# PBHs from Inflation and their Implications

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Based on 1611.06130, 1701.02544, 1711.08956 In collaboration with K.Inomata, M.Kawasaki, Y.Tada, T.T.Yanagida



# Introduction

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### Why Primordial Black Holes (PBHs)?

- Non-particle candidate of DM
- Candidate of gravitational wave events observed by LIGO
- Seeds of SMBHs
- Constrain **other** DM models; WIMP by UCMH, axion by super-radiance,...

### How do you produce them?

- Need Large density perturbations for Gravity > Pressure.
  - Collapse of localized configurations: bubble, DW, cosmic string, Q-ball,...
  - Collapse of **primordial** density perturbations: **inflation**, **curvaton**,...

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### Constraints independent of production mechanisms.

### ▶ Note: a **delta function** for PBH spectrum is assumed.



 Constraints from Neutron Star capture are evaded for a conservative value of DM inside the globular clusters. [See e.g. Kusenko+, 1310.8642; Carr+, 1607.06077]

### Hawking radiation EGy: 0912.5297

### **Gravitational lensing**

Femto: 1204.2056 HSC: 1701.02151 Kepler: PhysRevLett.111.181302 EROS/MACHO/OGLE: 0011506,0607207, 1106.2925 Dynamical

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UFD: 1605.03665, 1704.01668

#### Accretion

CMB: 1612.05644,1707.04206,...

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Assume a specific production mechanism (inflation). Are there any other ways to probe them? Implications on inflation models?

# Outline of Talk

### Introduction

## Constraints on PBHs from Inflation

- Inflation Models
- Summary

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# Constraints on PBHs from Inflation

### Need large $\delta \rho / \rho$ for Gravity > Pressure







► PBH mass (M)  $\Leftrightarrow$  scale of perturbation (k)  $M = \gamma \rho \frac{4\pi H^{-3}}{3} \simeq M_{\odot} \left(\frac{\gamma}{0.2}\right) \left(\frac{g_*}{3.36}\right)^{-\frac{1}{6}} \left(\frac{k/(2\pi)}{3 \times 10^{-9} \,\mathrm{Hz}}\right)^{-2}$ Carr, '75

• PBH abundance  $(\beta) \hookrightarrow$  amplitude of perturbation  $(\mathbf{P}_{\zeta})$ 

$$\beta(M) = \int_{\delta_c} \mathrm{d}\delta \frac{\mathrm{e}^{-\frac{\delta^2}{2\sigma^2(M)}}}{\sqrt{2\pi\sigma^2(M)}} \sim \sigma(M) \mathrm{e}^{-\frac{\delta_c^2}{2\sigma^2(M)}} \qquad \sigma^2(M(k)) = \int \mathrm{d}\ln q \, W^2(q \, k^{-1}) \frac{16}{81} (q \, k^{-1})^4 \, \mathscr{P}_{\zeta}(q) \\ \propto \mathscr{P}_{\zeta}(k)$$

✤ Loophole → Large non-Gaussianity

### **PBH production by high-\sigma tail:**

► 1% of DM @ O(10) solarmass  $\rightarrow \beta \sim 10^{-10} \ll I \leftrightarrow P_{\zeta} \sim O(0.01)$ 

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$$\frac{\delta \rho}{\rho}$$

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### **After reentry**



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### Energy injection from large small-scale perturbs.

How are they dissipated among background? → Depends on Era.











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## Large density perturbation as a source of GWs

Tensor perturbation obeys...

Saito, Yokoyama,'09; Bugaev, Klimai,'10

Talk by J.R. Espinosa

$$h_{ij}'' + 2\mathcal{H}h_{ij}' - \nabla^2 h_{ij} = -4\hat{\mathcal{T}}_{ij;kl}S_{kl}$$

Depends on the **density perturb.**,  $\Psi \sim \zeta$ 

$$S_{ij} \equiv 4\Psi \partial_i \partial_j \Psi + 2\partial_i \Psi \partial_j \Psi - \frac{4}{3(1+w)} \partial_i \left(\frac{\Psi'}{\mathcal{H}} + \Psi\right) \partial_j \left(\frac{\Psi'}{\mathcal{H}} + \Psi\right)$$



Production of GW by second order effects  $h_{i\,i} \propto \Psi^2 \sim \zeta^2$ 

$$\Omega_{\rm GW}(k)h^2 \sim 10^{-9} \left(\frac{\mathscr{P}_{\zeta}(k)}{10^{-2}}\right)^2$$
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### Large density perturbation as a source of GWs

Current and future observations of GWs



### Large density perturbation as a source of GWs

• GW has a corresponding peak at the same k.



$$\Omega_{\rm GW}(k)h^2 \sim 10^{-9} \left(\frac{\mathscr{P}_{\zeta}(k)}{10^{-2}}\right)^2$$

# Enhanced Curvature Perturb.

### Constraints on the **Power spectrum** ( $P_{\zeta}$ )



## Assume Gaussian Pz

### Constraints on $P_{\zeta} \leftrightarrow$ abundance of PBHs



# Enhanced non-Gaussianity

### We can **enhance** the tail while the width is fixed.



Can enhance the PBH formation for a fixed # of over-densities.



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# Inflation Models



### Flatten your potential

Single-field inflation for the total e-folds of N = 50-60





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- Single-field inflation for the total e-folds of N = 50-60
  - Single-field **slow-roll** inflation for PBH DM (M > 10<sup>15</sup> g) is **ruled out!**

Motohashi+1706.06784

### **Slow-roll must be violated** for PBHs with $M > 10^{15}$ g.

Quantum diffusion, non-Ganssianity?



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Pattison+ 1707.00537, Biagetti+ 1804.07124

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## Multi-fields dynamics

• Gauge preheating, Curvaton,...

 $\phi F ilde{F}$ Domcke+1704.03464

Higgs: Espinosa+ 1710.11196, Axion-like: **KM**+ 1711.08956 Enhanced **non-Gaussianity** can be obtained.

### PBHs for LIGO or DM from **Double Inflation**



- PBHs for LIGO  $\rightarrow$  **SKA** and future CMB observation.
- PBHs for  $DM \rightarrow eLISA$  and LISA.

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## Axion-like Curvaton

### PBHs for LIGO from Axion-like Curvaton

Blue-tilted spectrum of "axion" from "saxion" dynamics



### Curvature perturb.

$$\zeta = -H \frac{\delta \rho}{\dot{\rho}} \simeq \frac{\delta \rho_a}{4\rho_r + 3\rho_a} = \frac{r}{4 + 3r} \frac{\delta \rho_a}{\rho_a} \quad \text{w/} \ r = \frac{\rho_a}{\rho_r}$$

**Blue-tilted** via  $\varphi: M_{\rm Pl} \to f_a$  during inflation  $\rho_a \propto a^2$ ,  $\delta \rho_a \propto 2a\delta a$  w/  $a = \varphi \theta$ ,  $\delta a = \frac{H_{\rm inf}}{2\pi}$ 

$$\mathscr{P}_{\zeta} = \left(\frac{2r}{4+3r}\right)^2 \left(\frac{H_{\text{inf}}}{2\pi\varphi\theta}\right)^2 \propto r^2\varphi^{-2}$$

Non-Gaussianity

$$f_{\rm NL} = \frac{5}{12} \left( -3 + \frac{4}{r} + \frac{8}{4+3r} \right) \sim \frac{5}{3r}$$

### Tension...

PBH formation  $\rightarrow$  Large r **v.s.** 

Suppress GWs/ $\mu$ -dist.  $\rightarrow$  non-Gaussianity  $\rightarrow$  Small r



## urvaton

### ce Curvaton

▶ non-Gaussianity cannot be so large... KM+ 1711.08956

 $f_{\rm NL} \propto 1/r$  v.s.  $\mathscr{P}_{\zeta} \propto r^2$ 



## Axion-like Curvaton

### PBHs for LIGO from Axion-like Curvaton

• non-Gaussianity cannot be so large... км+ 1711.08956







# Summary

## Inflation for PBHs needs LARGE $\delta \rho / \rho \gg 10^{-5}$ .

### Many over-densities are generated per one PBH:

- CMB spectral distortion @ 10<sup>4</sup>-1 Mpc<sup>-1</sup>; BBN @ 10<sup>5</sup>-10<sup>4</sup> Mpc<sup>-1</sup>
- Induced GWs: PTA @ ~10<sup>6</sup> Mpc<sup>-1</sup>; eLISA @ 10<sup>11</sup>-10<sup>13</sup> Mpc<sup>-1</sup>
- UCMHs...depends on models and DM profile.





### Almost Gaussian

- → PBHs for DM @ ~10<sup>20</sup> g → eLISA/LISA; @ O(10) solarmass → marginal... PTA and spectral distortion.
- → PBHs for LIGO → same as PBHs for DM @ O(10) solarmass

**KM**+1611.06130, 1701.02544

### Large non-Gaussianity → currently safe, model building?

e.g., Domcke+1704.03464, Espinosa+ 1710.11196, **KM**+ 1711.08956



### PBHs for LIGO or DM from **Double Inflation**

$$V(\varphi,\chi) = V_{\text{pre}}(\chi) -2\sqrt{2}c v^{2}\varphi - \frac{\kappa}{2}v^{4}\varphi^{2} + \left(v^{2} - \frac{g}{2^{\frac{3}{2}}}\varphi^{3}\right)^{2} \qquad \mathcal{L}_{\text{kin}} = -\frac{1}{2}\partial_{\mu}\chi\partial^{\mu}\chi - \frac{1}{2}\partial_{\mu}\varphi\partial^{\mu}\varphi + \frac{c_{\text{kin}}}{4}\left(\partial_{\mu}\chi\partial^{\mu}\chi\right)\varphi^{2} + \cdots + \frac{1}{2}c_{\text{pot}}V_{\text{pre}}(\chi)\varphi^{2}$$



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![](_page_61_Figure_3.jpeg)

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![](_page_62_Figure_3.jpeg)

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Total e-folds (N=50-60) = Ist-inflation + 2nd inflation

![](_page_63_Figure_3.jpeg)

## PBHs for LIGO or DM from **Double Inflation**

### Total e-folds (N=50-60) = Ist-inflation + 2nd inflation

- SUGRA: discrete R symmetry breaking model.

[Kawasaki+1606.07631, Inomata+1611.06130]

$$W = mX\Psi \qquad K = \frac{1}{2} \left( \Psi + \Psi^{\dagger} \right)^{2} + |X|^{2} + |\Phi|^{2} + \frac{\kappa}{4} |\Phi|^{4} \qquad \frac{\|\Psi\|X\|\Phi\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{2}\|v^{$$

### $\Psi$ : Ist-inflaton; X: stabilizer; $\Phi$ : R-breaking field, 2nd-inflaton

![](_page_64_Figure_7.jpeg)

![](_page_64_Figure_8.jpeg)