Geant4 simulation of positron annihilation into 2 and 3 gamma

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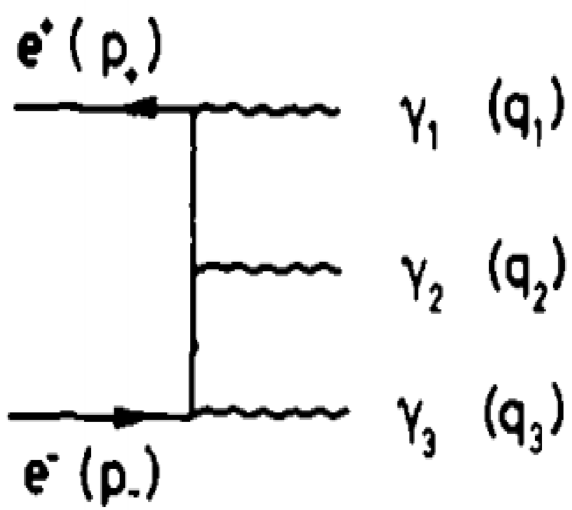
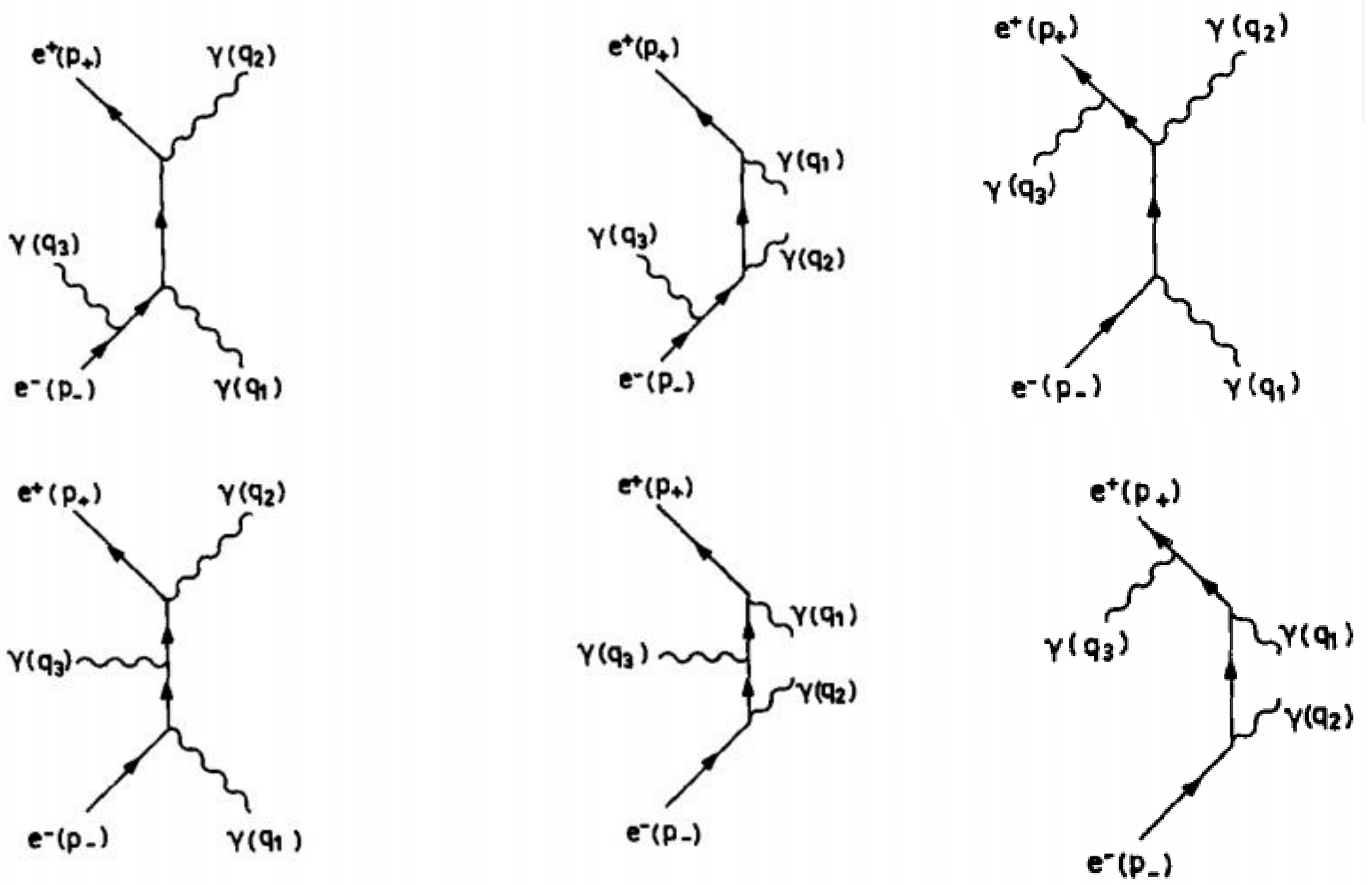
Outline

- Motivation
- Cross sections of 2γ and 3γ-annihilation
- Energy scales and role of the thresholds
- Simulation algorithm
- Summary



Main diagrams



Motivations

- 3γ-annihilation process at high energy affects high energy shower shape in EM and hadronic calorimeters
- 3γ-annihilation process at rest affects simulation for positron tomography
- This process may provide background for search of light dark matter particles

A new type of the Dark matter search experiments are considered for positron, electron or gamma beams. Presence of additional photons in the annihilation process has to be considered when the energy of an extra photon is higher than the beam energy spread or the energy resolution of the calorimeter.

V. Kozhuharov: «Background in the search for dark photon in e+e− annihilation»
EPJ Web of Conferences 142, 01018 (2017)

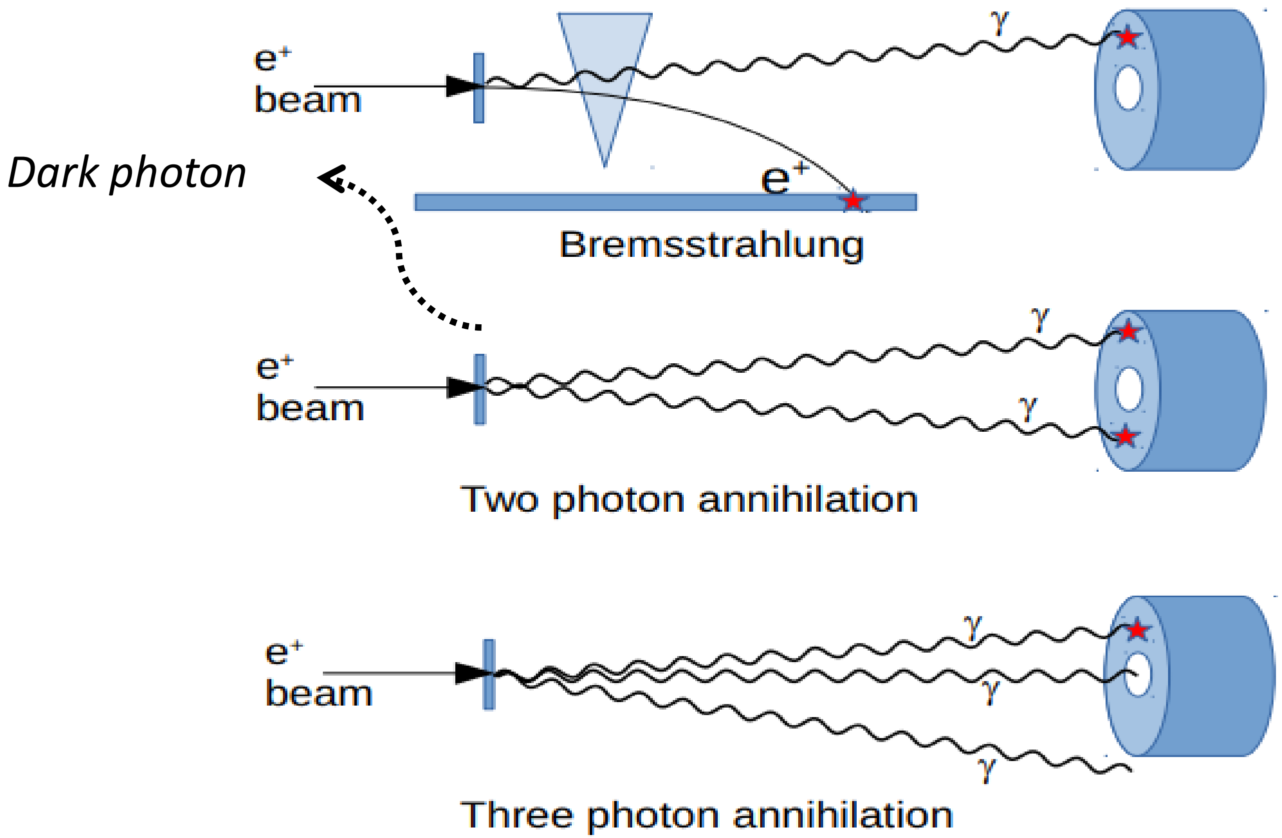


Figure 1. Dominant processes contributing to the search for invisible dark photon production in e^+e^- annihilation

Base formulas

$$e_+(p_+) + e_-(p_-) \to \gamma(q_1) + \gamma(q_2) + \gamma(q_3) . \tag{7}$$

In the Born approximation, the cross section of (7) summed and averaged over spin states is

$$d\sigma = \frac{(4\pi\alpha)^3}{s} \overline{s|M|^2} \frac{(2\pi)^4 \delta^{(4)}(p_+ + p_- - q_1 - q_2 - q_3)}{3!(2\pi)^9 2s} \frac{d^3q_1 d^3q_2 d^3q_3}{2w_1 2w_2 2w_3} , \tag{8}$$

$$\frac{s}{m^2} \overline{|M|^2} = \frac{s^2(k_1^2 + k_1'^2)}{m^4 k_2 k_3 k_2' k_3'} - \frac{2s}{m^2}\left[\frac{1}{k_1^2}\left(\frac{k_3'}{k_2'} + \frac{k_2'}{k_3'}\right) + \frac{1}{k_1'^2}\left(\frac{k_3}{k_2} + \frac{k_2}{k_3}\right)\right] + \text{two cyclic permutations} .$$

$$k_i = p_- q_i/m^2 , \qquad k_i' = p_+ q_i/m^2 .$$

The phase space of final particles can be written as

$$\frac{1}{s} d\Gamma = \frac{d^3q_1 d^3q_2 d^3q_3}{s(2w_1)(2w_2)(2w_3)} \frac{(2\pi)^4}{(2\pi)^9} \delta^{(4)}(p_+ + p_- - q_1 - q_2 - q_3)$$

S. Eidelman, E. Kuraev and V. Panin, «e+e - annihilation into two and three photons at high energy» // Nucl. Phys. B143 (1978) 353-364

e^+e^- annihilation into three photons

$$\nu_i = \frac{\omega_i}{\varepsilon}; \quad \nu_1 + \nu_2 + \nu_3 = 2$$

- ***The distribution in photon energies:***

$$\frac{\Delta E}{E} \le \nu_i \le 1$$

$$\frac{d^2\sigma}{d\nu_1 d\nu_2} = \frac{2\alpha^3}{3s}[F_{123} + F_{312} + F_{231}] \; ;$$

$$1 - \nu_i \gg m^2/s$$

where

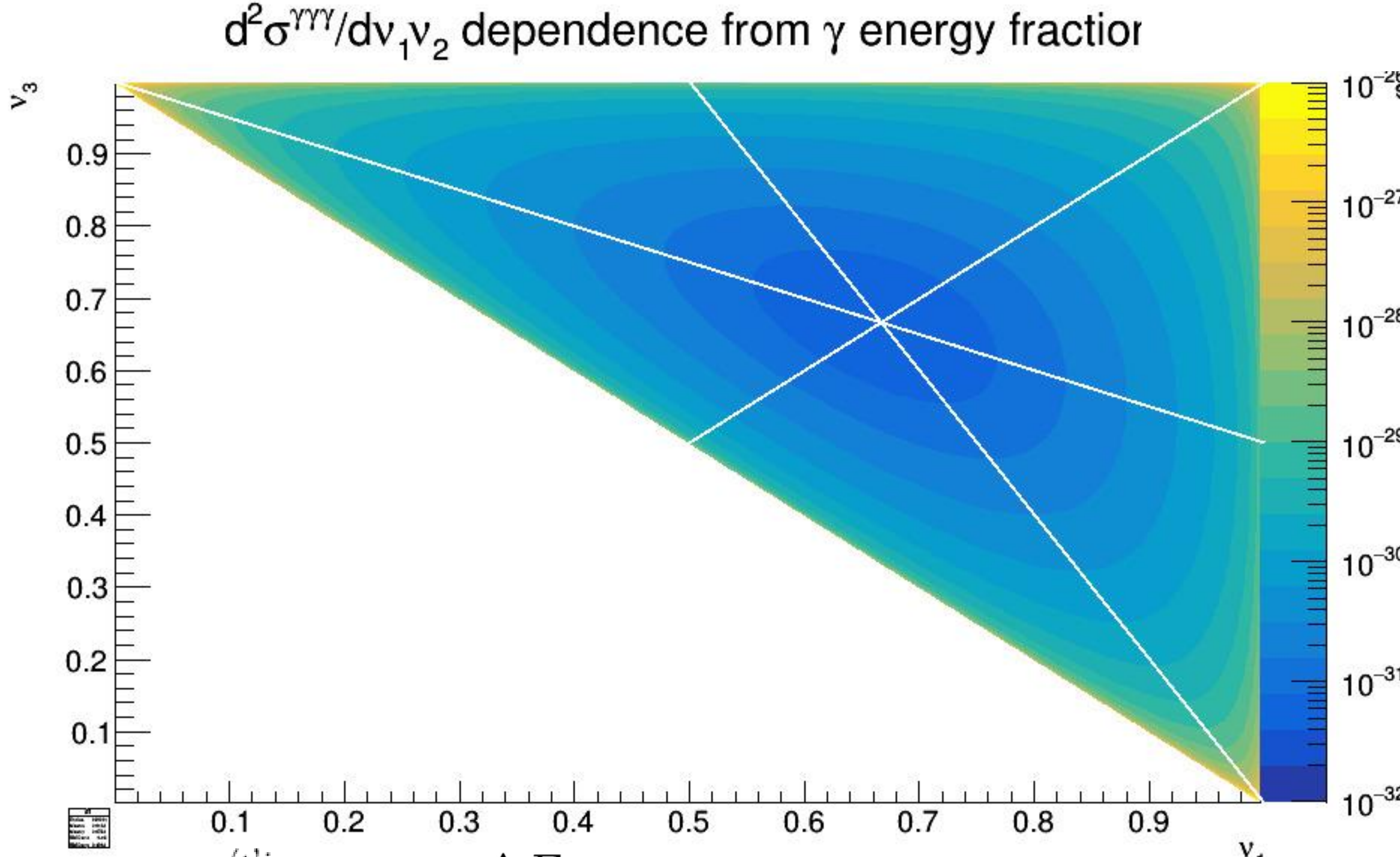
$$F_{123} = -(\frac{1}{\nu_1^2}+\frac{1}{\nu_2^2}) \ln \frac{s}{m^2} \quad + \quad \frac{\nu_3^2 + (\nu_2 - \nu_2)^2}{2\nu_1\nu_2(1-\nu_1)(1-\nu_2)}\times$$

$$\ln\left(\frac{s(1-\nu_1)(1-\nu_2)}{m^2\nu_1\nu_2}\right) \quad + \quad \frac{\nu_3^2 + (\nu_2 - \nu_2)^2}{2\nu_1\nu_2(1-\nu_3)}\times$$

$$\ln\left(\frac{s(1-\nu_3)}{m^2\nu_1\nu_2}\right) \quad - \quad \frac{(1-\nu_1)^2 + (1-\nu_2)^2}{\nu_3^2(1-\nu_1)(1-\nu_2)} \; ;$$

Integral becomes infinte on the borders
-> approximations

Same as *Baier, V. N.; Fadin, V. S.; Khoze, V. A.; Kuraev, E. A.: «Inelastic processes in high energy quantum electrodynamics» // Physics Reports 78 (1981), p. 293-393.*



$\nu_i = \dfrac{\omega_i}{\varepsilon};$ $\dfrac{\Delta E}{E} \leq \nu_i \leq 1$ $\nu_1 + \nu_2 + \nu_3 = 2$

Example of the differential cross section as a Dalitz-plot

Total cross sections in the lab system

- $$\sigma^{3\gamma} = \frac{r_e^2\alpha}{(\gamma+1)}\left(2(\rho-1)^2\ln\frac{E}{\Delta E} - (\rho-1)^2 + \xi(3) + 3\right) \;;$$

- $$\sigma^{2\gamma} = \frac{\pi r_e^2}{\gamma+1}(\rho-1) + \frac{r_e^2\alpha}{\gamma+1}\left(2(\rho-1)^2\ln\frac{\Delta E}{E} + \frac{1}{6}\rho^3 + \frac{3}{4}\rho^2 + \right.$$
$$\left. + \rho(\frac{1}{3}\pi^2 - 3) + 2\xi(3) - \frac{1}{3}\pi^2 + 1\right) \;;$$

The sum is finite because logarithms are mutually reduced

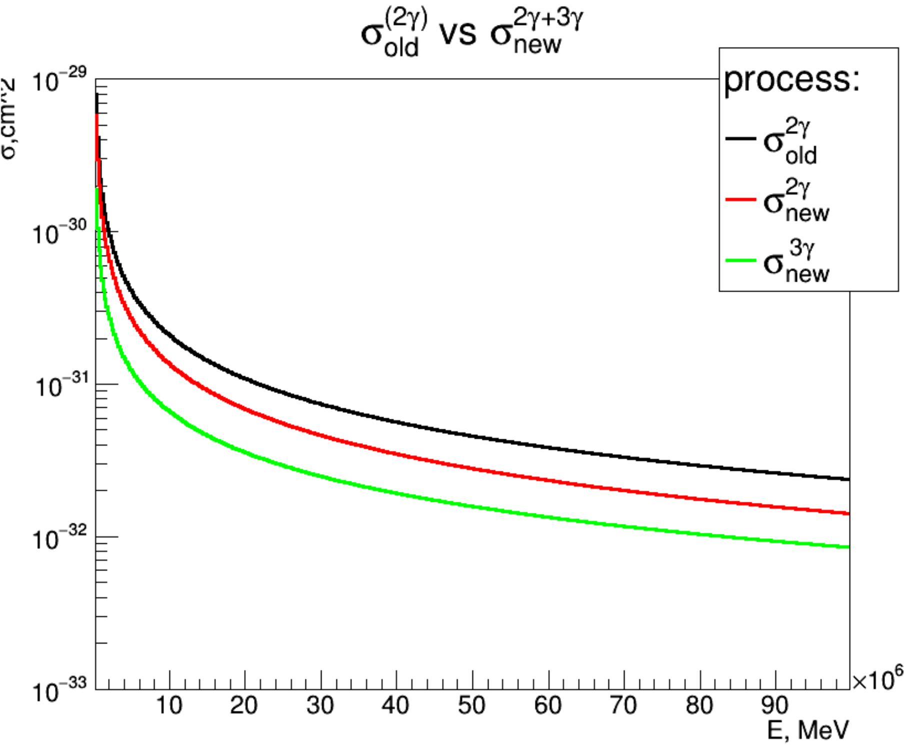
- $$\sigma^{2\gamma+3\gamma} = \frac{\pi r_e^2}{\gamma+1}(\rho-1) + \frac{r_e^2\alpha}{\gamma+1}\left(\frac{1}{6}\rho^3 - \frac{1}{4}\rho^2 + \rho(\frac{1}{3}\pi^2 - 3) + 3\xi(3) - \frac{1}{3}\pi^2 + 3\right);$$

- $$\rho \to \rho_{add} = \frac{\gamma^2+4\gamma+1}{\gamma^2-1}\ln\left(\gamma + \sqrt{\gamma^2-1}\right) - \frac{\gamma+3}{\sqrt{\gamma^2-1}} + 1;$$

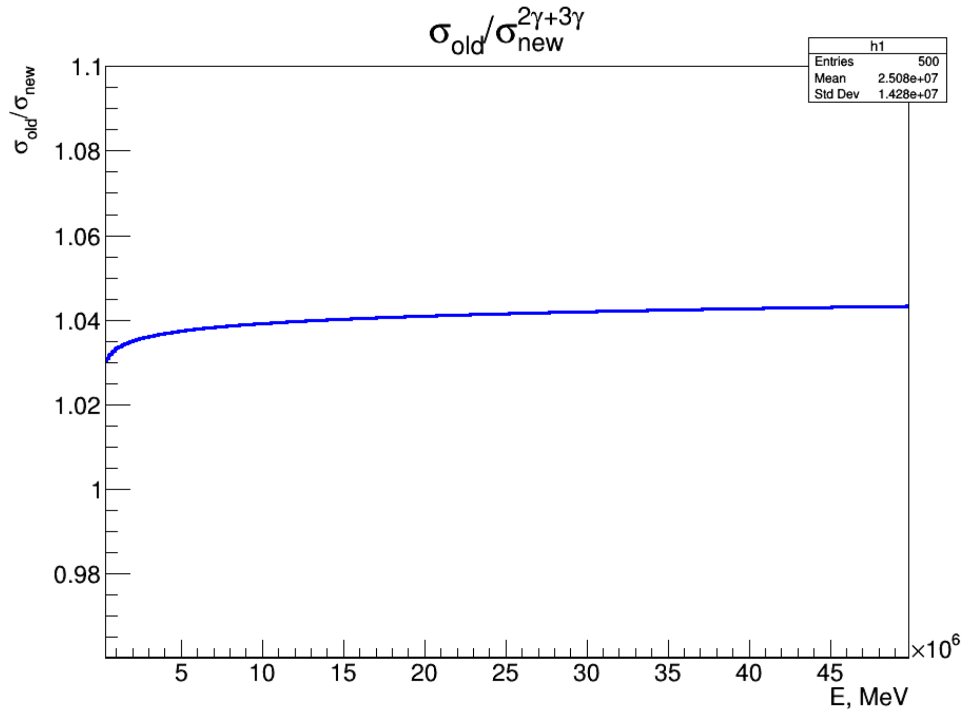
- $$\gamma = \frac{E^+}{mc^2}, \qquad \xi(3) = \sum_1^{\infty} n^{-3} = 1.202$$

Dirac P. A. M., Proc. Cambr. Phil. Soc, 26, 361 A930.

Total Cross sections



typical cross sections for e_+e_- annihilation to gamma quants



does not depend on dE/E

dE/E = 0.01

3 γ-annihilation channel changes total cross section for few %

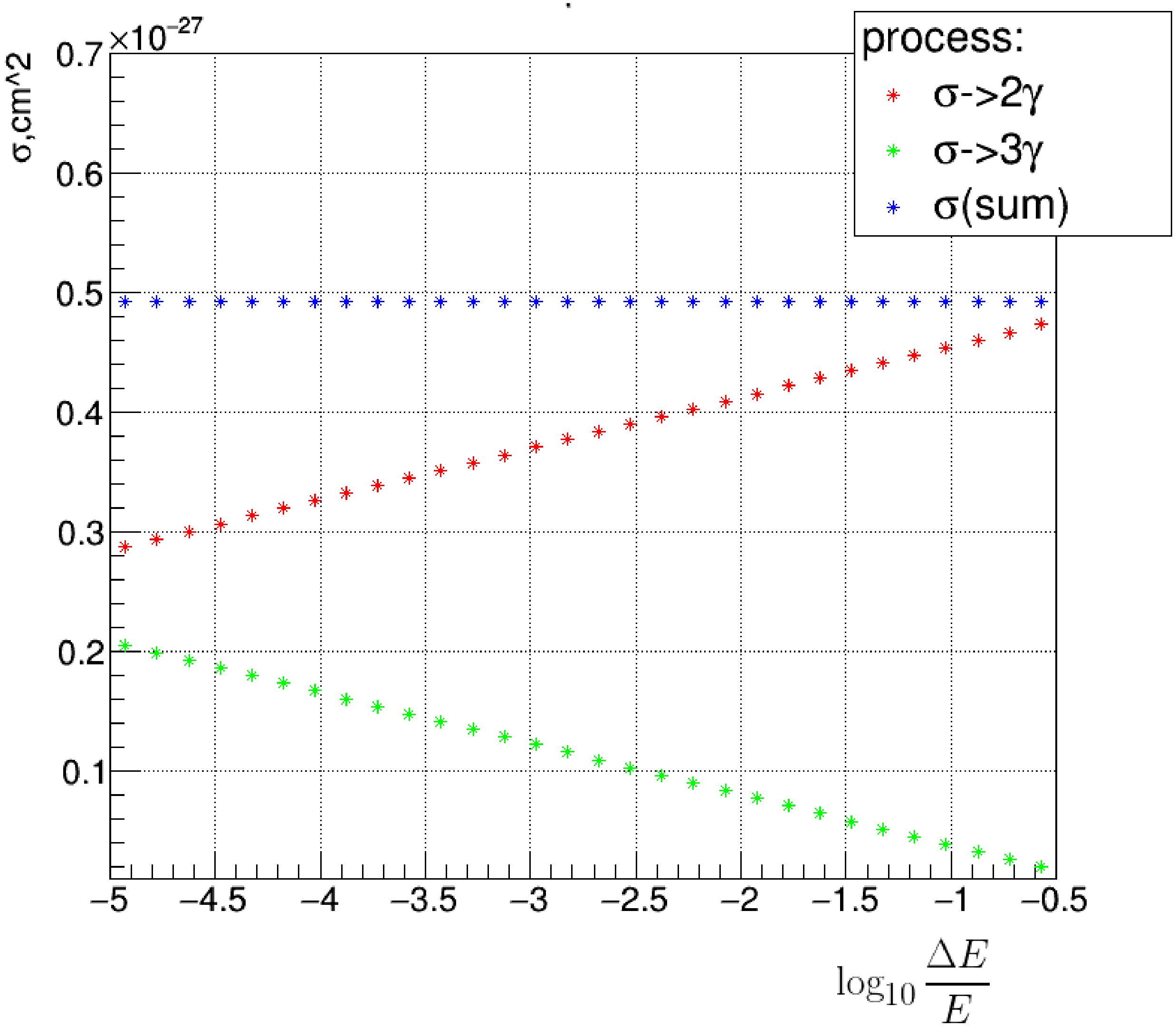
Role of ΔE/E factor

- $$\sigma^{2\gamma} = \frac{\pi r_e^2}{\gamma+1}(\rho-1) + \frac{r_e^2\alpha}{\gamma+1}\left(2(\rho-1)^2\ln\frac{\Delta E}{E} + \frac{1}{6}\rho^3 + \frac{3}{4}\rho^2 + \rho(\frac{1}{3}\pi^2 - 3) + 2\xi(3) - \frac{1}{3}\pi^2 + 1\right)$$
- $$\sigma^{3\gamma} = \frac{r_e^2\alpha}{(\gamma+1)}\left(2(\rho-1)^2\ln\frac{E}{\Delta E} - (\rho-1)^2 + \xi(3) + 3\right)$$

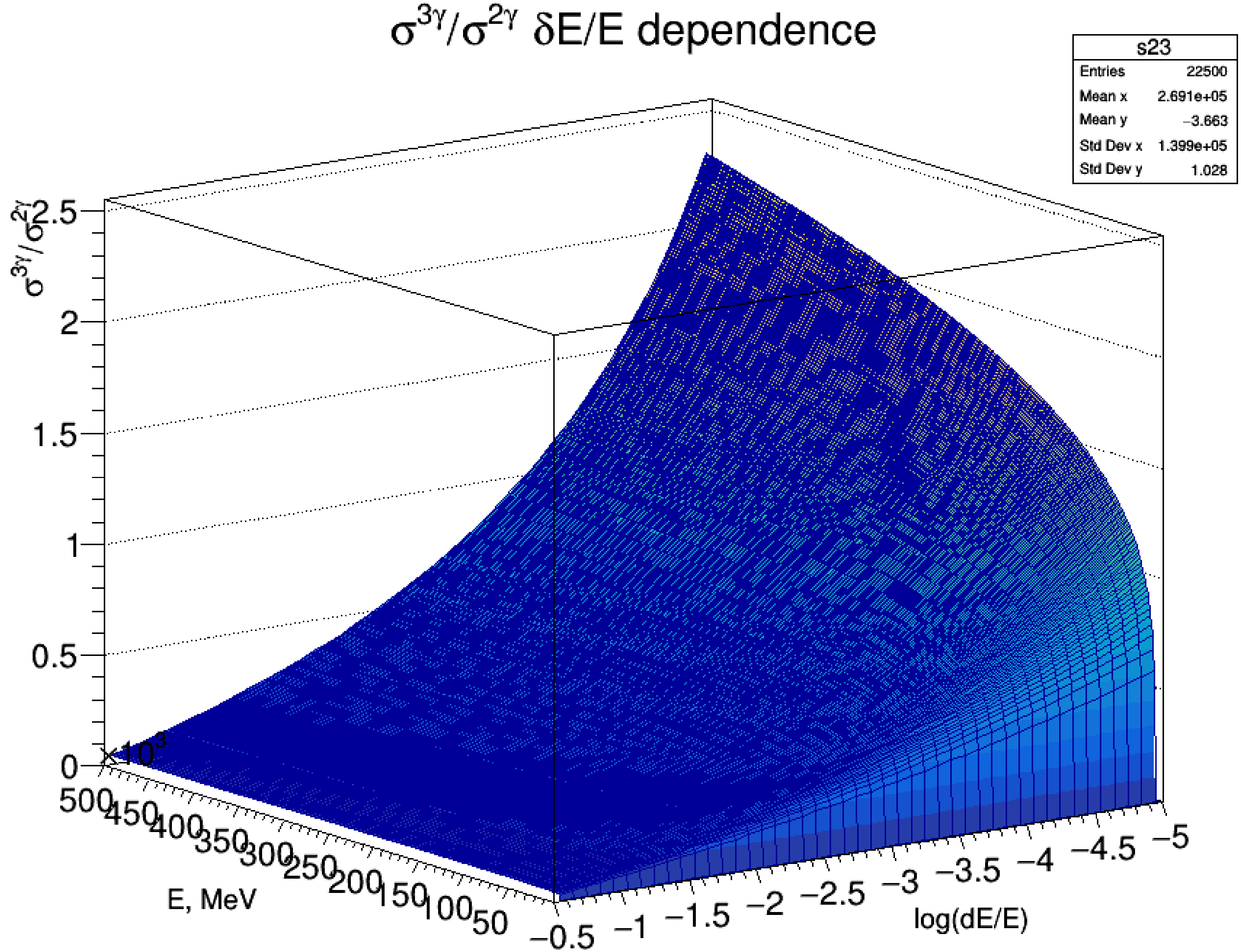
- Minimum photon energy
- The smaller the ΔE-threshold - the greater the proportion of events with 3γ
- Switch between importance of soft gamma carrying and computing resources

optimal threshold for user dE/E = 0.001
ability to set
non-standard threshold selection

Role of ΔE/E factor



- The smaller the ΔE-threshold - the greater the proportion of events with 3γ
- Total cross-section remains



default: dE/E = 0.001

Simulation algorithm

- Process (3γ or 2γ) selection using cross section ratio
  - depends of positron energy e^+ and treshold ΔE/E

$$P(3\gamma) = \frac{\sigma^{3\gamma}}{\sigma^{2\gamma+3\gamma}}$$

- Sampling of «weak» photon energy
- Sampling of the second photon energy fraction - - *rejection sampling*

(energies determines the angles between photons)

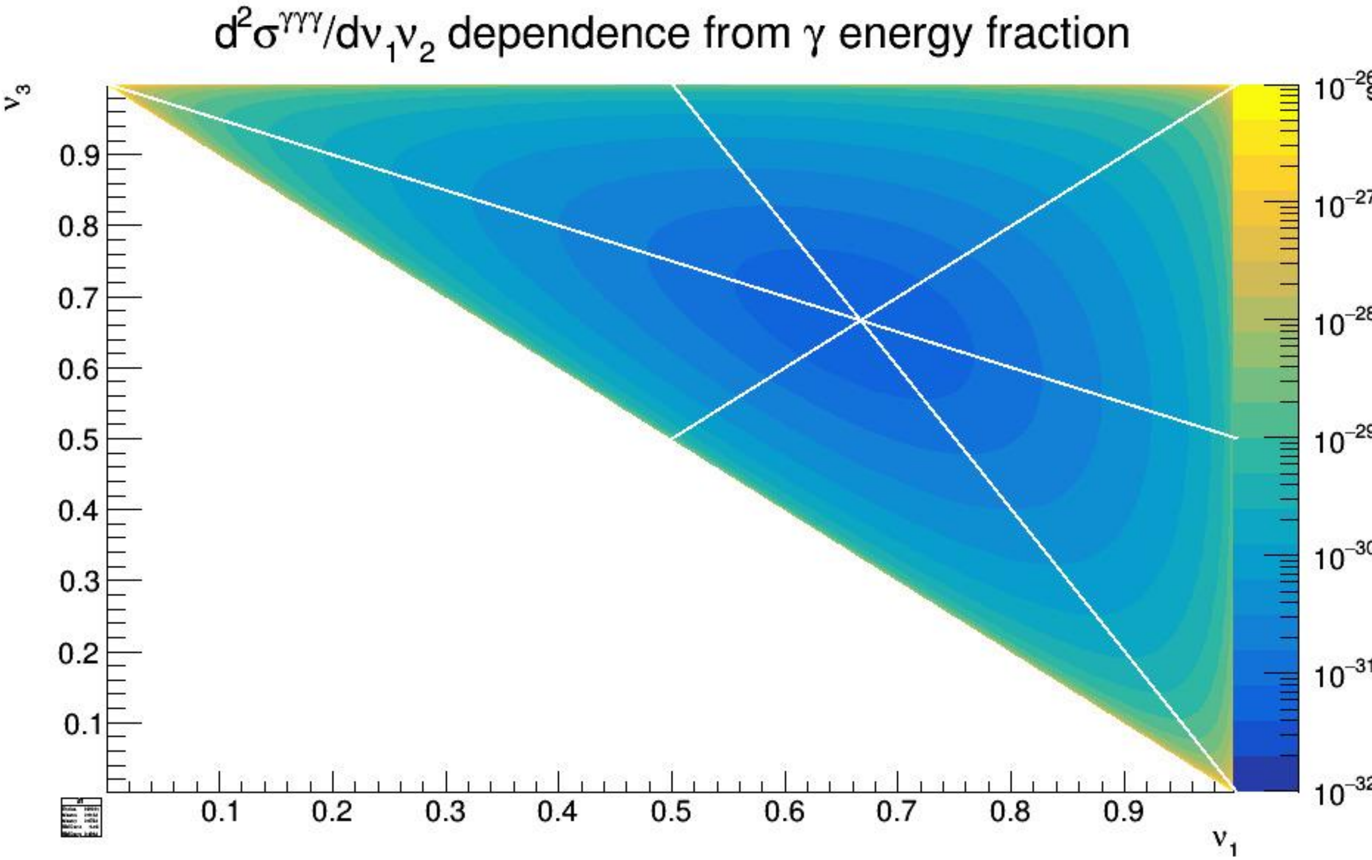
- Plane of produced photons
- Angle between two photons and their directions

(after that, the kinematics is completely determined)

- Boost to the lab reference system

Sampling of lowest photon energy fraction

- ν_1 sampling range: [0, 2/3];
- ν_2 sampling range: [2/3, 1];



Sampling of lowest photon energy fraction

$$\frac{d^2\sigma}{d\nu_1 d\nu_2} = \frac{2\alpha^3}{3s}\frac{1}{\nu_1^2}[\nu_1^2 F_{123} + \nu_1^2 F_{312} + \nu_1^2 F_{231}];$$

$$\frac{d\nu_1}{\nu_1^2} \to \frac{K_1}{\nu_1}$$

Putting the value $1/v^2_1$ of the brackets, and perferming the integration one can obtain the way of choosing v_1 through the random number q

$$\int_{\nu_{min}}^{\nu_{max}} P(\nu)d\nu = 1 \qquad q \in [0,1] = \int_{\nu_{min}}^{\nu} P(x)dx$$

$$K_1\left(\frac{1}{\nu_{min}} - \frac{1}{\nu_{max}}\right) = 1 \qquad q = \left(\frac{1}{\nu_{min}} - \frac{1}{\nu}\right)K_1$$

- Obtaining v_1 through the random number q
- Searching the maximum of expressions in brackets to determine v_2 by the rejection sampling

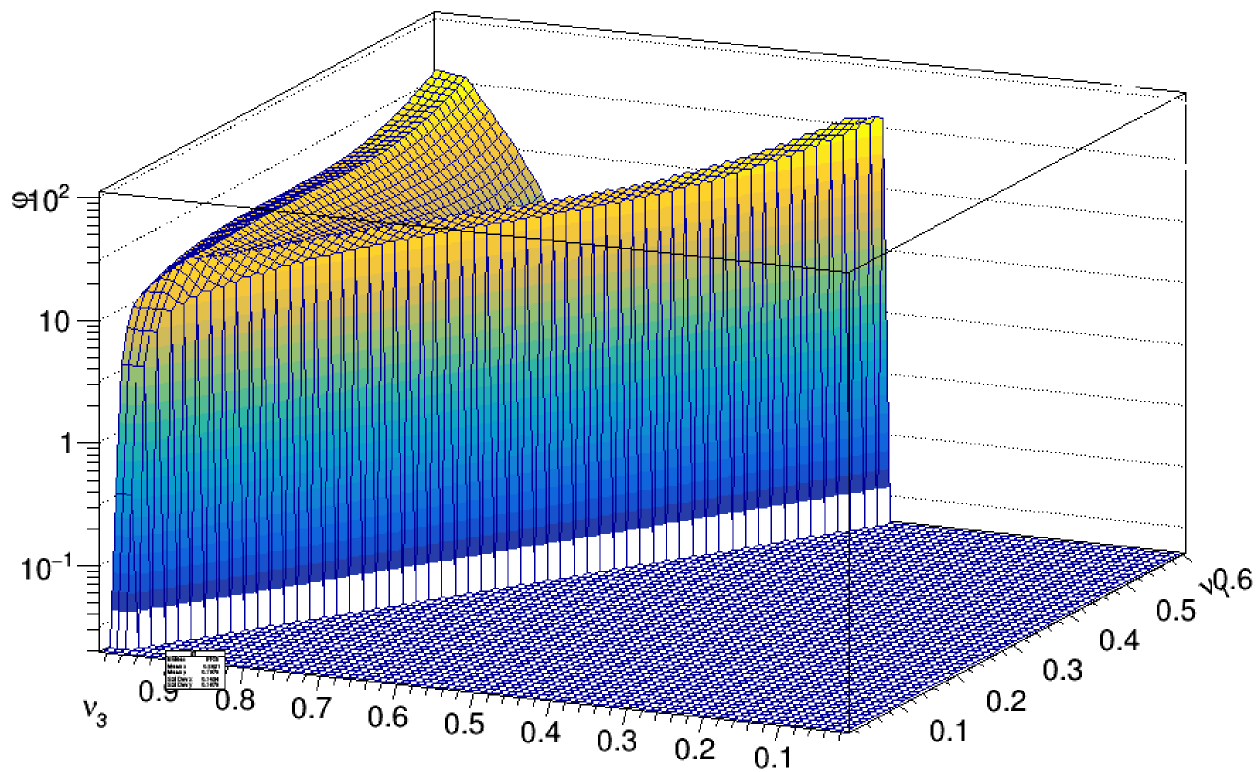
Sampling of the second photon energy fraction

$$\frac{d^2\sigma}{d\nu_1 d\nu_2} = \frac{2\alpha^3}{3s}\frac{1}{\nu_1^2}\varphi;$$

during the simulation the value of the rejection maximum may be computed only once at initialisation

This value will determine the line for rejection method of sampling of ν_2

φ dependence from γ energy fraction



$$\varphi = [\nu_1^2 F_{123} + \nu_1^2 F_{312} + \nu_1^2 F_{231}];$$

corresponds to the maximum of cross-section after sampling of the ν_1

BORDER! $1 - \nu_1 >> m^2/s$

Conclusions

A method of introducing the 3γ-annihilation of electron-positron pair as a next to leading order correction to 2γ-annihilation is proposed.

The 1st version of the model class and unit test to study cross section and final state generation are available.

We plan to deliver new model for Geant4 10.5.

BACKUP SLIDES

Main publications on 3γ-annihilation

- *Baier, V. N.; Fadin, V. S.; Khoze, V. A.; Kuraev, E. A.: «Inelastic processes in high energy quantum electrodynamics» // Physics Reports 78 (1981), p. 293-393.*

  Systematic discussion of inelastic processes in high energy QED, including analysis of 2γ and 3γ cross-sections, radiation corrections, corresponding angular and spectral distributions of the final particles, characteristics of 3rd and 4th order processes relevant for colliding beams

- *S. Eidelman, E. Kuraev and V. Panin, «e+e - annihilation into two and three photons at high energy» //* Nucl. Phys. B143 (1978) 353-364

- *S. Eidelman, E. Kuraev and V. Panin, «Processes $e^+e^- \to e^+e^- \gamma$, $\mu^+\mu^- \gamma$, $\gamma\gamma\gamma$ with emission of final particles at large angles » //Nucl. Phys. B148(1979),245.*

  Simplified analytical expressions for total and differential 3γ-cross-sections as a radiation correction for 2γ accurate to a given power energy

- *F. A. Berends, R. Kleiss «Distributions for electron-positron annihilation into two and three photons» //Nucl. Phys. B186 (1981) 22.*

  Similar study, simulation, numerical results

Standard formula for 2γ from Geant4 Physics Reference Manual:

$$\sigma(Z,E) = \frac{Z\pi r_e^2}{\gamma+1}\left[\frac{\gamma^2+4\gamma+1}{\gamma^2-1}\ln\left(\gamma+\sqrt{\gamma^2-1}\right)-\frac{\gamma+3}{\sqrt{\gamma^2-1}}\right]$$

Dirac P. A. M., Proc. Cambr. Phil. Soc, 26, 361 A930.

In high energy limit:

$$\sigma(Z,E) = \frac{Z\pi r_e^2}{\gamma+1}(\ln 2\gamma - 1)$$

And we have in this work:

$$\sigma^{2\gamma+3\gamma} = \frac{\pi r_e^2}{\gamma+1}(\rho-1)+\frac{r_e^2\alpha}{\gamma+1}\left(\frac{1}{6}\rho^3-\frac{1}{4}\rho^2+\rho(\frac{1}{3}\pi^2-3)+3\xi(3)-\frac{1}{3}\pi^2+3\right)$$

$\sigma^{2\gamma}$ $\sigma^{3\gamma}$

Rho definition:

$$\rho = \ln\frac{s}{\mu^2} = \ln\frac{2\mu(E_+ + \mu)}{\mu} = \ln 2\gamma$$

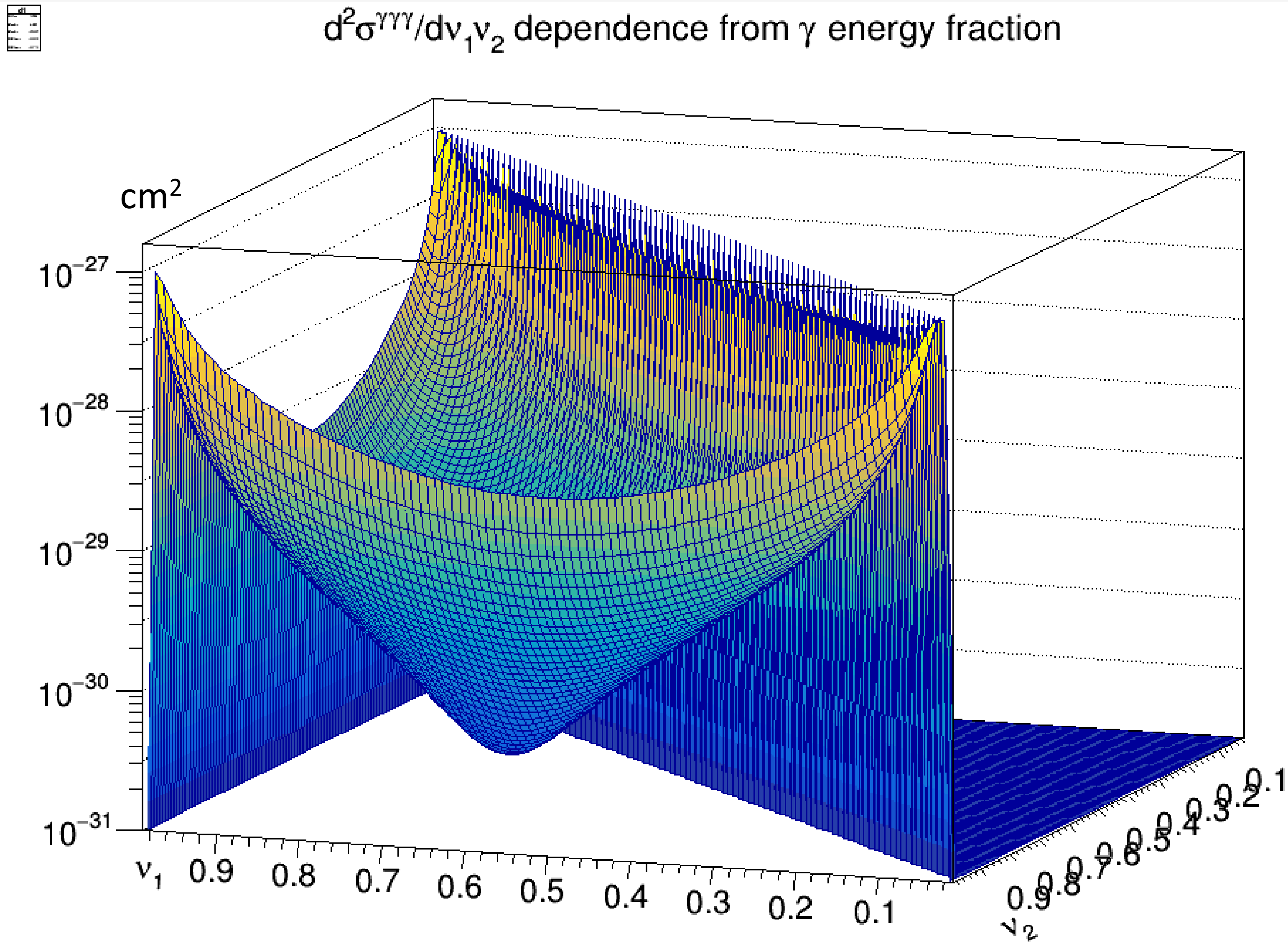
$$\boxed{\rho \to \rho_{add} = \frac{\gamma^2+4\gamma+1}{\gamma^2-1}\ln\left(\gamma+\sqrt{\gamma^2-1}\right)-\frac{\gamma+3}{\sqrt{\gamma^2-1}}+1;}$$

Translation into the lab system

…and into the notations similar to the standard.
Standard 2γ-annihilation is implemented in the lab system.

- $$E_{lab} \equiv E_+ = \frac{E_0 + \beta p_0}{\sqrt{1-\beta^2}} = \frac{E_0^2 + p_0^2}{\sqrt{E_0^2 - p_0^2}} = \frac{2E_0 - \mu^2}{\mu}$$
- $$s = 4E_0^2 = 2\mu(E_+ + \mu)$$
- $$\gamma = \frac{E_+}{\mu^2}$$
- $$\frac{2\pi\alpha^2}{s} = \frac{2\pi\alpha^2}{2\mu^2(E_+ + \mu)} = \frac{\pi r_e^2 \mu^2}{\mu^2(\gamma+1)(\hbar c)^2} = \frac{\pi r_e^2}{(\gamma+1)(\hbar c)^2}$$
- $$\frac{2\alpha^3}{s} = \frac{r_e^2 \alpha}{(\gamma+1)(\hbar c)^2}$$

The distribution in photon energies:



Angles

Energy-momentum conservation allows to determine kinematics of the final photons:

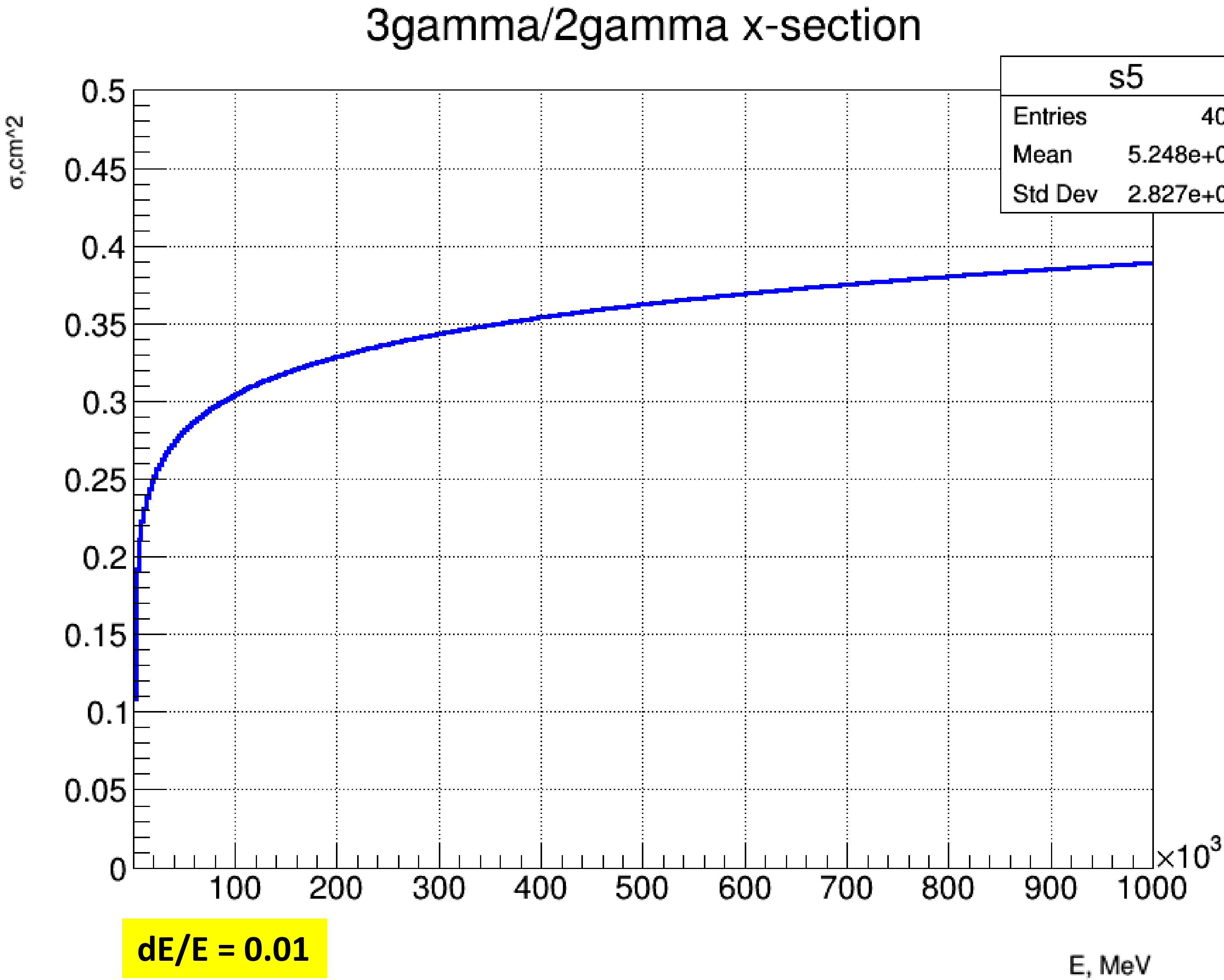
$$x_1 + x_2 + x_3 = 2, \qquad x_1 c_1 + x_2 c_2 + x_3 c_3 = 0,$$

$$x_1 = \frac{1 - x_3}{1 - x_3 \sin^2 \frac{\psi}{2}},$$

$$x_2 = \frac{\cos^2 \frac{\psi}{2} + (1 - x_3)^2 \sin^2 \frac{\psi}{2}}{1 - x_3 \sin^2 \frac{\psi}{2}},$$

$$c_2 = -\frac{x_1 c_1 + x_3 c_3}{x_2}, \qquad \psi = \widehat{\boldsymbol{k}_1 \boldsymbol{k}_3}. \qquad (4)$$

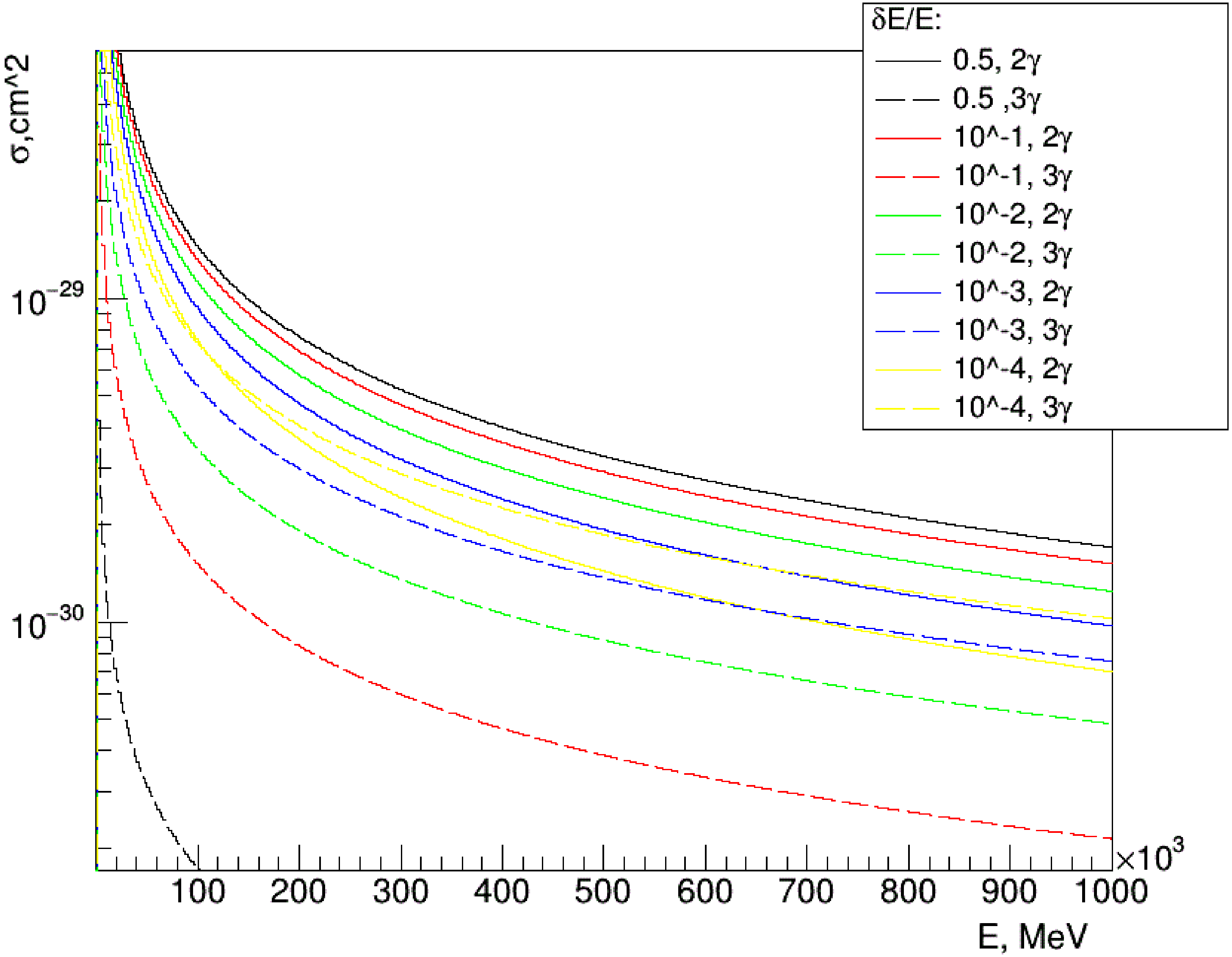
where $\chi_i = k_i p_-$, $\chi_i' = k_i p_+$, $i = 1, 2, 3$, $x_i = k_i^0/\varepsilon$, $c_i = \cos(\theta_i)$, $\theta_i = \widehat{\boldsymbol{p}_- \boldsymbol{k}_i}$.



dE/E = 0.01

every third process can be considered as annihilation on 3γ

Cross sections ΔE/E dependence



3γ events even exceeds 2γ if user set the threshold too small