Quark-Hadron Duality:

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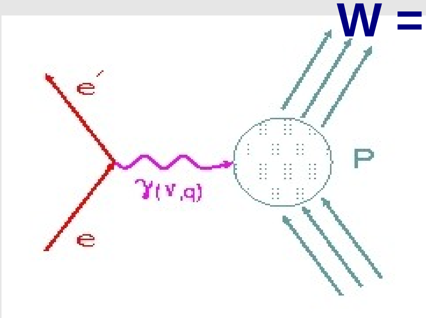
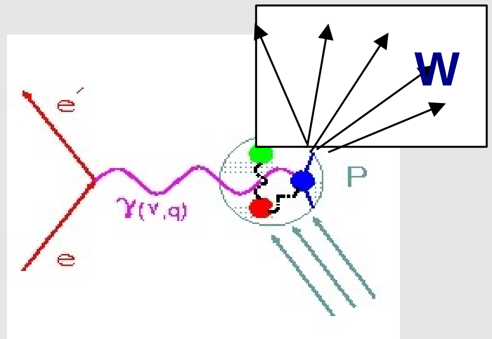
Inclusive Charged-Lepton Scattering

Elastic | Inelastic

W = M_p





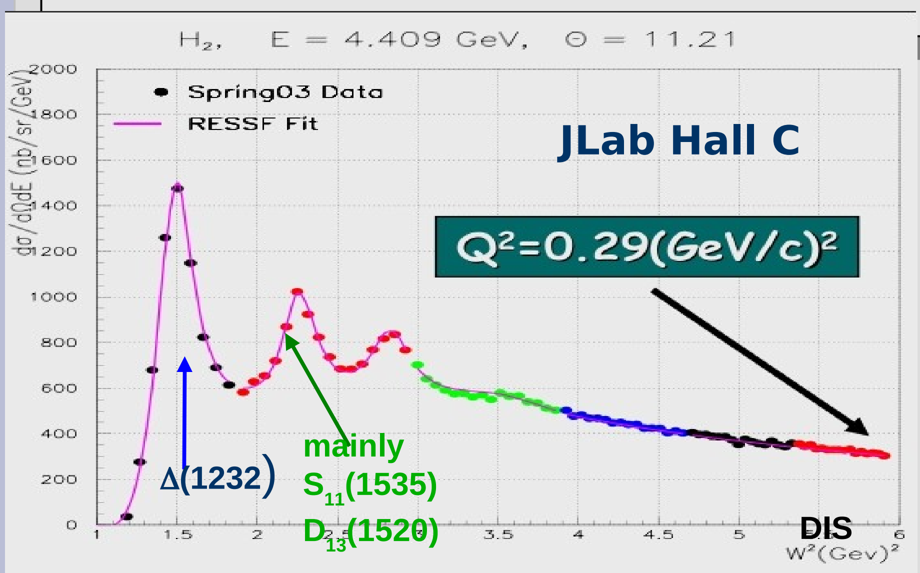
Q^2 : photon 4-momentum

ν: photon energy

W: Final state hadron mass

x: Bjorken variable

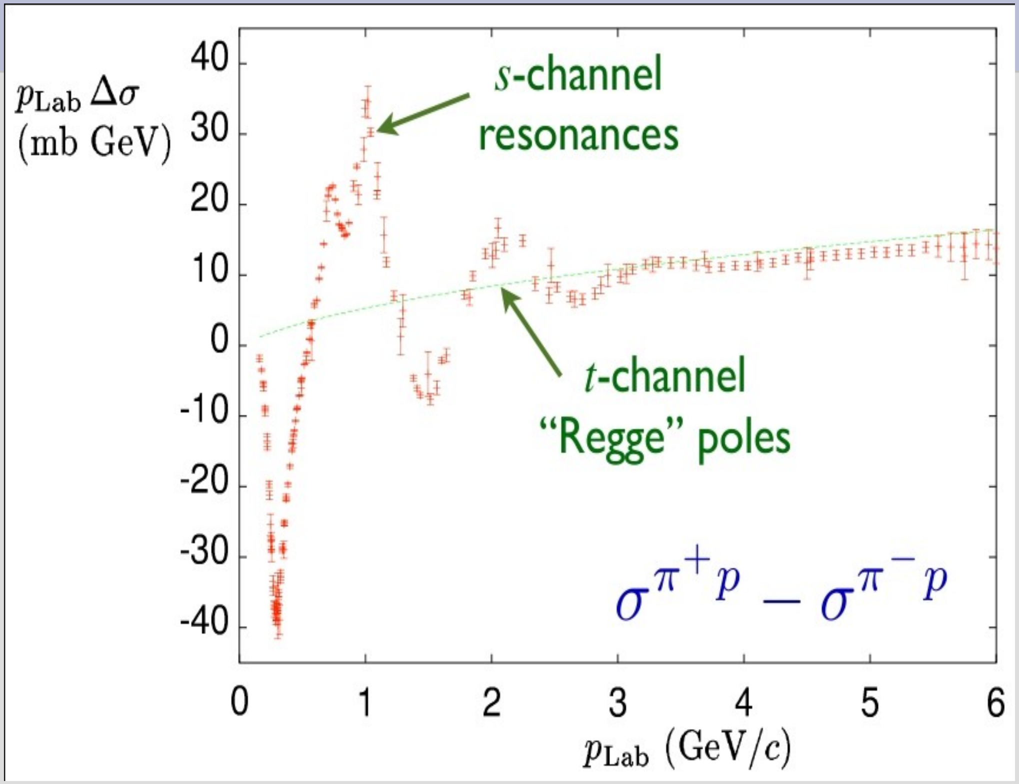
$$\frac{d\sigma}{d\Omega\, dE'} \propto \Gamma\, [2xF_1(x,Q^2) + \varepsilon\, F_L(x,Q^2)] \qquad F_2 = (2xF_1+F_L)/(1+\nu^2/Q^2)$$



Study the W (or x), Q^2 dependence of the structure functions from

Elastic → resonance → Continuum

Prior to Bloom-Gilman, a 'duality' was known from hadron-hadron scattering



→ Partial theoretical description provided by Finite Energy Sum Rules (FESR).

→ Provided relationship between t-channel Regge trajectories (high E) and s-channel resonance production (low E).

→ Developed in 1962 (Igi) and applied to charged pion-proton scattering in 1968 (Dolen, Horn, Schmidt).

→ Electroproduction is unique in that points at the same Bjorken *x* (ω') arising from different Q^2 at the same $s=W^2$, both in and outside the resonance region.

The Beginning: Bloom-Gilman duality

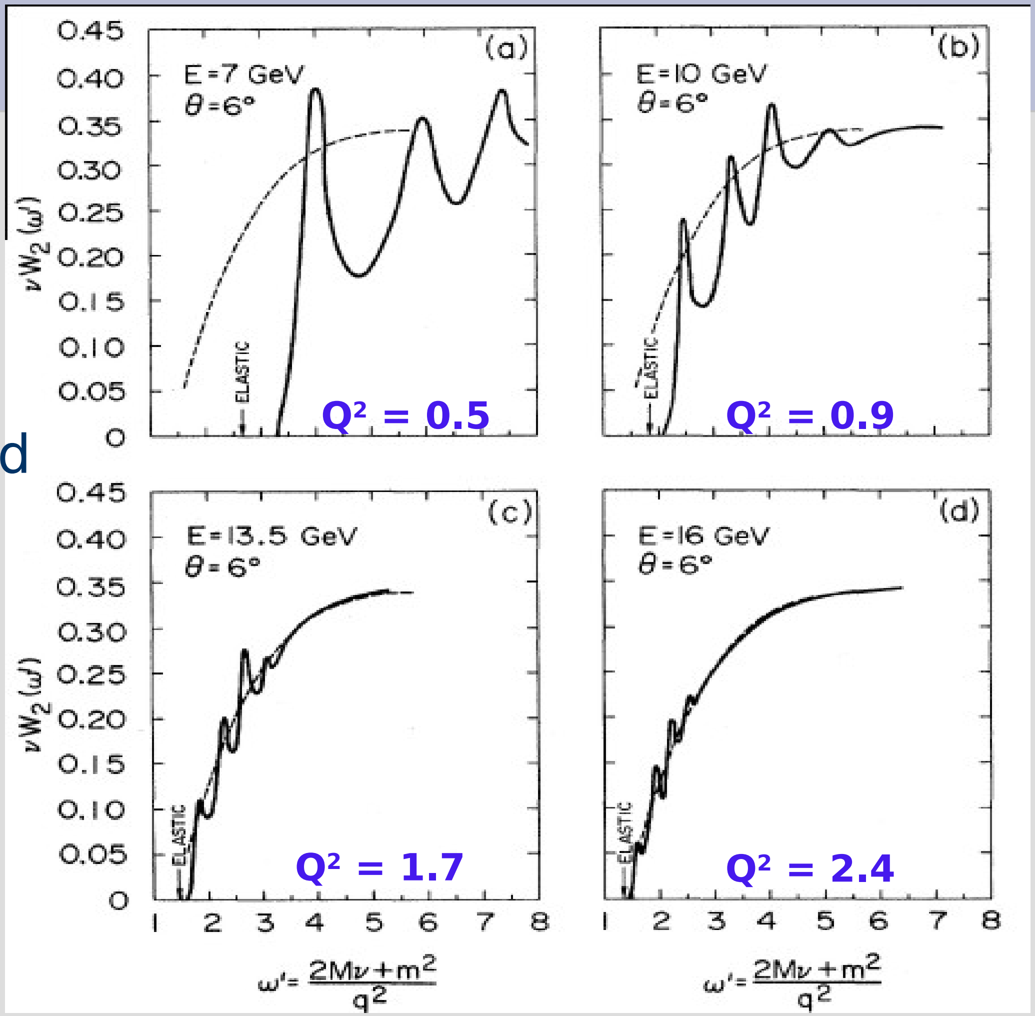
- Inclusive e-P scattering.

-Resonance excitation at low W,Q^2
-Continuum at larger W,Q^2

- First observed by Bloom and Gilman at SLAC *prior* to the development of QCD.
  Phys.Rev.Lett.25:1140,1970.

- Noted that resonances oscillate around a 'scaling' curve at all Q^2.

- hadrons excitations follow the DIS scaling behavior.



Bloom-Gilman Conclusions

- ✓ As Q^2 increased then resonances move toward $\omega' = 1$, ***each*** clearly following the smooth scaling-limit curve.
- ✓ The resonances are not a separate entity but are an intrinsic part of the scaling behavior.
- ✓ This connection between the behavior of resonances and scaling hints at a common origin in terms of a point-like substructure.

Novel observation that was generally left unstudied for next 30 years.

Local Duality allows us to relate structure Functions to Form Factors

For resonances, $F_2(W_{res} = M_{res}) \sim 2M\nu\, G^2(Q^2)$, where G is the resonance form

With $x_{res} = Q^2/(W^2_{res} - M^2 + Q^2) = Q^2/2M\nu_{res}$

If resonances slide down Q^2 independent F_2 scaling curve with

(1) $F_2 \sim (1-x)^{2n-1}$ for $x \to 1$

Then $G^2 \sim (1-x_{res})^{2n-1}/2M\nu_{res} = (1-x_{res})^{2n}$

And for $Q^2 >> W^2_{res} - M^2$

(2) $G \sim (1/Q^2)^n$

Relationship between (1) and (2) for elastic is the **Drell-Yan-West** relation
With 'n' the minimum # of gluons exchanged => pQCD counting rules.

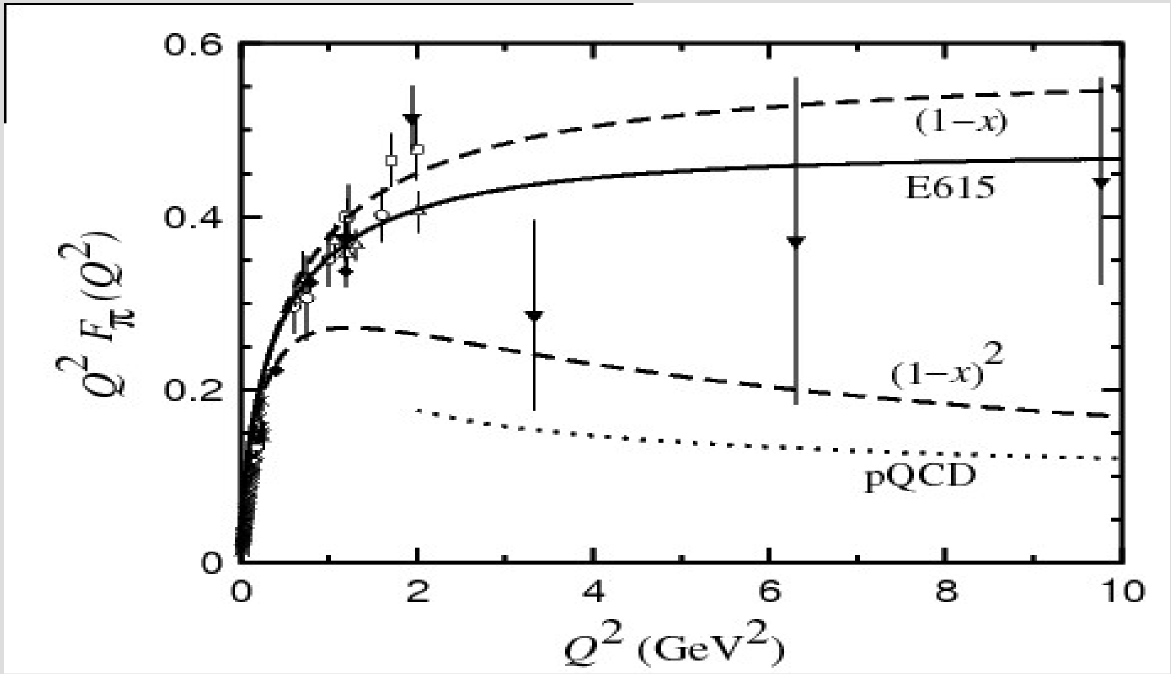
Conversely, DYW => 'local' duality for well isolated resonances.

Applicaton to pion structure function

$F_2^\pi \sim (1\text{-}x)^a$, with *a* from Drell-Yan **E615** data

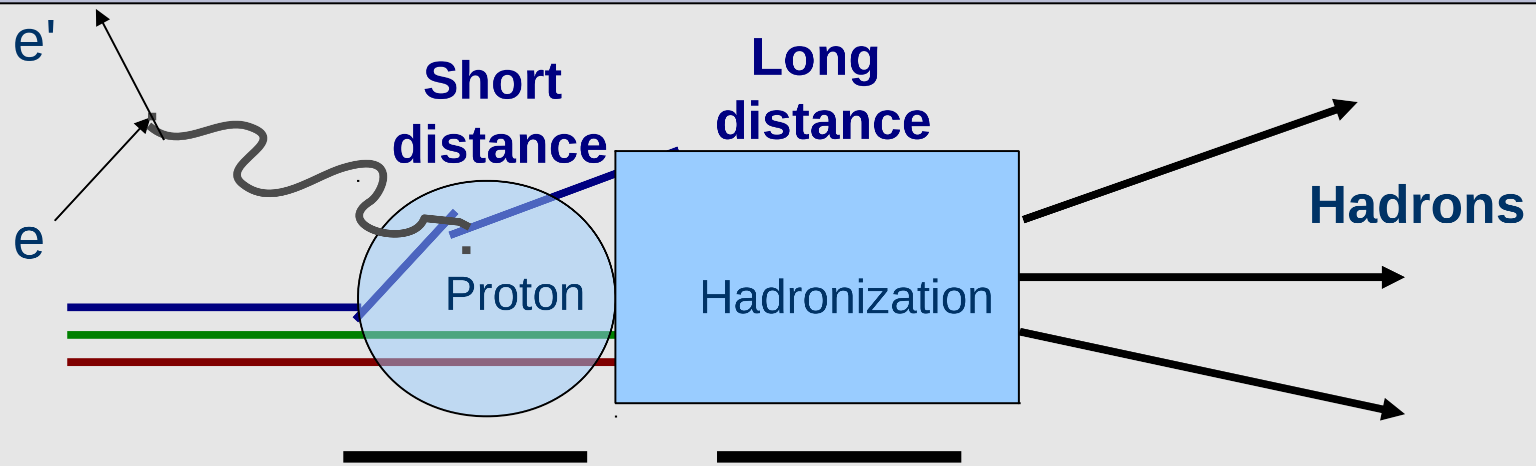
Assuming **local duality**, predict F_π form factor:

W. Melnitchouk, Eur.Phys.J.A. 17 (2003) 233.



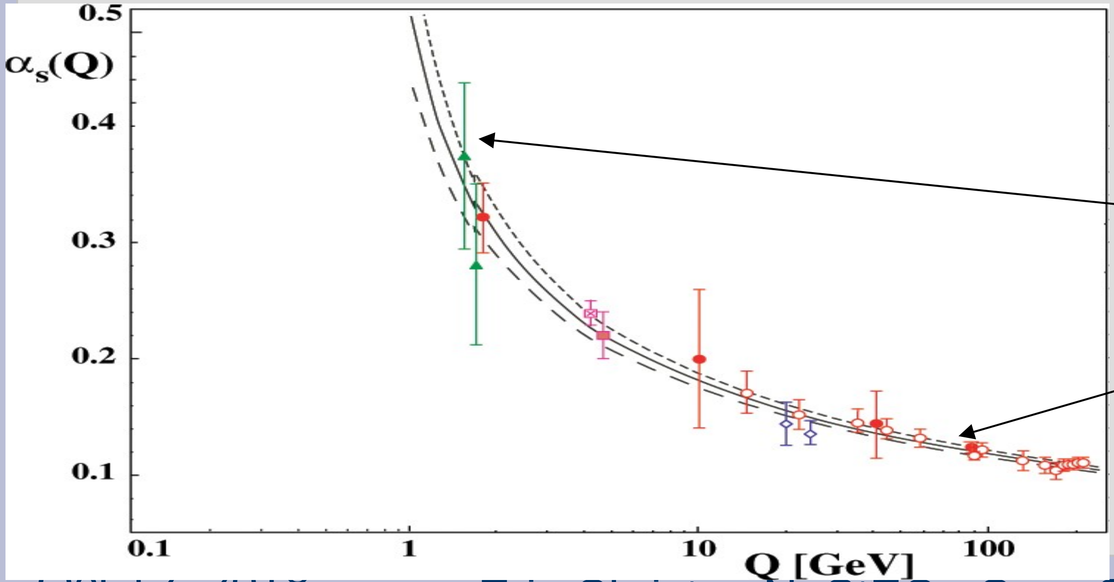
* Remarkable agreement with data

2 Defining Properties of QCD



Asymptotic Freedom ensures coupling to nearly free partons at high enough energies.

Confinement ensures that only hadrons are directly observed.



quarks are far apart

=> restoring force is large enough to pull q qbar pairs from vacuum.

quarks are close (~< 1fm)

=> strong coupling is small

When describing properties of hadrons:

1. At low energies effective theories with baryons and mesons as degrees of freedom often work well.

2. quarks and gluons are manifest at large energies as the fundamental constituents.

The transition between these 2 QCD regimes is *not* understood, and solutions to full QCD are primarily limited to the Lattice in the non-perturbative regime.

Quark-Hadron Duality

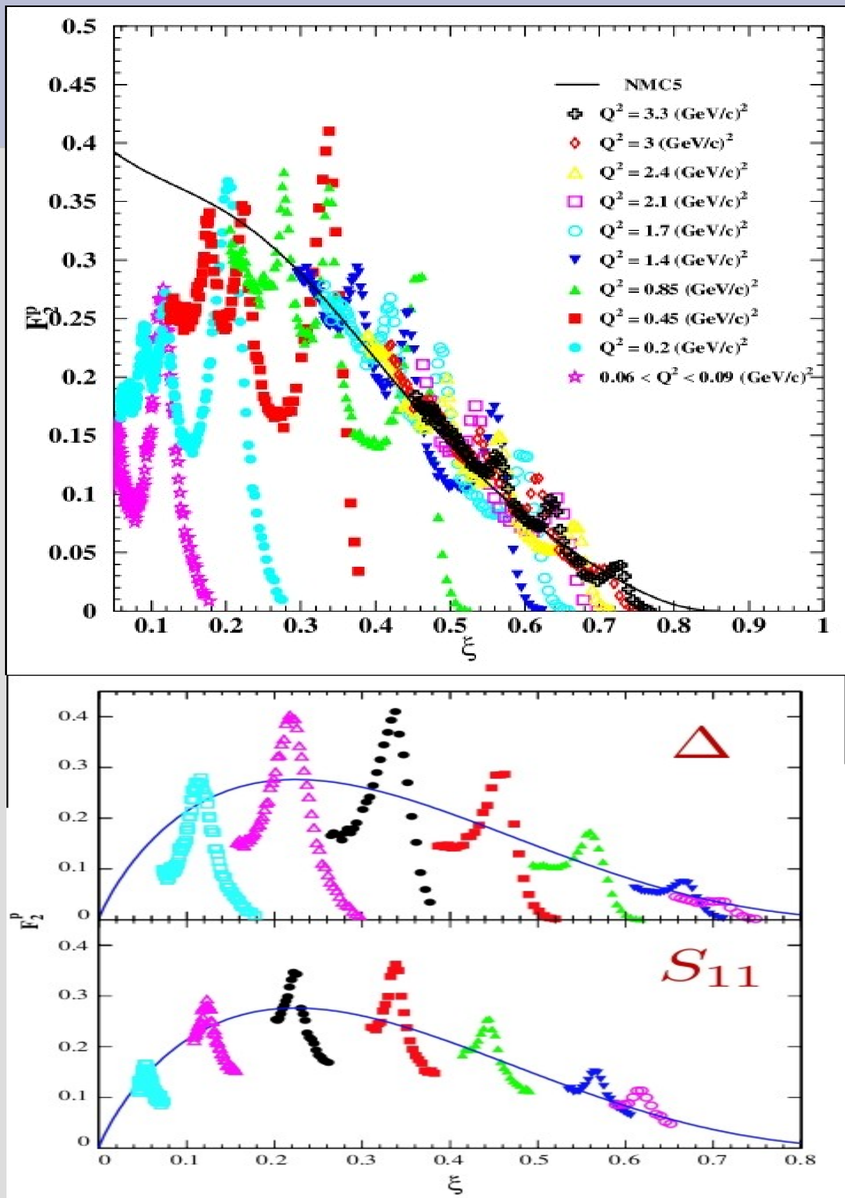
complementarity between quark and hadron descriptions of observables

At high enough energy:

Hadronic Cross Sections averaged over appropriate energy range	=	Perturbative (Quark-Gluon)
Σ_{hadrons}		Σ_{quarks}

Can use either set of complete basis states to describe physical phenomena **provided you sum over enough states**

First Hall C data



→ Confirmed Bloom-Gilman observation in spectacular fashion.

→ Observed that data trace out a *valence-like* curve when $Q^2 < 0.5$

→ *local* duality is observed.

To understand duality we need to determine when it works and when it doesn't.

Seems to work well for the proton F_2, but many open questions at start of JLab:

- Does if depend on the helicity of the photon exchanged?

  => Look at F_1 (transverse) and F_L (longitudinal)

- How does the nuclear environment affect duality?

  => Look at unpolarized scattering from nuclear targets

- What about spin structure functions from polarized scattering?

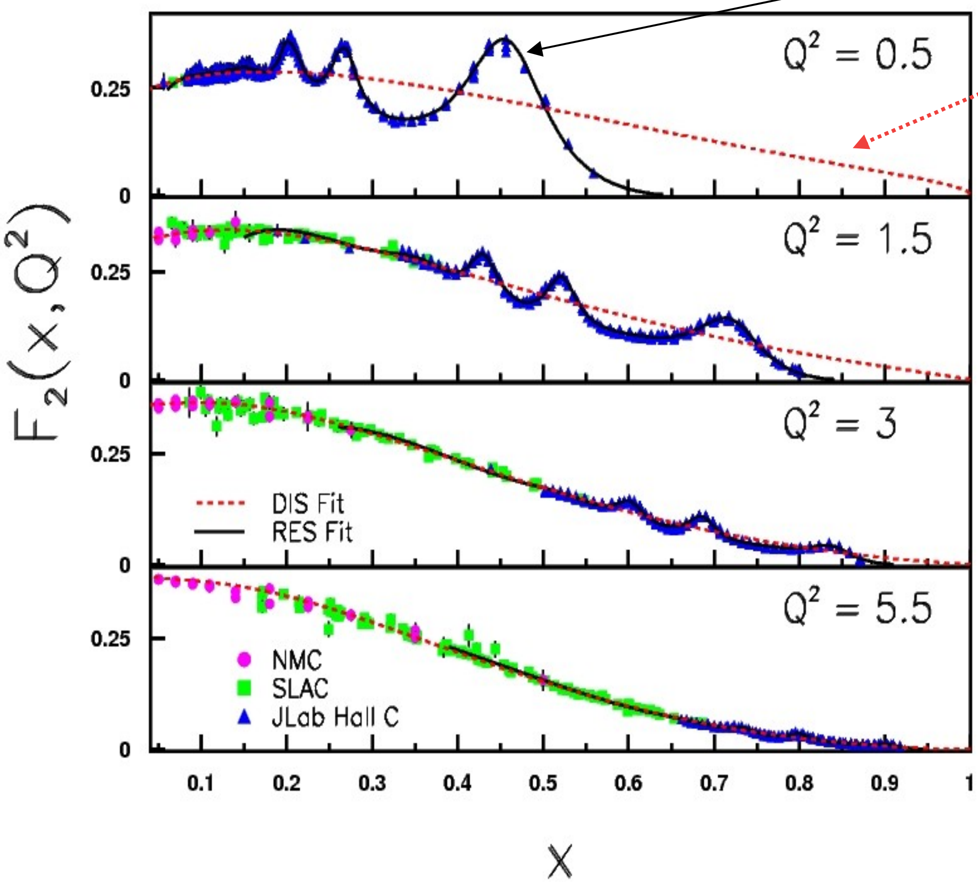
  => Look at g_1

- What about Semi-inclusive scattering?

Previously observed that resonance region proton F_2 *averages* to a scaling curve

Examine highest precision Resonance Region data on proton from Jlab Hall C E94-110

DIS fit – 'F2ALLM' H.Abramowicz and A.Levy, hep-ph/9712415
Res fit - E.C. and P.E. Bosted, PRC 81,055213



Resonance C-B fit reproduces data to ~3%

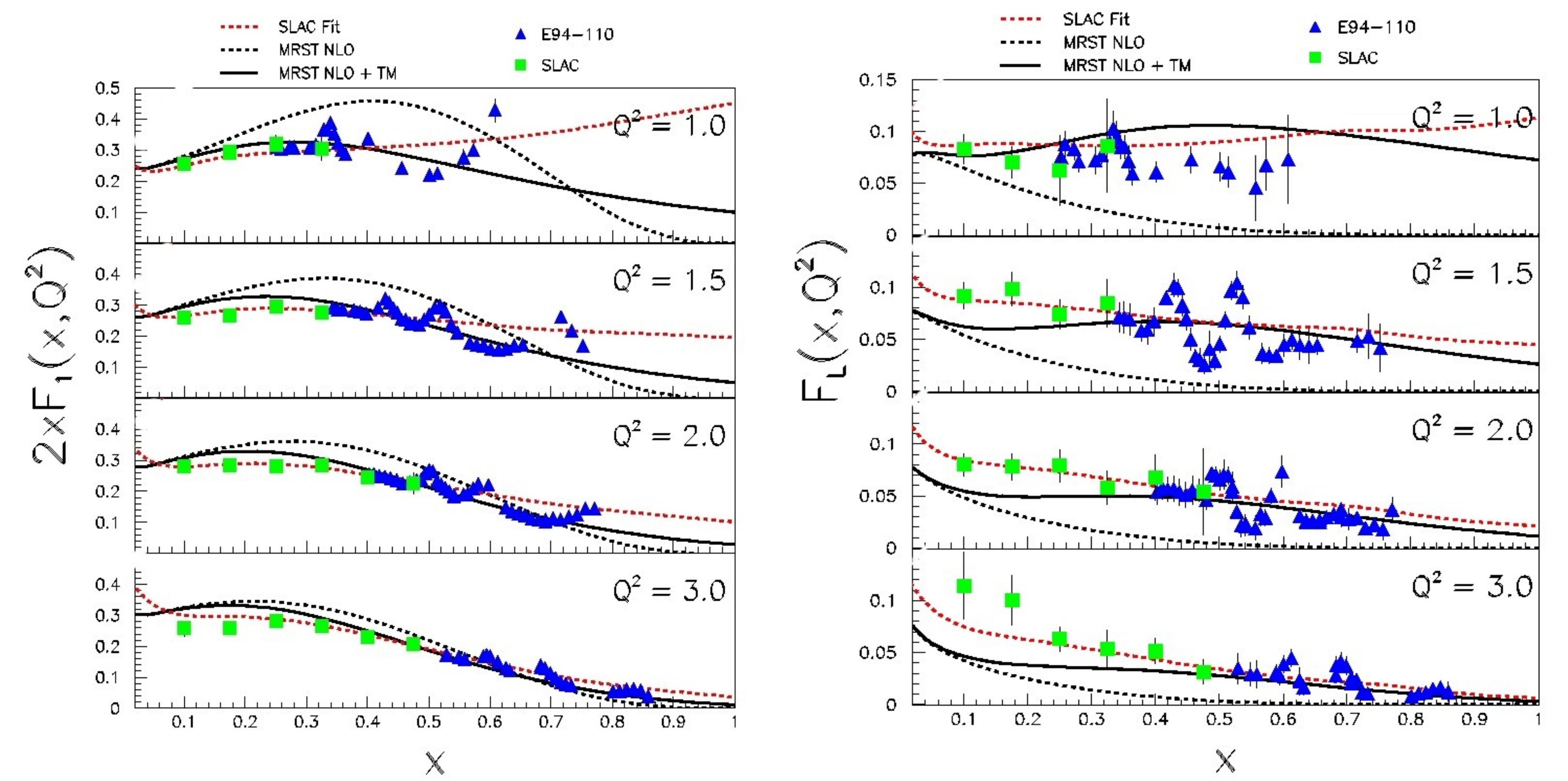
"DIS" fit is to larger W data all the way down to $Q^2 = 0$

→ Curve necessarily includes contributions beyond massless limit perturbative QCD:

I) Target Mass

II) Higher-Twist

Separated F_1 , F_L data from E94-110 provide more insight



→ Duality observed to hold at 10-20% level depending on the scaling curve chosen
SLAC F2global (Whitlow) + R1990 (Tao) or MRST2004 PDF

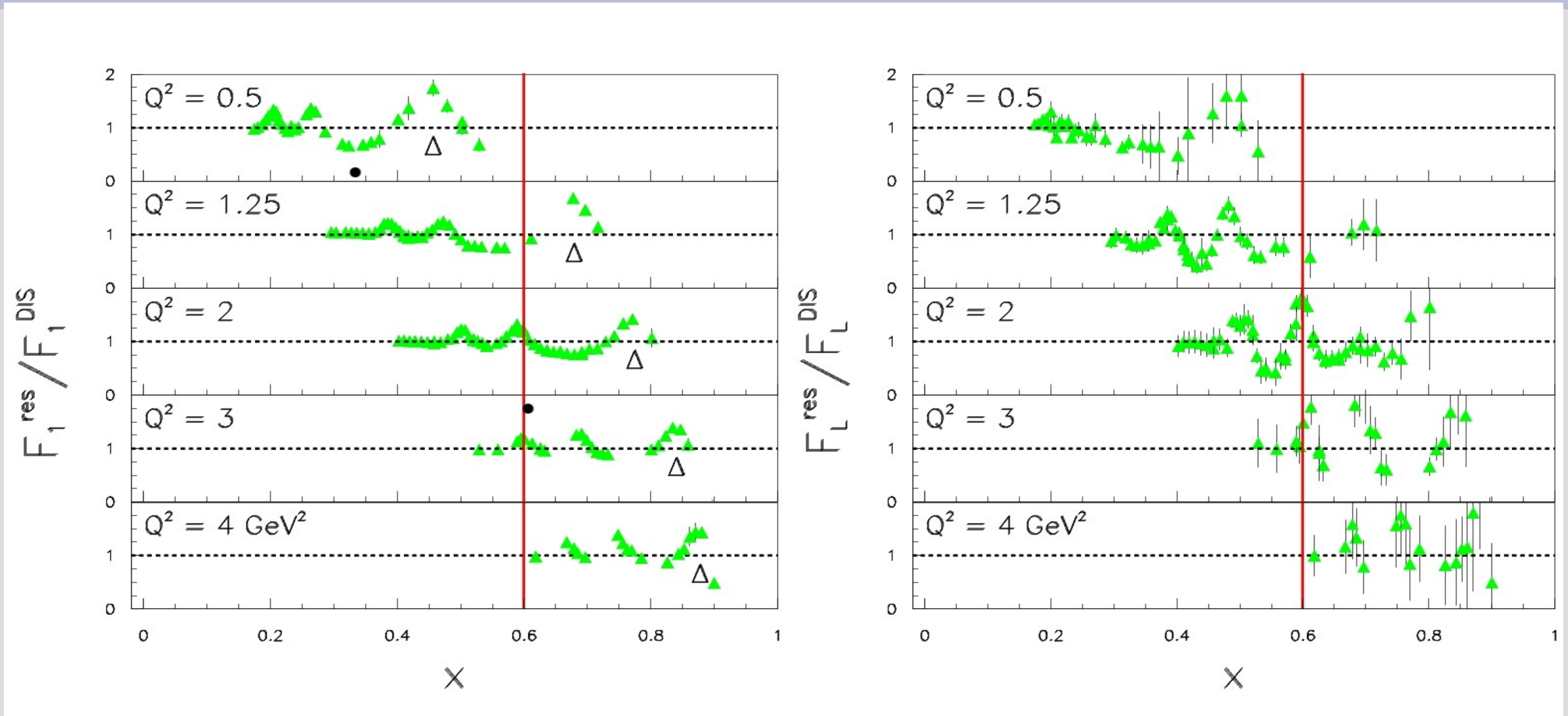
→ Target Mass (TM) contributions can be significant at low Q^2, especially in F_L
=> These are *necessary* for duality to hold at a reasonable level

Examine duality in F_1 and F_L relative to empirical DIS fit:

F_2ALLM fit to F_2 + $R = \sigma_L / \sigma_T$

H. Abramowicz and A.Levy, Hep-ph/9712415

K. Abe et.al Phys.Lett.B452:194-200,1999



Couple of important observations:

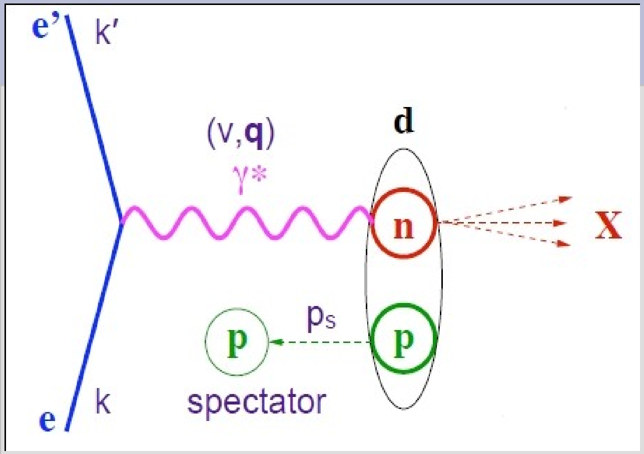
→ Many resonances pass though a given x for a large enough range of Q^2

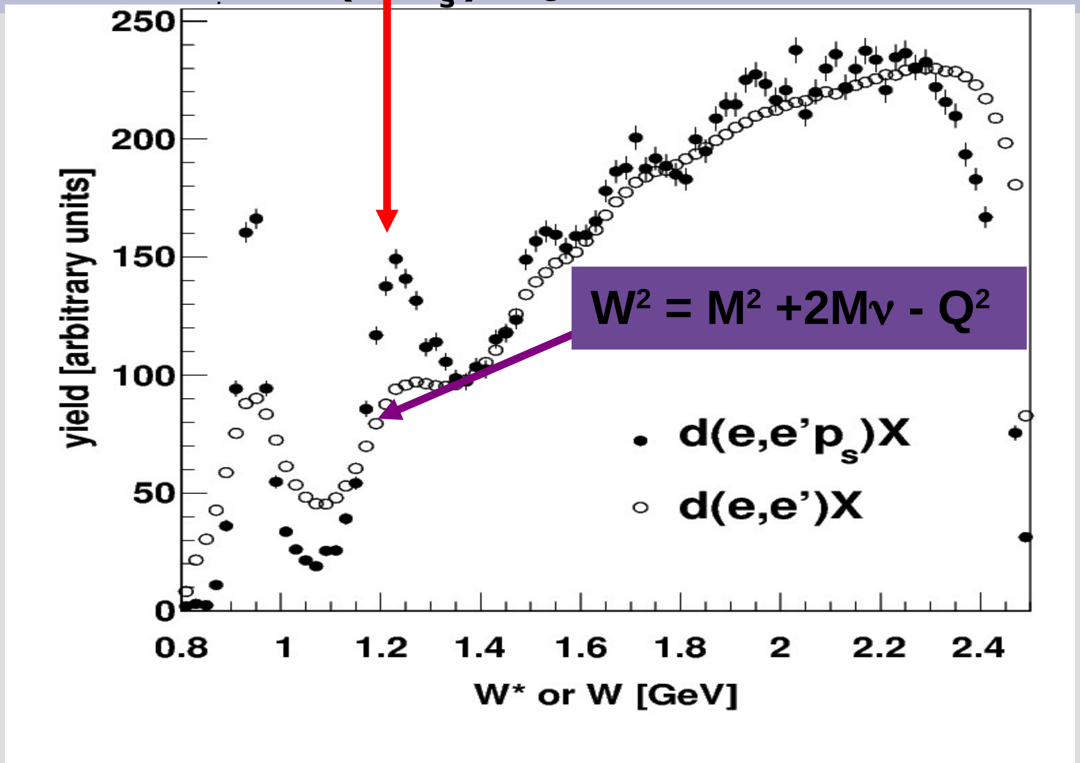
→ DIS fit describes well the average Q^2 dependence of resonant Region structure

What about neutron F_2?

Spectator Tagging (BoNuS)

$$W^2 = (p_n + q)^2 = p_n^{\mu}\, p_{n\mu} + 2([M_D - E_s]\nu - p_n \cdot q) - Q^2$$
$$\approx M^{*2} + 2M\nu(2 - \alpha_s) - Q^2$$





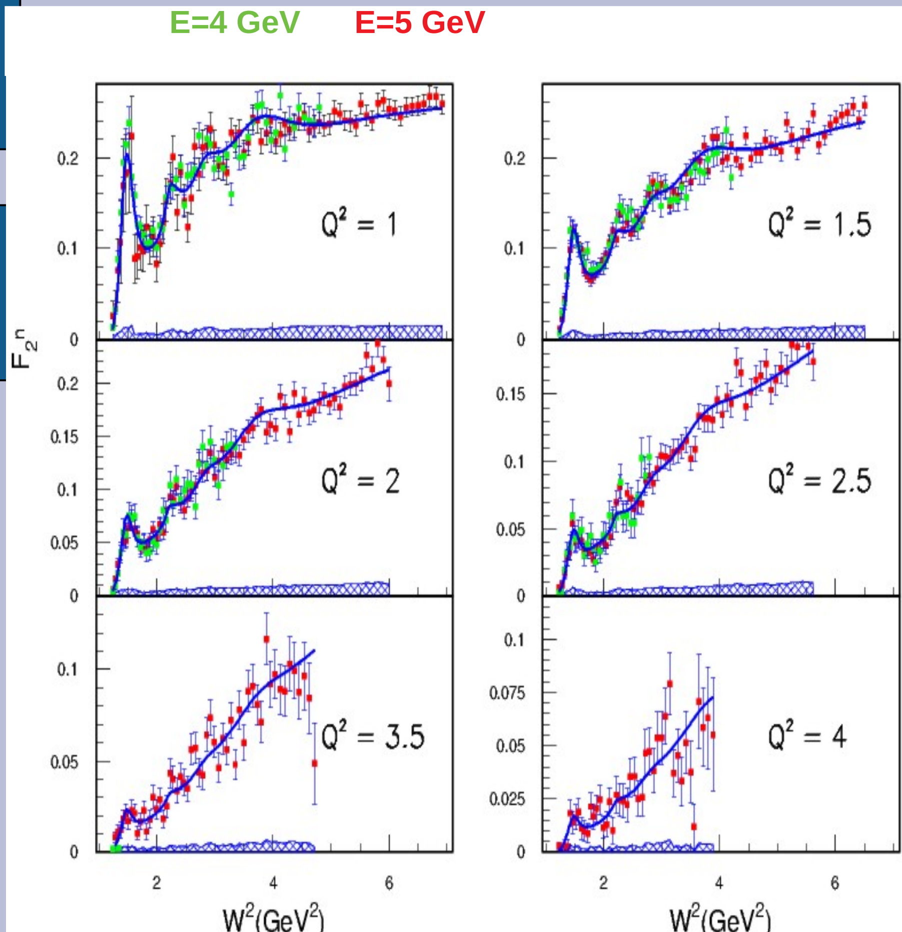
PWIA: → Backward P is spectator

→ Neutron is offshell

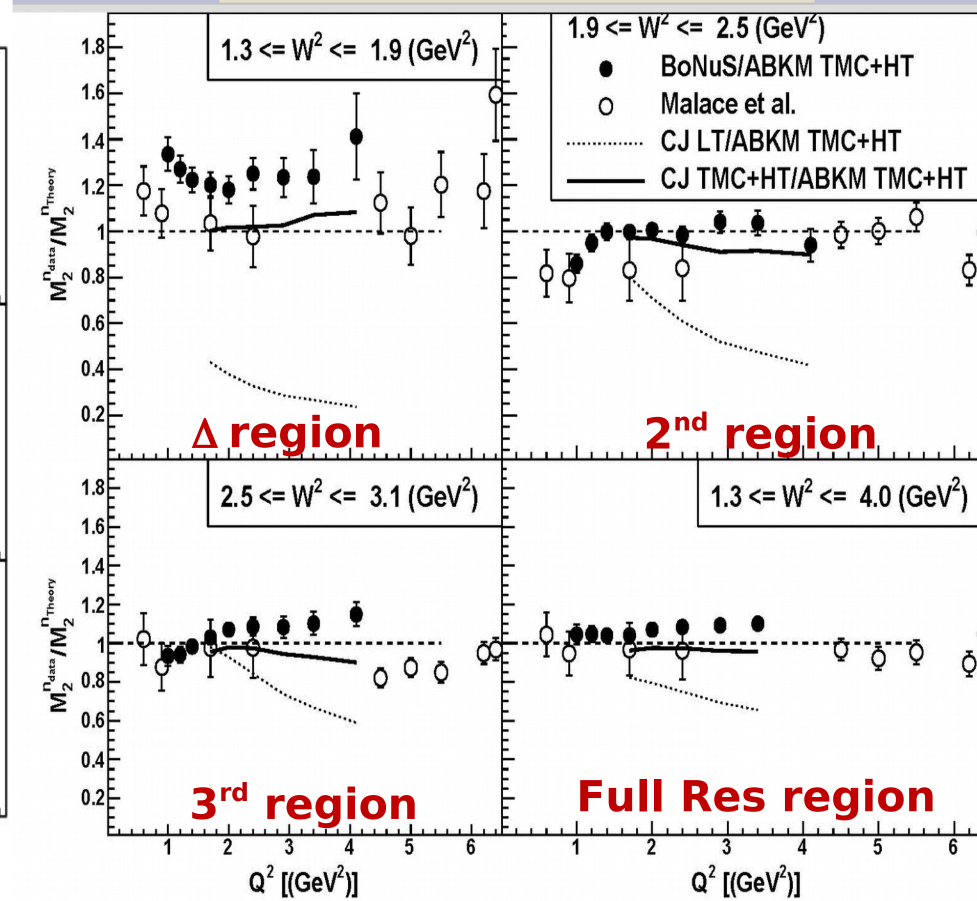
→ $\mathbf{p}_n = -\mathbf{p}_p$

=> correct for neutron momentum

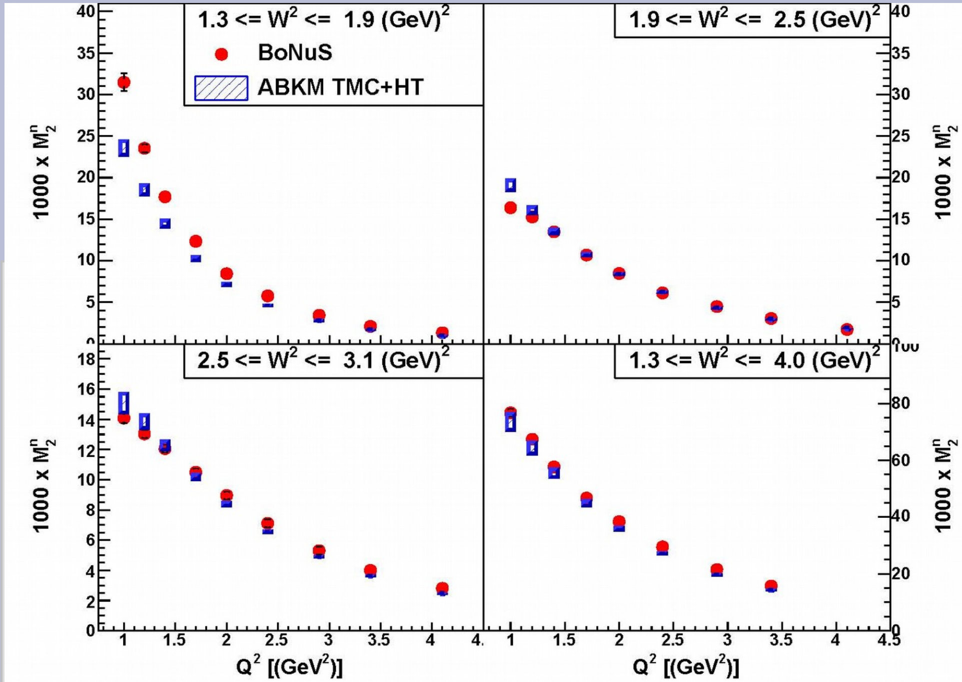
Neutron F_2 and duality tests



Local duality integrals compared to pQCD vs Q^2



- Duality observed for neutron locally within: ~30% for Δ and ~10% for higher W
- As for the proton, TM and H-T must be part of the scaling curve



Neutron moment ratios

data / scaling curve

Neutron:

→ Δ integral > scaling curve

→ higher W regions ~ scaling curve

Neutron to proton moment ratios

Resonance ~same for n and p

=>

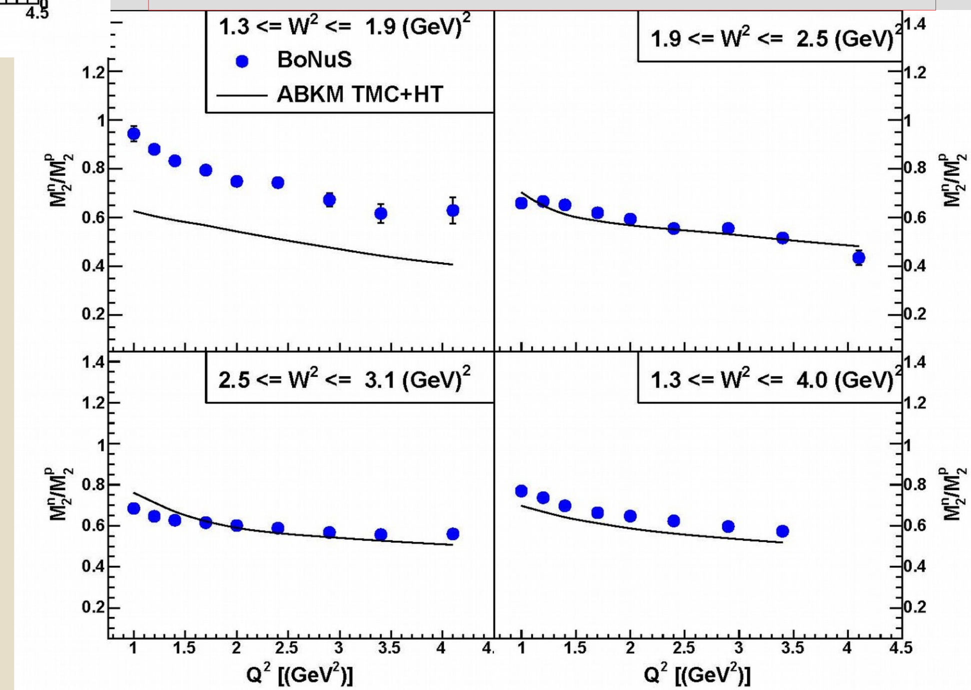
At low Q^2 resonance dominates

=> ratio → 1

At high Q2 non-res dominates

=> ratio → DIS n/p

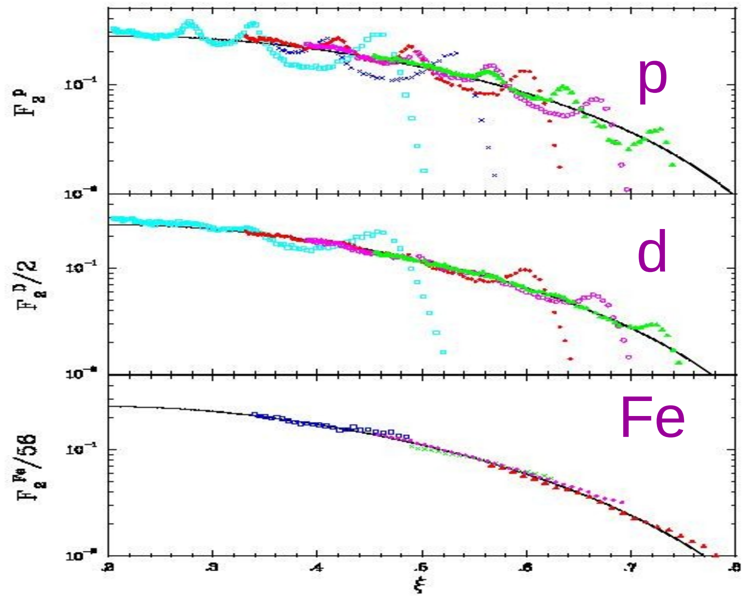
M_2 neutron / M_2 proton

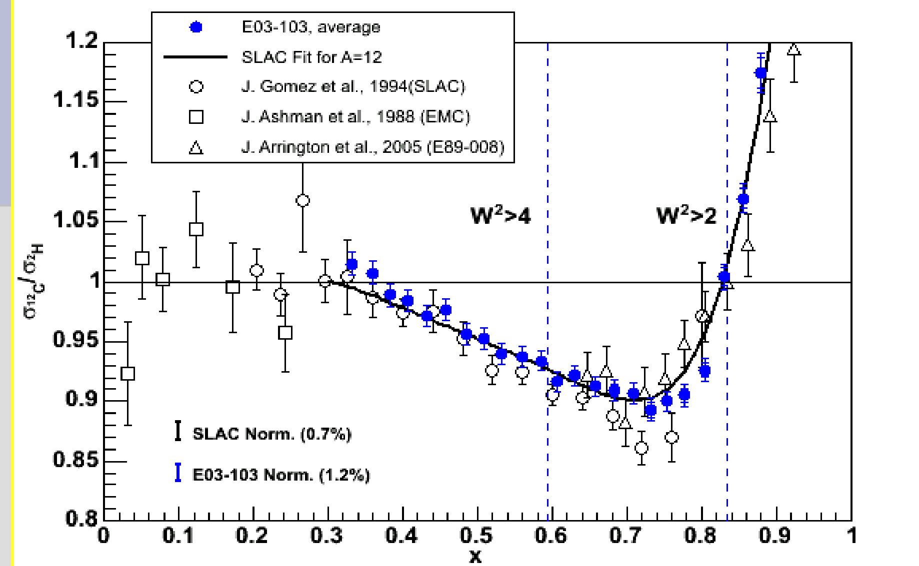


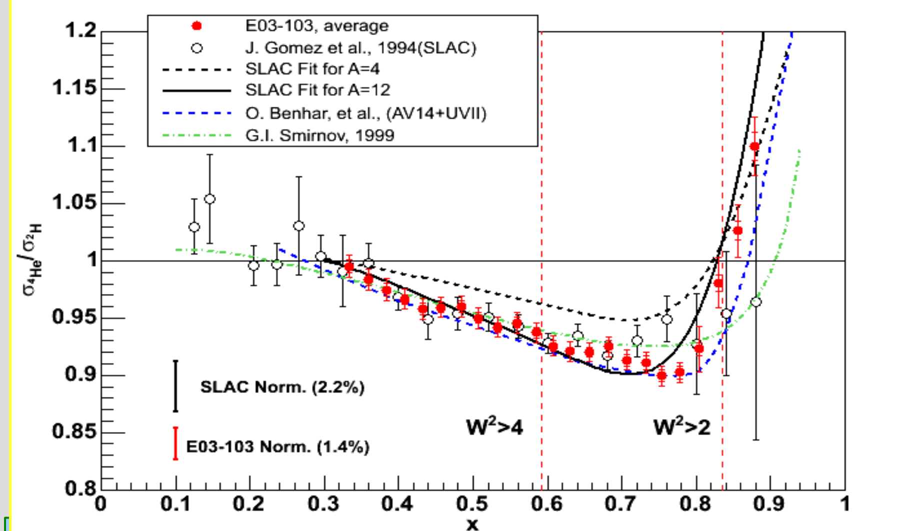
What about F_2 in nuclei?

Duality in Nuclei

$\xi = 2x / [1 + (1 + 4M^2x^2/Q^2)^{1/2}]$





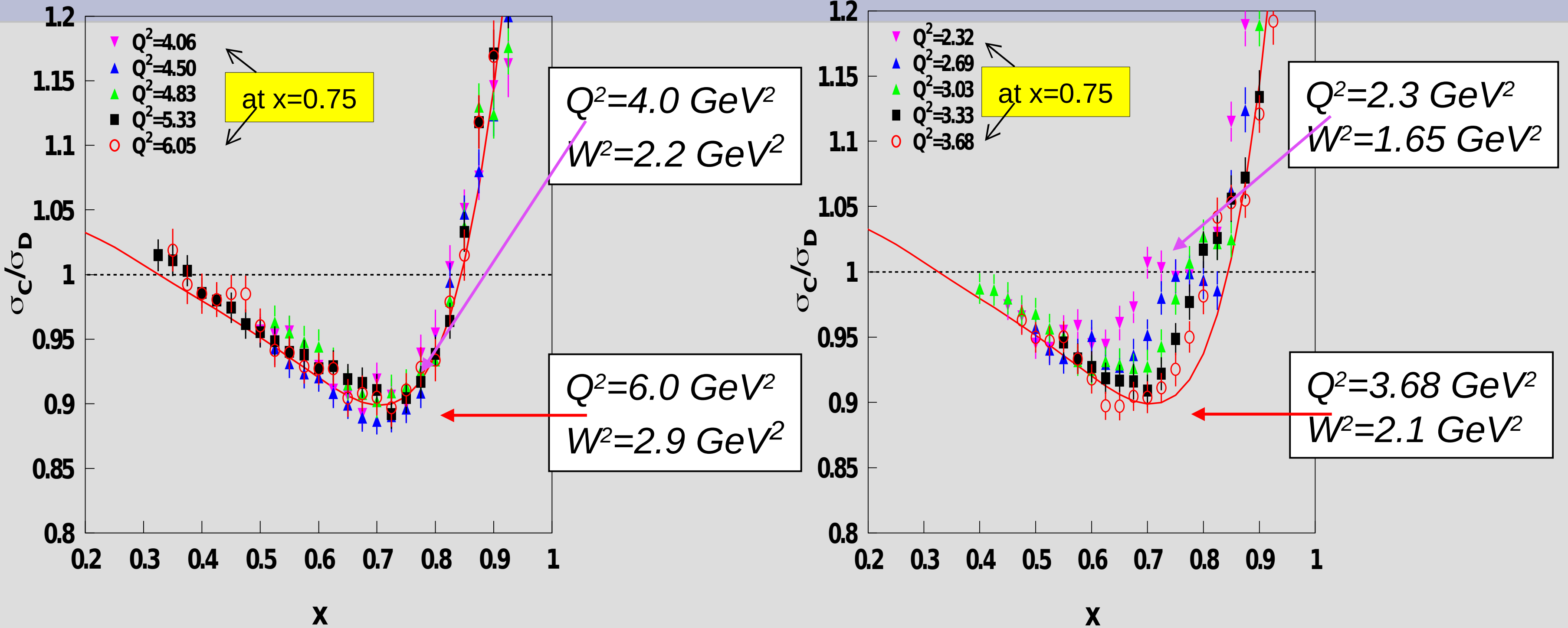


- Fermi motion in the nucleus accomplishes averaging in x, ξ.

=> Duality works even better in nuclei.

Duality is also observed in the EMC effect for ^{12}C and ^{4}He!

More Results from Hall C E03-103



→ Duality in EMC effect seems to work well for $W > W_{\Delta}$

→ Some deviation seen for Δ and $Q^2 < 2$

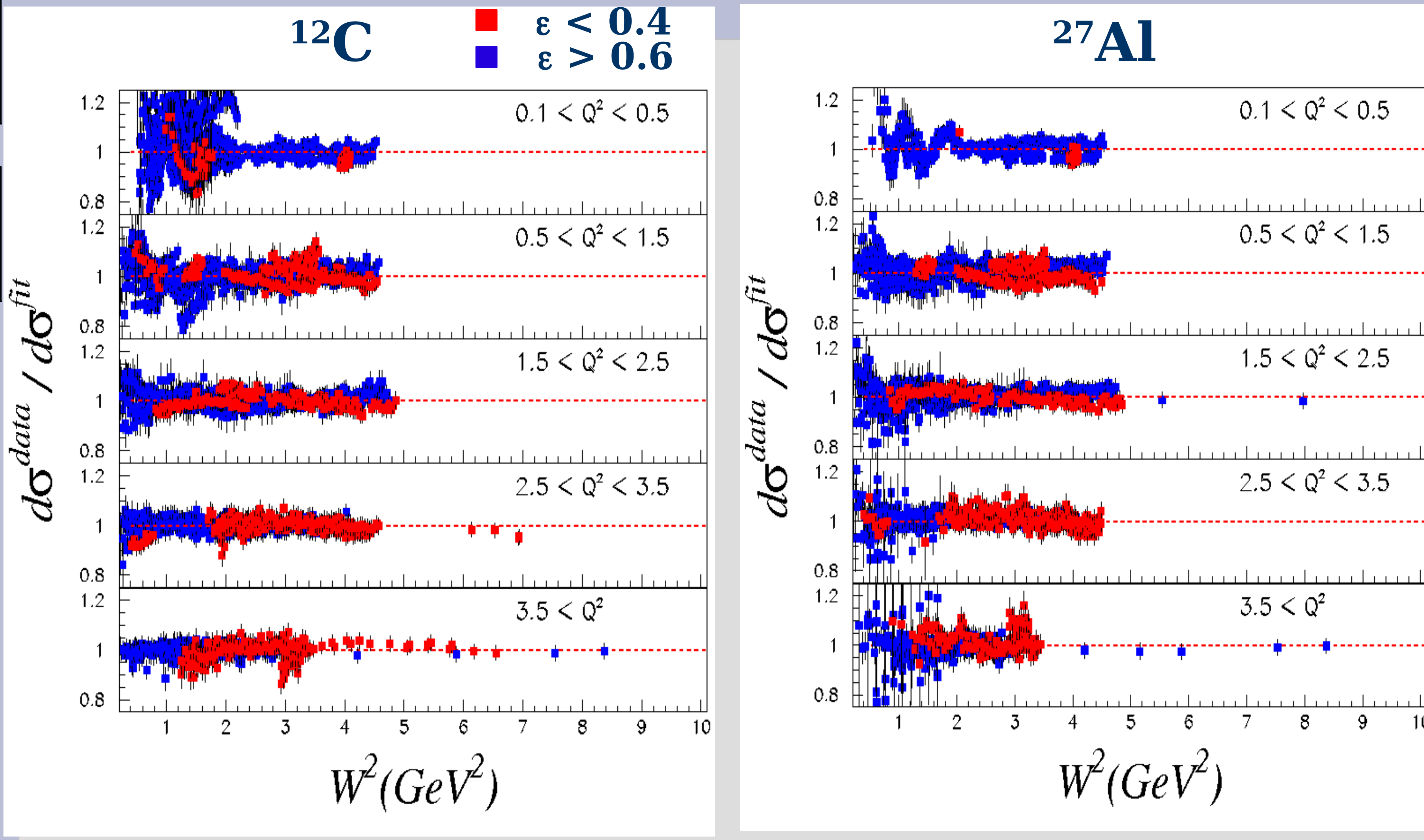
New inclusive cross section Fit for nuclei

- → **Include world cross section data set including quasielastic data from**

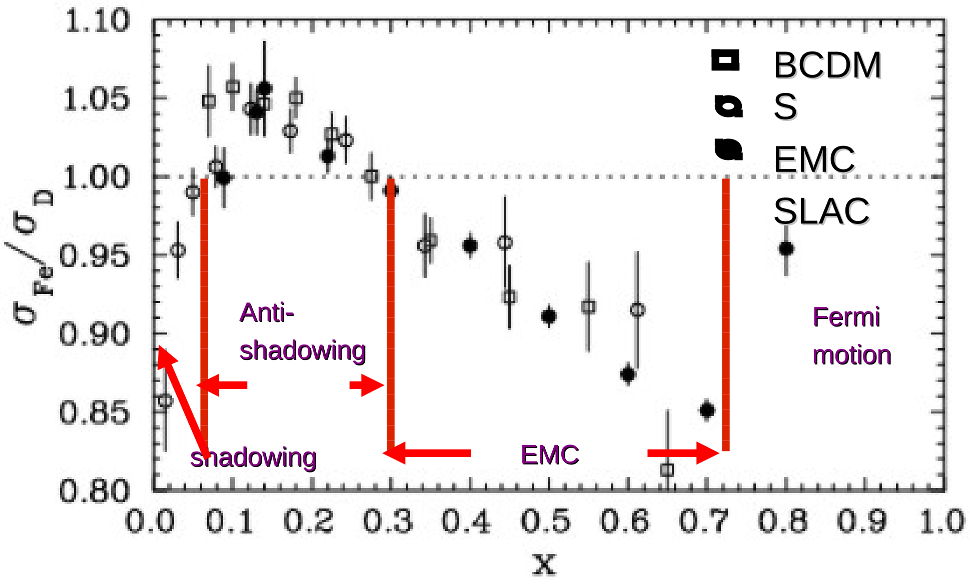
  Donal Day QE archive at http://faculty.virginia.edu/qes-archive/index.html

- → **Include Coulomb Corrections via effective momentum Approx.**
  A. Aste et. al 2005
- → **Update nucleon electromagnetic Form Factor parameterizations.**
- → **use new fits to proton and neutron inelastic structure functions as input**
- → **use Gaussian smearing from Bosted-Mamyan**
- → **Apply Pauli blocking**
- → **Include medium modification parameterization at nucleon level**
- → **Allow normalization factors for each data set with χ^2 penalty.**
- → **Allow for optimization of QE superscaling function**

Data / Fit Ratios



Medium Modification from fit



→ Included in the fit at the nucleon level (before smearing)

→ independent longitudinal and transverse modification

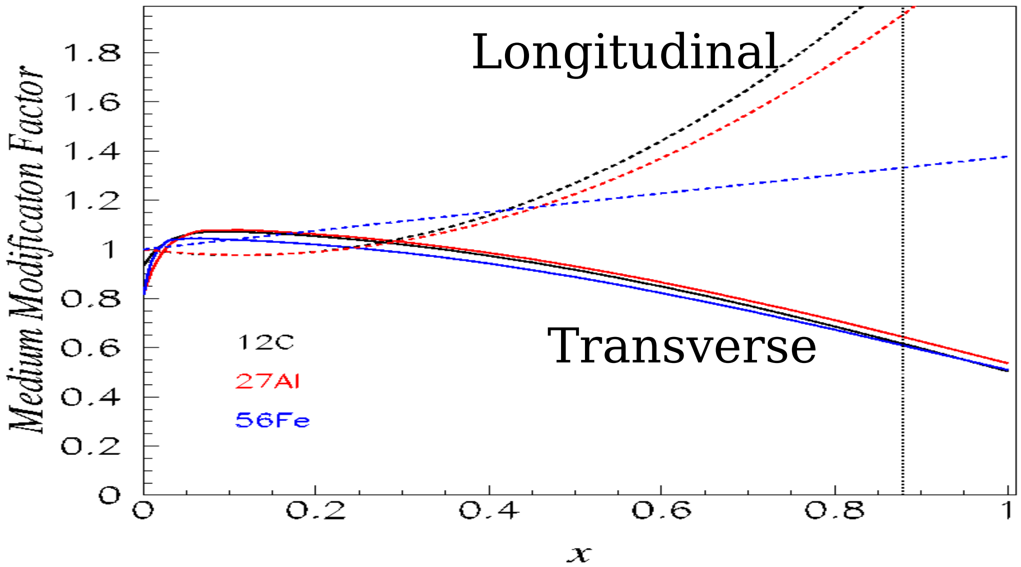
=> Allows for consistent integration into neutrino Monte Carlos without double counting (eg. fermi rise)

→ x-dependent modification works Reasonably well for both DIS and resonance region.

=> indication of duality in both longitudinal and transverse separated nuclear structure functions.

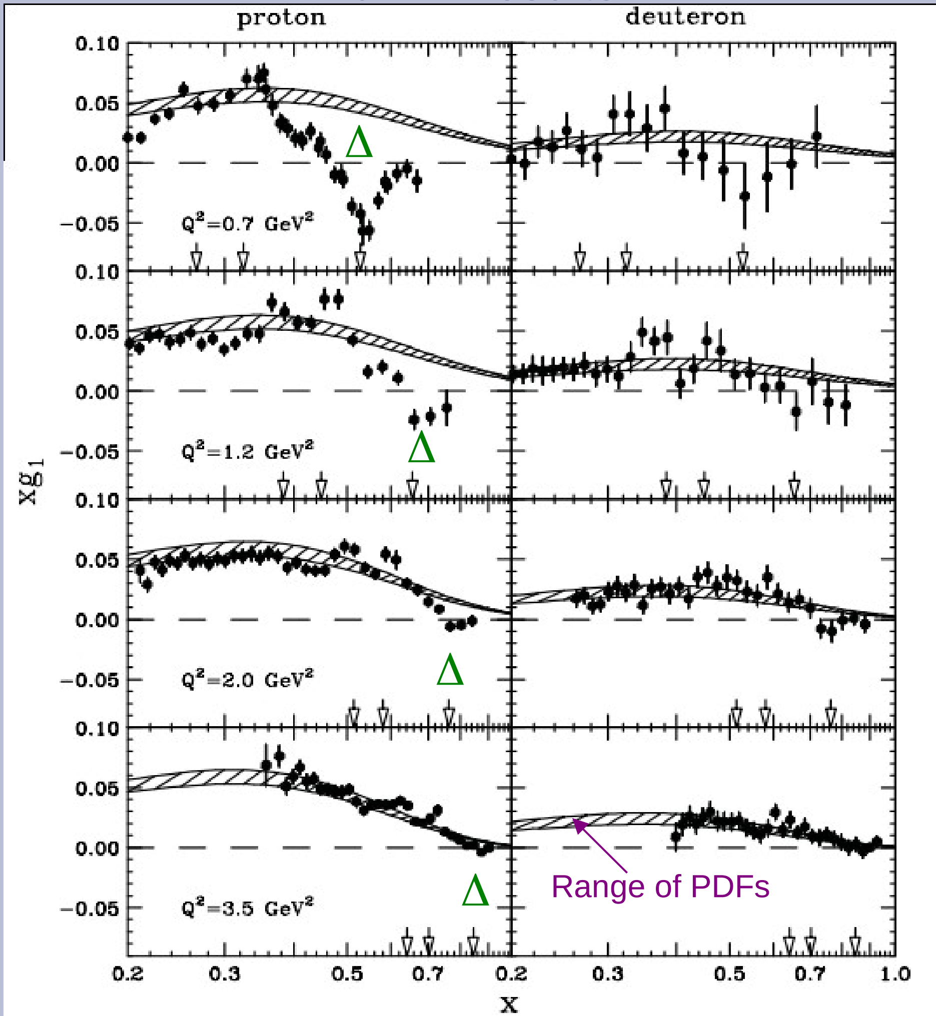
→ uncertainties not yet evaluated and will be significant in longitudinal modification.

Preliminary fit results



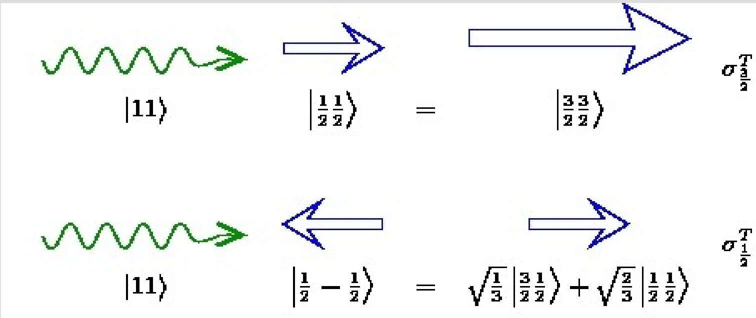
Duality in Spin Structure Functions

Hall B Results



P.E. Bosted, *et.al.* PRC 75, 035203 (2007)

Helicity asymmetries from polarized target



$g_1 \;\alpha\; (\sigma_{1/2} - \sigma_{3/2})$

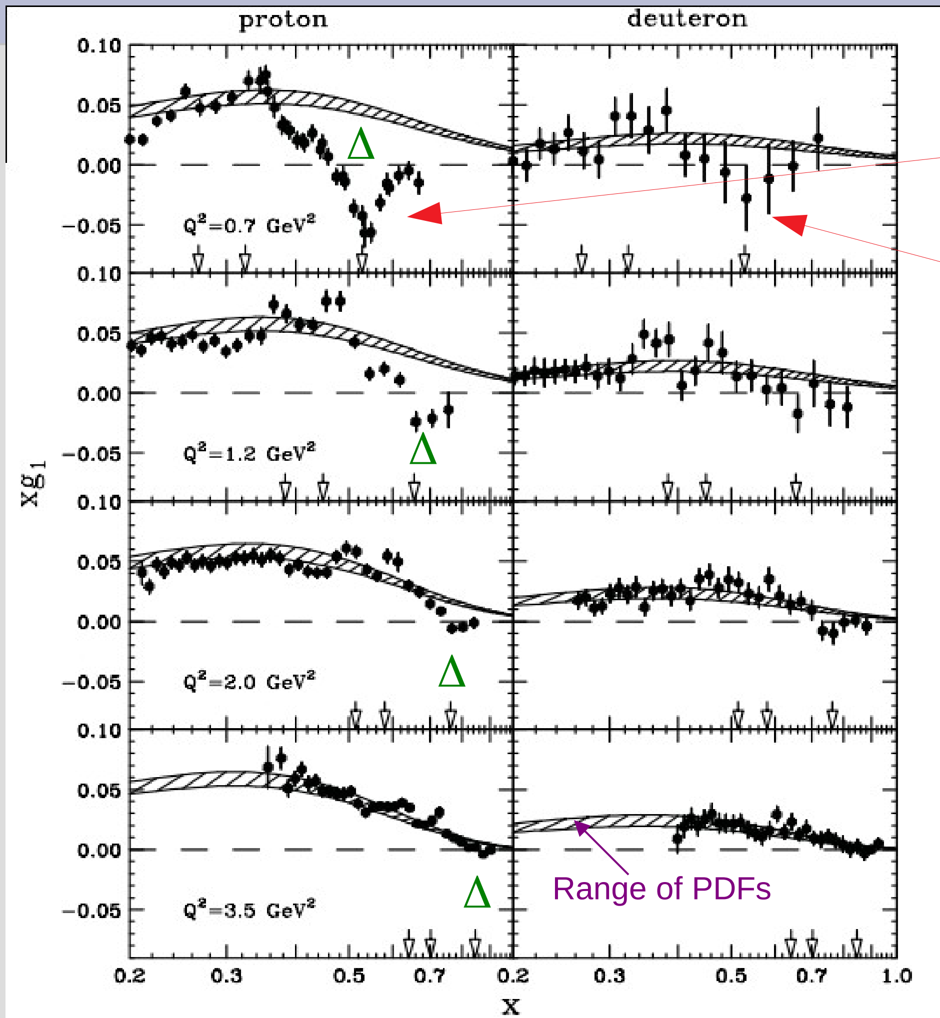
Sensitive to helicity of target particle

$F_1 \;\alpha\; (\sigma_{1/2} + \sigma_{3/2})$

Duality in both => duality in ***each*** helicity state.

Works well for $W > W_\Delta$

Some comments on duality in the Δ



Duality in Δ is broken in g_1

→ not the same in ½ vs 3/2 helicity amplitudes

→ duality in deuteron better than proton

At low Q^2 Δ is dominated by spin flip

of single quark =>

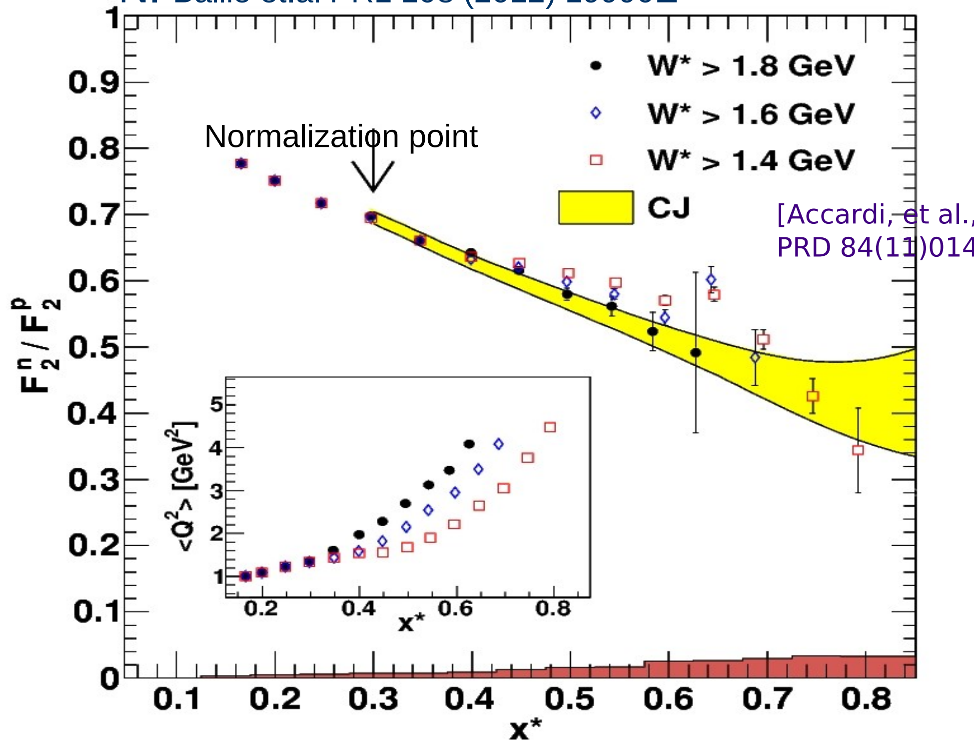
$$g_1 \;\alpha\; (\sigma_{1/2} - \sigma_{3/2})$$

- only $A_{3/2}$

=> g_1 **has to be negative**

Some comments on duality in the Δ

N. Ballie et.al PRL 108 (2012) 199902



Duality in Δ

Transition FF: n ~ p

DIS at large x: n << p

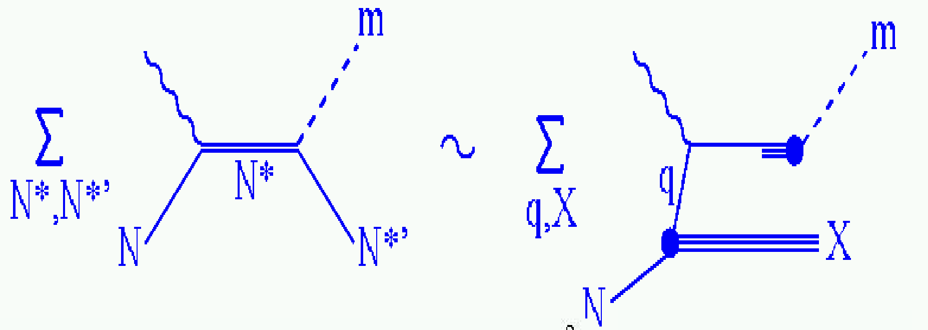
If duality holds for total: res+nonres

=> res / nonres ratio is very different

Between proton and neutron

Duality in semi-inclusive pion production

hadronic description quark-gluon



$$\sum_{N'^*}\left|\sum_{N^*} F_{\gamma^* N\to N^*}(Q^2,W^2)\,\mathcal{D}_{N^*\to N'^* M}(W^2,W'^2)\right|^2 = \sum_q e_q^2\, q(x)\, D_{q\to M}(z)$$

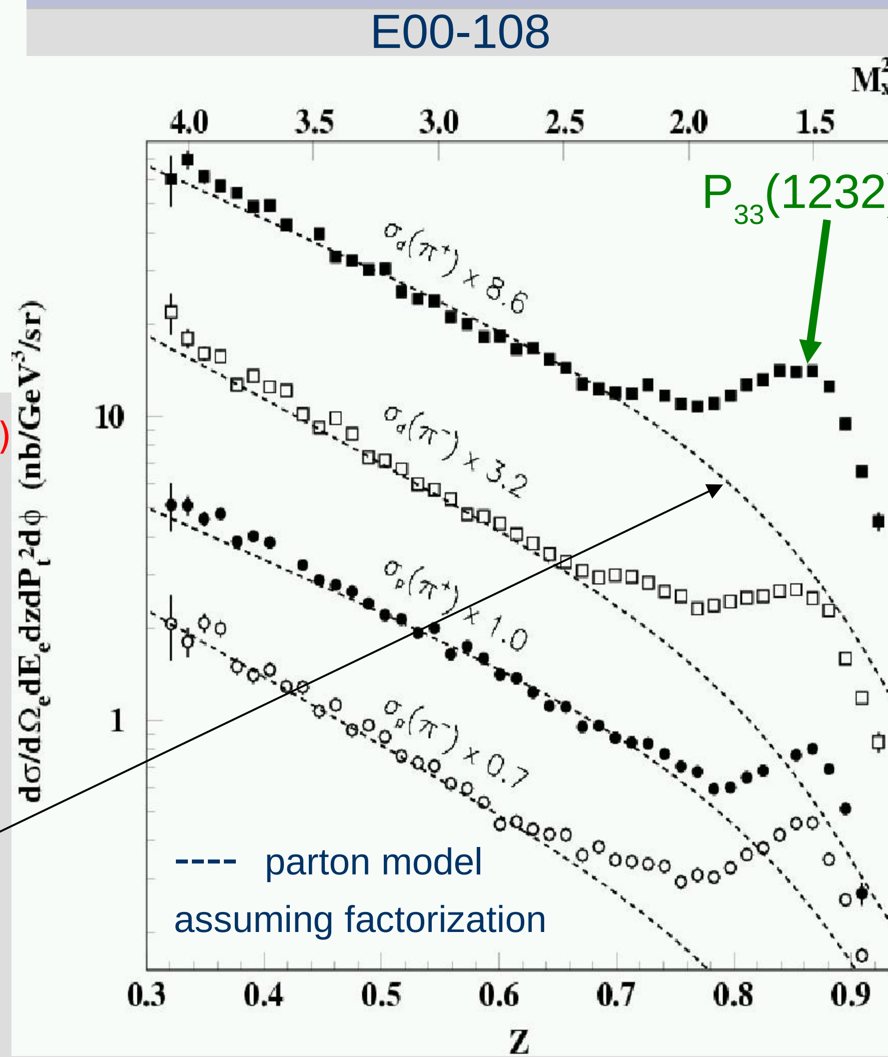
Transition Form Factor

Decay Amplitude

Quark distribution q(x) X Fragmentation Function. D(z)

$z = E_\pi / \nu$ is fractional energy carried by pion

Parton model using fragmentation functions from DIS generally describes data well away from Δ.



Verifying Factorization?

Neglect sea quarks and assume no k_t dependence to parton distribution functions

E00-108

→ Fragmentation function dependence drops out in Leading Order

→

$[\sigma_p(\pi^+) + \sigma_p(\pi^-)]/[\sigma_d(\pi^+) + \sigma_d(\pi^-)]$
$= [4u(x) + d(x)]/[5(u(x) + d(x))]$

$\sim \sigma_p/\sigma_d$ *independent of z and k_t*

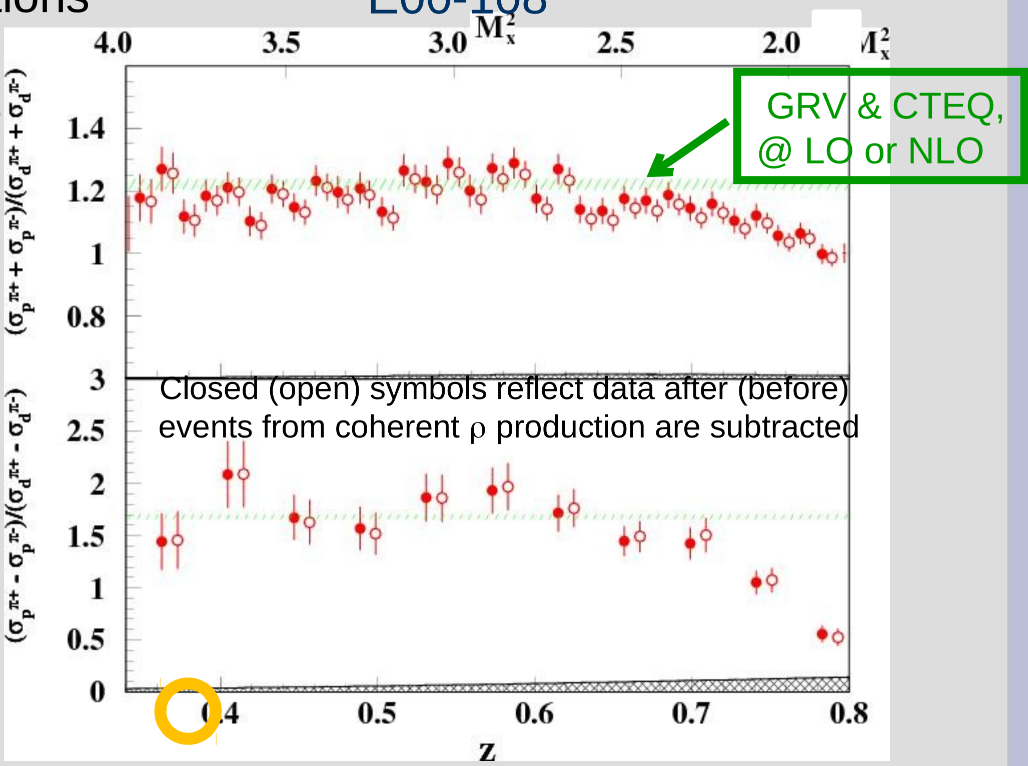
→

$[\sigma_p(\pi^+) - \sigma_p(\pi^-)]/[\sigma_d(\pi^+) - \sigma_d(\pi^-)]$

$= [4u(x) - d(x)]/[3(u(x) + d(x))]$

independent of z and k_t,

but more sensitive to assumptions



Good description for p and d targets for 0.4 < z < 0.65

(Note: z = 0.65 ~ M_x^2 = 2.5 GeV^2)

Duality and scaling

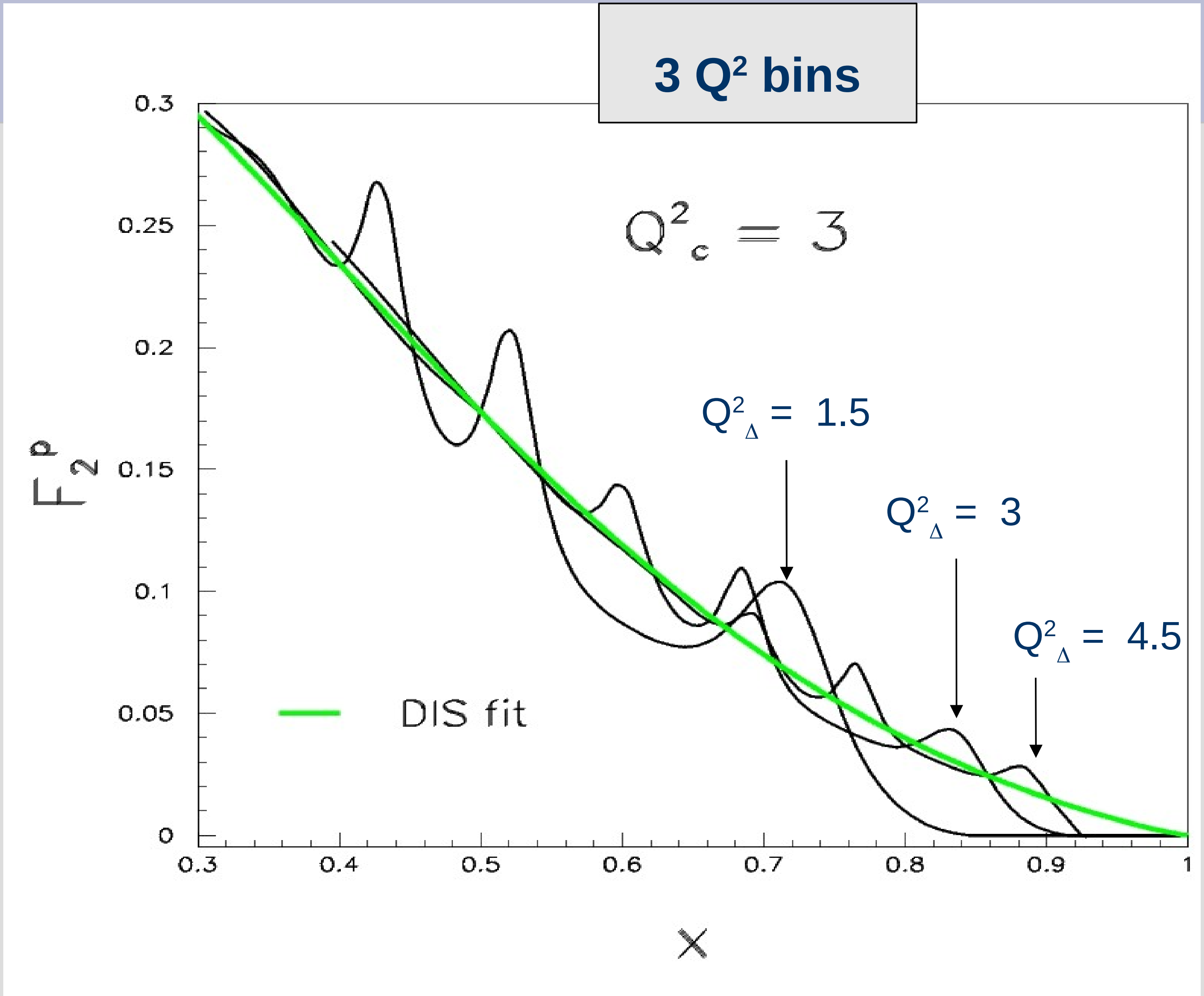
What does it mean?

Resonances have same Q^2 dependence as scaling curve.

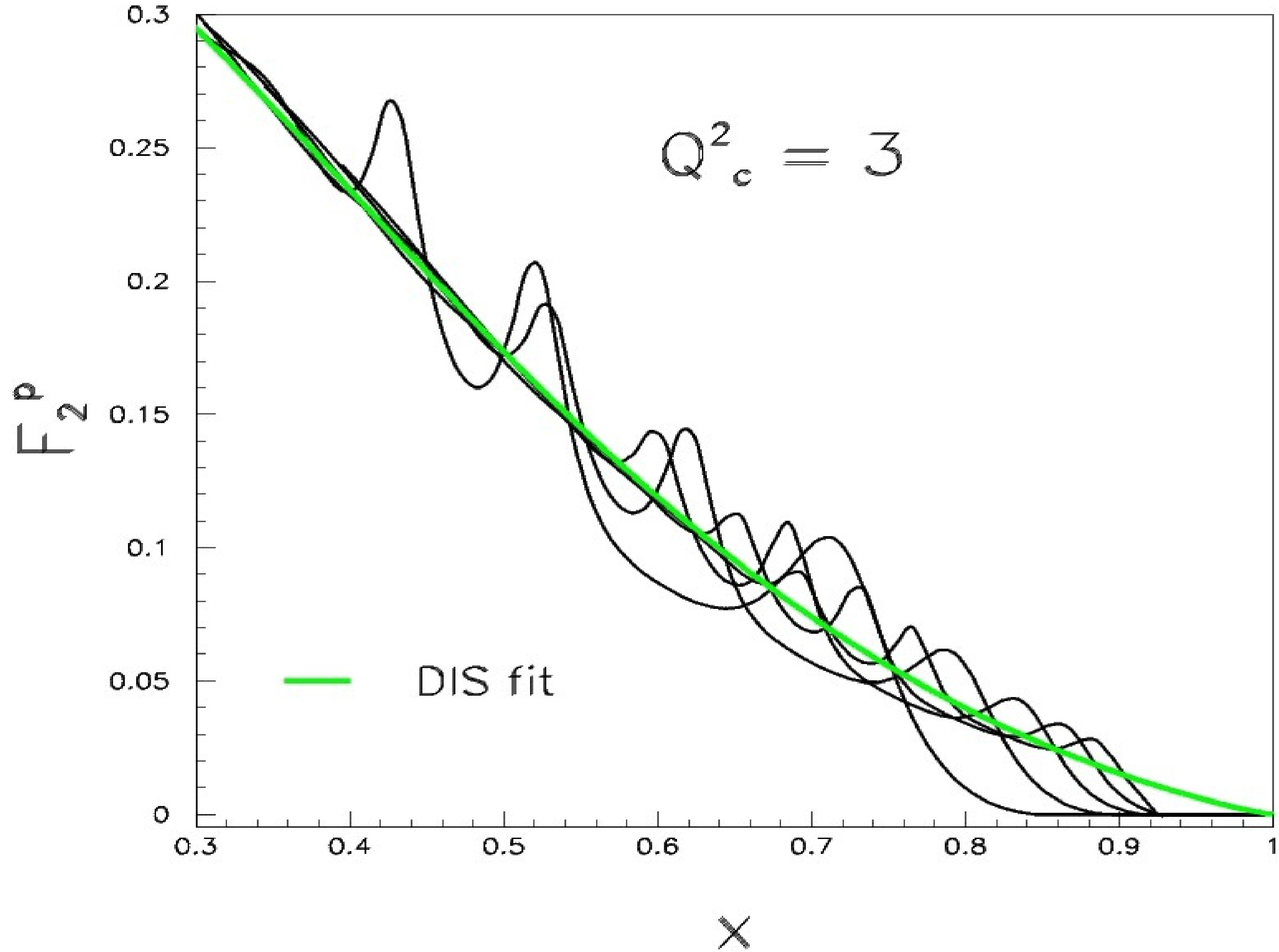
But what scaling curve?

A pure pQCD curve or that defined by data (LT + TM + HT)?

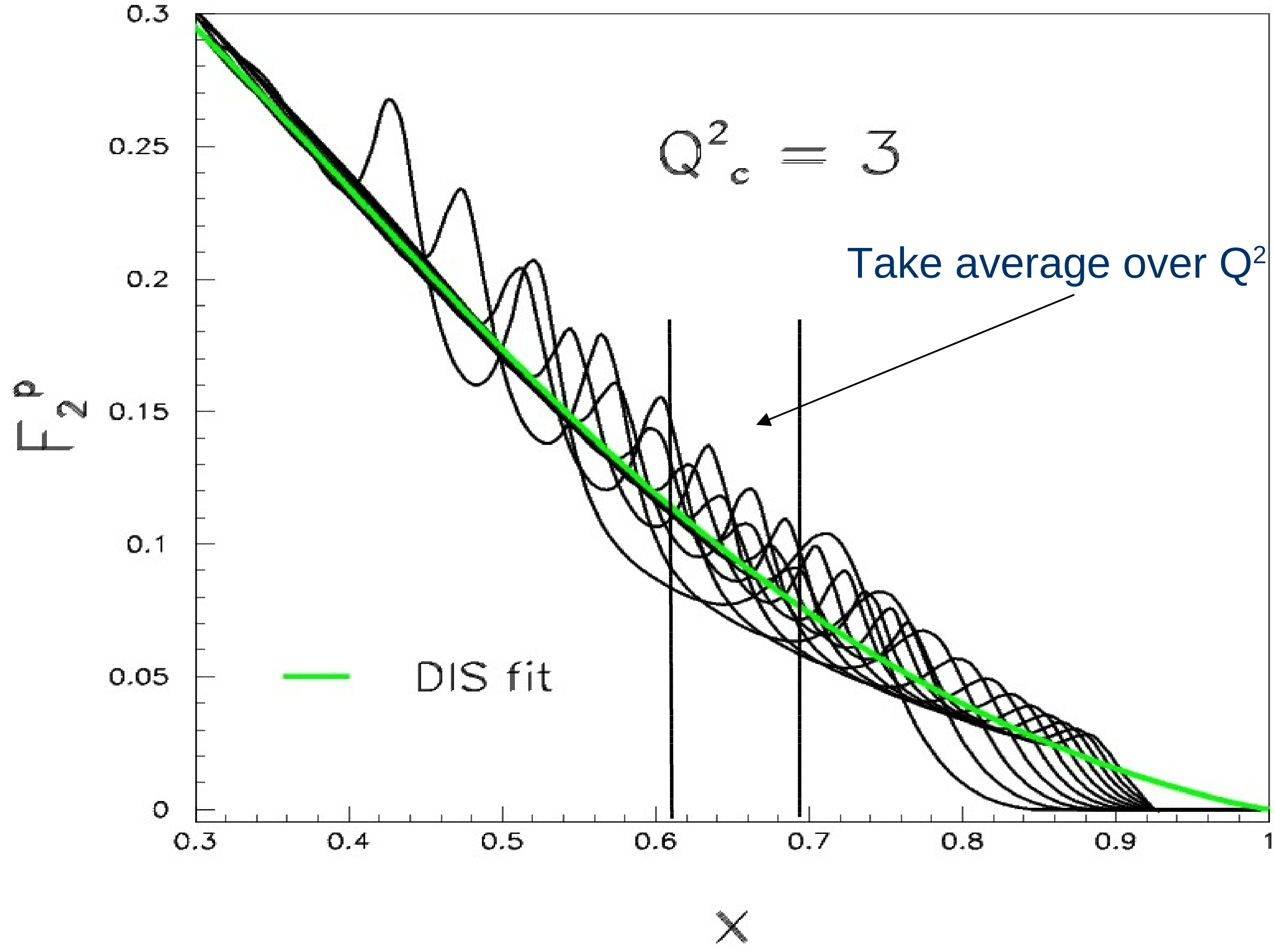
'DIS-like' duality averaging procedure



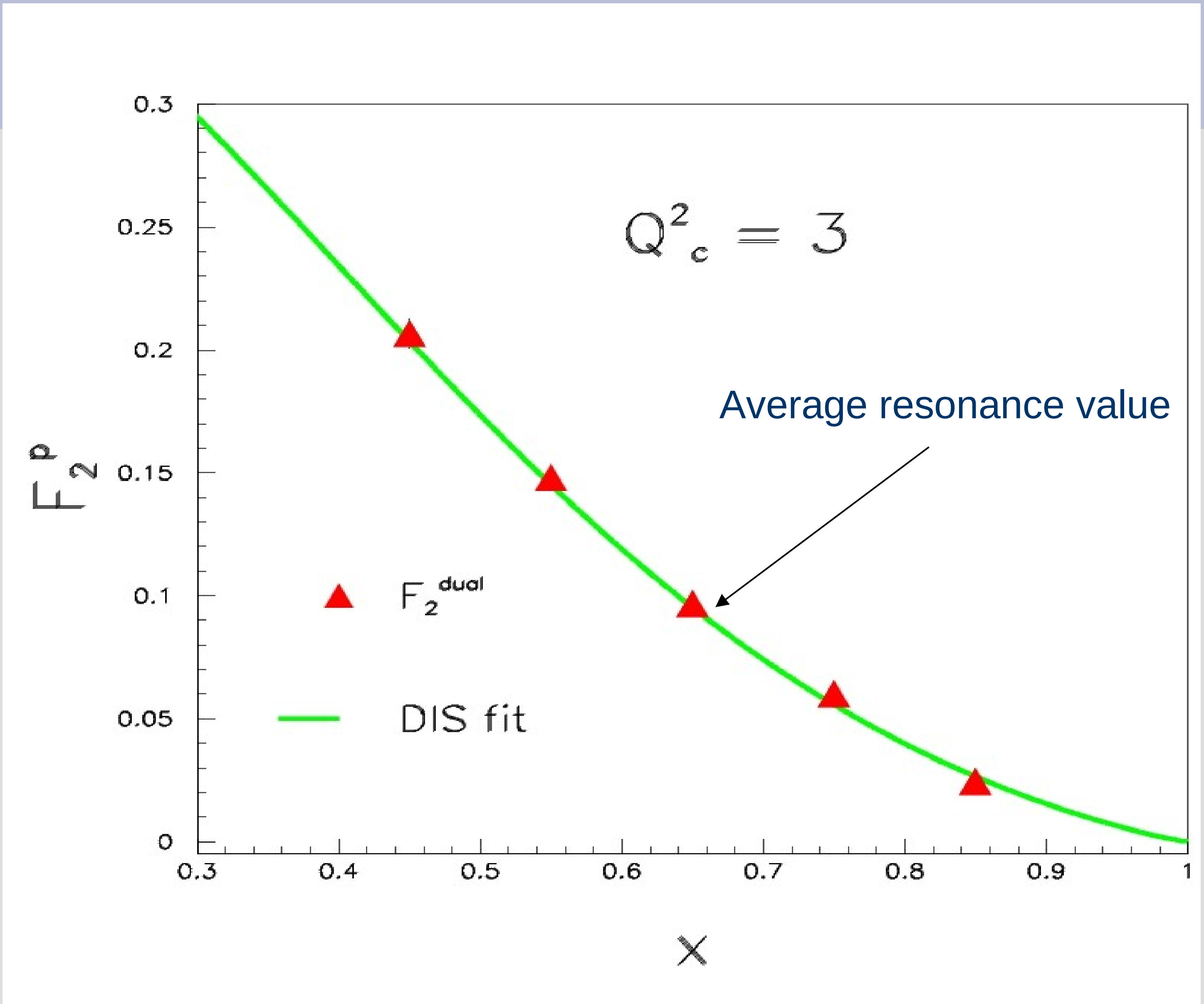
5 Q^2 bins



9 Q^2 bins

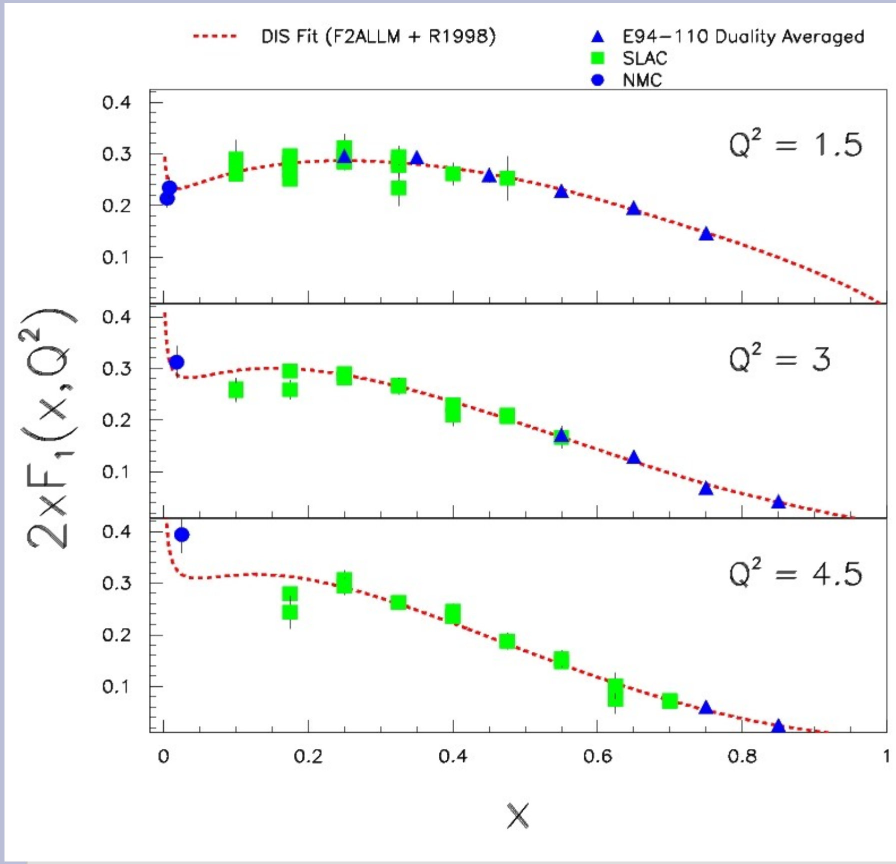


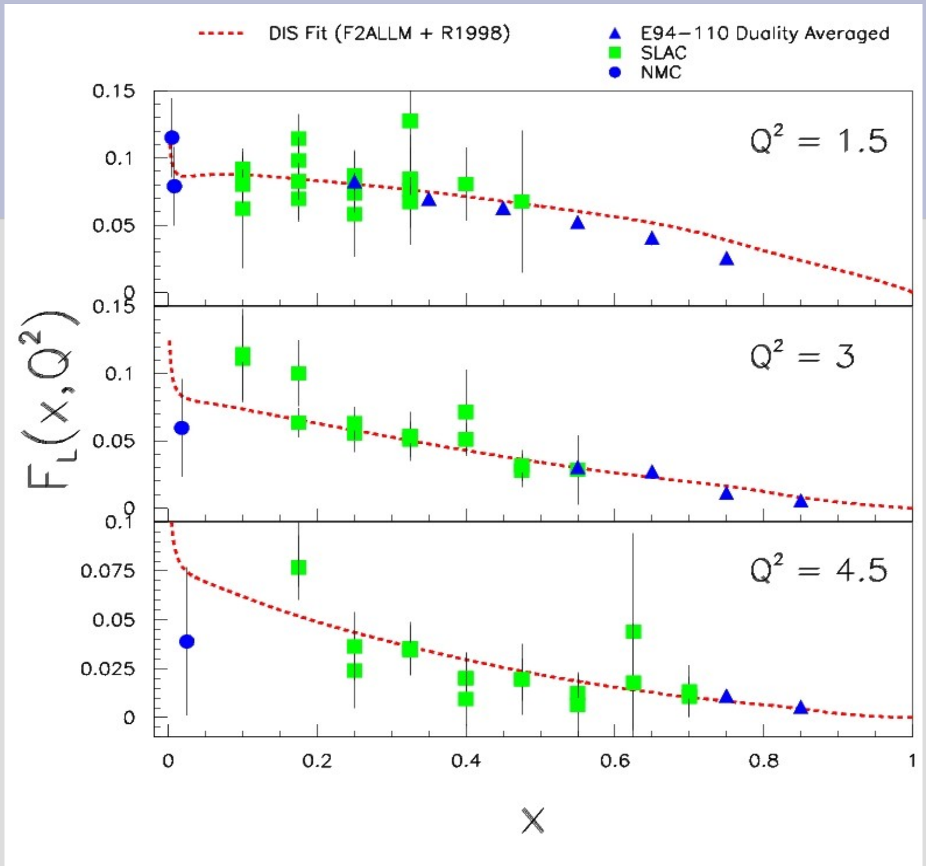
0.3
0.25
0.2
0.15
0.1
0.05
0
F_2^p
$Q^2_c = 3$
Take average over Q^2
DIS fit
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
X



$Q^2_c = 3$
Average resonance value
F_2^{dual}
DIS fit
F_2^p
x
0.3
0.25
0.2
0.15
0.1
0.05
0
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Duality averaging results for low Q^2 proton data





- **Good consistency with DIS and relatively smooth *x* dependence.**
- **Note different Q^2 dependence in averaged F_L from fit at lowest Q^2.**

Summary

- **Quark-hadron duality is a non-trivial property of QCD**
  → Soft-Hard Transition!
- **Duality has been shown to hold in many observables thus far, including:**

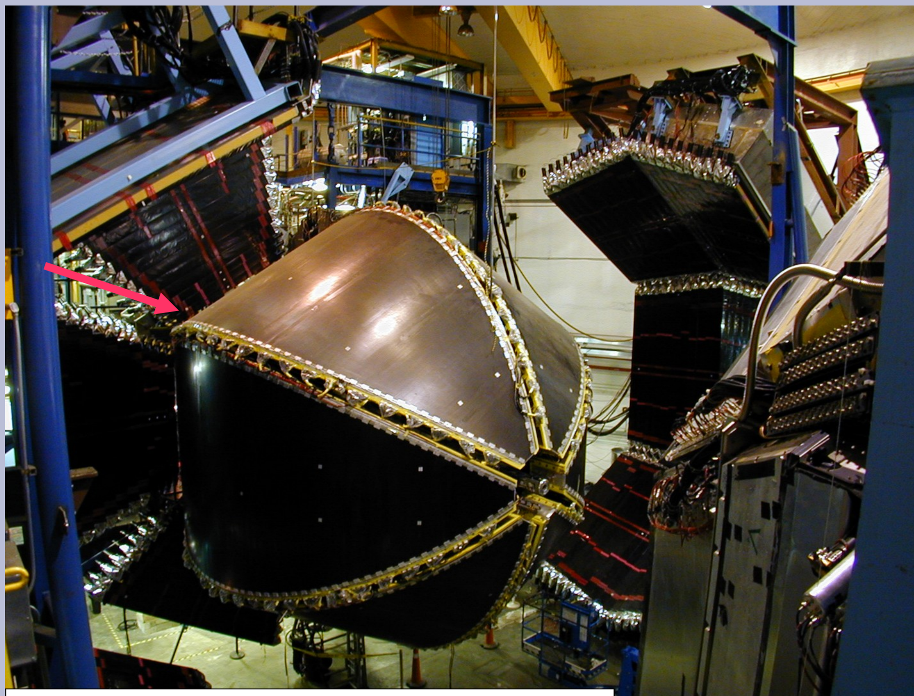
1. **All unpolarized structure functions (including Nuclei)**
2. **Polarized structure functions**
3. **Semi-inclusive**

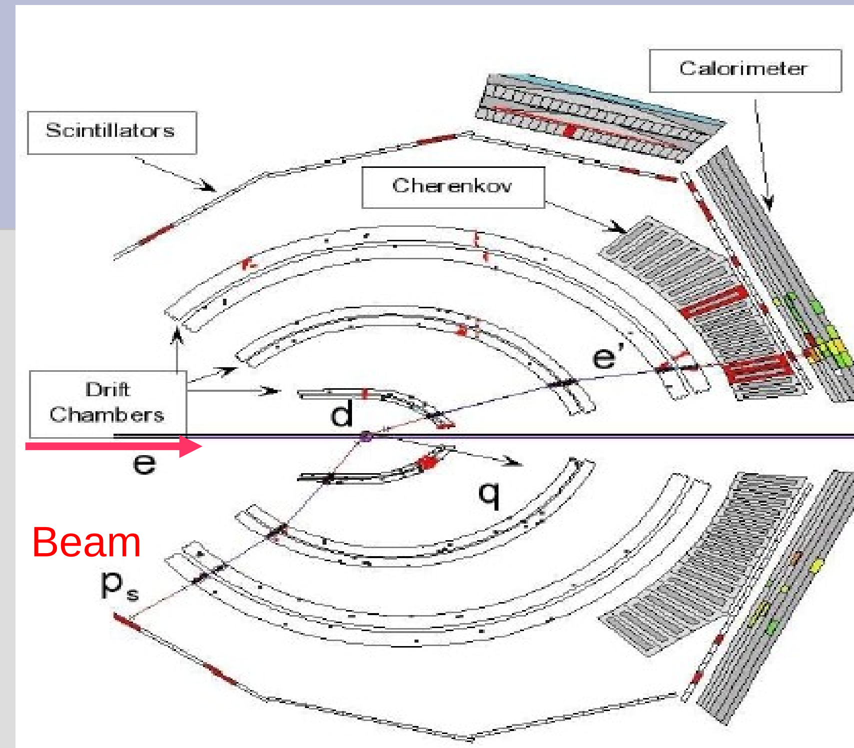
- **Models are being confronted with new data, including *free neutron***

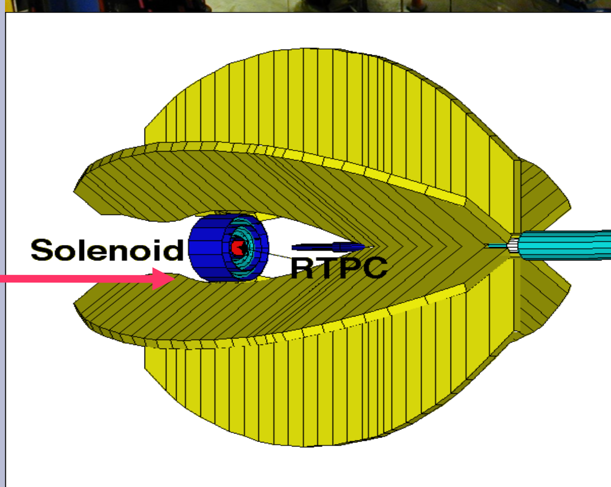
- **More experimental results are coming:**
  - → **E04-001 (L/T separated nuclear Sfs from 0.05 < Q2 < 4.5)**
  - → **Higher Q^2 data on p, d, and EMC effect from Hall C**

Extras

BoNuS Experiment

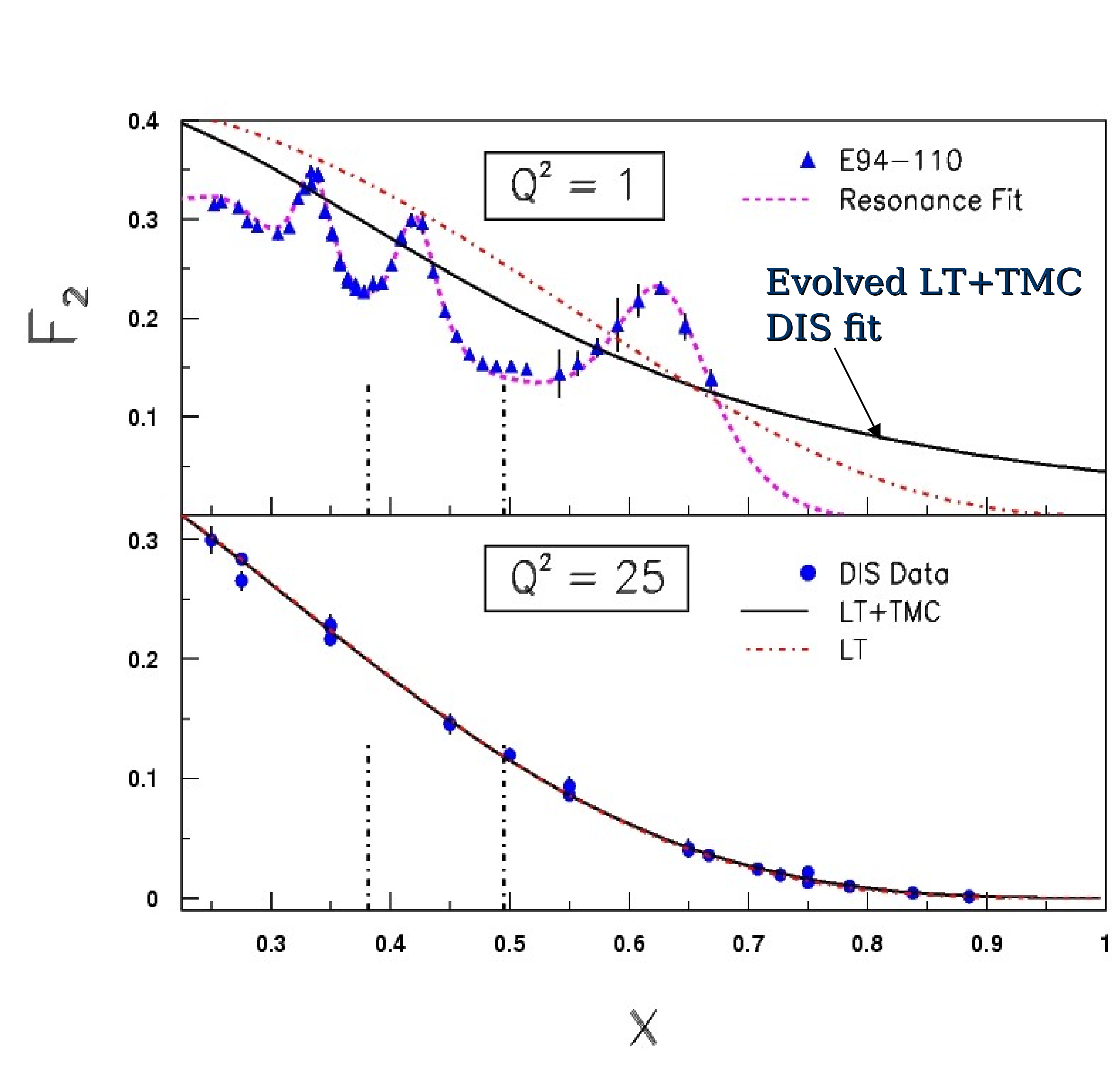






- Detect electrons in CLAS Spectrometer in Hall B
- Detect slow protons in radial time projection chamber (RTPC)
- Moller electrons bottled up by Solenoid field around target
- Solenoid field allows momentum determination

Truncated Moments - the basic idea



→ Compare integral over Select resonance regions to Evolved scaling curve + TM

Duality looks like it might be fundamental to QCD

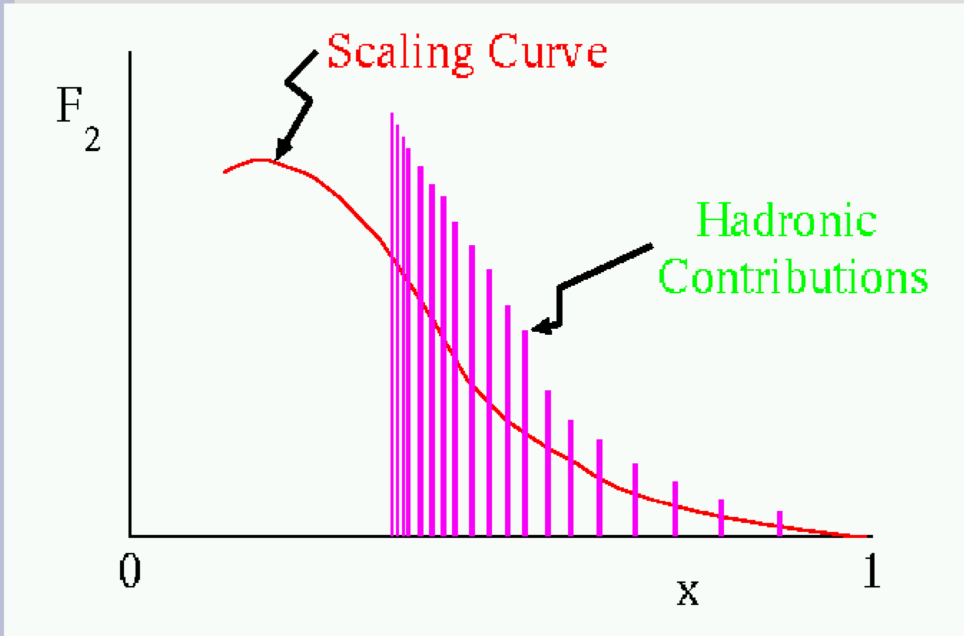
… but how do we understand it?

Theoretical progress has been made based on constituent quark models.

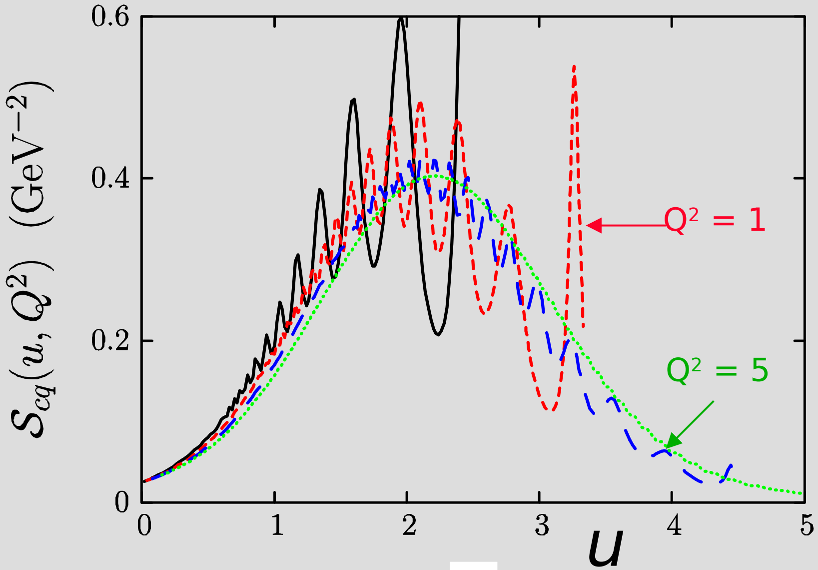
Close-Isgur Model: General observations

N. Isgur et al : $N_c \rightarrow \infty$

$q\bar{q}$ infinitely narrow resonances



One heavy quark, Relativistic HO

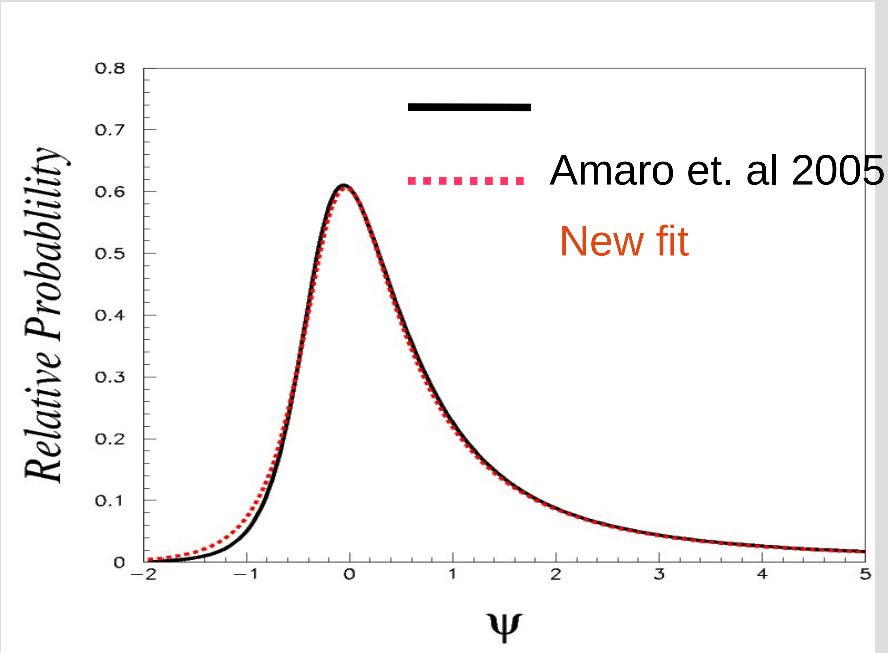


Scaling occurs rapidly!

- Illustrates how sum of resonance states can lead to scaling curve based on general properties of QCD
- Quark -Hadron Duality must be invoked even in the Bjorken Scaling region

12C Fit Results:

Updated ψ scaling distribution from fit



Amaro et. Al (2005)

$$F(\psi') = \frac{1.3429}{k_F}[1 + 1.7119^2(\psi' + 0.19525)^2](1 + e^{-1.69\psi'}) \quad (1)$$

fit using same functional form

$$F(\psi') = \frac{1.5576}{k_F}[1 + 1.7720^2(\psi' + 0.3014)^2)](1 + e^{-2.4291\psi'}) \quad (2)$$

Normalization of data sets (preliminary)

1.	Barreau	(1983)	0.979
2.	O'Connel	(1987)	0.968
3.	Sealock	(1989)	1.036
4.	Baran	(1988)	0.993
5.	Bagdasaryan	(1988)	0.983
6.	Zelllar	(1973)	Inconsistent
7.	Arrington	(1995)	0.975
8.	Day	(1993)	1.006
9.	Arrington	(1998)	0.985
10.	Gaskell	(2008)	0.992
11.	Whitney	(1974)	0.992
12.	E04-001 prelim low Q^2		0.996
13.	E04-001	high Q^2	1.000 (fixed)
14.	SLAC E139		1.012
15.	Fomin		1.006

→ Normalize included as penalty term on χ^2

→ Only Zeller data found to have large Inconsistencies.

Dynamical model of Close/Isgur PLB 509, 81 (2001)

→ Coupling to single quarks in baryon states in spin-flavor SU(6) model.

→ $F_2 \sim \Sigma e^2_q$ but Form factors ~ $(\Sigma e_q)^2$ How does square of sum become sum of squares

→ Need enough even and odd parity states for $\sim e_i e_j$ terms to cancel

N,Δ 2nd

$SU(6)$:	$[\mathbf{56},0^+]^2\mathbf{8}$	$[\mathbf{56},0^+]^4\mathbf{10}$	$[\mathbf{70},1^-]^2\mathbf{8}$	$[\mathbf{70},1^-]^4\mathbf{8}$	$[\mathbf{70},1^-]^2\mathbf{10}$	*total*
F_1^p	9	8	9	0	1	27
F_1^n	4	8	1	4	1	18

→ Similar calculations now available for semi-inclusive

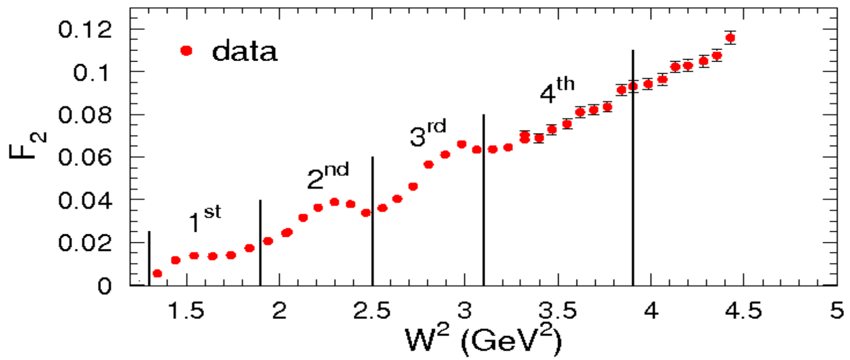
→ Duality is due to fortuitous cancellations in this model !!!

→ Duality obtained by end of second resonance region for proton, later for neutron => local duality different in neutron

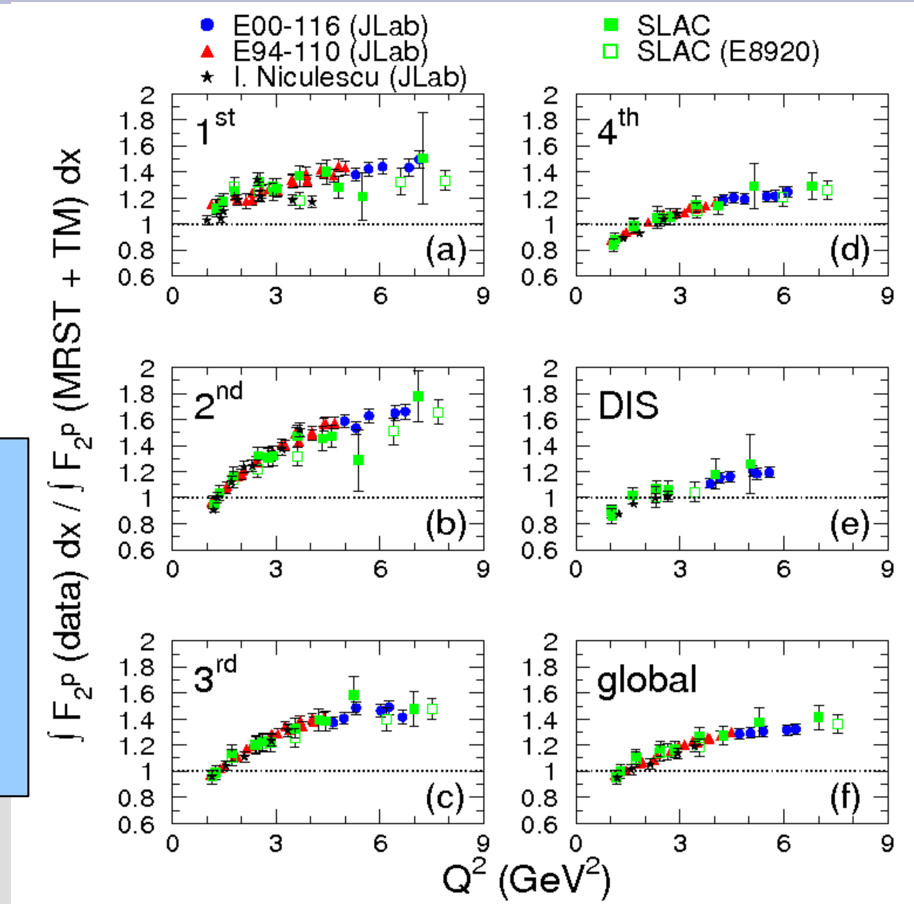
** Would like to test this with data **

Local Duality Quantification - I

S.P. Malace *et al.*, Phys. Rev. C 80 035207 (2009)



$$I = \frac{\int_{x_{min}}^{x_{max}} F_2^{data}(x,Q^2)dx}{\int_{x_{min}}^{x_{max}} F_2^{param.}(x,Q^2)dx}$$

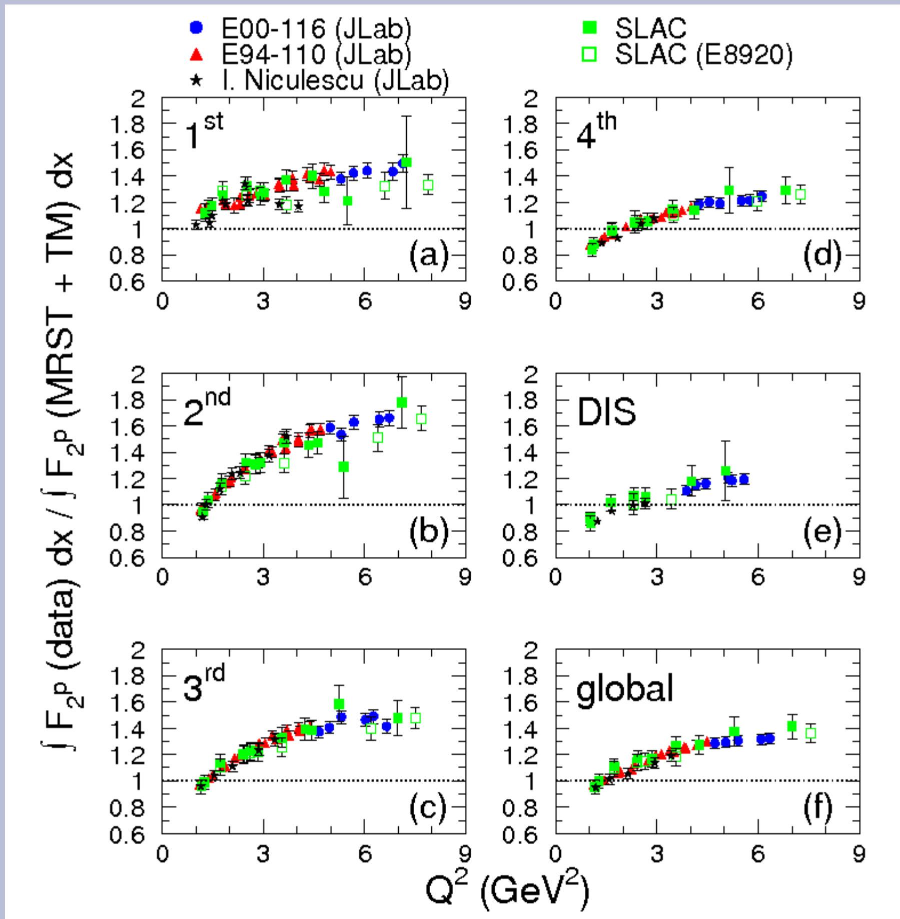


→ Data in all regions rise above PDF curve for $Q^2 > \sim 2$

→ largest for lower resonances which are at large *x*, where PDFs are well constrained.

Quantification - II

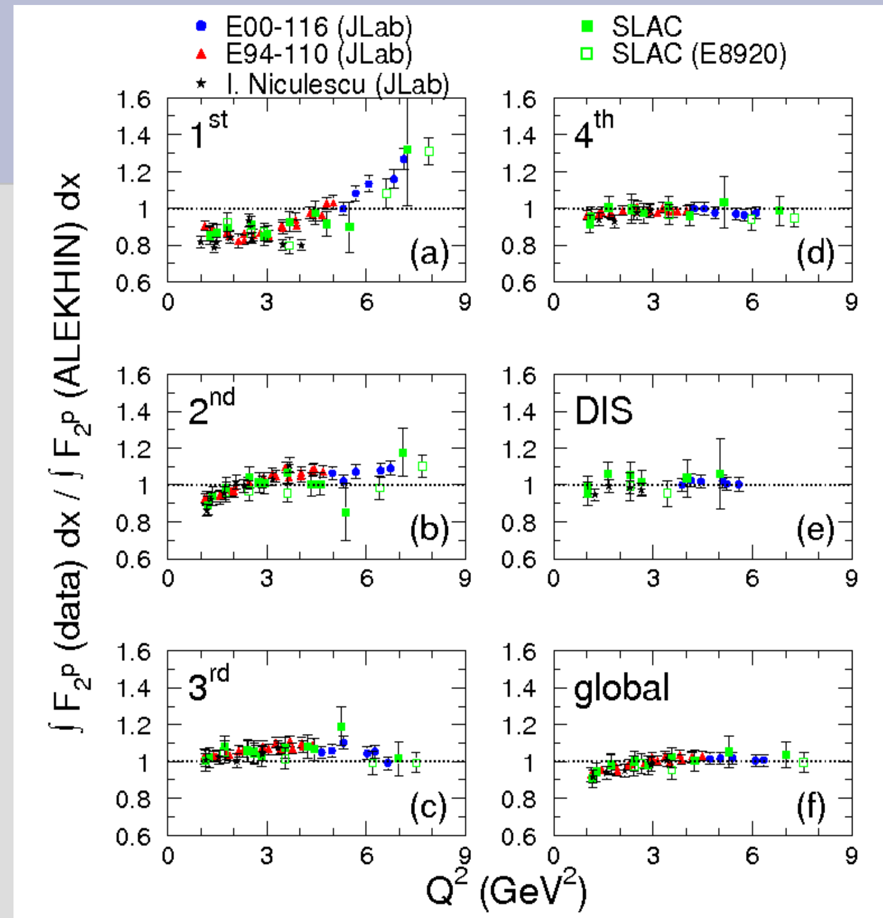
S.P. Malace *et al.*, Phys. Rev. C 80 035207 (2009)



MRST2004

→ tighter kinematic cuts excludes much large x

→ No TMC or HT included in fit.



Alehkin

→ looser kinematic cuts

→ TMC and HT included in fits.

→ **Older PDFs not enough strength at large *x***

=> ***looks*** **like larger duality violations (20-30%).**

→ **Not as much a failure of duality, but unconstrained PDFs at large *x***

→ **New efforts to relax kinematic constraints and include TMCs and HTs in PDF fits result in much smaller duality violations observed (< 10%, except at Δ(1232)).**

=> **telling us that *on average* resonance region H-T are the same as the DIS.**

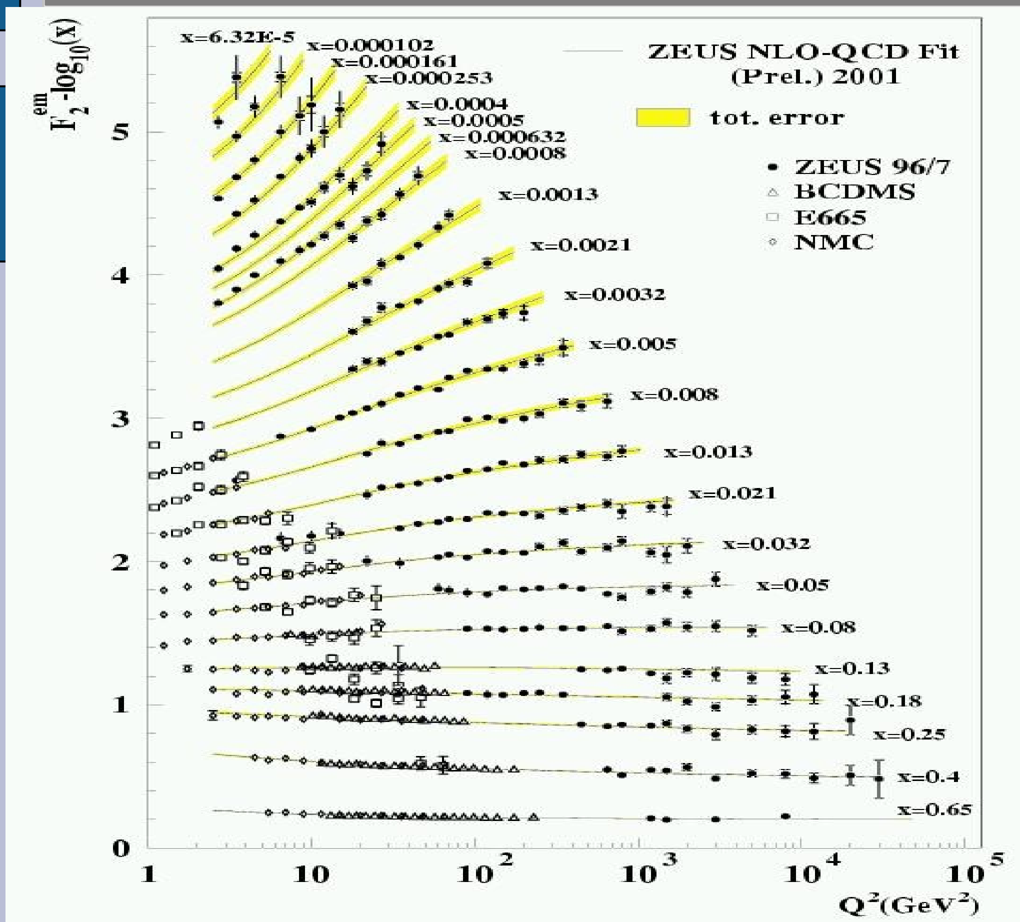
CTEQ6x

S. Alekhin, J. Blumlein, S. Klein, S. Moch, Phys. Rev. D 81, 014032 (2010).

Accardi, E.C, Keppel, Melnitchouk, Monaghan, Morfín, Owens, Phys. Rev. D 81, 034016 (2010).

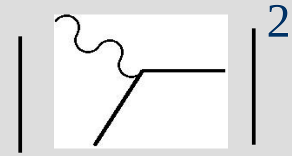
Separation of scale => Q^2 dependence of DIS structure functions governed by perturbative QCD

Scaling in F_2 measured to high precision over many orders of magnitude in *x* and Q^2,

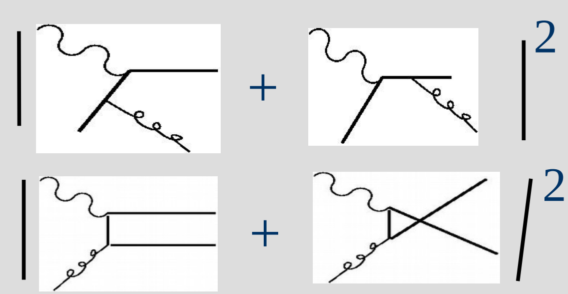


Single quark scattering (leading twist)

$$F_2(x,Q^2) = x\,\Sigma\, e_q^{\,2}\, \mathbf{q}(x,Q^2)$$



Where the $\mathbf{q}(x,Q^2)$ evolve via pQCD.
Order $\alpha_s(Q^2)$ corrections



but additional contributions at finite Q^2, e.g.

Kinematic 'Target Mass' Corrections':

Fractional nucleon momentum carried by the struck quark away from Bjorken limit

$$\xi = 2x/(1+r)$$

With

$$r = 1 + \nu^2/Q^2 = \sqrt{1 + \frac{4M^2x^2}{Q^2}}$$

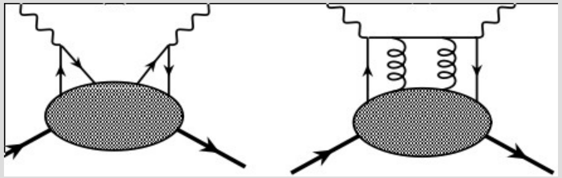
Note that $\xi \to x$ for $Q^2 \to \infty$ (or $M \to 0$) at fixed x

$$F_2^{TM}(x,Q^2) = \frac{x^2}{r^3}\frac{F_2^{(0)}(\xi,Q^2)}{\xi^2} + 6\frac{M^2}{Q^2}\frac{x^3}{r^4}\int_\xi^1 dx' \frac{F_2^{(0)}(x',Q^2)}{x'^2} + 12\frac{M^4}{Q^4}\frac{x^4}{r^5}\int_\xi^1 dx' \int_{x'}^1 dx'' \frac{F_2^{(0)}(x'',Q^2)}{x''^2}$$

'Massless' limit

Higher Twist contributions (H-T)':

Quark-Quark correlations: eg. gluon exchange between struck and spectator quarks.



Truncated Moments

Originally developed to address lack of low x data

Forte and Magnea, PLB 448, 295 (1999); Forte, Magnea, Piccione, and Ridolfi, NPB 594, 46 (2001); Piccione PLB 518, 207 (2001); Kotlorz and Kotlorz, PLB 644, 284 (2007).

Idea: construct doubly truncated moments from

$$\overline{M}_n(\Delta x, Q^2) = \int_{\Delta x} dx\; x^{n-2}\; F_2(x, Q^2)$$

Truncated moments follow ***DGLAP-like evolution*** equations.

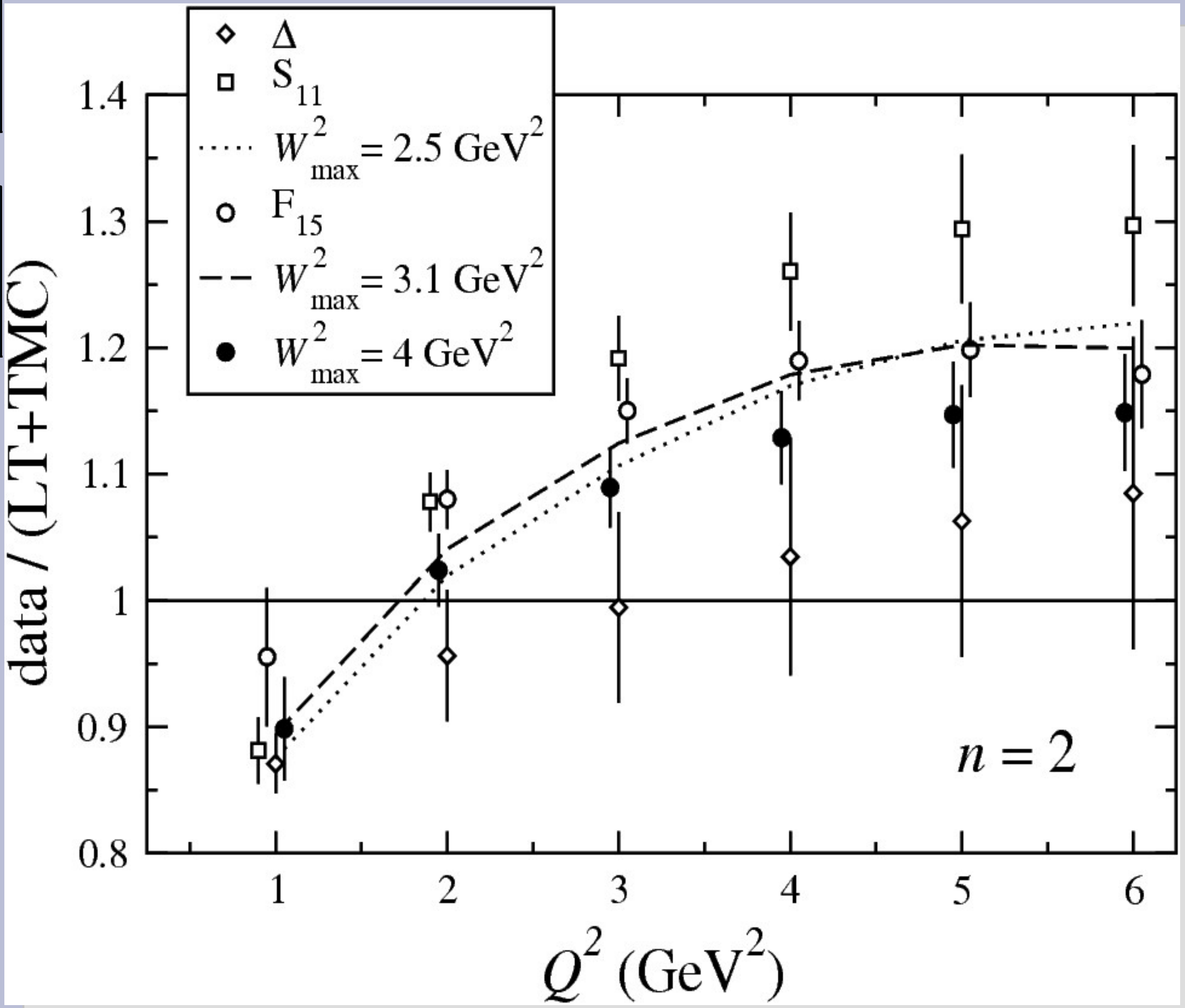
$$\frac{d\overline{M}_n(\Delta x, Q^2)}{d \log Q^2} = \frac{\alpha_s}{2\pi}\left(P'_{(n)} \otimes \overline{M}_n\right)(\Delta x, Q^2)$$

With modified splitting functions given by

$$P'_n(z, \alpha_S(Q^2)) = z^n P(z, \alpha_S(Q^2))$$

Allows study of ***regions in W*** within pQCD in well-defined, systematic way.

Q^2 Dependence of Truncated Moments, *x* Regions Defined by Resonances



- **Consider now individual and total resonance region**
- **Large Q^2 dependence below ~3 GeV^2 - decreases at higher Q^2**
- **Below Q^2 = 0.75 GeV^2 the applicability of pQCD analysis doubtful**
- **Facilitates careful Higher Twist analysis….**