New 5D gamma conversion model

D. Bernard & I. Semeniouk
LLR, Ecole Polytechnique and CNRS/IN2P3, France

23rd Geant4 Collaboration Meeting
August 2018, Lund, Sweden

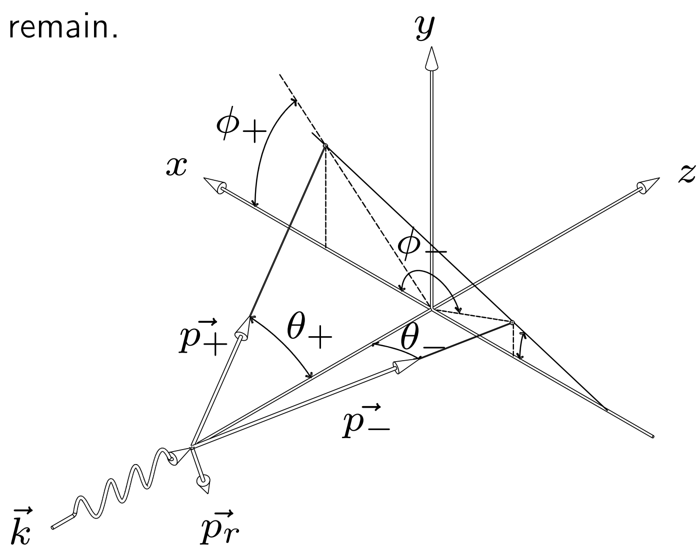
http://llr.in2p3.fr/∼dbernard/polar/harpo-t-p.html

γ conversions to e^+e^- pairs

- Target, final state — threshold
  - nucleus (nuclear conversion) $\gamma Z \rightarrow e^+e^-Z$ — $2mc^2$
  - electron (triplet conversion) $\gamma e^- \rightarrow e^+e^-e^-$ — $4mc^2$

  A three-particle final state, even when the trajectory of the recoiling target cannot be detected.

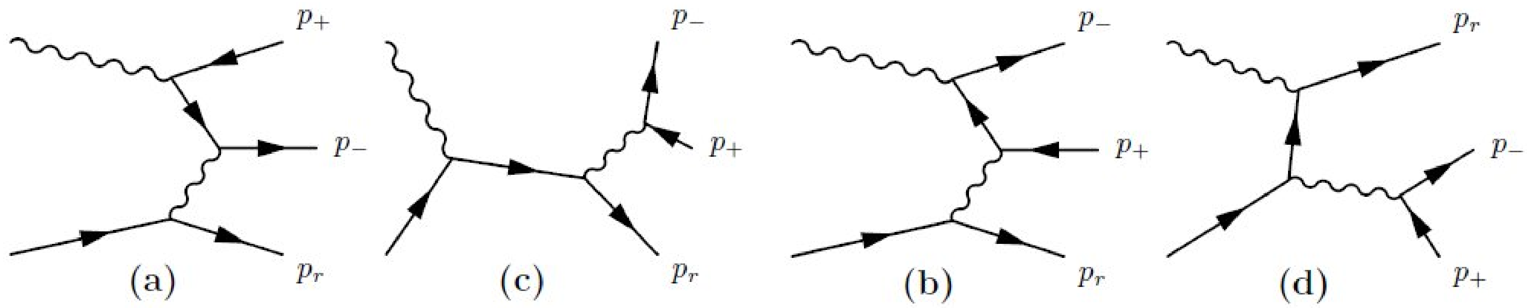
- A 5D phase space
  - 3 particle final state, $4 - 1 = 3$ free parameters for each one,
  - energy-momentum conservation fixes 4 of them.
  - $3 \times 3 - 4 = 5$ variables remain.



  - $+, -, r =$ positron, electron, recoil. ϕ azimuthal, θ polar angles.
  - $\Omega \equiv (\phi_+,\ \phi_-,\ \theta_+,\ \theta_-,\ x_+ \equiv E_+/E_\gamma)$

The Bethe-Heitler differential cross section

- Analytical 5D differential cross section



- First-order Born approximation: (a) and (b) diagrams. Excellent approximation:
  - for nuclear conversion ($M_{\text{nucleus}} \gg m$), m electron mass.
  - for high-energy triplet conversion ($E \gg mc^2$) Mork, Phys. Rev. 160 (1967) 1065.
- Non-polarised (Bethe and Heitler, Proc. R. Soc. Lond. A 146 (1934) 83),

  Polarised (Berlin and Madansky, Phys. Rev. 78 (1950) 623; May, Phys. Rev. 84 (1951) 265.)
  - Linear photon polarisation (not circular), no polarisation transferred to the final leptons
- Point-like target (extended-nucleus very-high-q^2 suppression neglected)
- Very-high-energy LPM (Landau-Pomeranchuk-Migdal) suppression neglected
- Coulomb (final state electrostatic interactions between leptons) corrections neglected
- At face value, assumes that the recoil energy is negligible $\Rightarrow$ $E_- = E - E_+$

Bethe-Heitler differential cross section: divergences

- Here the non-polarised expression

Bethe and Heitler, Proc. R. Soc. Lond. A 146 (1934) 83

$$d\sigma = \frac{-\alpha Z^2 r_0^2 m^2}{(2\pi)^2 \omega^3} dE_+ d\Omega_+ d\Omega_- \frac{|p_-||p_+|}{|\vec{q}|^4}$$

$$\left[\left(\frac{p_+ \sin\theta_+}{E_+ - p_+ \cos\theta_+} \right)^2 (4E_-^2 - q^2) + \left(\frac{p_- \sin\theta_-}{E_- - p_- \cos\theta_-} \right)^2 (4E_+^2 - q^2) + \right.$$

$$\left. \frac{2p_+p_- \sin\theta_+ \sin\theta_- \cos\phi}{(E_- - p_- \cos\theta_-)(E_+ - p_+ \cos\theta_+)} (4E_+E_- + q^2 - 2\omega^2) - 2\omega^2 \frac{(p_+ \sin\theta_+)^2 + (p_- \sin\theta_-)^2}{(E_+ - p_+ \cos\theta_+)(E_- - p_- \cos\theta_-)} \right]$$

with: $|\vec{q}|^2 = |\vec{p_+} + \vec{p_-} - \vec{k}|^2.$

- Divergences:
  - For e^+ and e^- $\frac{1}{(E - p\cos\theta)}$ forward divergence at high energies, the easy ones
  - $\frac{1}{q^4}$ small recoil divergence, involves several kinematic variables in a correlated way

Correlated divergences: Consequences for pre-existing physics models

- No attempt to sample the 5D differential cross-section
- No attempt to generate a target recoil momentum
  - The electron and the positron are generated in a plane that contains the photon
  - Azimuthal angles back to back, $\phi_- = \phi_+ \pm \pi$
- Single-particle polar angle θ distribution, decent.
- "Energy share", x_+ distribution, exact. $x_+ \equiv E_+/E_\gamma$
- Electron and positron polar angles θ_- and θ_+ generated independently
  - energy-momentum not conserved in the reaction !
  - some artificial transverse recoil apparently generated, but distribution wrong and direction in the "conversion plane"
- Polarised model wrong.

[polarisation **phase** bug corrected, though, in the 10.4beta release V. Ivantchenko & Ph. Gros, Sept. 2017.]

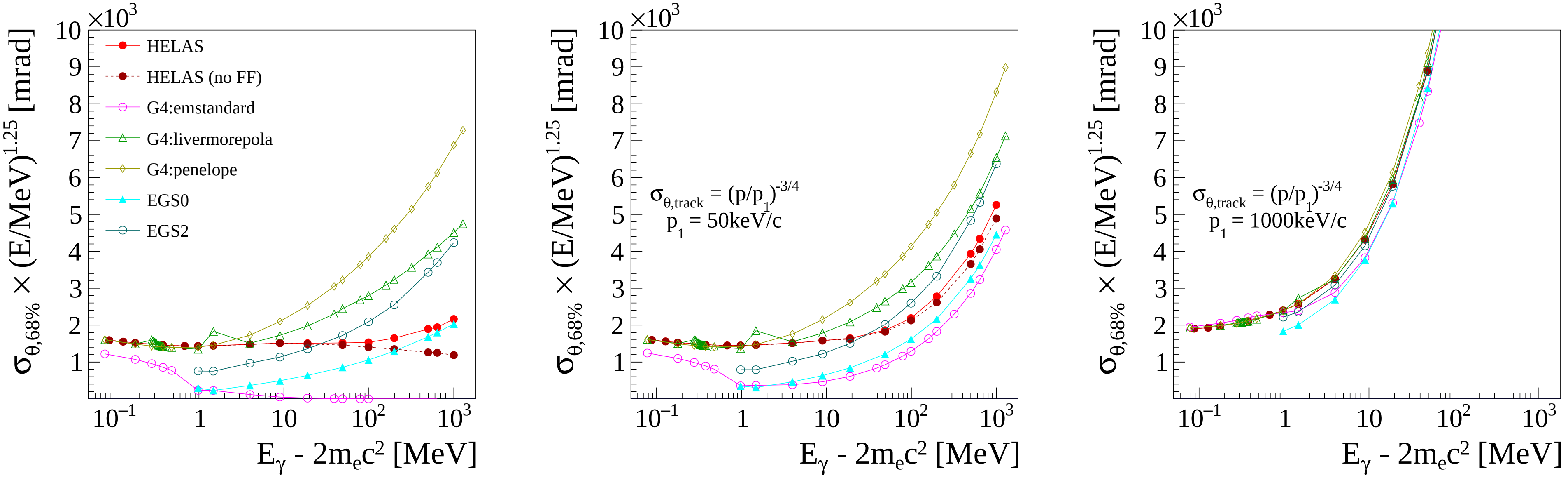
$(E < 1\,GeV)$ high-performance $\gamma \rightarrow e^+e^-$ telescopes: ongoing projects

Three active targets (i.e. combined converter + tracker) techniques being developed:

- Homogeneous low-density active targets (gas detectors): HARPO, Adept
- Tungsten-less, silicon-wafer stacks (e-ASTROGAM, AMEGO) and (so) many past other ones
- Homogeneous ultra-high-spatial-precision, high-density, active targets: emulsions (GRAINE)

Angular resolution due to unmeasured nucleus recoil: (normalized to $E^{-1.25}$)

no detector effect | high-performance telescope | Fermi-LAT-like telescope



Need an exact simulation of the conversion ! ("HELAS" is one of G4BetheHeitler5DModel's ancestors)

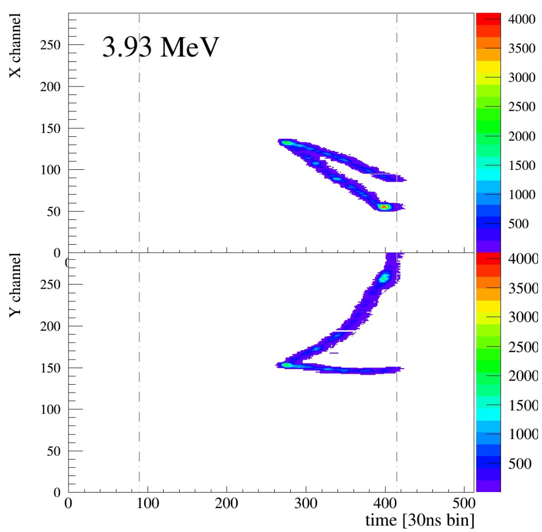
FF: atom electrons screening form-factor

P. Gros et al., Astropart. Phys. 88 (2017) 60

G4BetheHeitler5DModel development: Context: HARPO

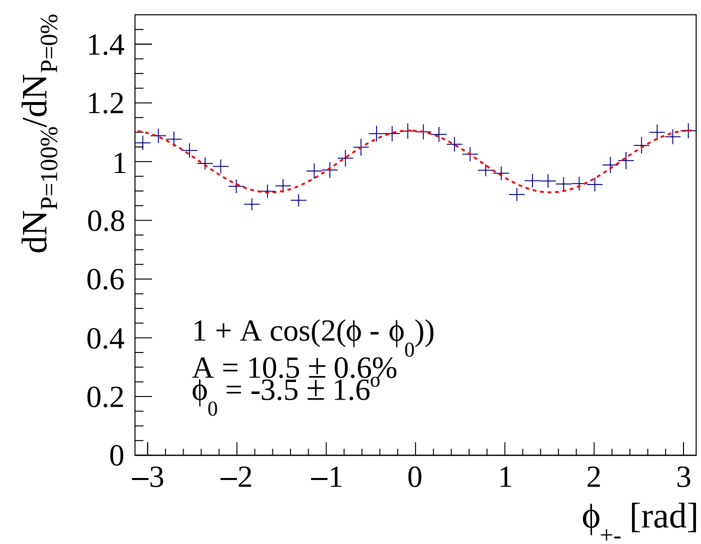
- Works on paper
  - Telescope performance — D. Bernard, Nucl. Instrum. Meth. A 701 (2013) 225
  - Polarimeter performance — D. Bernard, Nucl. Instrum. Meth. A 729 (2013) 765
- Experimental development
  - Gas (argon-isobutane) Time projection chamber (TPC) — SPIE 9144 (2014) 91441M
  - Validated in (polarised) gamma-ray beam, NewSUBARU, U. of Hyôgo, Japan

(x, t) and (y, t) signal maps
$\gamma \rightarrow e^+e^-$ in 2.1 bar Ar-iC4H10 95-5%



Ph. Gros *et al.*, SPIE (2016) 99052R

Azimuthal angle distribution
11.8 MeV γ in 2.1 bar Ar-iC4H10 95-5%



Ph. Gros *et al.*, Astropart. Phys. 97 (2018) 10

http://llr.in2p3.fr/∼dbernard/polar/harpo-t-p.html
http://llr.in2p3.fr/∼dbernard/polar/harpo_en/
arXiv:1805.10003 [astro-ph.IM]

G4BetheHeitler5DModel: Sampling Method: 1

- Mitigation of main, correlated, divergence: perform each step in appropriate Lorentz frame
  - Center-of-mass system (CMS) boost determined from photon energy E and target mass M.
  - Five variables are taken at random, (ℓ: lepton)

variable	name	Lorentz frame
θ	target and pair polar angle	CMS
μ	e^+e^- invariant mass	
θ_ℓ	electron and positron polar angle	pair frame
ϕ_ℓ	electron and positron azimuthal angle	pair frame
ϕ	target and pair azimuthal angle	CMS

  - In the CMS, target (mass M) and pair (mass μ) are back-to-back with opposite momenta.
  - "Decay" of the pair to an electron and a positron performed in the pair Lorentz frame.
  - The lepton 4-vectors are boosted "back" to the CMS.
  - The three final particle 4-vectors are boosted "back" to the laboratory Lorentz frame.
  - The Bethe-Heitler variables are obtained from the 4-vectors.
  - The probability density function (pdf) is computed.
- Final-state phase space normalization for this set of cascade decays:
  Review of Particle Physics (Particle Data Group) See eqs. (1)-(3) in Nucl. Instrum. Meth. A 899 (2018) 85
- Note that in contrast to Bethe-Heitler taken at face value, we **do conserve energy momentum**
  $E = E_- + E_+ + E_r$

G4BetheHeitler5DModel: Sampling Method: 2

Mitigation of residual, mostly uncorrelated, divergences:

- $[2012-2017]$: SPRING/BASES implementation of VEGAS method.

  (S. Kawabata, Comput. Phys. Commun. 88 (1995) 309.)

  Several seconds CPU overhead for each (photon-energy, target Z)

  Sytematic verification against known results (1D distributions, total cross sections ..)

  Nucl. Instrum. Meth. A 729 (2013) 765
  Astropart. Phys. 88 (2017) 60

- $[2018]$: change of variable — Nucl. Instrum. Meth. A 899 (2018) 85

  I. Semeniouk & D. Bernard, 14th Pisa Meeting on advanced detectors, 27 May - 02 June 2018, Isola d'Elba

- Check also previous Geant4 presentations:

Jan. 2018	CERN	Geant4 EM meeting	"stand-alone C++ version verified"
Sept. 2017	Wollongong	Geant4 User Workshop	"VEGAS-free, but still fortran"
April 2017	Guildford	Geant4 Space Users' Workshop	"de-VEGAS-ification in progress"

Implementation

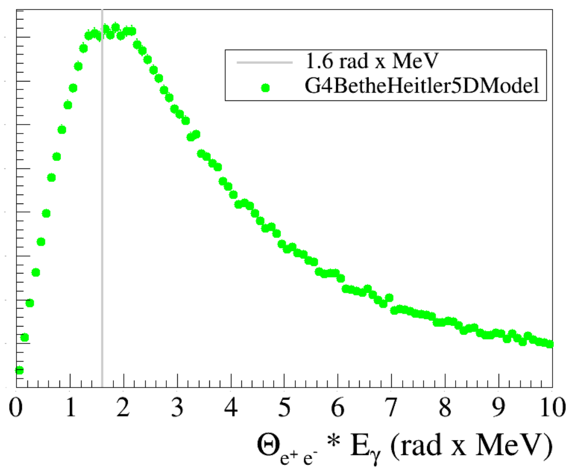
- G4EmLowEPPhysics physics list ($E < 80\,\mathrm{GeV}$)
- Gamma Conversion model G4BetheHeitler5DModel
- Inherit total cross section from G4BetheHeitlerModel
- Provide final state: SampleSecondaries
- Models flags provided via G4EmParameters flags and UI commands
  - Recoil particle (ion or electron) : Nuclear / triplet / (Z / 1) natural mixture
  - Isolated targets (QED checks), charged targets in atoms (detector simulation)
- Linearly polarised / non polarised photons (from norm of photon polarisation vector)
- SampleSecondaries CPU time (μs/event), argon, 1 GeV

G4BetheHeitler5DModel	G4BetheHeitlerModel
74.	0.9

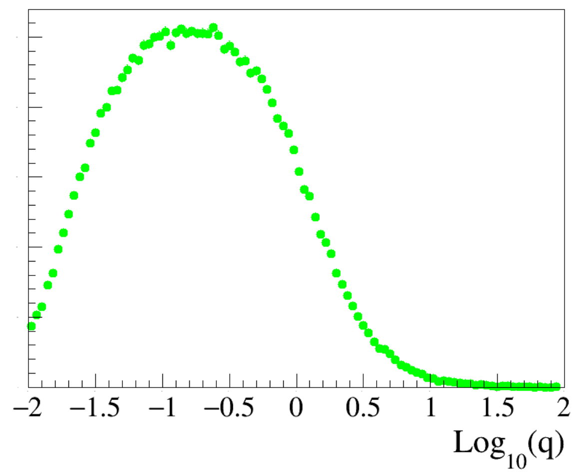
See also Sec. 5.4 and Fig. 8 of Nucl. Instrum. Meth., A 899 (2018) 85.

- No flag for primary interaction implemented yet.

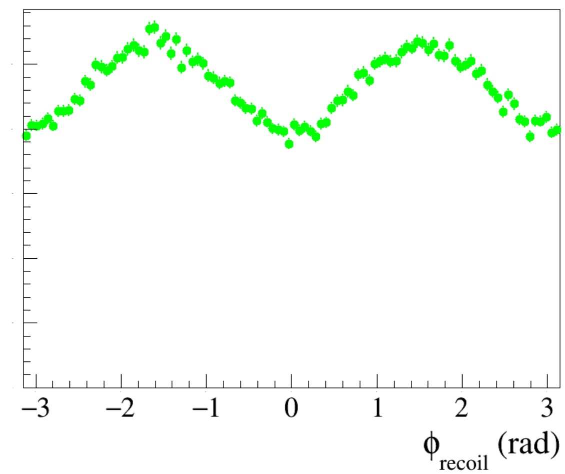
TestEm15



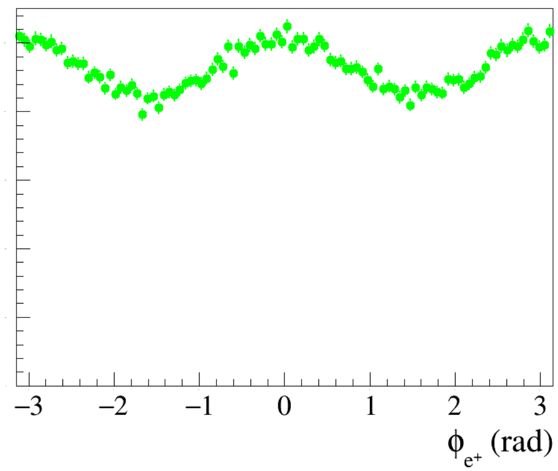
Olsen, Phys. Rev. 131 (1963) 406



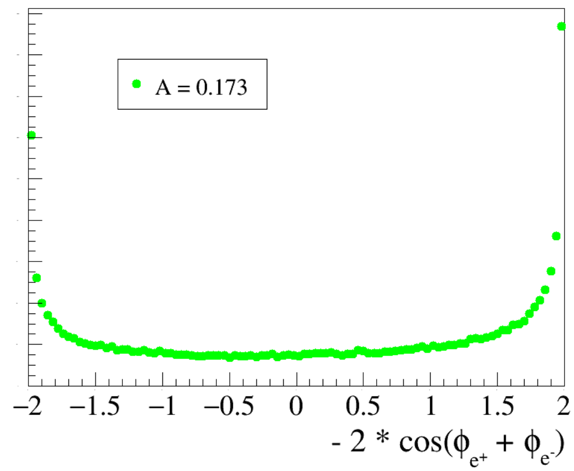
q (MeV/c) is recoil momentum



recoil preferentially $\perp$ pol

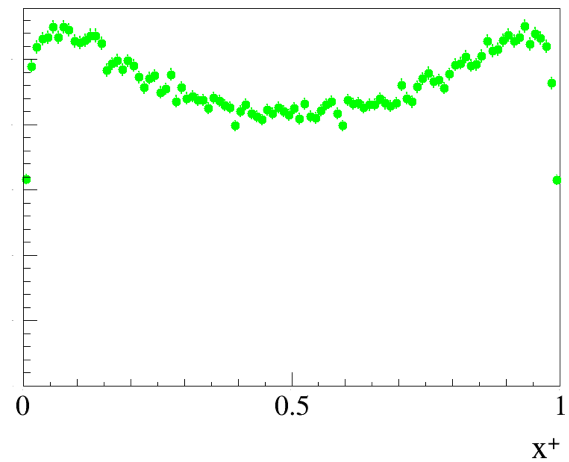


e^+ and e^- preferentially along pol



$A = \langle -2\cos 2\phi \rangle, \quad \phi = (\phi_+ + \phi_-)/2$

minus sign due to π e^+/e^- shift



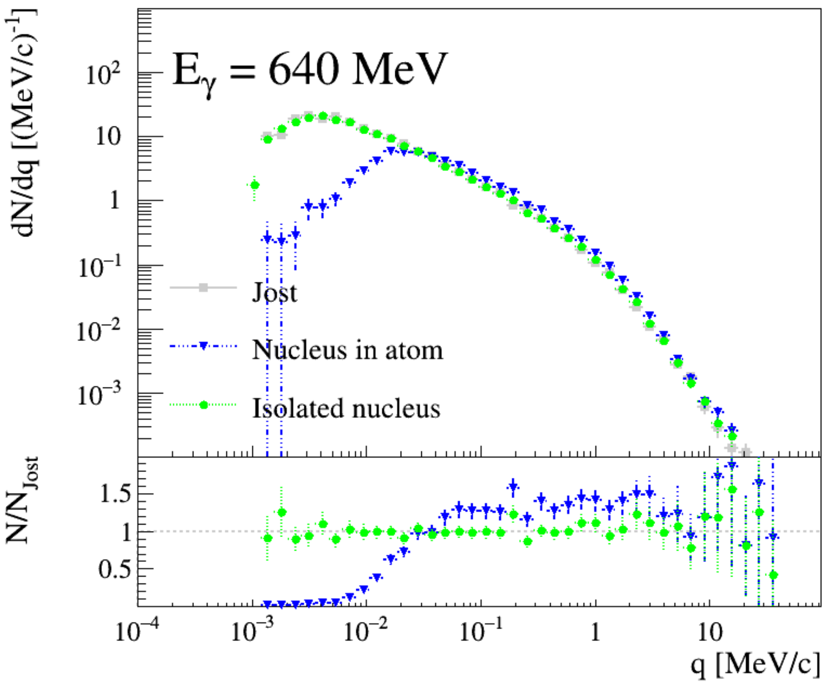
energy share $x_+ \equiv E_+/E_\gamma$

1 GeV, argon

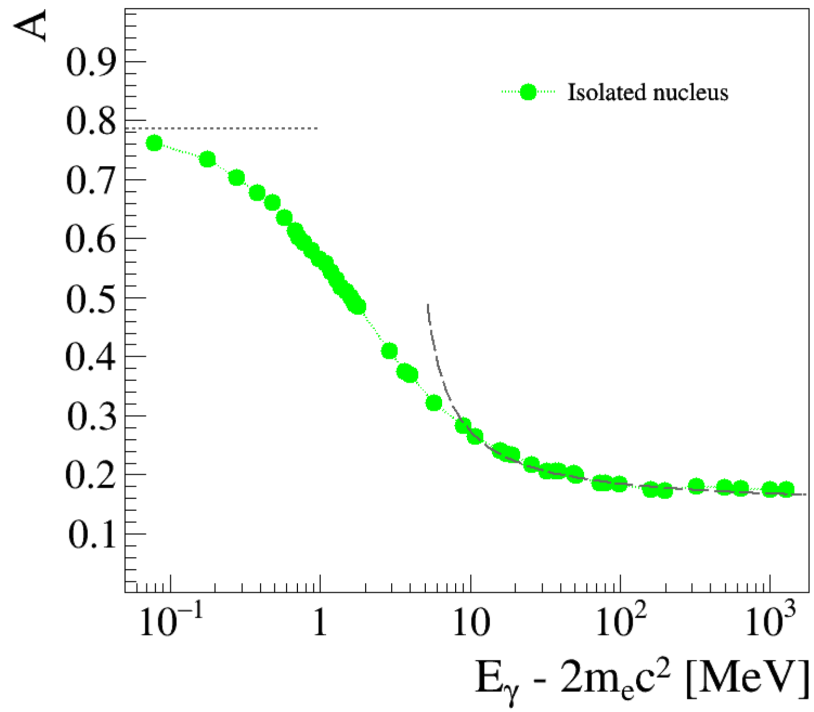
Astropart. Phys. 88 (2017) 30

TestEm15/README.gamma

G4BetheHeitler5DModel: Verifications again



Recoil momentum distribution compared to the analytical high-energy expression from
Jost, Phys. Rev. 80, 189 (1950).
The ratio plot is relative to Jost.



Polarisation asymmetry as a function of available energy, compared to published asymptotic expressions

I. Semeniouk & D. Bernard, 14th Pisa Meeting on advanced detectors, 27 May - 02 June 2018, Isola d'Elba

P. Gros et al., Astropart. Phys. 88 (2017) 60

Changes made since 10.5-BETA announcement

- TestEm15
  - Protection for case without gamma conversion
  - Use CLHEP::Hep3Vector::orthogonal() for coordinate axis definition
  - Use G4NistManager to allow use of predefined Geant4 materials
- G4BetheHeitler5DModel
  - Code clean up
  - Bug fix: Polarisation vector was used not normalized in coordinate transformation
  - Use inverted CDF for X generation (15 - 20% speed up)
  - Use CLHEP::Hep3Vector::orthogonal() for coordinate axis definition
  - Replace if with std::copysign in ϕ calculation

Committed to Geant4 trunk branch.

Documentation

Edition of Physics Reference Manual, Release 10.4 (April 2018)

- 6.5.4 Five-dimensional (5D) Bethe-Heitler gamma Conversion to e+e-
- 13.10 Pair production by Linearly Polarised Gamma Rays - Five-dimensional (5D) Bethe-Heitler Model

Exact reference numbers to be checked when 10.5 Physics Reference Manual is out.

Acknowledgements

And warm thanks to :

- Vladimir Ivantchenko
- Mihaly Novak
- Philippe Gros [The HARPO Collaboration]

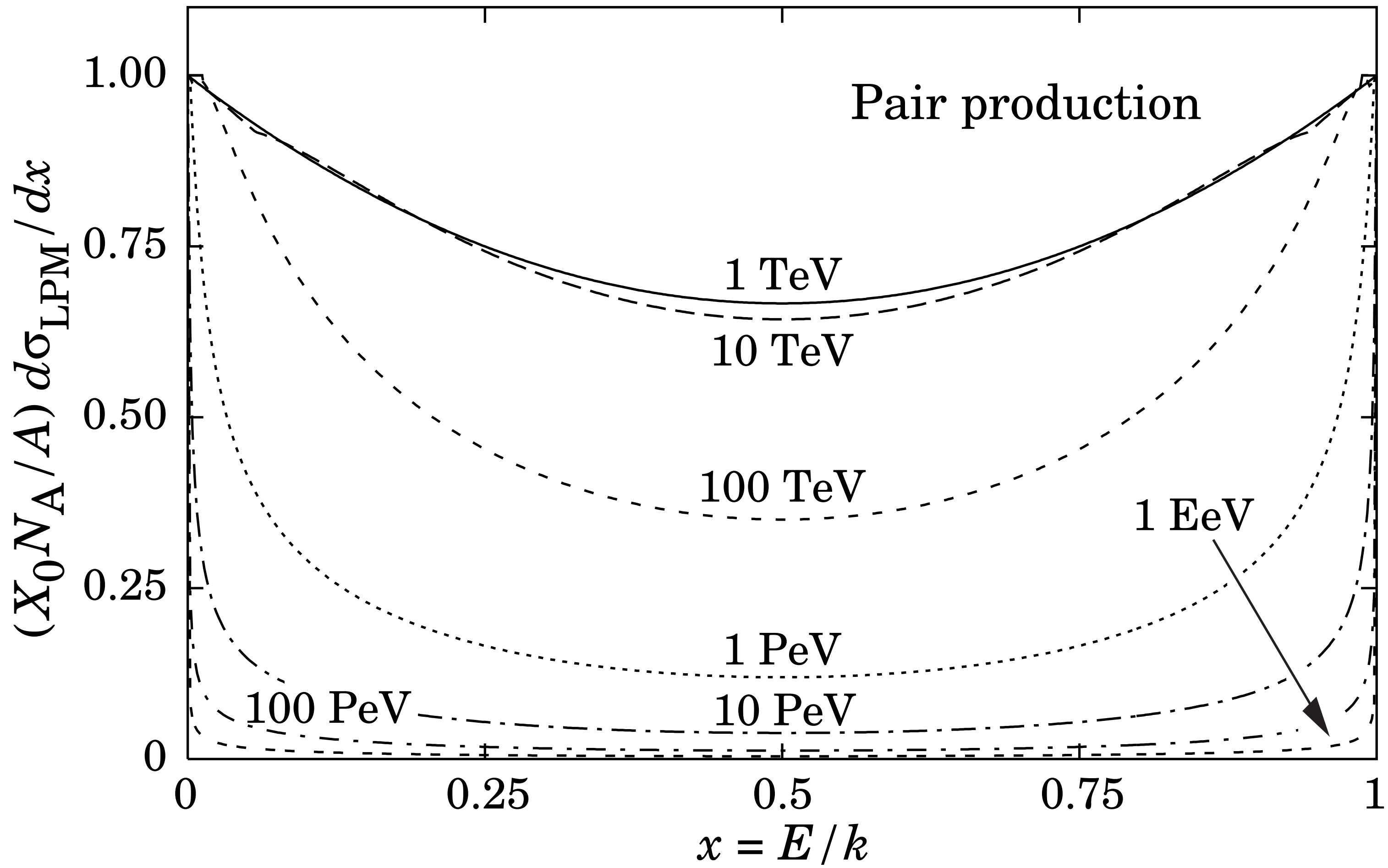
Back-up slides

Screening of target electric field by atomic electrons

small-q^2 screening suppression: $F(q^2)$ form factor

- coherent (nuclear conversion)
- incoherent (triplet conversion)

LPM



Landau-Pomeranchuk-Migdal Review of Particle Physics (Particle Data Group)