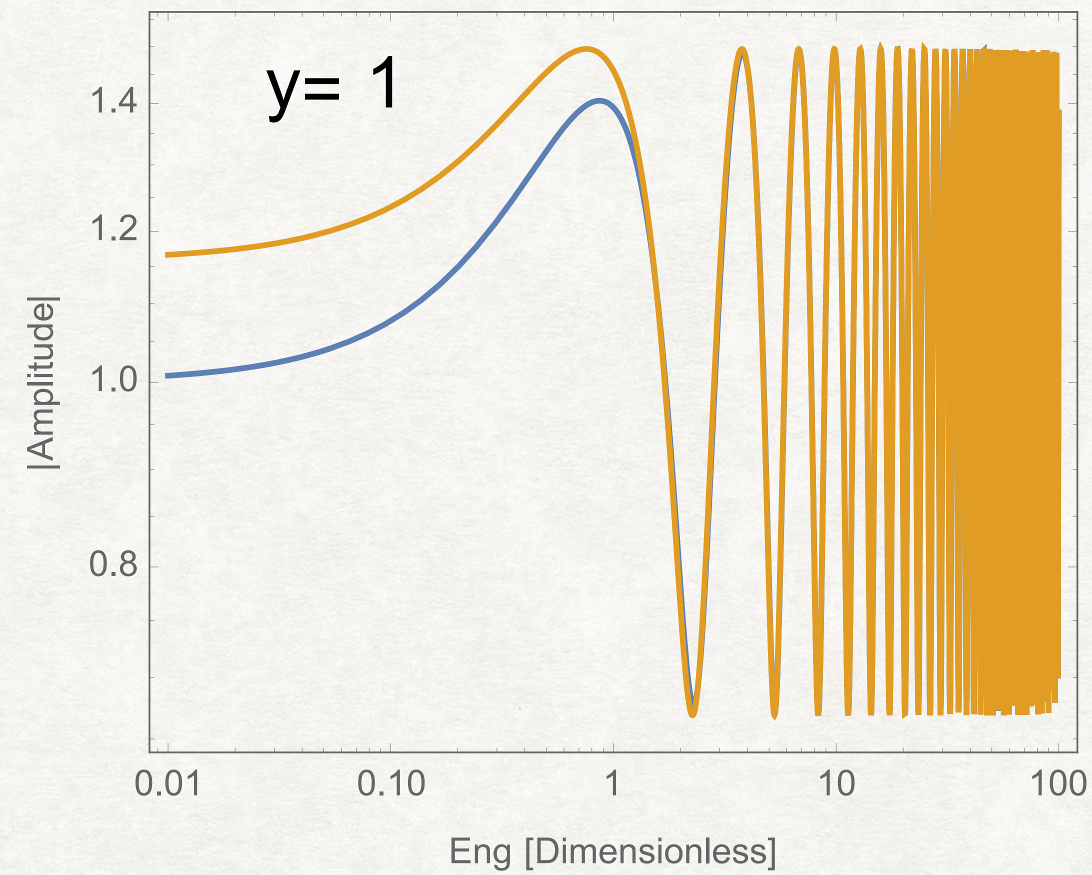
$$A \sim 1 + \mathcal{O}(1) \times \cos(\omega \delta t)$$

EFFECTS OF THE WAVE OPTICS

THE BLACK HOLES

For low frequencies as we saw the geometric optic approximation breaks down:

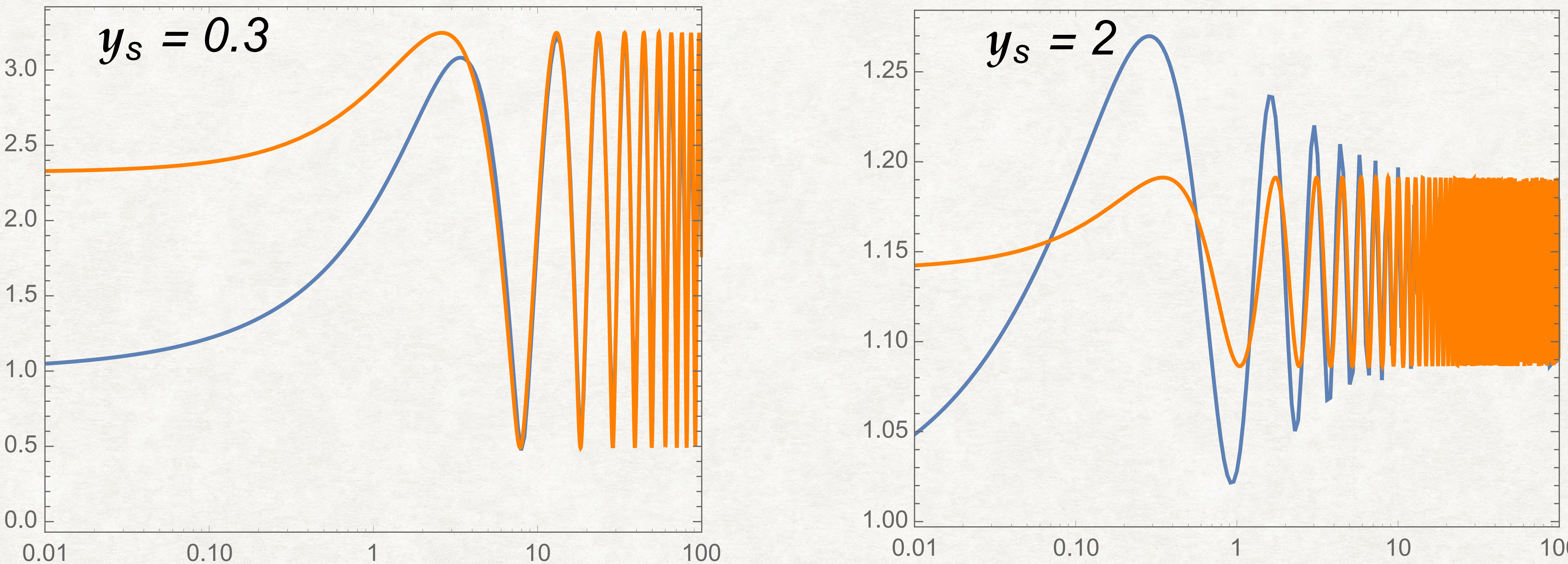




Essentially here the effect boils down to the height of the first peak. Deep in the wave optics regime there are no oscillations, as expected

EFFECTS OF THE WAVE OPTICS

SELF SIMILAR INFALL



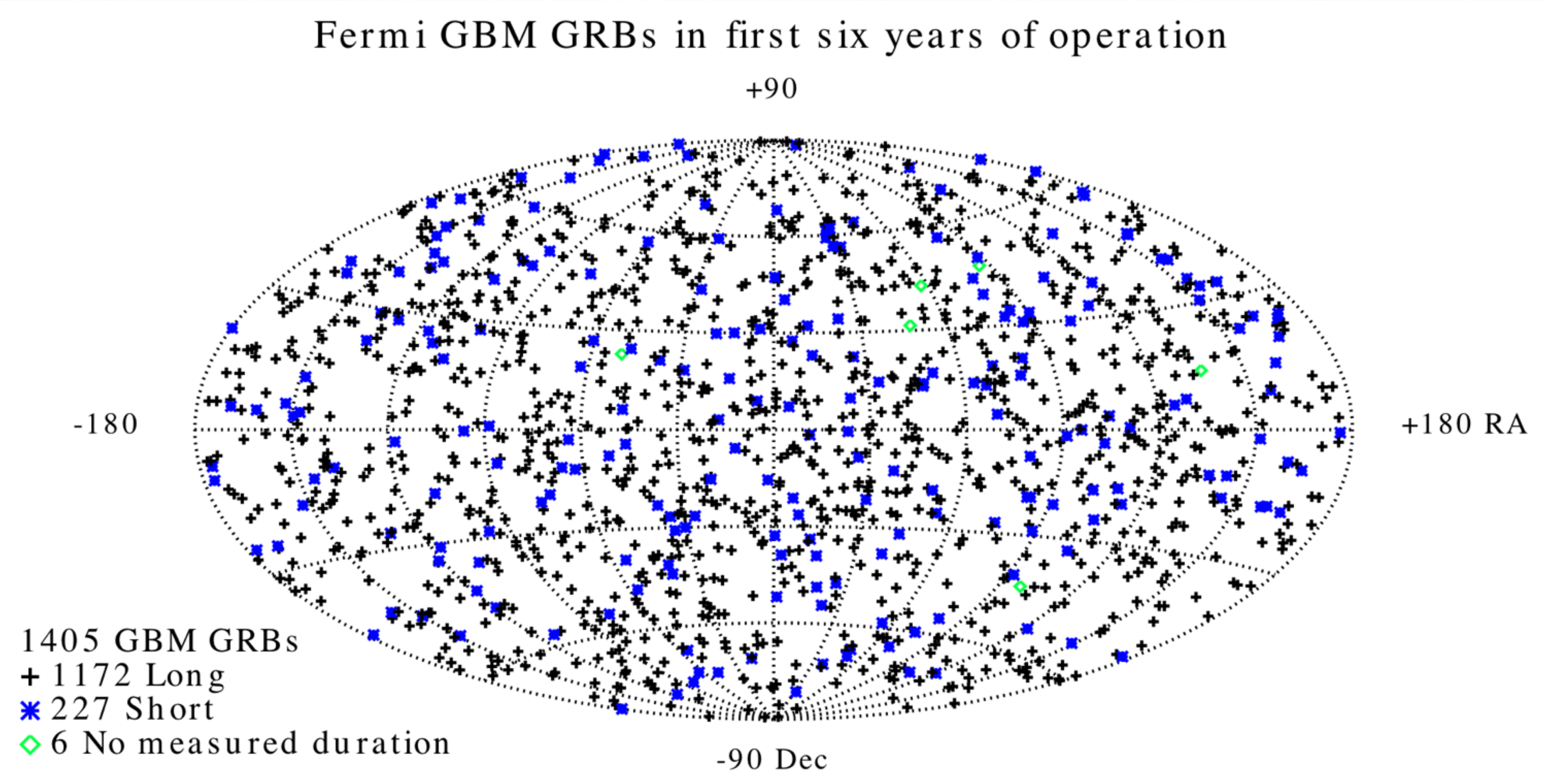
The effect is much more dramatic, especially for the objects “far away” from the center of the distribution in the lens plain. First 5-6 peaks can be easily affected by the wave optics effects

PROFILES SUMMARY

	Density	***image radius***	***time delay***
Point Mass	δ	r_E	$\propto r_E^2$
isothermal	$\rho_0(r_0 / r)^2$	$(r_E / r_0)\, r_E$	$\propto (r_E / r_0)^2 r_E^2$
free-falling	$\rho_0(r_0 / r)^{9/4}$	$(r_E / r_0)^{3/5}\, r_E$	$\propto (r_E / r_0)^{6/5} r_E^2$
general, not NFW	$\rho_0(r_0 / r)^n$	$(r_E / r_0)^{(3-n)/(n-1)}\, r_E$	$\propto (r_E / r_0)^{2(3-n)/(n-1)} r_E^2$

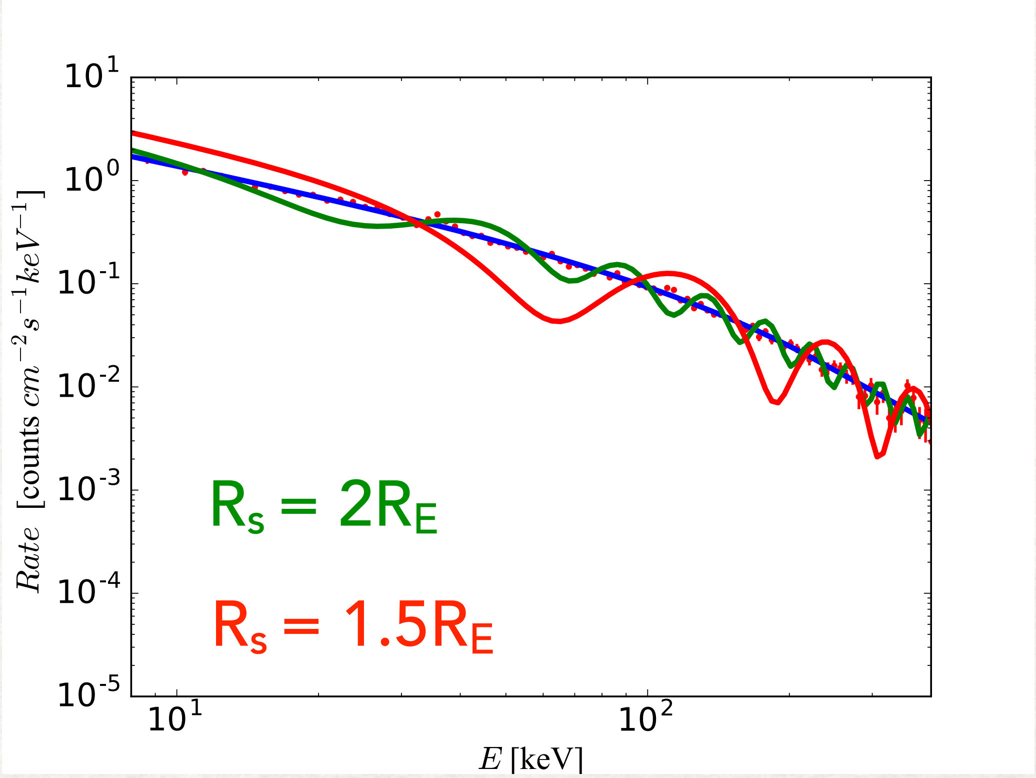
DATA ANALYSIS

GRB SOURCES



SOME ANALYSIS DETAILS

- Take the effects with well measured z (probably one can be more aggressive, not we are not pursuing this pass)
- Subtract the background (after 10s) from the burst
- Fit the GRB to one of the phenomenological models (all these bursts look pretty smooth)
- Compare to a fitted profile + lensing effect
- From these considerations calculate the effective cross section for each mass



SOME ANALYSIS DETAILS

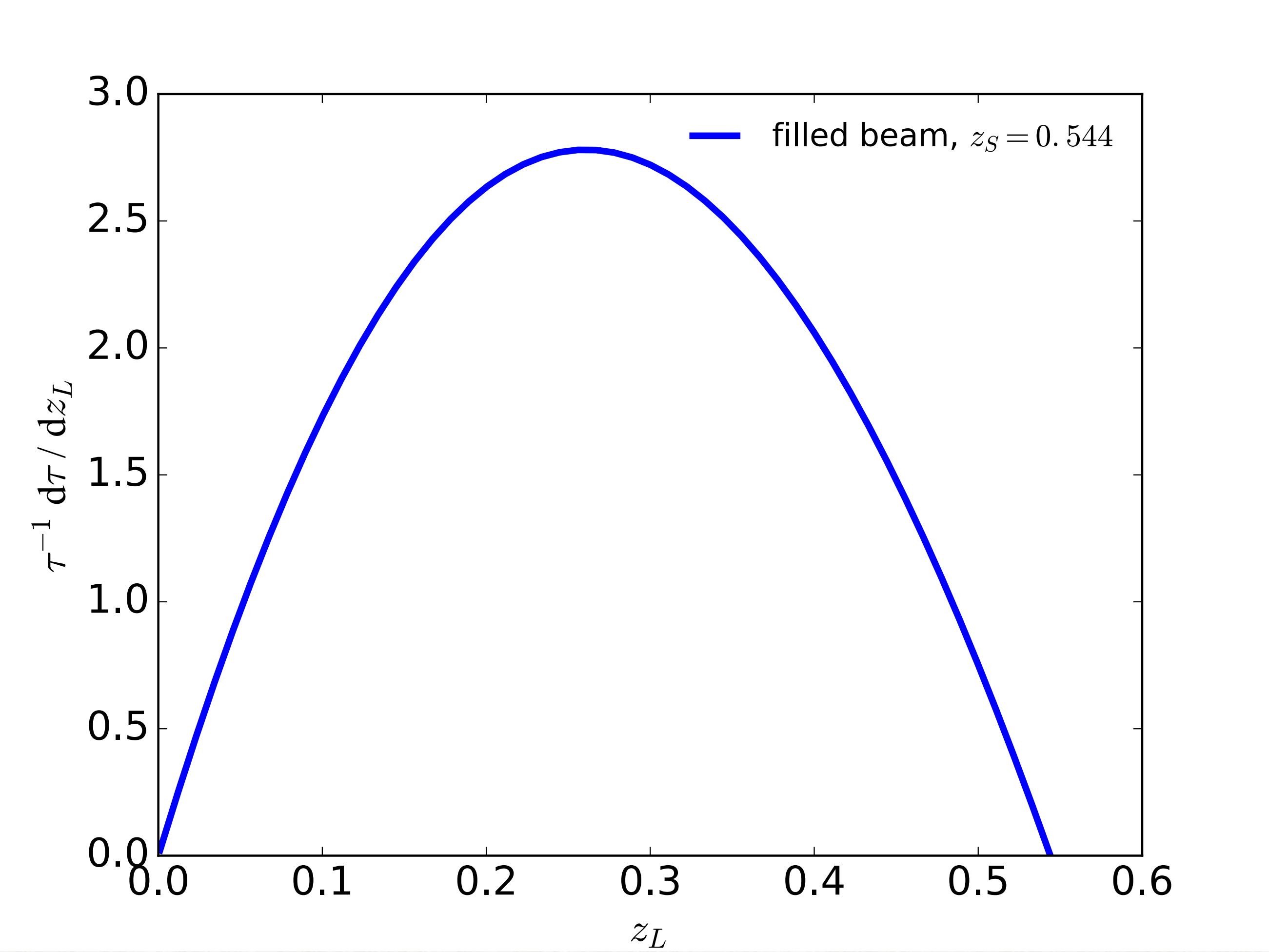
THE OPTICAL DEPTH

Differential optical depth:

$$d\tau = \frac{\rho_{halo}}{M_{halo}}(1+z_l)^3\sigma\frac{cdt}{dz_l}dz_l$$

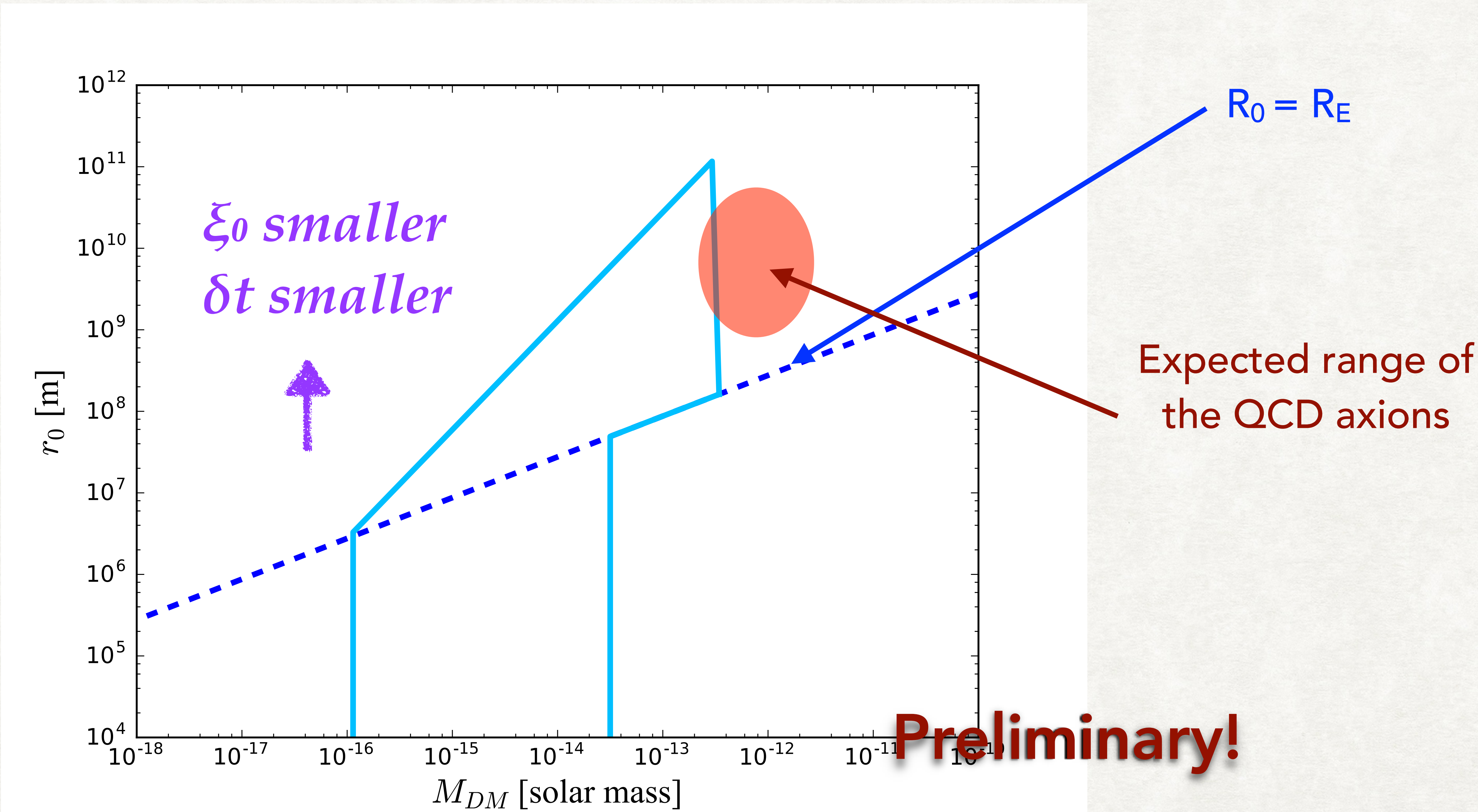
Usually, for small depth it is simply a probability to find a lens. In the full case the probability to find a lens is

$$P = 1 - e^{-\tau}$$



EXPECTED CONSTRAINTS

BHS VS SELF SIMILAR INFALL



FAST RADIO BURSTS

CAN WE PLAY THE SAME TRICK THERE?

- Wavelengths from 100 MHz to GHz. Relatively fine spectra are available (in 10s of MHz)
- Cosmological origin (z~1), verified by their dispersion measure ("DM")
- Short durations (order ms). Has been exploited to push the idea of lensing of "time-resolved images". 1ms ⇒ 10…100 $M_\odot$

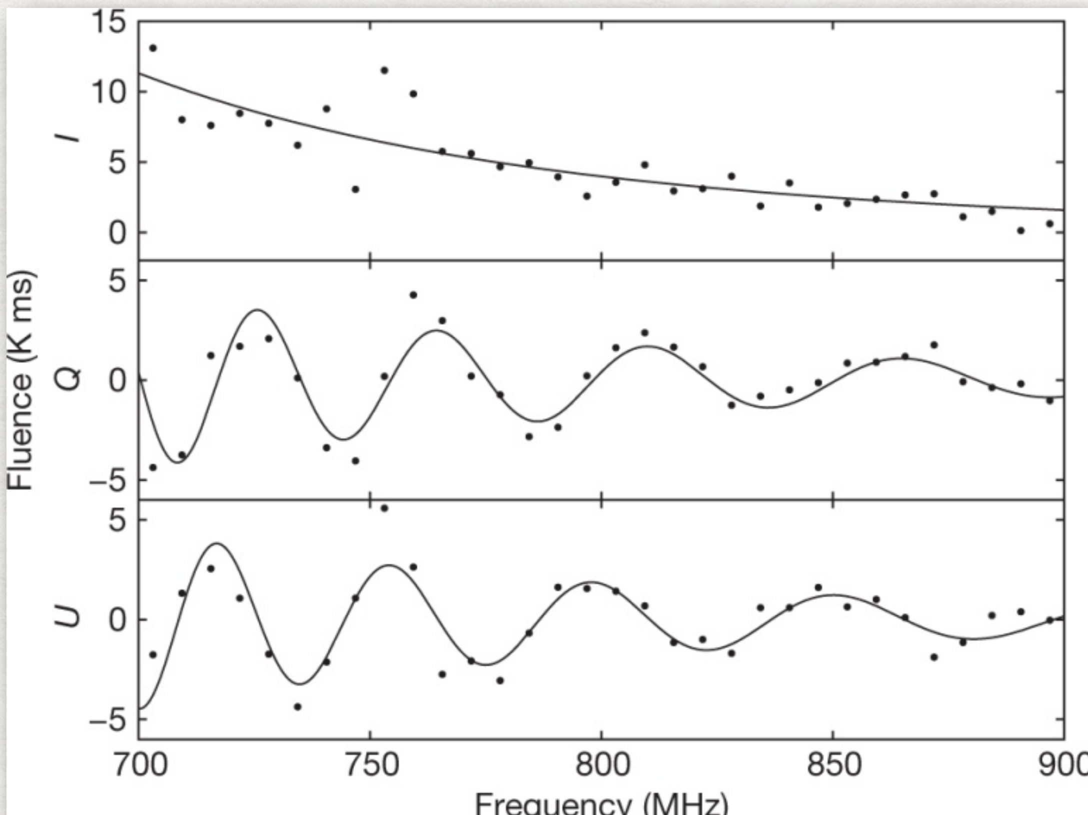
If we just blindly use the same logic as in the femtolensing, the preferred range will be around $10^{-1}…10^{-3}$ $M_\odot$

This range is already covered by the micro-lensing measurements, but this can potentially be the first probe of this range on the cosmological scale (rather than galactic / cluster scale)

SCINTILLATION

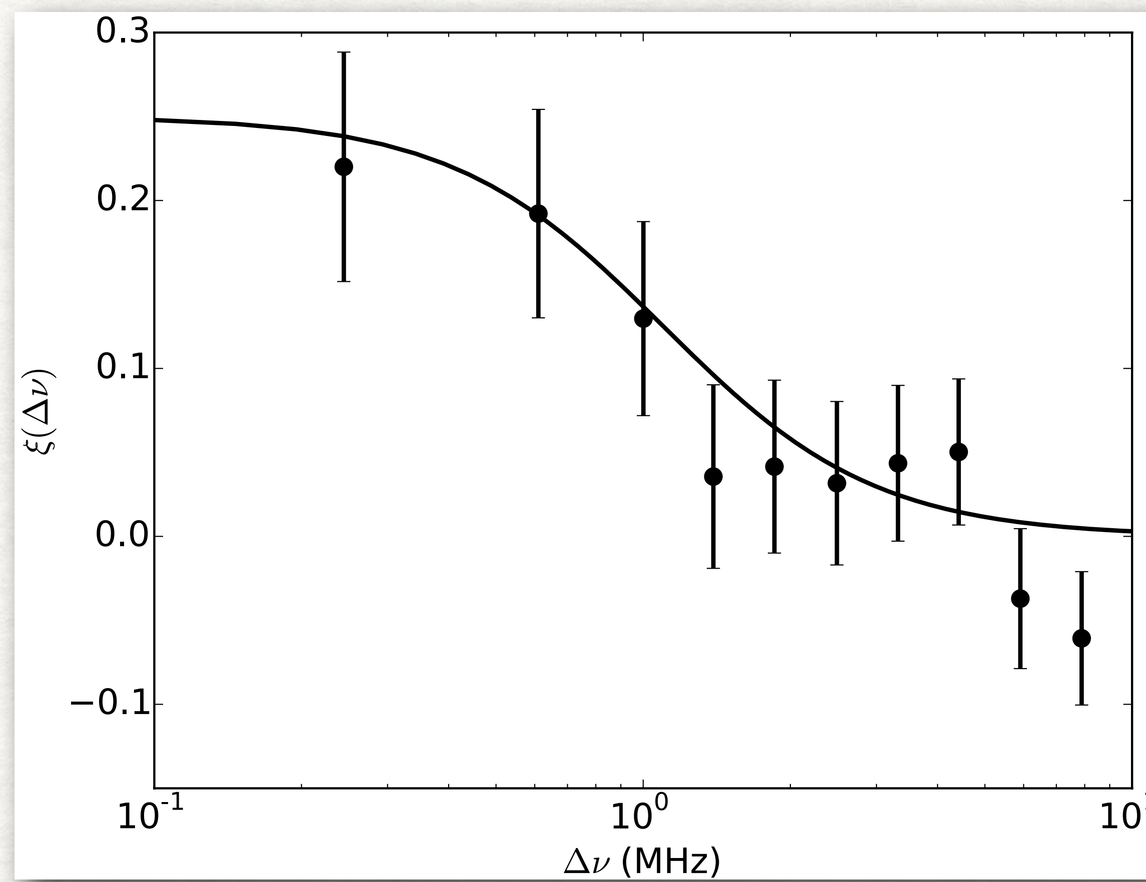
WHY DO THE STARS TWINKLE?

FRB 110523 spectrum:



Frequency separation less than 1.2 MHZ

Stars — effectively point-like sources; light emitted from the opposite edges has path length difference < λ. The lights propagates through a turbulent atmosphere, multi path interference picture changes with time.

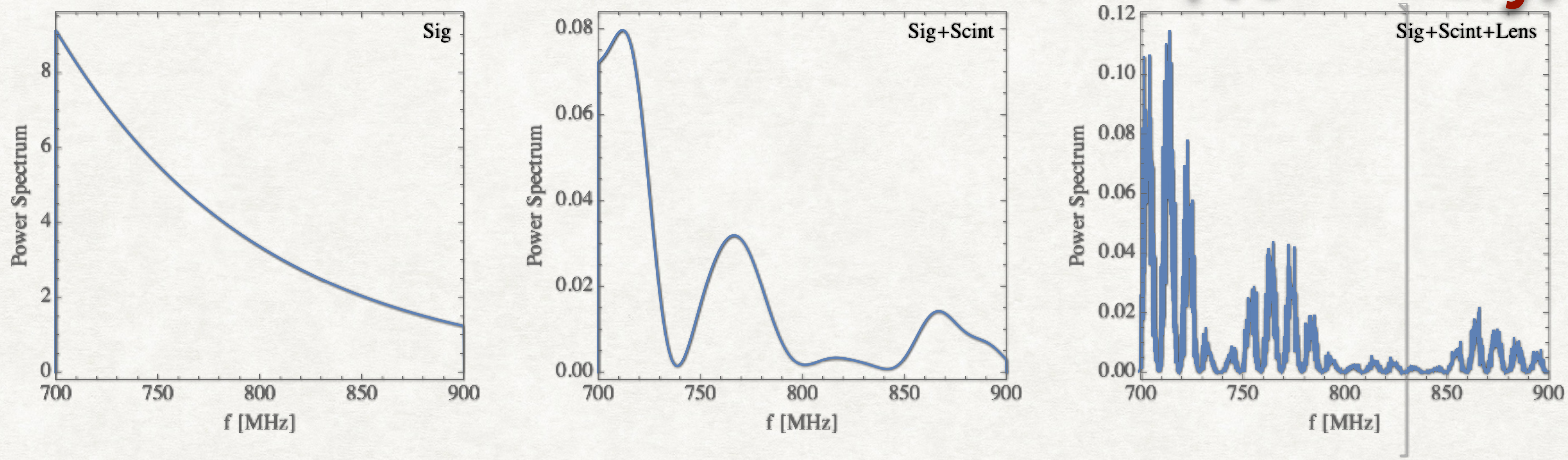


Extended Data Figure 3 | Spectral brightness correlation function of FRB 110523. The intensity spectrum has structure that is correlated for frequency separations less than $f_{dc} = 1.2$ MHz. Error bars are the standard deviation of 3,268 simulated measurements with 817 independent noise realizations and are correlated.

CAN WE RESOLVE THE SCINTILLATION VS LENSING?

MAYBE...

Preliminary!



Naively it looks possible if the scintillation frequency and the lensing frequency are sufficiently different! Of course, more input about the turbulence in the intergalactic medium is needed.

QUICK OUTLOOK

- DM in the Ultra compact mini-clusters is a reasonable possibility. QCD axions can be a nice benchmark point, but it is not the only option
- These objects (if extremely light) can in principle be probed by the femtolensing, but the details are rather different from the primordial BHs lensing
- Effects of the wave optics are almost always important in the femtolensing, and can rarely be completely neglected (essentially only on the edge of the distributions)
- Interesting to see how exactly this idea can be pushed in the FRB domain.

THANK YOU!