

False Vacuum Inflation

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Outline

False Vacuum
Inflation

Model

Einstein frame

Post-Inflation

Gravitational Waves

- 1 Model
 - Einstein frame

- 2 Post-Inflation
 - Gravitational Waves

Slow-roll

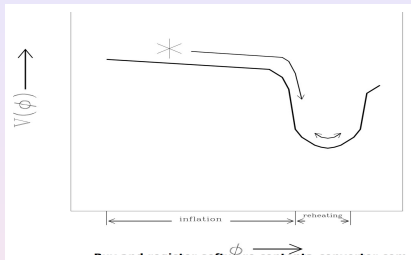
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- Slow-Roll on a very flat potential for $N > 60$ e-folds

Slow-roll

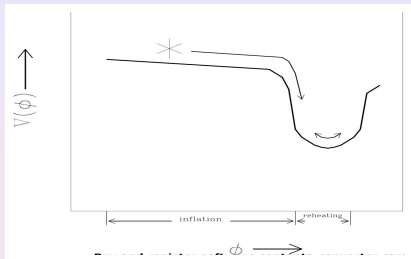
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- Slow-Roll on a very flat potential for $N > 60$ e-folds
- Then fast roll and decay to other particles ("Reheating")

Slow-roll

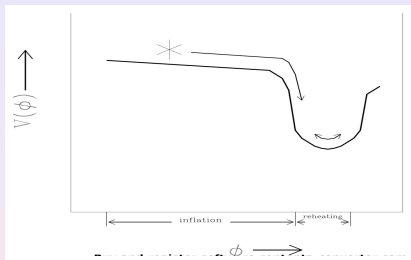
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- Slow-Roll on a very flat potential for $N > 60$ e-folds
- Then fast roll and decay to other particles ("Reheating")
- It also fluctuates \Rightarrow Density fluctuations

Slow-roll

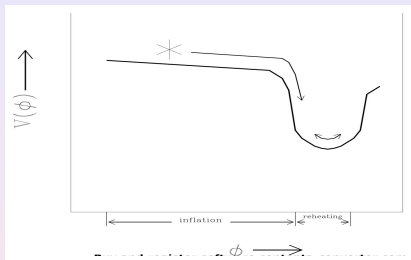
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- Then fast roll and decay to other particles ("Reheating")
- It also fluctuates \Rightarrow Density fluctuations

Inflation in a False Minimum

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- Can we end Inflation with a tunneling from a False vacuum?

Inflation in a False Minimum

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- Can we end Inflation with a tunneling from a False vacuum?
- ...It was the first way to introduce Inflation ("Old" Inflation, Guth, 1982).

Inflation in a False Minimum

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- Can we end Inflation with a tunneling from a False vacuum?
- ...It was the first way to introduce Inflation ("Old" Inflation, Guth, 1982).
- Old Inflation does **not** have **Graceful Exit**:
non-successful Bubble Nucleation

Nucleation of Bubbles

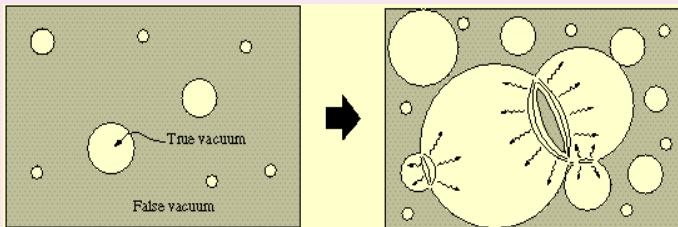
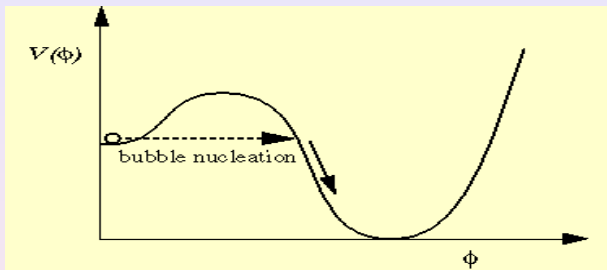
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What is the problem of False Vacuum Inflation?

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Requirements:

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Requirements:

- For sufficient inflation $\Gamma \ll H^4$

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Requirements:

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- For transition to radiation $\Gamma \simeq H^4$

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Requirements:

- For sufficient inflation $\Gamma \ll H^4$
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- Either Inflation too short or never ends.

What is the problem of False Vacuum Inflation?

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Gravitational Waves

Requirements:

- For sufficient inflation $\Gamma \ll H^4$
- For transition to radiation $\Gamma \simeq H^4$
- Either Inflation too short or never ends.

Way-out:

- **Start** with $\Gamma \ll H^4$
- And **then** $\Gamma \simeq H^4$

Get a Graceful exit

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If a field χ is trapped in a false vacuum:

Get a Graceful exit

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If a field χ is trapped in a false vacuum:

- Need additional degree of freedom ϕ to set **time dependence** in Γ/H^4

Get a Graceful exit

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If a field χ is trapped in a false vacuum:

- Need additional degree of freedom ϕ to set **time dependence** in Γ/H^4
- Extra Scalar ϕ

Get a Graceful exit

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If a field χ is trapped in a false vacuum:

- Need additional degree of freedom ϕ to set **time dependence** in Γ/H^4
- Extra Scalar ϕ
- Two possibilities
 - Make **H variable** (couple ϕ to gravity)
 - Make **Γ variable** (couple ϕ to χ)

Variable H

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- If H decreases with time
- At some point $\Gamma^{1/4} = H$

Modifying \mathcal{L}_G ...

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Adding a Non-Minimally coupled scalar field (ϕ)

\Rightarrow Graceful exit

Modifying $\mathcal{L}_G...$

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Adding a Non-Minimally coupled scalar field (ϕ)

\Rightarrow Graceful exit

False Vacuum Inflation, or First order Inflation, similar to "Extended Inflation"

F.Di Marco & A.N., Phys.Rev.D '05,

T. Biswas & A.N. Phys.Rev.D '06

In the context of Higgs potential:

I. Masina & A.N., Phys.Rev. D85 (2012); I. Masina & A.N., Phys.Rev.Lett.
108 (2012); A.N., Phys.Rev. D91 (2015)

An initial Lagrangian

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Gravitational Waves

- As a starting point we take the action

$$S_1 = \int d^4x \sqrt{-g} \left[-\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - f(\phi) R + V(\chi_0) \right]$$

An initial Lagrangian

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Gravitational Waves

- As a starting point we take the action

$$S_1 = \int d^4x \sqrt{-g} \left[-\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - f(\phi) R + \Lambda^4 \right]$$

An initial Lagrangian

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Gravitational Waves

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$$S_1 = \int d^4x \sqrt{-g} \left[-\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - f(\phi) R + \Lambda^4 \right] \\ + [U(\phi)] ,$$

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$$S_1 = \int d^4x \sqrt{-g} \left[-\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - f(\phi) R + \Lambda^4 \right] \\ + [U(\phi)] ,$$

- We assume

$$f(\phi) = M^2 + \beta \phi^2 + \gamma_4 \frac{\phi^4}{M^2} + \gamma_6 \frac{\phi^6}{M^4} + \dots$$

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- **Two cases:** β dominant or γ_n dominant

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- **Two cases:** β dominant or γ_n dominant
- Assume $U(\phi)$ to be negligible ($U \lesssim \Lambda^4$) during Inflation
(..but important later)

Early Time Evolution

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Gravitational Waves

$$S_1 = \int d^4x \sqrt{-g} \left[\frac{1}{2} M^2 R + \frac{1}{2} \gamma \frac{\phi^n}{M^{n-2}} R - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \Lambda^4 \right],$$

where $\gamma > 0$, $n > 2$.

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1. Start with small $\phi \Rightarrow$

Early Time Evolution

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1. Start with small $\phi \Rightarrow f(\phi) \simeq M^2 \Rightarrow$ **Exponential Inflation:**

$$H_I^2 = \frac{\Lambda^4}{3M^2},$$

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3. When $\gamma \frac{\phi^n}{M^{n-2}} \gtrsim M^2$ **power-law expansion** $\bar{a} \sim \bar{t}^{3/4}$.

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3. When $\gamma \frac{\phi^n}{M^{n-2}} \gtrsim M^2$ **power-law expansion** $\bar{a} \sim \bar{t}^{3/4}$.

4. $H \propto \frac{1}{\bar{t}}$ and when $H = \Gamma^{1/4} \Rightarrow$ **Graceful Exit.**

Evolution of H

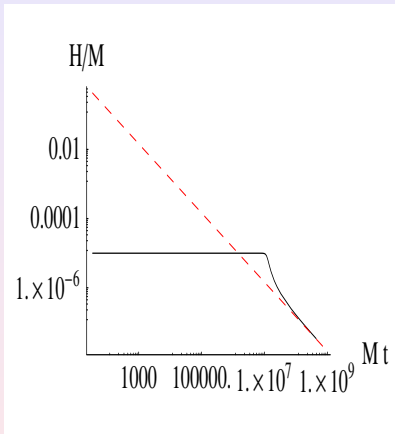
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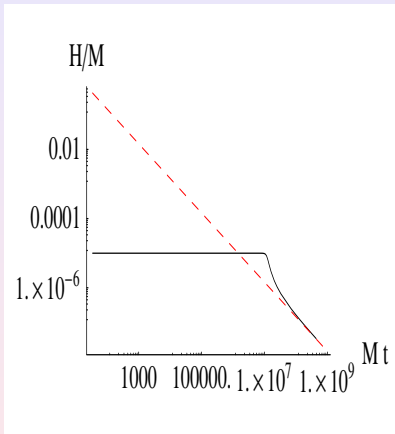
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Crucially

Evolution of H

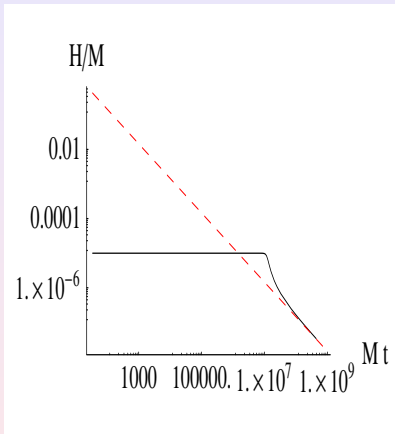
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Crucially

- *Phase I*: Perturbations that we see

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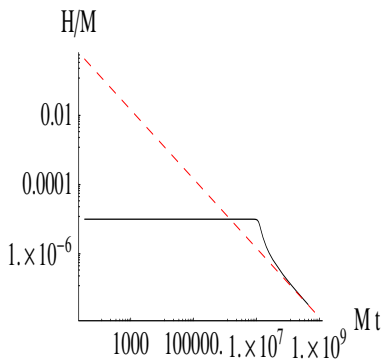
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Crucially

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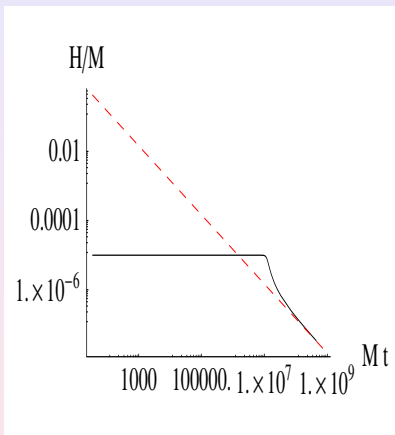
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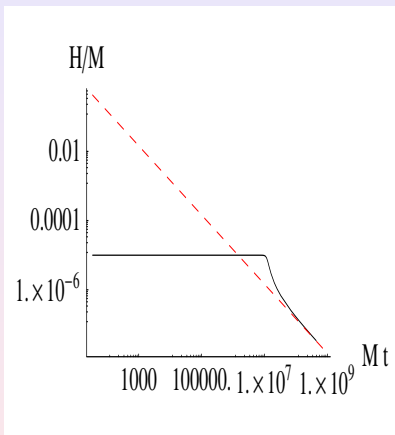
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Difference with "Extended Inflation" (Mathiazhagan, C. and Johri, V. B. , CQG, 1984, "La, D. and Steinhardt, P., PRL, 1989": only $\beta\phi^2$. Steinhardt, P. and Accetta F., PRL, 1990": used $M = 0$.

General Lagrangian

False Vacuum
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Gravitational Waves

- If we regard M as the fundamental scale of the theory
- The full theory has operators like

$$S = \int d^4x \sqrt{-g} \left[M^2 + \beta \phi^2 + \gamma_4 \frac{\phi^4}{M^2} + \gamma_6 \frac{\phi^6}{M^4} + \dots \right] R,$$

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- Higher order operators **important** at $\phi \gg M$

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- The transition is strong enough (**decelerated** expansion), for any $f(\phi) > \phi^2$!

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- Higher order operators **important** at $\phi \gg M$
- The transition is strong enough (**decelerated** expansion), for any $f(\phi) > \phi^2$!
- Without knowing exactly $f(\phi)$
(...an infinite number of couplings)!

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Going to the Einstein frame

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Gravitational Waves

- It is convenient to transform

$$\bar{g}_{\mu\nu} = f(\phi)g_{\mu\nu},$$

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Gravitational Waves

- It is convenient to transform

$$\bar{g}_{\mu\nu} = f(\phi)g_{\mu\nu},$$

- Get canonical gravity:

$$S_E = \frac{1}{2} \int d^4x \sqrt{-\bar{g}} [M^2 \bar{R} - K(\phi)(\bar{\partial}\phi)^2] + S_0,$$

Going to the Einstein frame

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Gravitational Waves

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and the false vacuum energy, in this frame

$$-S_0 = \int d^4x \sqrt{-\bar{g}} \frac{\Lambda^4}{f^2(\phi)} \equiv \int d^4x \sqrt{\bar{g}} \bar{V}(\chi_0, \phi).$$

becomes a potential (but it disappears after tunneling)

Phase I: $\phi \ll M$

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Gravitational Waves

- Expand

$$f(\phi) \approx 1 + \gamma_n \left(\frac{\phi}{M} \right)^n .$$

Phase I: $\phi \ll M$

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Gravitational Waves

- Expand

$$f(\phi) \approx 1 + \gamma_n \left(\frac{\phi}{M} \right)^n .$$

- Therefore:

$$K(\phi) \approx 1, \quad \bar{V} \approx \Lambda^4 \left[1 - 2\gamma_n \left(\frac{\phi}{M} \right)^n \right] .$$

Phase I: $\phi \ll M$

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$$f(\phi) \approx 1 + \gamma_n \left(\frac{\phi}{M} \right)^n .$$

- Therefore:

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- it looks like slow roll on top of a hill

Phase I: $\phi \ll M$

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- So in slow-roll approximation:

Phase I: $\phi \ll M$

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- So in slow-roll approximation:

$$\epsilon \equiv \frac{M^2}{2} \left| \frac{1}{V} \frac{dV}{d\phi} \right|^2 = 32\gamma_4^2 \left(\frac{\phi}{M} \right)^6,$$

$$\eta \equiv M^2 \frac{1}{V} \frac{d^2 V}{d\phi^2} = -24\gamma_4 \left(\frac{\phi}{M} \right)^2.$$

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$$\epsilon \equiv \frac{M^2}{2} \left| \frac{1}{V} \frac{dV}{d\phi} \right|^2 = 32\gamma_4^2 \left(\frac{\phi}{M} \right)^6,$$

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- When ϕ of order M : end of slow-roll

Phase II: $\phi \gg M$

- In this phase:

$$K(\phi) \equiv \frac{2f(\phi) + 3M^2 f'^2(\phi)}{2f^2(\phi)} \approx \frac{3M^2}{2} \left(\frac{f'}{f} \right)^2, \quad \bar{V}(\phi) \approx \frac{\Lambda^4}{f(\phi)^2}.$$

Using $f(\phi) > \phi^2$

Phase II: $\phi \gg M$

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Using $f(\phi) > \phi^2$

- So we introduce a **canonical variable** via

$$\Phi \equiv \sqrt{\frac{3}{2}} M \ln f(\phi),$$

Phase II: $\phi \gg M$

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- In this phase:

$$K(\phi) \equiv \frac{2f(\phi) + 3M^2 f'^2(\phi)}{2f^2(\phi)} \approx \frac{3M^2}{2} \left(\frac{f'}{f}\right)^2, \quad \bar{V}(\phi) \approx \frac{\Lambda^4}{f(\phi)^2}.$$

Using $f(\phi) > \phi^2$

- So we introduce a **canonical variable** via

$$\Phi \equiv \sqrt{\frac{3}{2}} M \ln f(\phi),$$

- The kinetic term is canonical and the potential becomes:

$$\bar{V}(\Phi) = \frac{\Lambda^4}{f(\phi)^2} = \Lambda^4 \exp\left(-2\sqrt{\frac{2}{3}} \frac{\Phi}{M}\right).$$

Phase II: $\phi \gg M$

- The exponential potential is well-known to lead to power-law expansion

$$\bar{a} \sim \bar{t}^p \text{ with } p = \frac{3}{4}.$$

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$$\bar{a} \sim \bar{t}^p \text{ with } p = \frac{3}{4}.$$

- The end of this phase when

$$\bar{H}^2 \simeq \bar{\Gamma}^{1/2}$$

Model

Einstein frame

Post-Inflation

Gravitational Waves

Phase II: $\phi \gg M$

False Vacuum
Inflation

Model

Einstein frame

Post-Inflation

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Einstein frame

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 $M_{Pl}^2/M^2 = f(\phi) \gg 1.$

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- M_{Pl} at the end of Inflation can be huge if $\bar{\Gamma}_{vac}$ tiny:
 $M_{Pl}^2/M^2 = f(\phi) \gg 1.$
- Can address the **Hierarchy problem!** (if ϕ stabilized at ϕ_F)

Transition to radiation and Stabilization

False Vacuum
Inflation

Model

Einstein frame

Post-Inflation

Gravitational Waves

- When $H^4 \simeq \Gamma$ many bubbles of $\chi_{out} \neq \chi_0$ are **nucleated**
- They **collide** producing a nearly homogeneous field χ_{out} + radiation

Transition to radiation and Stabilization

False Vacuum
Inflation

Model

Einstein frame

Post-Inflation

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- χ rolls down, produces more radiation and relaxes to true minimum

Transition to radiation and Stabilization

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Einstein frame

Post-Inflation

Gravitational Waves

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- They **collide** producing a nearly homogeneous field χ_{out} + radiation
- χ rolls down, produces more radiation and relaxes to true minimum
- During radiation ϕ **slows down**:

$$R = 6(2H^2 + \dot{H}) \approx 0.$$

Stabilization of ϕ

- Nonetheless we need to stabilize ϕ at late times:

False Vacuum
Inflation

Model

Einstein frame

Post-Inflation

Gravitational Waves

Stabilization of ϕ

- Nonetheless we need to stabilize ϕ at late times:
 - variation of G_N during matter domination.

False Vacuum
Inflation

Model

Einstein frame

Post-Inflation

Gravitational Waves

Stabilization of ϕ

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False Vacuum
Inflation

Model

Einstein frame

Post-Inflation

Gravitational Waves

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False Vacuum
Inflation

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Einstein frame

Post-Inflation

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Inflation

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Inflation

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Post-Inflation

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- Screening mechanisms?

Flat spectrum of ϕ fluctuations

False Vacuum
Inflation

Model

Einstein frame

Post-Inflation

Gravitational Waves

- Fluctuations in ϕ that ends inflation.

Flat spectrum of ϕ fluctuations

False Vacuum
Inflation

Model

Einstein frame

Post-Inflation

Gravitational Waves

- Fluctuations in ϕ that ends inflation.
- In Einstein frame there is a potential \Rightarrow **almost flat spectrum**

$$\bar{V} \approx \Lambda^4 \left[1 - 2\gamma \left(\frac{\phi}{M} \right)^{n\gamma} \right].$$

$$\left\{ \begin{array}{l} n_S - 1 = 2\eta - 6\epsilon \\ \Delta_R^2 = \left(\frac{\bar{H}_I}{M} \right)^2 \frac{1}{8\pi^2\epsilon} \Big|_{\phi=\phi(\bar{N} \approx \bar{N}_{3000h^{-1} \text{Mpc}})} \end{array} \right.$$

Parameter values (quadratic term absent)

False Vacuum
Inflation

Model

Einstein frame

Post-Inflation

Gravitational Waves

- Predicted spectral index:

$$n_s \simeq 1 + 2\eta \simeq 1 - \frac{2}{\mathcal{N}_{3000h^{-1}\text{Mpc}}} \left(\frac{n-1}{n-2} \right) = 0.95 - \frac{0.04}{n-2}$$

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- For large n : $n_s \simeq 0.94 - 0.95$ (close to measured central value)

T.Biswas & A.N., PRD '05.

Outline

False Vacuum
Inflation

Model

Einstein frame

Post-Inflation

Gravitational Waves

- 1 Model
 - Einstein frame
- 2 Post-Inflation
 - Gravitational Waves

Gravity waves

False Vacuum
Inflation

Model

Einstein frame

Post-Inflation

Gravitational Waves

- Reheating proceeds through bubble collisions.

Gravity waves

False Vacuum
Inflation


Model

Einstein frame

Post-Inflation

Gravitational Waves

- Reheating proceeds through bubble collisions.
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¹ M. S. Turner and F. Wilczek, A. Kosowsky, R. Watkins, M. Kamionkowski (1990-1995) 

Gravity waves

False Vacuum
Inflation

Model

Einstein frame

Post-Inflation

Gravitational Waves

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Amplitude of Gravity waves

False Vacuum
Inflation

Model

Einstein frame

Post-Inflation

Gravitational Waves

- $$\Omega_{GW} \approx \Omega_{CMB} \left(\frac{H_*}{\beta} \right)^2 f^2$$

²A. Kosowsky, M. S. Turner , R. Watkins 1992, A. Kosowsky,
M. S. Turner 1993.

Amplitude of Gravity waves

False Vacuum
Inflation

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Einstein frame

Post-Inflation

Gravitational Waves

- $\Omega_{GW} \approx \Omega_{CMB} \left(\frac{H_*}{\beta} \right)^2 f^2 \simeq 10^{-6} \left(\frac{H_*}{\beta} \right)^2$

- f fraction of vacuum energy that goes into GW vs. total²

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- Amplitude at the peak is big enough to be **detectable**:

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$$\left\{ \begin{array}{ll} \text{Expected value at peak} & \Omega_{GW} h^2 \approx 10^{-7} \\ \text{LIGO bound} & \Omega_{GW} h^2 \lesssim \mathcal{O}(10^{-6}) \\ \text{aLIGO sensitivity} & \Omega_{GW} h^2 \approx 10^{-9} \\ \text{LISA sensitivity} & \Omega_{GW} h^2 \approx 10^{-11} - 10^{-12} \end{array} \right.$$

- * Assuming **all** energy converted into bubbles

²A. Kosowsky, M. S. Turner, R. Watkins 1992, A. Kosowsky, M. S. Turner 1993.

GW detectors

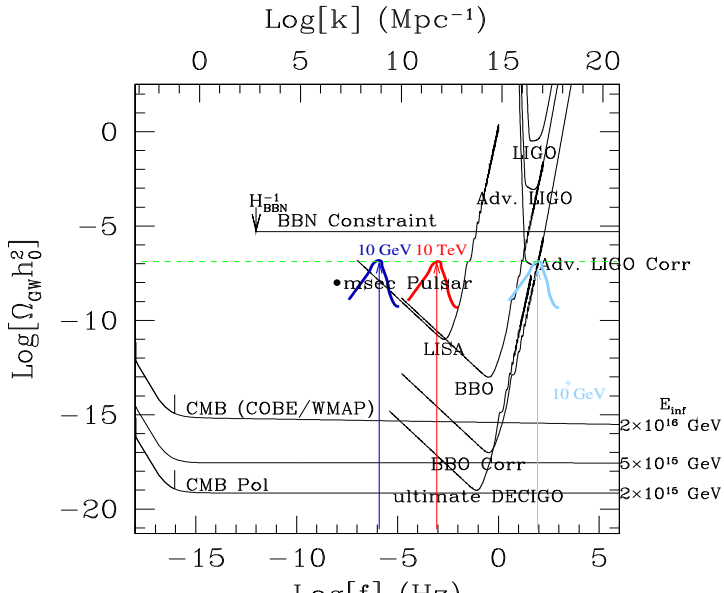
False Vacuum
Inflation

Model

Einstein frame

Post-Inflation

Gravitational Waves



Higgs False Vacuum Inflation

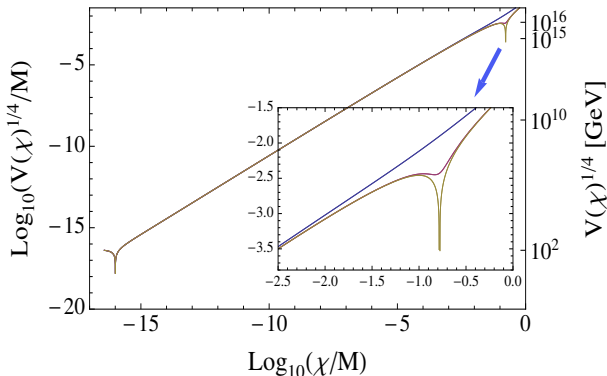
False Vacuum
Inflation

Model

Einstein frame

Post-Inflation

Gravitational Waves



- $m_H = 125.2, 125.158, 125.157663$ GeV,
 $m_t = 171.8$ GeV

I. Masina & A.N., Phys.Rev. D85 (2012); I. Masina & A.N., Phys.Rev.Lett.
108 (2012); A.N., Phys.Rev. D91 (2015)

Primordial Black Holes?

False Vacuum
Inflation

Model

Einstein frame

Post-Inflation

Gravitational Waves

- Bubbles produce $\mathcal{O}(1)$ density fluctuations on scales close to H^{-1}
- Convert into black holes³
- If Energy scale $> 10^8 \text{ GeV}$ such Black Holes evaporate

³S.Hsu, Phys.Lett.B, 1990