Higher Twist and Duality in the SIS/DIS Transition Region

Huma Haider : AMU Group

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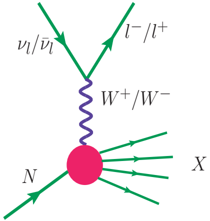
Outline

General process for the deep inelastic scattering is
$l(k) + N(p) \longrightarrow l'(k') + X(p'), \quad l, l' = e^{\pm}, \mu^{\pm}, \nu_l, \bar{\nu}_l, \; N = n, p$

Kinematic variables

- Incoming neutrino energy:
  $E_\nu = E_{had} + E_\mu$.
- Square of the four momentum transfer: $Q^2 = 4E_\nu E_\mu sin^2(\frac{\theta}{2})$.
- Invariant mass of the hadronic shower:
  $W = \sqrt{M_N^2 + 2E_{had}M_N - Q^2}$.
- Bjorken-x of the event: $x = \frac{Q^2}{2M_N E_{had}}$
- Inelasticity : $y = \frac{E_{had}}{E_\nu}$



Differential Scattering Cross Section

$\nu(\bar{\nu}) - N$ scattering The differential cross section:

$$\frac{d^2\sigma^{\nu(\bar{\nu})}}{dx\ dy} = \frac{G_F^2 M E_\nu}{\pi(1+Q^2/M_W^2)^2}\Bigg(\left[y^2 x + \frac{m_l^2 y}{2E_\nu M}\right] F_{1N}(x,Q^2) + \left[(1-\frac{m_l^2}{4E_\nu^2}) - (1+\frac{Mx}{2E_\nu})y\right] F_{2N}(x,Q^2) \pm \left[xy(1-\frac{y}{2}) - \frac{m_l^2 y}{4E_\nu M}\right] F_{3N}(x,Q^2)\Bigg)$$

Longitudinal Structure Function:

$$F_{LN} = \left(1+\frac{4M^2x^2}{Q^2}\right) F_{2N}(x,Q^2) - 2xF_{1N}(x,Q^2)$$

$$F_{1N}(x,Q^2) = \frac{1}{2x}\left[\left(1+\frac{4M^2x^2}{Q^2}\right) F_{2N}(x,Q^2) - F_{LN}(x,Q^2)\right]$$

For charged lepton scattering on protons

$$\begin{aligned} F_2^{lp}(x, Q^2) &= \frac{4}{9}x(u(x, Q^2) + \bar{u}(x, Q^2) + c(x, Q^2) + \bar{c}(x, Q^2)) \\ &\quad + \frac{1}{9}x(d(x, Q^2) + \bar{d}(x, Q^2) + s(x, Q^2) + \bar{s}(x, Q^2)) \end{aligned}$$

for a neutron target isospin swapping gives

$$\begin{aligned} F_2^{ln}(x, Q^2) &= \frac{4}{9}x(d(x, Q^2) + \bar{d}(x, Q^2) + c(x, Q^2) + \bar{c}(x, Q^2)) \\ &\quad + \frac{1}{9}x(u(x, Q^2) + \bar{u}(x, Q^2) + s(x, Q^2) + \bar{s}(x, Q^2)) \end{aligned}$$

For charged current anti(neutrino) scattering on protons:

$$\begin{aligned} F_2^{\nu p} &= 2x(d(x,Q^2) + s(x,Q^2) + \bar{u}(x,Q^2) + \bar{c}(x,Q^2)), \\ xF_3^{\nu p} &= 2x(d(x,Q^2) + s(x,Q^2) - \bar{u}(x,Q^2) - \bar{c}(x,Q^2)), \end{aligned}$$

$$\begin{aligned} F_2^{\bar{\nu} p} &= 2x(u(x,Q^2) + c(x,Q^2) + \bar{d}(x,Q^2) + \bar{s}(x,Q^2)), \\ xF_3^{\bar{\nu} p} &= 2x(u(x,Q^2) + c(x,Q^2) - \bar{d}(x,Q^2) - \bar{s}(x,Q^2)), \end{aligned}$$

For charged current anti(neutrino) scattering on neutrons:

$$\begin{aligned} F_2^{\nu n} &= 2x(u(x,Q^2) + s(x,Q^2) + \bar{d}(x,Q^2) + \bar{c}(x,Q^2)), \\ xF_3^{\nu n} &= 2x(u(x,Q^2) + s(x,Q^2) - \bar{d}(x,Q^2) - \bar{c}(x,Q^2)), \end{aligned}$$

$$\begin{aligned} F_2^{\bar{\nu} n} &= 2x(d(x,Q^2) + c(x,Q^2) + \bar{u}(x,Q^2) + \bar{s}(x,Q^2)), \\ xF_3^{\bar{\nu} n} &= 2x(d(x,Q^2) + c(x,Q^2) - \bar{u}(x,Q^2) - \bar{s}(x,Q^2)) \end{aligned}$$

The structure functions for (anti)neutrino scattering on isoscalar targets are given by

$$F_2^{\nu N}(x,Q^2) = x(u(x,Q^2) + d(x,Q^2) + \bar{u}(x,Q^2) + \bar{d}(x,Q^2) + 2s(x,Q^2)$$

$$F_2^{\bar{\nu} N}(x,Q^2) = x(u(x,Q^2) + d(x,Q^2) + \bar{u}(x,Q^2) + \bar{d}(x,Q^2) + 2\bar{s}(x,Q^2)$$

and

$$xF_3^{\nu N}(x,Q^2) = x(u(x,Q^2) + d(x,Q^2) - \bar{u}(x,Q^2) - \bar{d}(x,Q^2) + 2s(x,Q^2$$

$$xF_3^{\bar{\nu} N}(x,Q^2) = x(u(x,Q^2) + d(x,Q^2) - \bar{u}(x,Q^2) - \bar{d}(x,Q^2) - 2\bar{s}(x,Q^2$$

F_2 charged lepton-nucleon scattering on an isoscalar target may be written as

$$F_2^{lN} = \frac{5}{18}x(u + \bar{u} + d + \bar{d}) + \frac{1}{9}x(s + \bar{s}) + \frac{4}{9}x(c + \bar{c})$$

The neutrino and lepton structure functions may be related as follows. Assuming $s = \bar{s}, c(x) = \bar{c}$,

$$F_2^{lN} = \frac{5}{18}F_2^{\nu N} - \frac{1}{3}xs(x,Q^2) + \frac{1}{3}xc(x,Q^2) \approx \frac{5}{18}F_2^{\nu N}$$

Next-to-Leading Order(NLO) (Important at low x and Moderate Q^2)

NLO corrections are the Q^2 evolution of partons.

- In the naive parton model(No dynamics between the partons),

$$F_{1N}(x, Q^2) \xrightarrow[Q^2\to\infty,\nu\to\infty]{x\to finite} F_{1N}(x)$$
$$F_{2N}(x, Q^2) \xrightarrow[Q^2\to\infty,\nu\to\infty]{x\to finite} F_{2N}(x)$$

- The QCD improved naive parton model allows quarks to interact among themselves via the gluon exchange.
- Gluon emission introduces the Q^2 dependence in structure functions.
- As Q^2 increases more and more gluons are emitted, which in turn split in to $q\bar{q}$ pairs.

Ref: Vermaseren et al. NPB 724(2005) 3
Van Neerven and Vogt NPB 568(2000) 263

Target Mass Correction(Effective at high x and low Q^2)

Contribution which are suppressed by power of $\frac{1}{Q^2}$

- At finite Q^2 heavy quarks are produced like charm.
- Heavy quarks production modifiy the scattering kinematics, therefore

$$x \to \xi$$

$$\xi = \frac{2x}{1 + \sqrt{1 + 4\mu x^2}}, \qquad \mu = \frac{M_N^2}{Q^2}$$

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$$\xi = \frac{2x}{1+\sqrt{1+4\mu x^2}}, \qquad \mu = \frac{M_N^2}{Q^2}$$

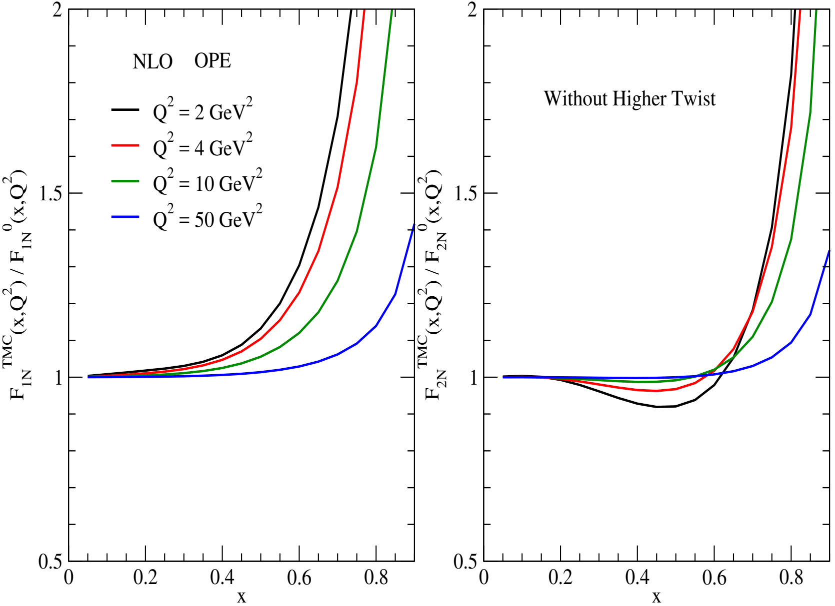
- Also know as Kinematic Higher Twist Effect.

TMC for details: I. Schienbein et al., J. Phys. G 35, 053101 (2008)

$$
\begin{aligned}
F_j^{TMC}(x,Q^2) &= \sum_{i,j=1}^{5} [A_j^i\ F_i^{(0)}(\xi,Q^2) + B_j^i\ h_i^(\xi,Q^2) + C_j\ g_2(\xi,Q^2)] \\
F_1^{TMC}(x,Q^2) &= \frac{x}{\xi r}\ F_1^{(0)}(\xi,Q^2) + \frac{M_N^2 x^2}{Q^2 r^2}\ h_2(\xi,Q^2) + \frac{2M_N^4 x^3}{Q^4 r^3} g_2(\xi,Q^2) \\
F_2^{TMC}(x,Q^2) &= \frac{x^2}{\xi^2 r^3} F_2^{(0)}(\xi,Q^2) + \frac{6M_N^2 x^3}{Q^2 r^4} h_2(\xi,Q^2) + \frac{12M_N^4 x^4}{Q^4 r^5} g_2(\xi,Q^2), \\
F_L^{TMC}(x,Q^2) &= \frac{x^2}{\xi^2 r} F_L^{(0)}(\xi,Q^2) + \frac{4M_N^2 x^3}{Q^2 r^2} h_2(\xi,Q^2) + \frac{8M_N^4 x^4}{Q^4 r^3} g_2(\xi,Q^2),
\end{aligned}
$$

where

$$
\begin{aligned}
\xi &= \frac{2x}{1+r} \\
r &= \sqrt{1+\frac{4M^2x^2}{Q^2}}
\end{aligned}
$$


NLO OPE
Q^2 = 2 GeV^2
Q^2 = 4 GeV^2
Q^2 = 10 GeV^2
Q^2 = 50 GeV^2
F_1N^TMC(x,Q^2) / F_1N^0(x,Q^2)
F_2N^TMC(x,Q^2) / F_2N^0(x,Q^2)
Without Higher Twist
x
0
0.2
0.4
0.6
0.8
0.5
1
1.5
2

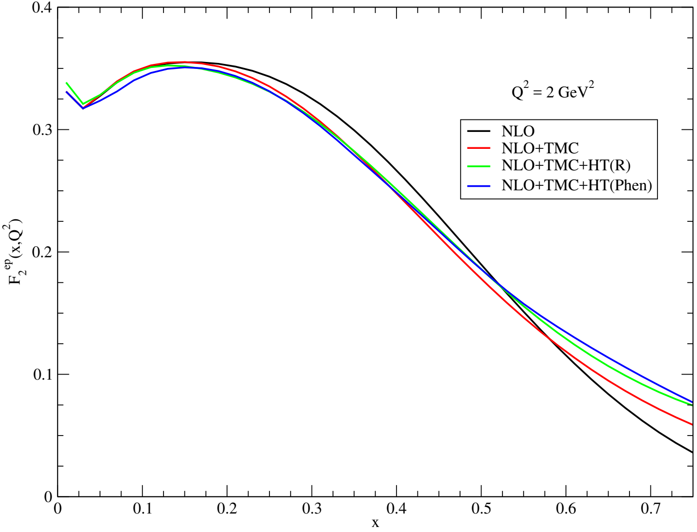
Dynamical Higher Twist Effect (Significant at High x and low Q^2)

Higher twist corrections include multiparton correlation in the nucleon

1. At low Q^2 Lepton/Neutrino-nucleon scattering can involve multiple partons.
2. Compared to the leading twist diagrams (pQCD) the higher twist diagrams are suppressed by powers of $\frac{1}{Q^2}$, so they are important at low Q^2.
3. This is also the region where the strong coupling constant is large and pQCD is invalid

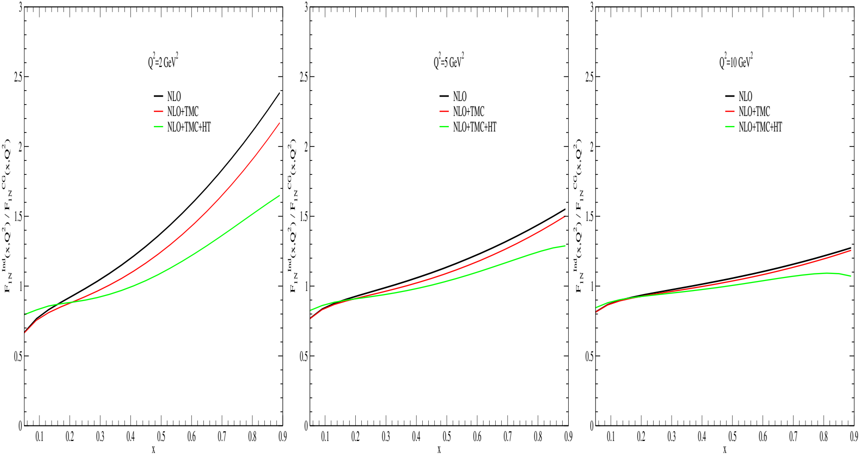
$$F_2(x, Q^2) = F^{LT+TMC}(x, Q^2)\left(1 + \frac{D_2(x, Q^2)}{Q^2}\right),$$

For Details: E. Stein et al. Nucl. Phys. B 536, 318 (1998)
M. Dasgupta et al. Phys. Lett. B 382, 273



Q^2 = 2 GeV^2
NLO
NLO+TMC
NLO+TMC+HT(R)
NLO+TMC+HT(Phen)
F_2^ep(x,Q^2)
x
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7

Ratio of $\frac{F_{1N}^{Ind}(x,Q^2)}{F_{1N}^{CG}(x,Q^2)}$

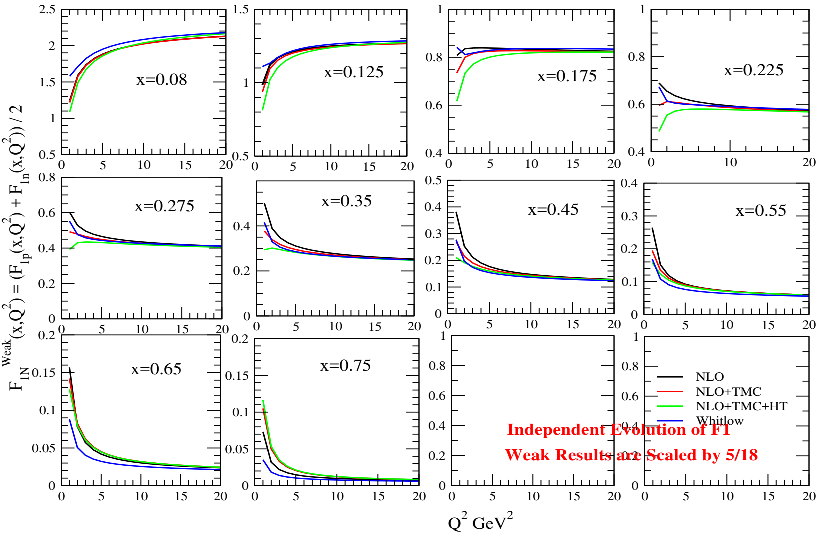


Whitlow's parameterization: Phys. Lett. B 250, 193(1990)

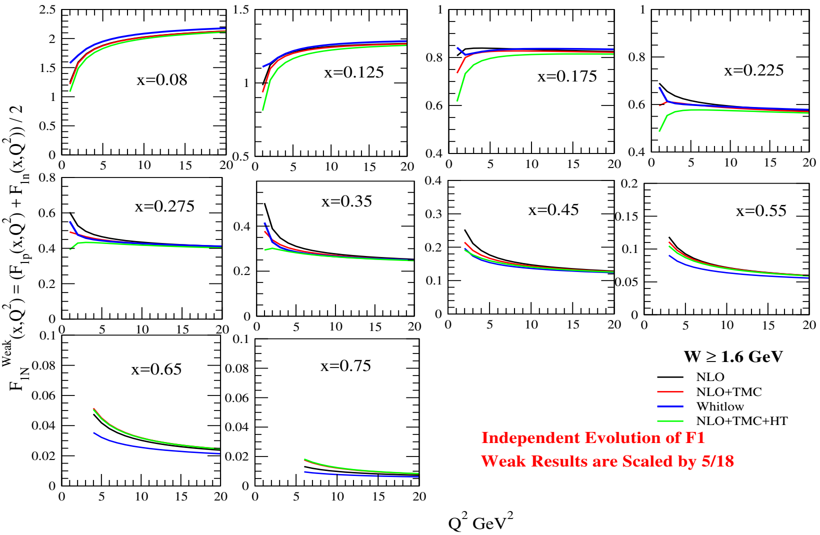
$$2xF_{1N}(x,Q^2) = \frac{\gamma^2\ F_{2N}(x,Q^2)}{1+R(x,Q^2)}; \qquad \gamma^2 = 1 + \frac{4M_N^2x^2}{Q^2}$$

where

$$\begin{aligned} R(x,Q^2) &= \frac{0.0635}{ln(Q^2/0.04)}\,\Theta(x,Q^2) + \frac{0.5747}{Q^2} - \frac{0.3534}{Q^4+0.3^2} \\ \Theta(x,Q^2) &= 1 + 12\left(\frac{Q^2}{Q^2+1}\right)\left(\frac{0.125^2}{0.125^2+x^2}\right) \end{aligned}$$

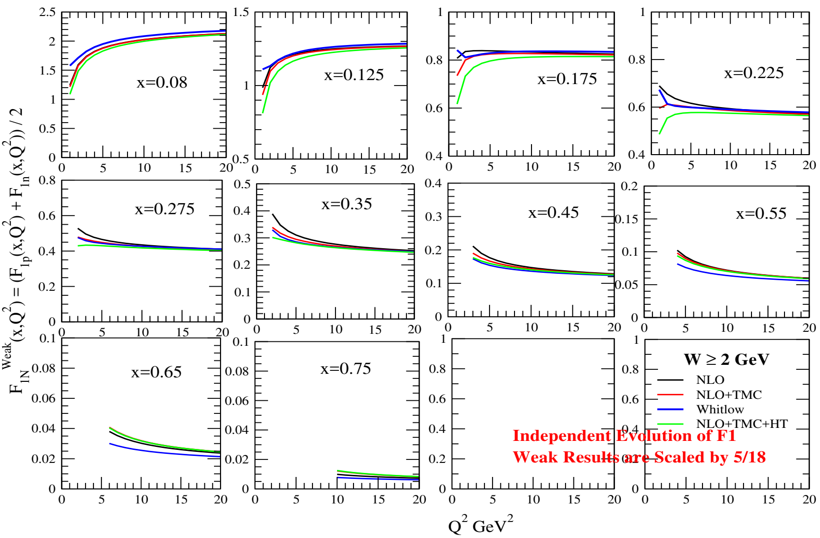


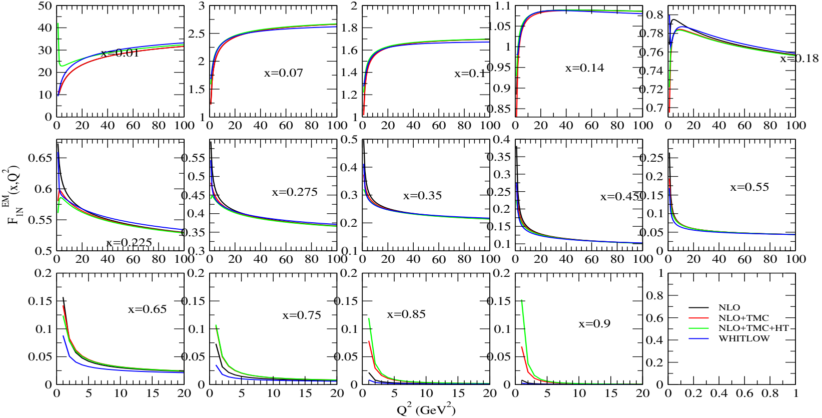
x=0.08
x=0.125
x=0.175
x=0.225
x=0.275
x=0.35
x=0.45
x=0.55
x=0.65
x=0.75
NLO
NLO+TMC
NLO+TMC+HT
Whitlow
Independent Evolution of F1
Weak Results are Scaled by 5/18
$F_{1N}^{Weak}(x,Q^2) = (F_{1p}(x,Q^2) + F_{1n}(x,Q^2)) / 2$
Q^2 GeV^2



Independent Evolution of F1

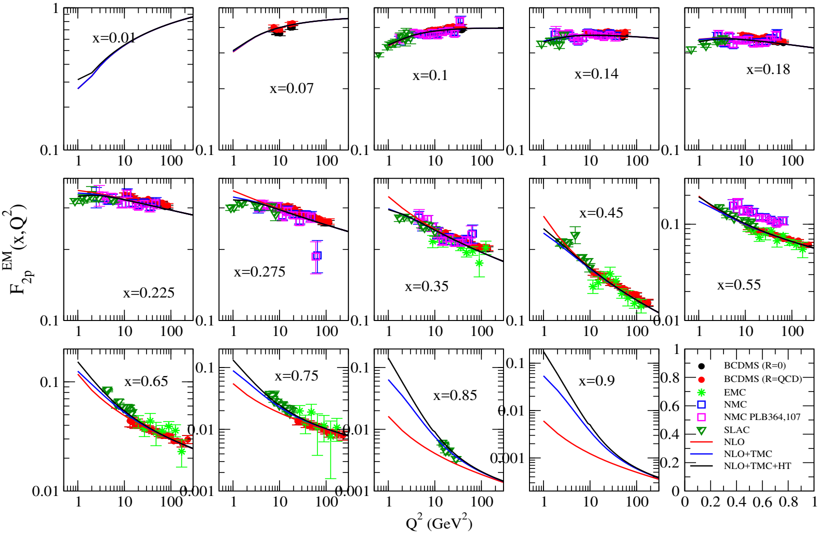
Weak Results are Scaled by 5/18

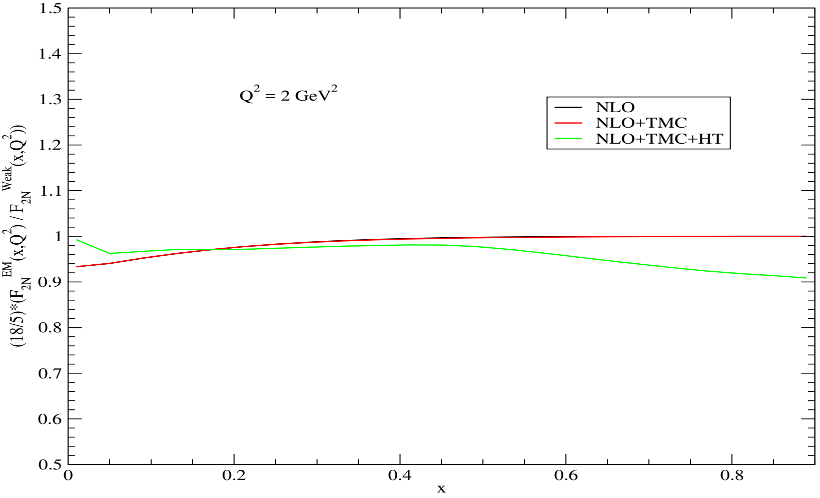

x=0.08
x=0.125
x=0.175
x=0.225
x=0.275
x=0.35
x=0.45
x=0.55
x=0.65
x=0.75
W ≥ 2 GeV
NLO
NLO+TMC
Whitlow
NLO+TMC+HT
Independent Evolution of F1
Weak Results are Scaled by 5/18
$F_{1N}^{Weak}(x,Q^2) = (F_{1p}(x,Q^2) + F_{1n}(x,Q^2)) / 2$
Q^2 GeV^2



x=0.01
x=0.07
x=0.1
x=0.14
x=0.18
x=0.225
x=0.275
x=0.35
x=0.45
x=0.55
x=0.65
x=0.75
x=0.85
x=0.9
NLO
NLO+TMC
NLO+TMC+HT
WHITLOW
$F_{1N}^{EM}(x,Q^2)$
Q^2 (GeV^2)

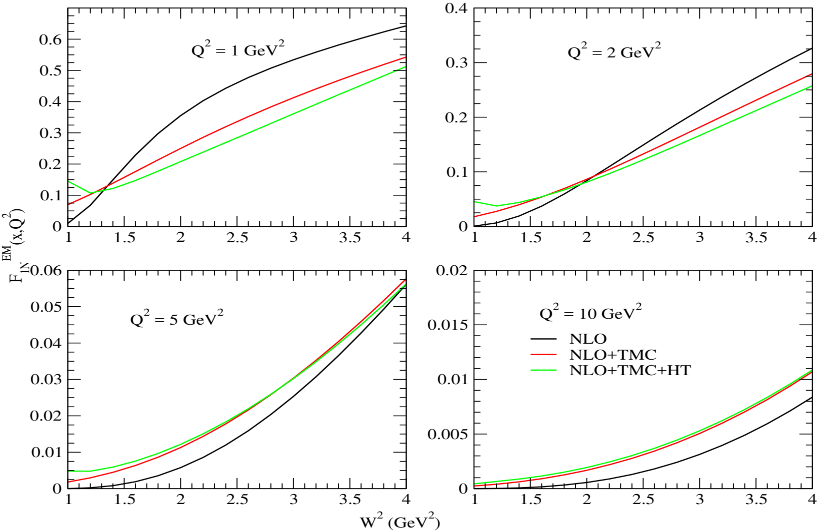
Comparison of $F_{2p}^{EM}(x, Q^2)$ with experimental data



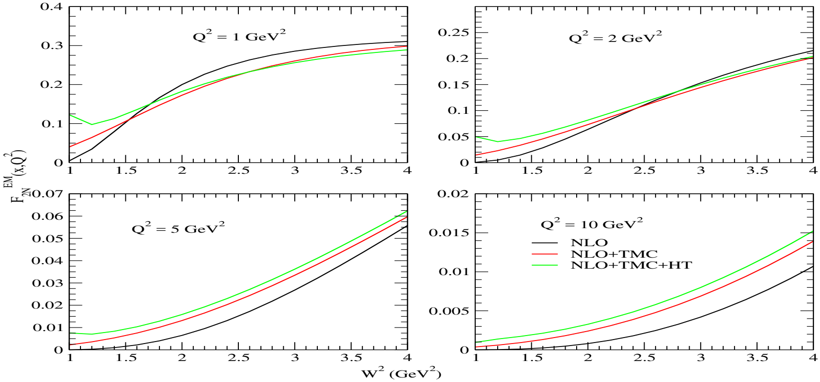


Q^2 = 2 GeV^2
NLO
NLO+TMC
NLO+TMC+HT
(18/5)*(F_2N^EM(x,Q^2) / F_2N^Weak(x,Q^2))
x
1.5
1.4
1.3
1.2
1.1
1
0.9
0.8
0.7
0.6
0.5
0
0.2
0.4
0.6
0.8

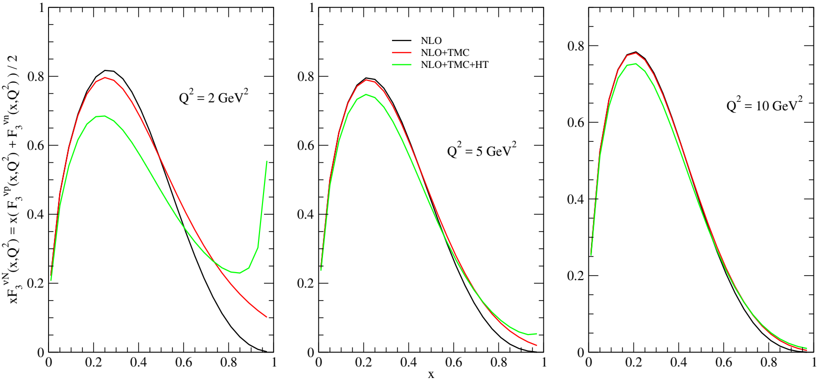
Results of $F_{1N}^{EM}(x, Q^2)$ obtained independently vs W^2



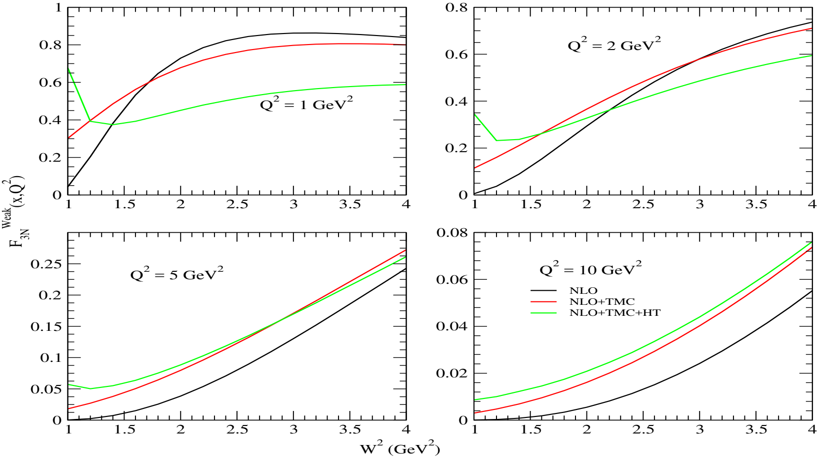
Results of $F_{2N}^{EM}(x, Q^2)$ vs W^2



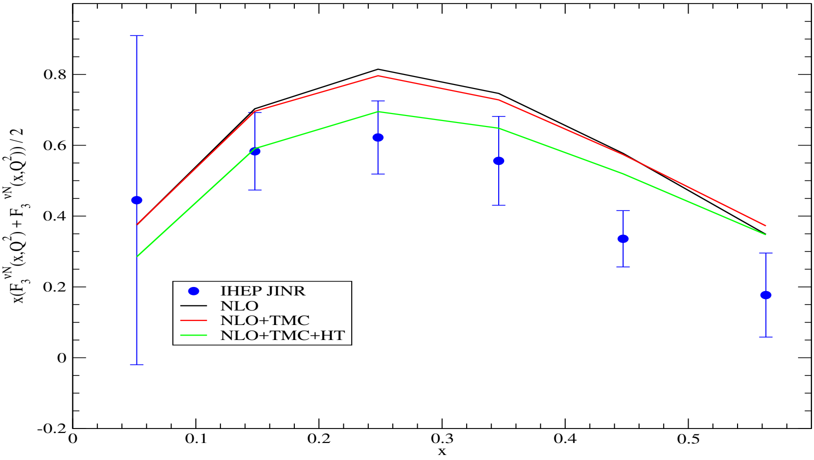
Results of $xF_3^{\nu N}(x, Q^2)$



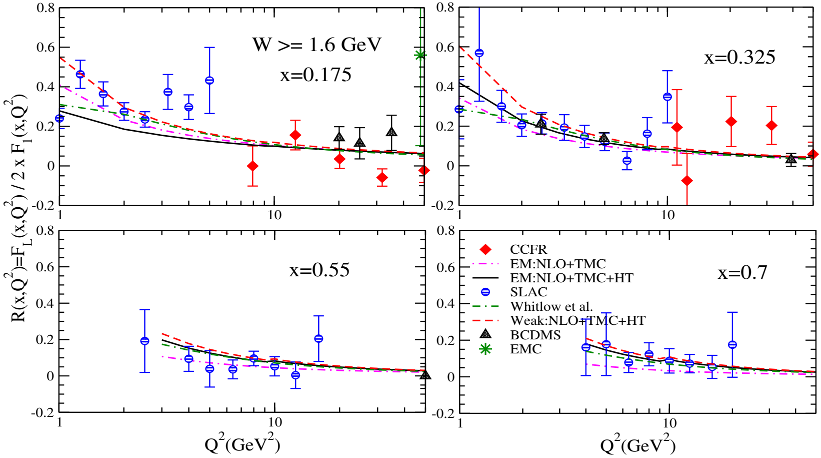
Results of $F_{3N}^{Weak}(x, Q^2)$ vs W^2



Comparison of $\frac{xF_3^{\nu N}(x,Q^2)+xF_3^{\bar{\nu} N}(x,Q^2)}{2}$ with the experimental data

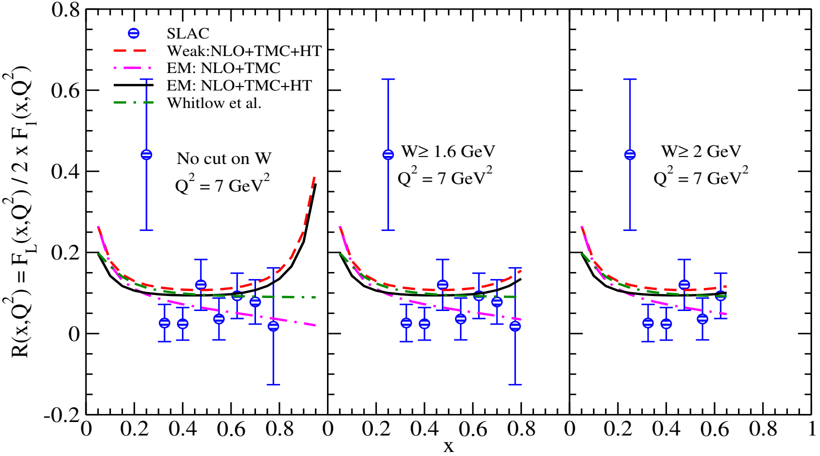


$R(x, Q^2)$ vs Q^2



W >= 1.6 GeV
x=0.175
x=0.325
x=0.55
x=0.7
$R(x,Q^2)=F_L(x,Q^2) / 2 x F_1(x,Q^2)$
$Q^2(GeV^2)$
CCFR
EM:NLO+TMC
EM:NLO+TMC+HT
SLAC
Whitlow et al.
Weak:NLO+TMC+HT
BCDMS
EMC

$R(x, Q^2)$ vs x


SLAC
Weak:NLO+TMC+HT
EM: NLO+TMC
EM: NLO+TMC+HT
Whitlow et al.
No cut on W
Q2 = 7 GeV2
W≥ 1.6 GeV
Q2 = 7 GeV2
W≥ 2 GeV
Q2 = 7 GeV2
R(x,Q2) = FL(x,Q2) / 2 x F1(x,Q2)
x
0.8
0.6
0.4
0.2
0
-0.2
0
0.2
0.4
0.6
0.8
1

Conclusions

1. It is important to understand uncertainties in the evolution of nucleon structure function in the wide range of x and Q^2 before we conclude the magnitude of nuclear medium effects in lepton nucleus and neutrino nucleus DIS events.
2. For the evaluation of nucleon structure function we have performed the calcultions at NLO using various PDFs available in the literature and included TMC and dynamical higher twist correction.
3. Our results would be useful both at low Q^2 as well as in the wide energy region of experiments being performed using nuclear targets.

Conclusions

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2. For the evaluation of nucleon structure function we have performed the calcultions at NLO using various PDFs available in the literature and included TMC and dynamical higher twist correction.
3. Our results would be useful both at low Q^2 as well as in the wide energy region of experiments being performed using nuclear targets.
4. This is not the end: Stay tuned for Sajjad Athar's talk where he will take this nucleon in to nuclear enviroment.