nPDFs from $\ell^{\pm}A$ and νA scattering

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based on: PRD 77 (2008) 054013; PRD 80 (2009) 094004; PRL 106 (2011) 122301

NuSTEC: *Workshop on Shallow- and Deep-Inelastic Scattering*
Gran Sasso Science Institute,
L'Aquila, Italy

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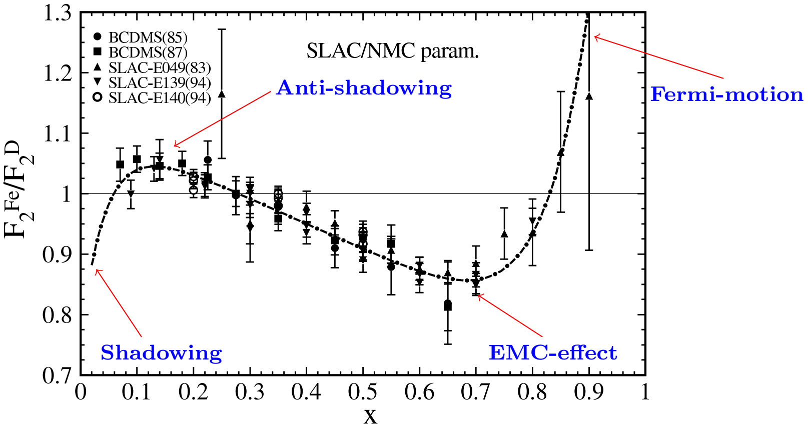
Outline

- Basics of global nPDF analysis
- nPDFs from neutrino DIS data
- nPDFs from charged-lepton DIS data
- Are the nPDFs form charged-lepton and neutrino DIS data compatible?
- Other analysis of $\ell^{\pm}A$ and νA data
- Summary

Introduction

- Cross-sections in nuclear collisions are modified

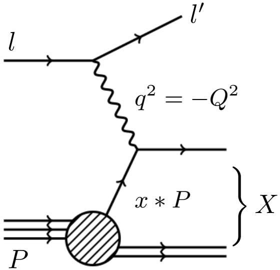
$$F_2^A(x) \neq Z F_2^p(x) + N F_2^n(x)$$



- Can we translate this modifications into **universal** **nuclear PDFs**?

Introduction

- Factorization in case of deep inelastic scattering (DIS)



$$\frac{d^2\sigma}{dxdQ^2} = \sum_i f_i^A(x, Q^2) \otimes d\hat{\sigma}_{il\to l'X} + \mathcal{O}\left(\frac{\Lambda^2_{\mathrm{QCD}}}{Q^2}\right)$$

- We assume that the nuclear effects can be absorbed into the universal nPDFs $[f_i^A(x, Q^2)]$.
- Do not consider any cold nuclear matter effects (e.g. energy loss).
- Include kinematic cuts to suppress non-leading terms, e.g.
  - nCTEQ: $\begin{cases} Q > 2 \text{ GeV} \\ W > 3.5 \text{ GeV} \end{cases}$ EPS: $Q > 1.3$ GeV

Assumptions entering the nuclear PDF analysis

1. **Factorization** & DGLAP evolution
   - allow for definition of **universal PDFs**
   - make the formalism **predictive**
   - needed even if it is broken
2. Isospin symmetry $\left\{ \begin{array}{l} u^{n/A}(x) = d^{p/A}(x) \\ d^{n/A}(x) = u^{p/A}(x) \end{array} \right.$ $\quad f_i^{(A,Z)} = \frac{Z}{A} f_i^{p/A} + \frac{A-Z}{A} f_i^{n/A}$
3. The *bound proton* PDFs have the *same evolution equations* and sum rules as the free proton PDFs *provided we neglect any contributions from the region* $x > 1$ (which is expected to have negligible contribution [PRC 73, 045206 (2006), `arXiv:hep-ph/0509241`])

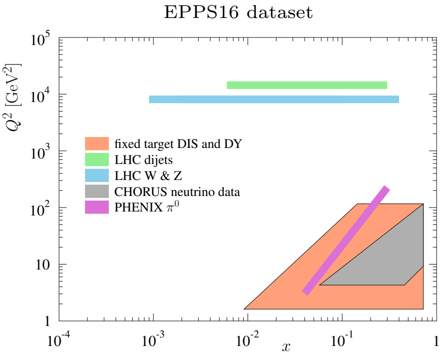
Then observables $\mathcal{O}^A$ can be calculated as:

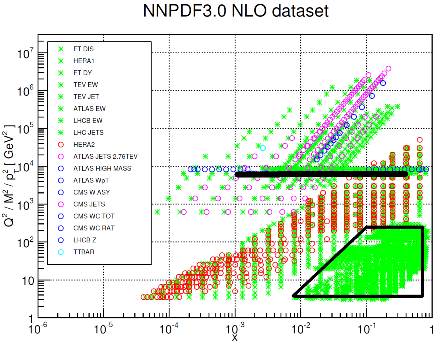
$$\mathcal{O}^A = Z\,\mathcal{O}^{p/A} + (A-Z)\,\mathcal{O}^{n/A}$$

With the above assumptions we can use the free proton framework to analyze nuclear data

Differences with the free-proton PDFs

- Theoretical status of Factorization
- Parametrization – more parameters to model A-dependence
- Different data sets – much less data:





- Less data → less constraining power → **more assumptions** (fixing) about fitting parameters

Schematics of Global Analysis

1. Choose experimental data (e.g. DIS, DY, inclusive jet prod., etc.)
2. Parametrize **bound proton PDFs** at low initial scale $\mu = Q_0 = 1.3\text{GeV}$:

$$f_i^{(A,Z)} = \frac{Z}{A} f_i^{p/A} + \frac{A-Z}{A} f_i^{n/A}$$

$$f_i^{p/A}(x, Q_0) = f_i^{p/A}(x; a_0, a_1, \dots) = a_0 x^{a_1} (1-x)^{a_2} P(x; a_3, \dots)$$

   with $a_j = a_j(A)$ depending on the nuclei.
3. Use DGLAP equation to evolve $f_i(x, \mu)$ from $\mu = Q_0$ to $\mu = Q_{\max}$.
4. Calculate theory predictions corresponding to the data (σ_{DIS}, σ_{DIS}, etc.).
5. Calculate appropriate χ^2 function – compare data and theory

$$\chi^2(\{a_i\}) = \sum_{\text{experiments}} w_n \chi_n^2(\{a_i\})$$

$$\chi_n^2(\{a_i\}) = \sum_{\text{data points}} \left(\frac{\text{data} - \text{theory}(\{a_i\})}{\text{uncertainty}} \right)^2$$

   (by default $w_n = 1$)
6. Minimize χ^2 function with respect to parameters $a_0, a_1, \dots$

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nCTEQ global analyses framework [PRD 93 (2016) 085037]

- PDFs for nucleus (A, Z)

$$f_i^{(A,Z)}(x,Q) = \frac{Z}{A} f_i^{p/A}(x,Q) + \frac{A-Z}{A} f_i^{n/A}(x,Q)$$

(bound neutron PDF $f_i^{n/A}$ by isospin symmetry)

- Functional form of the bound proton PDF same as for the free proton (`CTEQ6M`, x restricted to $0 < x < 1$)

$$x f_i^{p/A}(x,Q_0) = c_0 x^{c_1}(1-x)^{c_2} e^{c_3 x}(1+e^{c_4}x)^{c_5}, \qquad i = u_v, d_v, g, \ldots$$
$$\bar{d}(x,Q_0)/\bar{u}(x,Q_0) = c_0 x^{c_1}(1-x)^{c_2} + (1+c_3 x)(1-x)^{c_4}$$

- A-dependent fit parameters (reduces to free proton for $A = 1$)

$$c_k \to c_k(A) \equiv c_{k,0} + c_{k,1}\left(1 - A^{-c_{k,2}}\right), \quad k = \{1, \ldots, 5\}$$

- Calculations:
  - NLO in QCD including heavy quark mass effects (ACOT scheme),
  - include Target Mass Corrections [PRD 69 (2004) 034002; J.Phys.G 35 (2008) 053101].

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nPDFs from neutrino DIS data [PRD 77 (2008) 054013]

In the following paper: Schienbein *et al.* PRD 77 (2008) 054013 from the nCTEQ group a set of nuclear PDFs (nPDFs) for iron have been extracted in a global analysis of neutrino DIS data from the NuTeV experiment [PRD 74 (2006) 012008, PRD 64 (2001) 112006].

- Main motivation: provide flexible parametrization of nuclear corrections for neutrino data in proton PDF fits.
- Extract iron nPDFs – no need to parametrize A-dependence
- Data on *cross-section level* was used (and dimuon data):

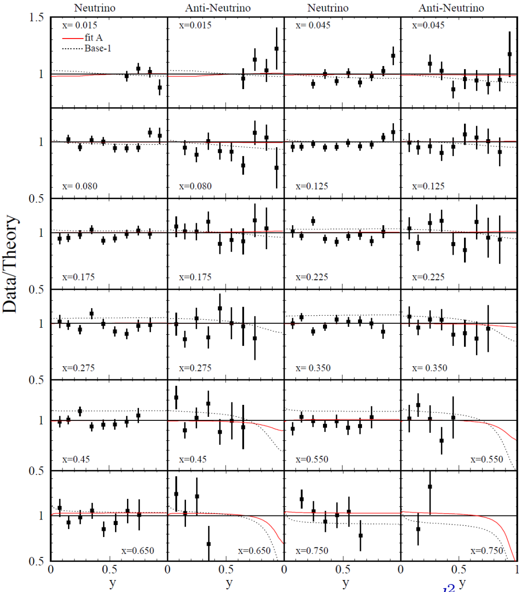
$$\frac{d^2\sigma^{\nu(\bar\nu)A}}{dx\,dy} = \frac{G^2 ME}{\pi}\left[\left(1-y-\frac{Mxy}{2E}\right)F_2^{\nu(\bar\nu)A} + y^2 x F_1^{\nu(\bar\nu)A} \pm y\left(1-\frac{y}{2}\right)xF_3^{\nu(\bar\nu)A}\right]$$

$$F_i^A(x,Q) = \frac{Z}{A}F_i^{p/A}(x,Q) + \frac{A-Z}{A}F_i^{n/A}(x,Q)$$

  including **correlated errors**.

- Theory predictions in NLO QCD including heavy quark mass effects (ACOT scheme) and TMCs.

nPDFs from neutrino DIS data [PRD 77 (2008) 054013]



Base-1: free proton PDFs (E=245 GeV) $\frac{d^2\sigma}{dxdy}$

- kinematical cuts:

  A2 fit : $\begin{cases} Q > 2 \text{ GeV} \\ W > 3.5 \text{ GeV} \end{cases}$

  A fit: $Q > 1.3$ GeV
- x range: $0.015 \le x \le 0.75$
- No of $\nu + \bar{\nu}$ data points: 3111
- χ^2/pts(A2 fit) = 1.35

  χ^2/pts(A fit) = 1.37

✓ $x = 0.75$: σ^{nuc} enhanced
$\rightarrow$ Fermi motion

✓ $x \in (0.225, 0.65)$: suppression
$\rightarrow$ EMC effect

✗ $x \in (0.125, 0.175)$:
NO clear anti-shadowing

✗ $x \in (0.045, 0.08)$:
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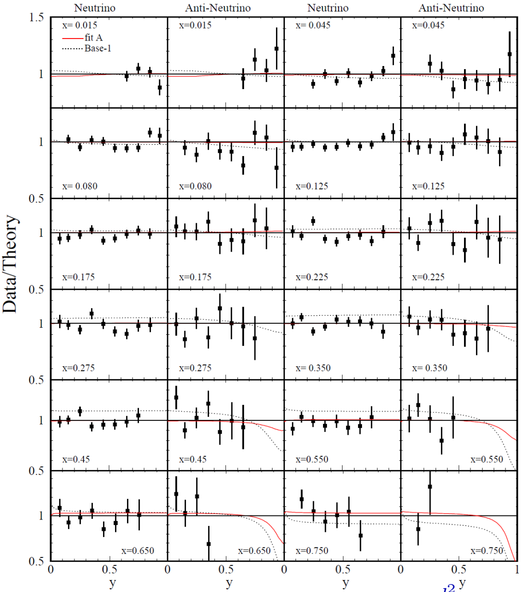
nPDFs from neutrino DIS data [PRD 77 (2008) 054013]



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nPDFs from neutrino DIS data [PRD 77 (2008) 054013]

Using the obtained nPDFs one can calculate **nuclear corrections**, R, for any observable, e.g. for the neutrino cross-section $\frac{d^2\sigma^{\nu(\bar{\nu})\mathrm{Fe}}}{dxdy}$, where

$$R[\frac{d^2\sigma^{\nu(\bar{\nu})\mathrm{Fe}}}{dxdy}] = \frac{d^2\sigma^{\nu(\bar{\nu})\mathrm{Fe}}}{dxdy}(\frac{Z}{A}f_i^{p/A} + \frac{A-Z}{A}f_i^{n/A}) / \frac{d^2\sigma^{\nu(\bar{\nu})\mathrm{Fe}}}{dxdy}(\frac{Z}{A}f_i^{p} + \frac{A-Z}{A}f_i^{n})$$

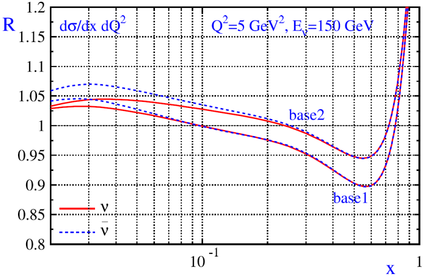

FIG. 7: Nuclear correction factor R according to Eq. (25) for the differential cross section $d^2\sigma/dx\,dQ^2$ in charged current νFe scattering at $Q^2 = 5$ GeV^2 and $E_\nu = 150$ GeV. Results are shown using the 'A2' fit for the charged current neutrino (solid lines) and anti-neutrino (dashed lines) scattering from iron. The upper (lower) pair of curves shows the result of our analysis with the Base-2 (Base-1) free-proton PDFs. The correction factors shown here are for an iron target which has been corrected for the neutron excess.

Base-1: includes deuteron corrections

Base-2: NO deuteron corrections

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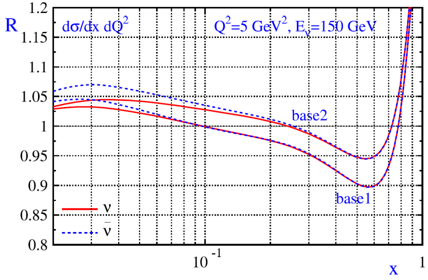

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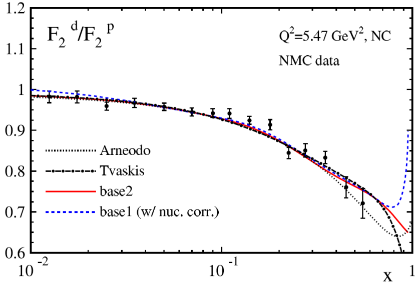


FIG. 5: NMC data for F_2^D/F_2^p [46] at $Q^2 = 5.47$ GeV^2 in comparison with the theory prediction for F_2^D/F_2^p computed using free-proton Base-2 PDFs. The dashed line shows the structure function ratio obtained with the Base-1 PDFs; in this case a nuclear correction factor for deuterium has been applied (*cf.*, Refs. [9, 10]). For comparison, we also show the parameterizations of Arneodo et al. [46] and Tvaskis et al. [47, 48].

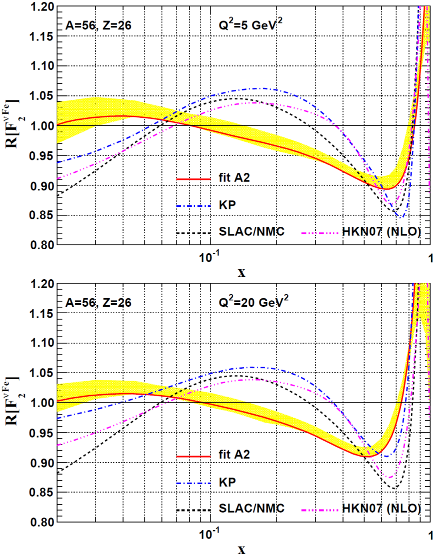
Base-1: includes deuteron corrections

Base-2: NO deuteron corrections

nPDFs from neutrino DIS data [PRD 77 (2008) 054013]

Nuclear corrections for the structure functions.

$R[F_2^{\nu \mathrm{Fe}}]$ $R[F_2^{\bar{\nu} \mathrm{Fe}}]$



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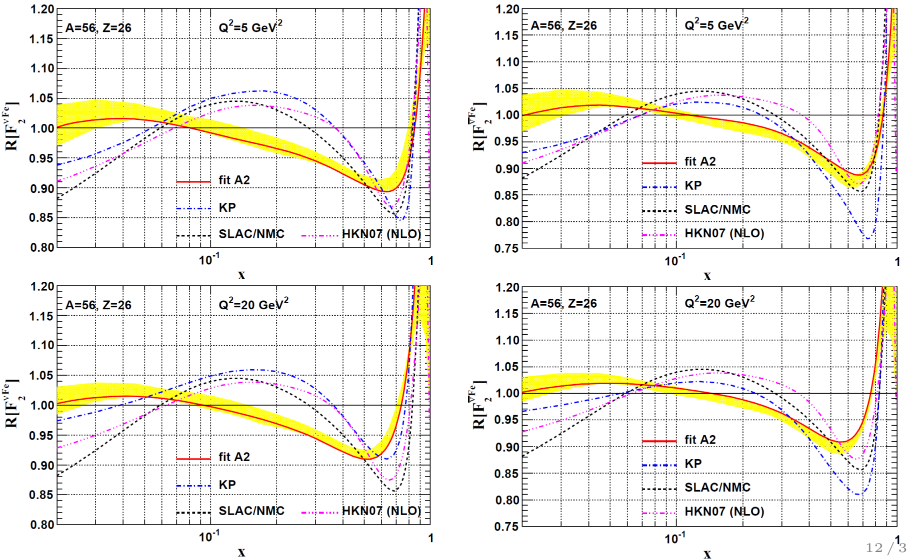

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nPDFs from charged-lepton DIS data [PRD 80 (2009) 094004]

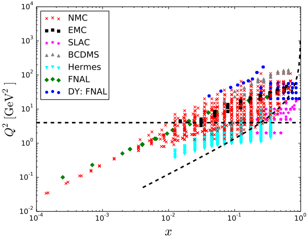
F_2^A/F_2^D : Observable	Experiment	Ref.	# data
D	NMC-97	[31]	275
He/D	SLAC-E139	[18]	18
	NMC-95,re	[32]	16
	Hermes	[33]	92
Li/D	NMC-95	[34]	15
Be/D	SLAC-E139	[18]	17
C/D	EMC-88	[35]	9
	EMC-90	[36]	2
	SLAC-E139	[18]	7
	NMC-95,re	[32]	16
	NMC-95	[34]	15
	FNAL-E665-95	[37]	4
N/D	BCDMS-85	[19]	9
	Hermes	[33]	92
Al/D	SLAC-E049	[38]	18
	SLAC-E139	[18]	17
Ca/D	EMC-90	[36]	2
	SLAC-E139	[18]	7
	NMC-95,re	[32]	15
	FNAL-E665-95	[37]	4
Fe/D	BCDMS-85	[19]	6
	BCDMS-87	[20]	10
	SLAC-E049	[21]	14
	SLAC-E139	[18]	23
	SLAC-E140	[22]	6
Cu/D	EMC-88	[35]	9
	EMC-93(addendum)	[39]	10
	EMC-93(chariot)	[39]	9
Kr/D	Hermes	[33]	84
Ag/D	SLAC-E139	[18]	7
Sn/D	EMC-88	[35]	8
Xe/D	FNAL-E665-92(em cut)	[40]	4
Au/D	SLAC-E139	[18]	18
Pb/D	FNAL-E665-95	[37]	4
Total:			862

$F_2^A/F_2^{A'}$: Observable	Experiment	Ref.	# data
Be/C	NMC-96	[41]	15
Al/C	NMC-96	[41]	15
Ca/C	NMC-95	[32]	20
	NMC-96	[41]	15
Fe/C	NMC-95	[41]	15
Sn/C	NMC-96	[42]	144
Pb/C	NMC-96	[41]	15
C/Li	NMC-95	[32]	20
Ca/Li	NMC-95	[32]	20
Total:			279

$\sigma_{DY}^{pA}/\sigma_{DY}^{pA'}$: Observable	Experiment	Ref.	# data
C/D	FNAL-E772-90	[43]	9
Ca/D	FNAL-E772-90	[43]	9
Fe/D	FNAL-E772-90	[43]	9
W/D	FNAL-E772-90	[43]	9
Fe/Be	FNAL-E866-99	[44]	28
W/Be	FNAL-E866-99	[44]	28
Total:			92

In the following paper:
Schienbein *et al.* PRD 80 (2009) 094004
from the nCTEQ group a set of
A-dependent nPDFs have been extracted
in a global analysis of *charged-lepton DIS*
and *Drell-Yan* (DY) data.

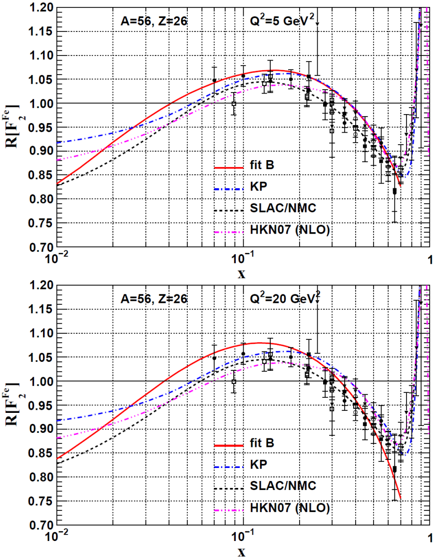
- kinematical cuts: $\begin{cases} Q > 2\ \text{GeV} \\ W > 3.5\ \text{GeV} \end{cases}$
- No of data points (after cuts): 708



- $\chi^2/\text{dof} = 0.946$

nPDFs from charged-lepton DIS data [PRD 80 (2009) 094004]

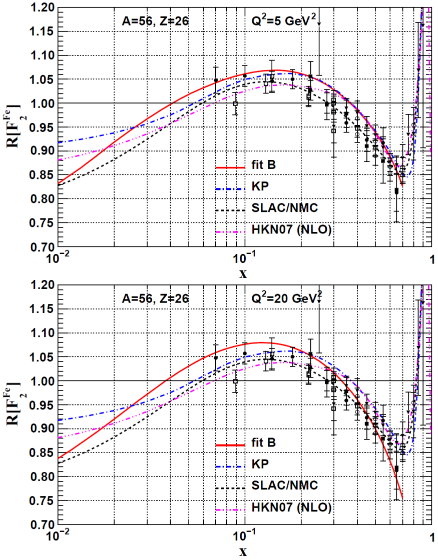
$F_2^{\ell^\pm \mathrm{Fe}} / F_2^{\ell^\pm \mathrm{D}}$



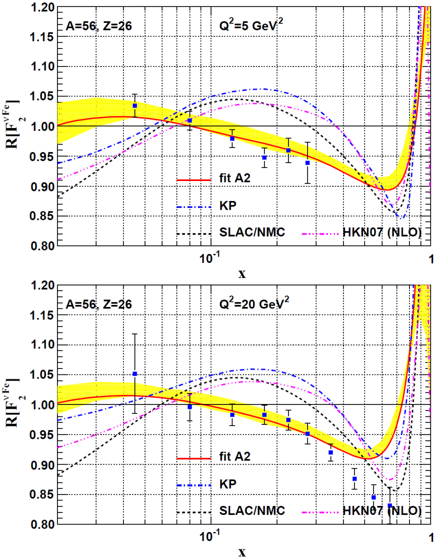
$F_2^{\nu \mathrm{Fe}} / F_2^{\nu \mathrm{D}}$ [PRD 77 (2008) 054013]

nPDFs from charged-lepton DIS data [PRD 80 (2009) 094004]

$F_2^{\ell^\pm \mathrm{Fe}} / F_2^{\ell^\pm \mathrm{D}}$



$F_2^{\nu \mathrm{Fe}} / F_2^{\nu \mathrm{D}}$ [PRD 77 (2008) 054013]



nPDFs from charged-lepton DIS data [PRD 80 (2009) 094004]

Disclaimer

$$F_2^{\ell^\pm A} \neq F_2^{\nu A}$$

at LO with $f_i^{(A,Z)} = \frac{Z}{A} f_i^{p/A} + \frac{A-Z}{A} f_i^{n/A}$ and $\Delta = \frac{1}{2} - \frac{Z}{A}$ we have:

$$\frac{1}{x} F_2^{\ell^\pm A} = \frac{5}{18}\left(f_u^{p/A} + f_{\bar{u}}^{p/A} + f_d^{p/A} + f_{\bar{d}}^{p/A}\right) + \frac{1}{9}\left(f_s^{p/A} + f_{\bar{s}}^{p/A}\right) + \frac{4}{9}\left(f_c^{p/A} + f_{\bar{c}}^{p/A}\right) - \frac{1}{3}\Delta\left(f_u^{p/A} + f_{\bar{u}}^{p/A} - f_d^{p/A} - f_{\bar{d}}^{p/A}\right) + \cdots$$

$$\frac{1}{2x} F_2^{\nu A} + \frac{1}{2x} F_2^{\bar{\nu} A} = f_u^{p/A} + f_{\bar{u}}^{p/A} + f_d^{p/A} + f_{\bar{d}}^{p/A} + f_s^{p/A} + f_{\bar{s}}^{p/A} + f_c^{p/A} + f_{\bar{c}}^{p/A} + \cdots$$

$$\frac{1}{2x} F_2^{\nu A} + \frac{1}{2x} F_2^{\bar{\nu} A} - \frac{18}{5}\frac{1}{x} F_2^{\ell^\pm A} = \frac{3}{5}\left(f_s^{p/A} + f_{\bar{s}}^{p/A}\right) - \frac{3}{5}\left(f_c^{p/A} + f_{\bar{c}}^{p/A}\right) + \frac{6}{5}\Delta\left(f_u^{p/A} + f_{\bar{u}}^{p/A} - f_d^{p/A} - f_{\bar{d}}^{p/A}\right) + \cdots$$

- Nuclear corrections for $F_2^{\ell^\pm A}$ and $F_2^{\nu A}$ are NOT the same, similarly for $\frac{d^2\sigma^{\nu A}}{dx\,dy}$ (additional differences will also enter at NLO).
- Instead one should compare the **universal** nPDFs.

Models predicting differences between neutrinos and charged-leptons

A list of some of the models predicting differences:

- S. Brodsky, I. Schmidt, J. Yang PRD 70 (2004) 116003
- J. Qiu, I. Vitev PLB 587 (2004) 52
- S. Kulagin, R. Petti PRD 76 (2007) 094023
- ...

Outline

- Basics of global nPDF analysis
- nPDFs from neutrino DIS data
- nPDFs from charged-lepton DIS data
- Are the nPDFs form charged-lepton and neutrino DIS data compatible?
- Other analysis of $\ell^{\pm}A$ and νA data
- Summary

Compatibility of νA and $\ell^{\pm} A$ DIS data [PRL 106 (2011) 122301]

ID	Observable	$A/A'(A)$	Experiment	# data
1	$F_2^A/F_2^{A'}$	He/D	SLAC-E139, NMC-95 re	15
2	$F_2^A/F_2^{A'}$	Li/D	NMC-95	11
3	$F_2^A/F_2^{A'}$	Be/D	SLAC-E139	3
4	$F_2^A/F_2^{A'}$	C/D	EMC-88,90, SLAC-E139	
			NMC-95,95 re, FNAL-E665-95	38
5	$F_2^A/F_2^{A'}$	N/D	BCDMS-85	9
6	$F_2^A/F_2^{A'}$	Al/D	SLAC-E049,E139	3
7	$F_2^A/F_2^{A'}$	Ca/D	EMC-90, SLAC-E139	
			NMC-95,re, FNAL-E665-95	17
8	$F_2^A/F_2^{A'}$	Fe/D	BCDMS-85,87	
			SLAC-E049,E139,E140	24
9	$F_2^A/F_2^{A'}$	Cu/D	EMC-88,93	27
10	$F_2^A/F_2^{A'}$	Ag/D	SLAC-E139	2
11	$F_2^A/F_2^{A'}$	Sn/D	EMC-88	8
12	$F_2^A/F_2^{A'}$	Xe/D	FNAL-E665-92	2
13	$F_2^A/F_2^{A'}$	Au/D	SLAC-E139	3
14	$F_2^A/F_2^{A'}$	Pb/D	FNAL-E665-95	3
15	$F_2^A/F_2^{A'}$	Be/C	NMC-96	14
16	$F_2^A/F_2^{A'}$	Al/C	NMC-96	14
17	$F_2^A/F_2^{A'}$	Ca/C	NMC-95,96	29
18	$F_2^A/F_2^{A'}$	Fe/C	NMC-95	14
19	$F_2^A/F_2^{A'}$	Pb/C	NMC-96	14
20	$F_2^A/F_2^{A'}$	C/Li	NMC-95	7
21	$F_2^A/F_2^{A'}$	Ca/Li	NMC-95	7
22	$F_2^A/F_2^{A'}$	He/D	Hermes	17
23	$F_2^A/F_2^{A'}$	Kr/D	Hermes	12
24	$F_2^A/F_2^{A'}$	Sn/C	NMC-96	111
25	$F_2^A/F_2^{A'}$	N/D	Hermes	19
32	$F_2^A/F_2^{A'}$	D	NMC-97	201
26	$\sigma_{DY}^{pA}/\sigma_{DY}^{pA'}$	C/D	FNAL-E772	9
27	$\sigma_{DY}^{pA}/\sigma_{DY}^{pA'}$	Ca/D	FNAL-E772	9
28	$\sigma_{DY}^{pA}/\sigma_{DY}^{pA'}$	Fe/D	FNAL-E772	9
29	$\sigma_{DY}^{pA}/\sigma_{DY}^{pA'}$	W/D	FNAL-E772	9
30	$\sigma_{DY}^{pA}/\sigma_{DY}^{pA'}$	Fe/Be	FNAL-E866	28
31	$\sigma_{DY}^{pA}/\sigma_{DY}^{pA'}$	W/Be	FNAL-E866	28
			$l^{\pm}A$ DIS & DY Total:	**708**
33	$d\sigma^{\nu A}/dx\,dy$	Pb	CHORUS ν	824
34	$d\sigma^{\nu A}/dx\,dy$	Pb	CHORUS $\bar{\nu}$	412
35	$d\sigma^{\nu A}/dx\,dy$	Fe	NuTeV ν	1170
36	$d\sigma^{\nu A}/dx\,dy$	Fe	NuTeV $\bar{\nu}$	966
37	$d\sigma^{\nu A}/dx\,dy$	Fe	CCFR di-μ	44
38	$d\sigma^{\nu A}/dx\,dy$	Fe	NuTeV di-μ	44
39	$d\sigma^{\nu A}/dx\,dy$	Fe	CCFR di-μ	44
40	$d\sigma^{\nu A}/dx\,dy$	Fe	NuTeV di-μ	42
			νA Total:	**3134**

In Kovarik *et al.* PRL 106 (2011) 122301 a combined fit of *neutrino* and *charged lepton DIS* data and *DY* data has been performed to study their compatibility.

- kinematical cuts: $\begin{cases} Q > 2 \text{ GeV} \\ W > 3.5 \text{ GeV} \end{cases}$
- No of data points (after cuts)
  - $\ell^{\pm} A$ DIS & DY: 708
  - νA DIS: 3134
- Family of fits with different weights $w = \{0, \frac{1}{7}, \frac{1}{2}, 1, \infty\}$

$$\chi^2 = \sum_{\ell^{\pm} A \text{ data}} \chi_i^2 + \sum_{\nu A \text{ data}} w \chi_i^2$$

Table II. Summary table of a family of compromise fits.

w	$l^{\pm}A$	χ^2 (/pt)	νA	χ^2 (/pt)	total χ^2(/pt)
0	708	638 (0.90)	-	-	638 (0.90)
1/7	708	645 (0.91)	3134	4710 (1.50)	5355 (1.39)
1/2	708	680 (0.96)	3134	4405 (1.40)	5085 (1.32)
1	708	736 (1.04)	3134	4277 (1.36)	5014 (1.30)
∞	-	-	3134	4192 (1.33)	4192 (1.33)

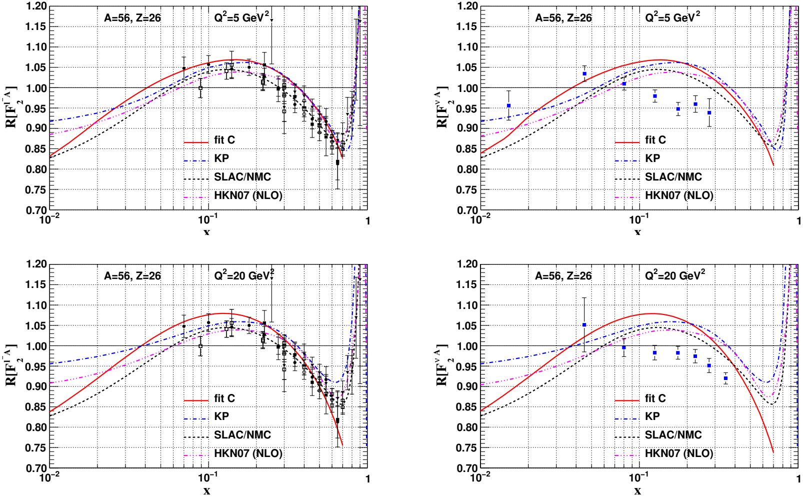
Compatibility of νA and $\ell^{\pm} A$ DIS data [PRL 106 (2011) 122301]

- $\ell^{\pm} A$ DIS and DY data sets as before
- 8 Neutrino DIS data sets:
  - NuTeV cross section data: νFe, $\bar{\nu}$Fe
  - CHORUS cross section data: νPb, $\bar{\nu} Pb$
  - NuTeV dimuon data: νFe, $\bar{\nu}$Fe
  - CCFR dimuon data: νFe, $\bar{\nu}$Fe
- Problem: Neutrino data sets have much higher statistics. Systematically study fits with different weights.

Weight	Fit name	ℓ data	χ^2 (/pt)	ν data	χ^2 (/pt)	total χ^2 (/pt)
$w = 0$	decut3	708	639 (0.90)	-	-	639 (0.90)
$w = 1/7$	glofac1a	708	645 (0.91)	3134	4710 (1.50)	5355 (1.39)
$w = 1/4$	glofac1c	708	654 (0.92)	3134	4501 (1.43)	5155 (1.34)
$w = 1/2$	glofac1b	708	680 (0.96)	3134	4405 (1.40)	5085 (1.32)
$w = 1$	global2b	708	736 (1.04)	3134	4277 (1.36)	5014 (1.30)
$w = \infty$	nuanua1	-	-	3134	4192 (1.33)	4192 (1.33)

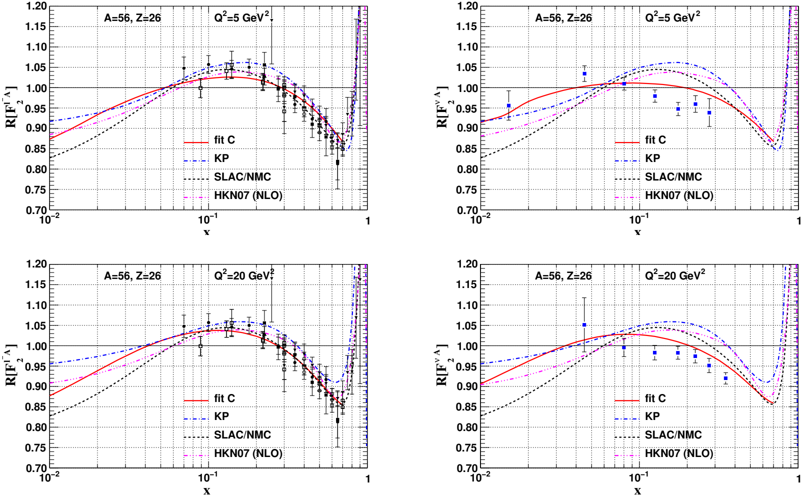
$R[F_2^{\ell^\pm \mathrm{Fe}}]$ (left) vs $R[F_2^{\nu \mathrm{Fe}}]$ (right)

decut3 $(w = 0)$



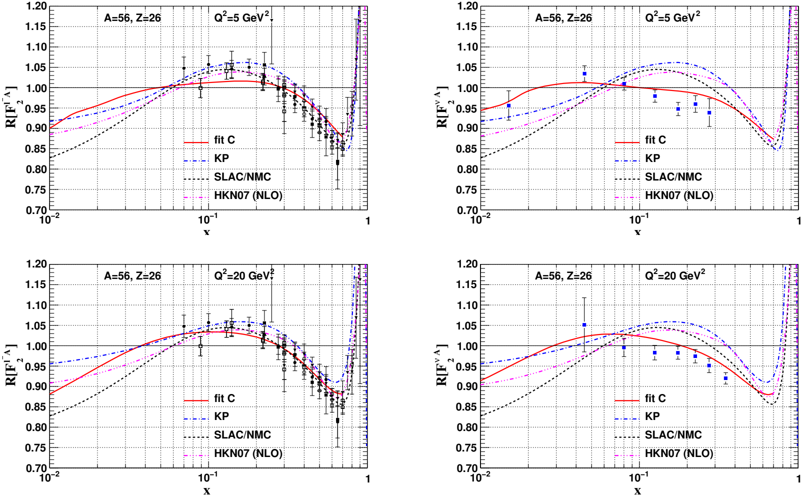
$R[F_2^{\ell^\pm \mathrm{Fe}}]$ (left) vs $R[F_2^{\nu \mathrm{Fe}}]$ (right)

glofac1a ($w = 1/7$)



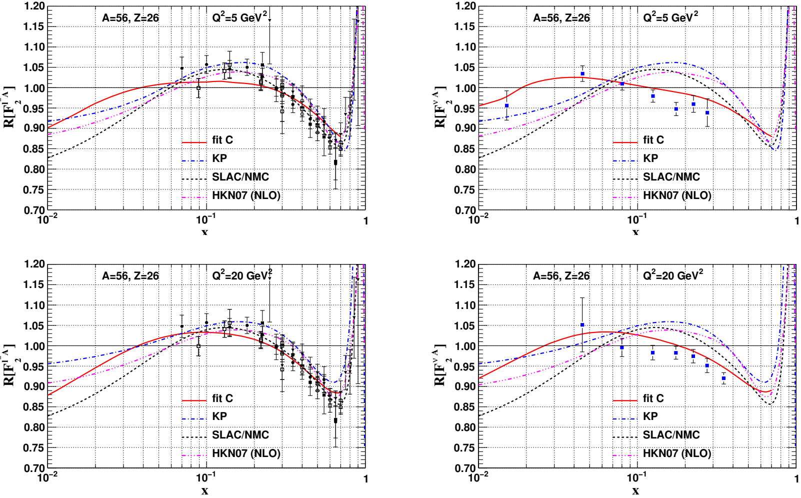
$R[F_2^{\ell^\pm \mathrm{Fe}}]$ (left) vs $R[F_2^{\nu \mathrm{Fe}}]$ (right)

glofac1c ($w = 1/4$)



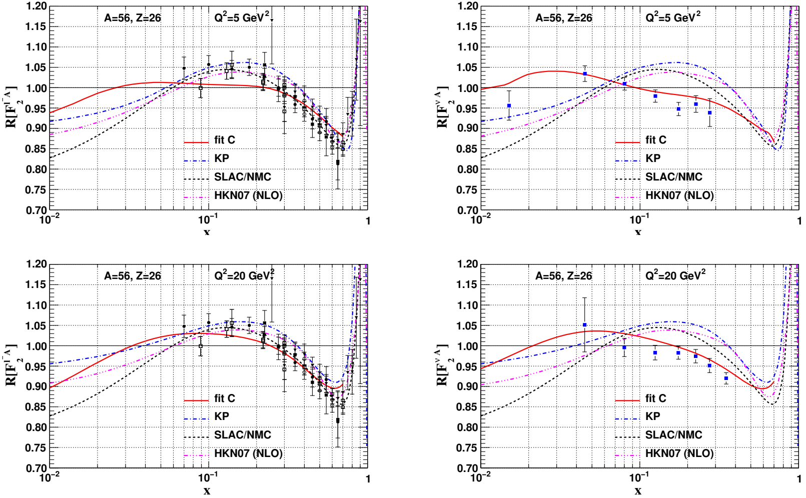
$R[F_2^{\ell^\pm \mathrm{Fe}}]$ (left) vs $R[F_2^{\nu \mathrm{Fe}}]$ (right)

glofac1b ($w = 1/2$)



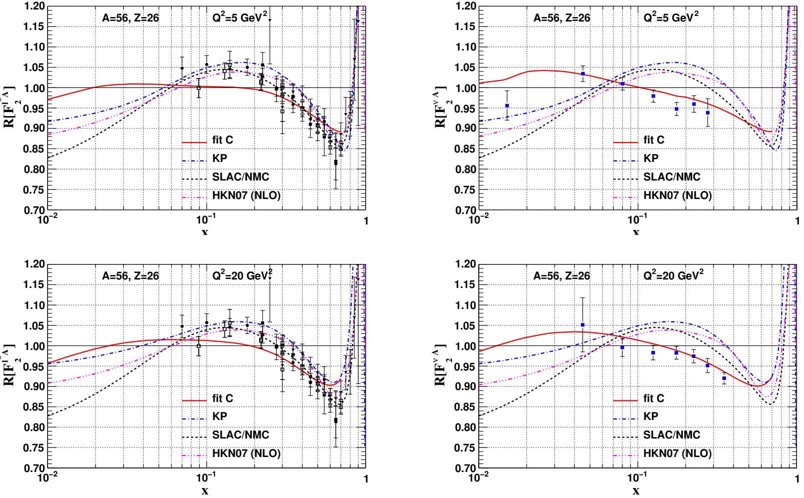
$R[F_2^{\ell^\pm \mathrm{Fe}}]$ (left) vs $R[F_2^{\nu \mathrm{Fe}}]$ (right)

global2b ($w = 1$)



$R[F_2^{\ell^\pm \mathrm{Fe}}]$ (left) vs $R[F_2^{\nu \mathrm{Fe}}]$ (right)

nuanua1 $(w = \infty)$



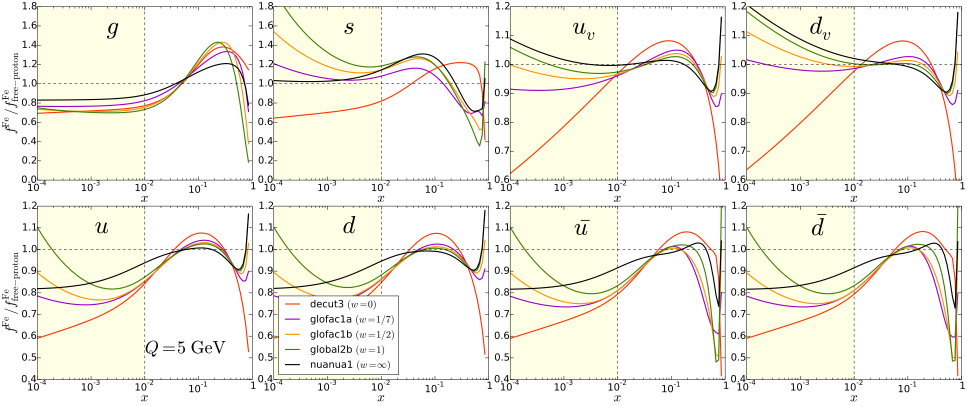
Is there a reasonable compromise fit?

Weight	Fit name	ℓ data	χ^2 (/pt)	ν data	χ^2 (/pt)	total χ^2 (/pt)
$w = 0$	decut3	708	639 (0.90)	-	-	639 (0.90)
$w = 1/7$	glofac1a	708	645 (0.91)	3134	4710 (1.50)	5355 (1.39)
$w = 1/4$	glofac1c	708	654 (0.92)	3134	4501 (1.43)	5155 (1.34)
$w = 1/2$	glofac1b	708	680 (0.96)	3134	4405 (1.40)	5085 (1.32)
$w = 1$	global2b	708	736 (1.04)	3134	4277 (1.36)	5014 (1.30)
$w = \infty$	nuanua1	-	-	3134	4192 (1.33)	4192 (1.33)

- $w = 0$: No. Problem: $R[F_2^{\nu \mathrm{Fe}}]$
- $w = 1/7$: No. Problem: $R[F_2^{\nu \mathrm{Fe}}]$
- $w = 1/4, 1/2$: Possible candidates for compromise solution but...
  - $Q^2 = 5$: Undershoots $R[F_2^{\ell^\pm \mathrm{Fe}}]$ for $x < 0.2$. Overshoots $R[F_2^{\nu \mathrm{Fe}}]$ for $x \in [0.1, 0.3]$
  - $Q^2 = 20$: $R[F_2^{\ell^\pm \mathrm{Fe}}]$ still ok. Overshoots $R[F_2^{\nu \mathrm{Fe}}]$.
- $w = 1$: No. Possibly there is a compromise if more strict Q^2 cut?
  - $Q^2 = 5$: Undershoots $R[F_2^{\ell^\pm \mathrm{Fe}}]$ for $x < 0.2$. $R[F_2^{\nu \mathrm{Fe}}]$ ok.
  - $Q^2 = 20$: $R[F_2^{\ell^\pm \mathrm{Fe}}]$ still ok. $R[F_2^{\nu \mathrm{Fe}}]$ ok.
- $w = \infty$: No. Problem: $R[F_2^{\ell^\pm \mathrm{Fe}}]$

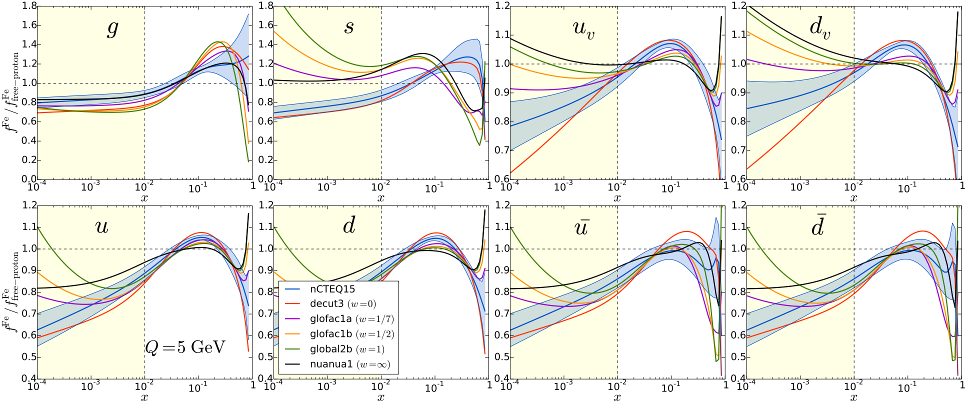
How does it look at the level of nPDFs?

$$f_i^{\mathrm{Fe}}(x,Q) = \frac{26}{56} f_i^{p/\mathrm{Fe}}(x,Q) + \frac{56-26}{56} f_i^{n/\mathrm{Fe}}(x,Q)$$



How does it look at the level of nPDFs?

$$f_i^{\rm Fe}(x,Q) = \frac{26}{56} f_i^{p/{\rm Fe}}(x,Q) + \frac{56-26}{56} f_i^{n/{\rm Fe}}(x,Q)$$



nCTEQ15 PDFs PRD 93 (2016) 085037

Discussion/Intermediate Conclusion

Discussion based on the comparison of the nuclear correction factors $R[F_2^{\ell^\pm A}]$ and $R[F_2^{\nu A}]$

- There is definitely a tension between the NuTeV and the charged lepton data
  - There is a clear dependence on the weight.
  - Theory curves for $R[F_2^{\ell^\pm A}]$ and $R[F_2^{\nu A}]$ are both shifted down with increasing weight of the neutrino data.
- Preliminary conclusion: At the level of the (high) precision there doesn't seem to be a good compromise fit of the combined $\ell^\pm A$, DY and νA data.
- However one has to be careful:
  - These are precision effects
  - For each weight, the curves have uncertainty bands not considered
  - The figures show the comparison to only few (representative) data
- Comparison of the corresponding nPDFs with the newer nCTEQ15 nPDFs including error bands also show tension for the valence distributions.

Consider next quantitative criterion based on χ^2

Tolerance criterion

Probability distribution for the χ^2 function with N degrees of freedom

$$P_N(\chi^2) = \frac{(\chi^2)^{N/2-1} e^{-\chi^2/2}}{2^{N/2}\Gamma(N/2)}$$

Determine percentile ξ_{50} and ξ_{90}, ξ_{99} (i.e. $p = 50$, $p = 90$, $p = 99$):

$$\int_0^{\xi_p} d\chi^2 P_N(\chi^2) = p/100$$

Account for the deviations of minimal χ^2 compared to the mean value ξ_{50}:

$$C_{90} = \chi_0^2 \frac{\xi_{90}}{\xi_{50}} \qquad C_{99} = \chi_0^2 \frac{\xi_{99}}{\xi_{50}}$$

A fit with a given χ^2 is compatible with the best fit with χ_0^2 at 90% (99%) C.L. if $\chi^2 < C_{90}$ ($\chi^2 < C_{99}$).

Tolerance criterion – total χ^2

Tolerance conditions for the **charged lepton** χ^2 and the **neutrino** χ^2 at 90% (99%) C.L.

- decut3 ($w = 0$): $C_{90}^{\ell^\pm A} = 684$ and $C_{99}^{\ell^\pm A} = 722$
- nuanua1 ($w = \infty$): $C_{90}^{\nu A} = 4330$ and $C_{99}^{\nu A} = 4445$

Is there a compromise fit compatible to both, decut3 **and** nuanua1 at 90% CL.?

Weight	Fit name	$\ell^\pm$ data	χ^2	ν data	χ^2	total χ^2 (/pt)
$w = 0$	decut3	708	639	-	nnnn **NO**	639 (0.90)
$w = 1/7$	glofac1a	708	645 **YES**	3134	4710 **NO**	5355 (1.39)
$w = 1/4$	glofac1c	708	654 **YES**	3134	4501 **NO**	5155 (1.34)
$w = 1/2$	**glofac1b**	708	680 **YES**	3134	4405 **NO***	5085 (1.32)
$w = 1$	global2b	708	736 **NO**	3134	4277 **YES**	5014 (1.30)
$w = \infty$	nuanua1	-	nnn **NO**	3134	4192	4192 (1.33)

Observations:

- There is no good compromise fit based on the 90% C.L. criterion.
- Our best candidate is **glofac1b** which is *marginally* compatible: $4405 - 4192 = 213 \simeq 1.5 \times (C_{90}^{\nu A} - \chi_0^2)$
- Observations in agreement with the previous conclusions based on $R[F_2^{\ell^\pm \mathrm{Fe}}]$ and $R[F_2^{\nu \mathrm{Fe}}]$.

Tolerance criterion – total χ^2

Tolerance conditions for the **charged lepton** χ^2 and the **neutrino** χ^2 at 90% (99%) C.L.

- decut3 ($w = 0$): $C_{90}^{\ell^\pm A} = 684$ and $C_{99}^{\ell^\pm A} = 722$
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Is there a compromise fit compatible to both, decut3 **and** nuanua1 at 99% CL.?

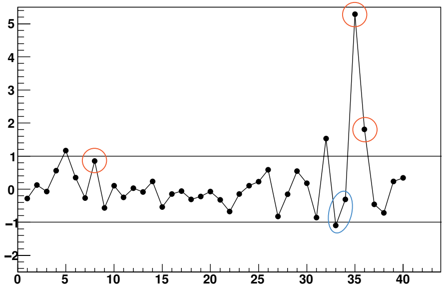
Weight	Fit name	$\ell^\pm$ data	χ^2	ν data	χ^2	total χ^2 (/pt)
$w = 0$	decut3	708	639	-	nnnn **NO**	639 (0.90)
$w = 1/7$	glofac1a	708	645 **YES**	3134	4710 **NO**	5355 (1.39)
$w = 1/4$	glofac1c	708	654 **YES**	3134	4501 **NO**	5155 (1.34)
$w = 1/2$	**glofac1b**	708	680 **YES**	3134	4405 **YES**	5085 (1.32)
$w = 1$	global2b	708	736 **NO**	3134	4277 **YES**	5014 (1.30)
$w = \infty$	nuanua1	-	nnn **NO**	3134	4192	4192 (1.33)

Observations:

- At the 99% C.L. there is only one compromise fit **glofac1b** ($w = 1/2$).

Tolerance criterion – individual data sets

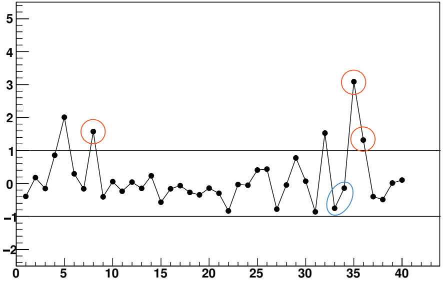
glofac1a ($w = 1/7$)



- **Y-axis**: $\frac{\Delta\chi^2}{\Delta C_{90}} = \frac{\chi_i^2 - \chi_{i,0}^2}{C_{90,i} - \chi_{i,0}^2}$; **X-axis**: Number of the data set ($n = 1, \ldots, 40$)
  $\frac{\Delta\chi^2}{\Delta C_{90}} = 1$: 90% C.L. $\frac{\Delta\chi^2}{\Delta C_{90}} \simeq 2$: 99% C.L.
- Important data sets:
  - $n = 8$ (red circle): Fe/D charged lepton data
  - $n = 33, 34$ (blue ellipse): CHORUS νPb, $\bar{\nu}$Pb cross section data
  - $n = 35, 36$ (red ellipse): NuTeV νFe, $\bar{\nu}$Fe cross section data

Tolerance criterion – individual data sets

glofac1c ($w = 1/4$)

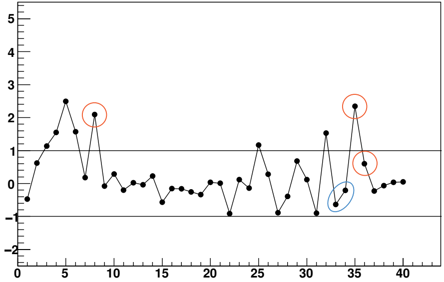


- **Y-axis**: $\frac{\Delta\chi^2}{\Delta C_{90}} = \frac{\chi_i^2 - \chi_{i,0}^2}{C_{90,i} - \chi_{i,0}^2}$; **X-axis**: Number of the data set ($n = 1, \ldots, 40$)

  $\frac{\Delta\chi^2}{\Delta C_{90}} = 1$: 90% C.L. $\frac{\Delta\chi^2}{\Delta C_{90}} \simeq 2$: 99% C.L.
- Important data sets:
  - $n = 8$ (red circle): Fe/D charged lepton data
  - $n = 33, 34$ (blue ellipse): CHORUS νPb, $\bar{\nu}$Pb cross section data
  - $n = 35, 36$ (red ellipse): NuTeV νFe, $\bar{\nu}$Fe cross section data

Tolerance criterion – individual data sets

glofac1b ($w = 1/2$)

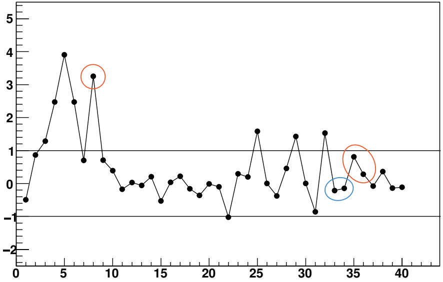


- **Y-axis**: $\frac{\Delta\chi^2}{\Delta C_{90}} = \frac{\chi_i^2 - \chi_{i,0}^2}{C_{90,i} - \chi_{i,0}^2}$; **X-axis**: Number of the data set ($n = 1, \ldots, 40$)

  $\frac{\Delta\chi^2}{\Delta C_{90}} = 1$: 90% C.L. $\frac{\Delta\chi^2}{\Delta C_{90}} \simeq 2$: 99% C.L.
- Important data sets:
  - $n = 8$ (red circle): Fe/D charged lepton data
  - $n = 33, 34$ (blue ellipse): CHORUS νPb, $\bar{\nu}$Pb cross section data
  - $n = 35, 36$ (red ellipse): NuTeV νFe, $\bar{\nu}$Fe cross section data

Tolerance criterion – individual data sets

global2b ($w = 1$)



- **Y-axis**: $\frac{\Delta\chi^2}{\Delta C_{90}} = \frac{\chi_i^2 - \chi_{i,0}^2}{C_{90,i} - \chi_{i,0}^2}$; **X-axis**: Number of the data set ($n = 1, \ldots, 40$)

  $\frac{\Delta\chi^2}{\Delta C_{90}} = 1$: 90% C.L. $\frac{\Delta\chi^2}{\Delta C_{90}} \simeq 2$: 99% C.L.
- Important data sets:
  - $n = 8$ (red circle): Fe/D charged lepton data
  - $n = 33, 34$ (blue ellipse): CHORUS νPb, $\bar{\nu}$Pb cross section data
  - $n = 35, 36$ (red ellipse): NuTeV νFe, $\bar{\nu}$Fe cross section data

Tolerance criterion – individual data sets

Observations:

- CHORUS data (blue ellipse) always compatible; little dependence on weight w
- $w = 1/7$: $\Delta\chi^2/\Delta C_{90} > 5$ for NuTeV νFe; $\Delta\chi^2/\Delta C_{90} \simeq 1.8$ for NuTeV $\bar{\nu}$Fe
- increasing weight: NuTeV cross section data improve; charged lepton Fe/D data get worse
- our best candidate ($w = 1/2$)
  - Fe/D ($n = 8$): $\Delta\chi^2/\Delta C_{90} \simeq 2$
  - NuTeV νFe ($n = 35$): $\Delta\chi^2/\Delta C_{90} \simeq 2.2$
  - NuTeV $\bar{\nu}$Fe ($n = 36$): $\Delta\chi^2/\Delta C_{90} < 1$
  - some other data sets $n = 3, 4, 5, 6, 32$ with $\Delta\chi^2/\Delta C_{90} > 1$
- $w = 1$: Fe/D ($n = 8$): $\Delta\chi^2/\Delta C_{90} > 3$
- Confirms and quantifies observations based on R plots

Summary of the nCTEQ studies

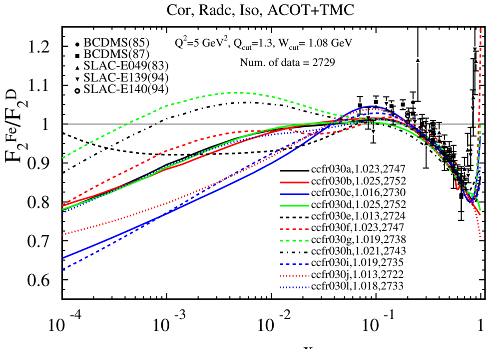
Based on nuclear correction factors R and the 90% C.L. tolerance criterion $\Delta\chi^2/\Delta C_{90} < 1$:

- There is no good compromise fit to the $\ell^\pm A$ DIS + DY + νA DIS data.
- Most problematic: tension between NuTeV νFe cross section data and Fe/D data in charged lepton DIS.
- The NuTeV $\bar{\nu}$Fe data are less problematic. They have larger errors.
- The CHORUS νPb and $\bar{\nu} Pb$ data are compatible with both, the $\ell^\pm A$-DIS+DY and the NuTeV νFe and $\bar{\nu}$Fe data, as is well known. They also have larger errors.

- Relaxing the tolerance criterion to 99% C.L. ($\Delta\chi^2/\Delta C_{90} \lesssim 2$) the fit with weight $w = 1/2$ would be *marginally* acceptable.
- This can also (qualitatively) be verified with the R-plots.
- A larger Q^2-cut, say $Q^2 > 5$ GeV^2, could also help to reduce the tension. (In particular, this would remove some of the rather precise NuTeV cross section data at small x.)

Is that all?

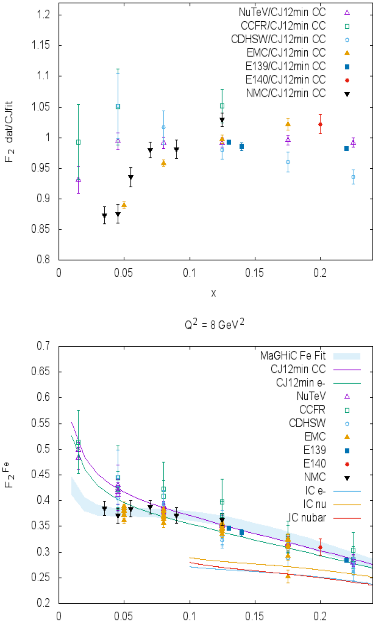
- PROBLEM: this conclusions are based only on the NuTeV experiment in particular including correlated errors! (if errors added in quadrature still slight tensions exist).
  - On top of this fitting just the NuTeV data the obtained $\chi^2/\text{dof} \geq 1.3$
  - Even dividing the NuTeV data into neutrino and anti-neutrino and fitting them separately it still gives $\chi^2/\text{dof} \geq 1.3$ (neutrinos).
- CHORUS is generally compatible.
- How about CCFR? If we analyses CCFR data instead of the NuTeV we are able to obtain a compromise fit:



Is that all?

- How about other analysis?

Direct use of absolute cross-sections/structure functions

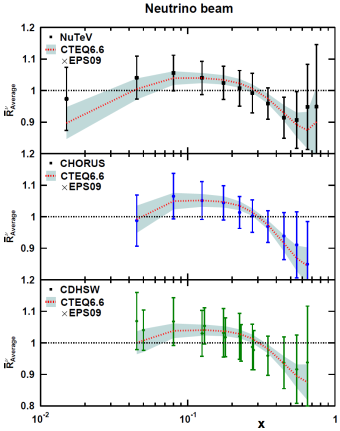


Analysis from JLAB by Cynthia [PRC 96 (2017) 032201]

- Check compatibility of the charged-lepton and neutrino DIS data on Fe target.
- Work on the level of absolute structure functions instead of ratios – removes deuteron problem ($F_2^{\ell^\pm A}$ and $F_2^{\nu A}$ compared using 18/5 scaling).
- **Main findings**: discrepancy between $F_2^{\ell^\pm A}$ and $F_2^{\nu A}$ data on the order of 15% in the low-x region.

Other global nPDF analysis

- DSSZ [PRD 85 (2012) 074028]:
  - No problem seen by the Authors.
  - Use of structure functions instead of cross-sections.
  - No correlated errors.
- EPS [JHEP 1007 (2010) 032; PRL 110 (2013) 212301]:
  - Use of cross-section data (no correlated errors).
  - Notice problems with energy dependence of the NuTeV data.
  - It is attributed to problems with the data and dealt with by normalizing data to individual energy bins.
  - Then the NuTeV data is find to be fully compatible with the charged lepton data.



Normalize the data "by itself"

[H. Paukkunen, DIS2013,
22-26 April 2013, Marseille, France]

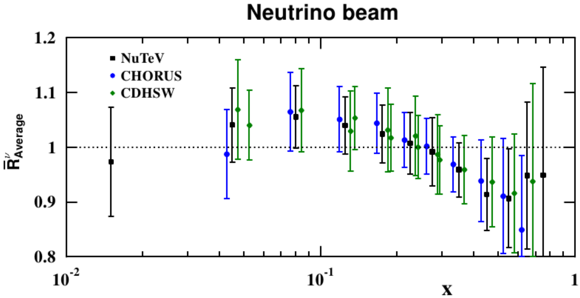
- **Try to account for the differences in the absolute normalization. Define**

$$I^{\nu}_{\text{exp}}(E) \equiv \sum_{i \in \text{fixed } E} \sigma_{\text{exp},i}(x,y,E) \times B_i(x,y)$$

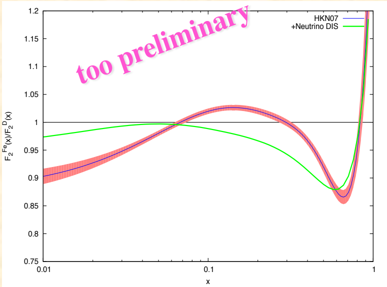
Size of the experimental bin

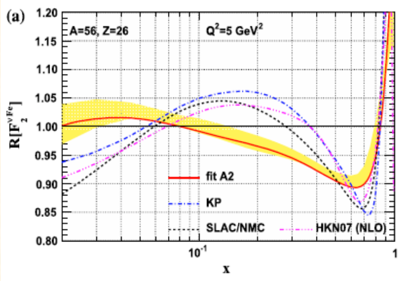
- **Instead of "bare" cross-section ratios, consider ratios of *normalized* cross-sections**

$$\overline{R}^{\nu}(x,y,E) \equiv \frac{\sigma^{\nu}_{\text{exp}}(x,y,E)/I^{\nu}_{\text{exp}}(E)}{\sigma^{\nu}_{\text{CTEQ6.6}}(x,y,E)/I^{\nu}_{\text{CTEQ6.6}}(E)}.$$



Our research in progress (M. Hirai, SK, K. Saito)





We are getting a similar modification to the nCTEQ one.

Questions

- What is actually the correct interpretation
  - are the $\ell^{\pm}A$ and νA data compatible?
  - or is it only the NuTeV data that has problems?
  - (in the nPDF analyses the incompatibilities originate only from the NuTeV data)
- Should we trust the NuTeV data and use it in the PDF/nPDF fits?
- Should we rather use CCFR instead of NuTeV?
- Is there any hope for results on cross-sections from NOMAD?

Thank You

Nuclear CTEQ parto ×

ncteq.hepforge.org

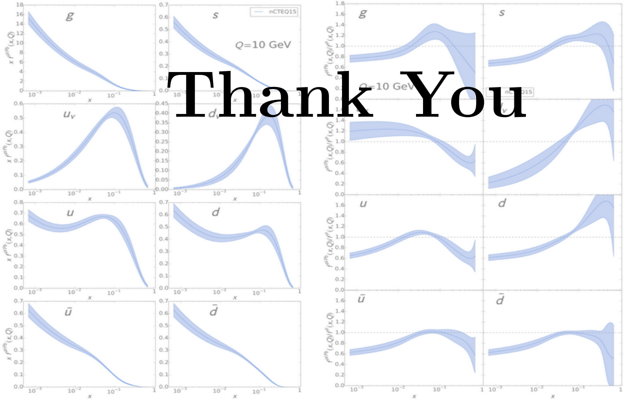
nCTEQ is hosted by Hepforge, IPPP Durham

nCTEQ

nuclear parton distribution functions

- Home
- PDF grids & code
  - nCTEQ15
  - previous PDF grids
- Papers & Talks
- Subversion
- Tracker
- Wiki

nCTEQ project is an extension of the CTEQ collaborative effort to determine parton distribution functions inside of a free proton. It generalizes the free-proton PDF framework to determine densities of partons in bound protons (hence nCTEQ which stands for nuclear CTEQ). All details on the framework and the first complete results can be found in arXiv:15????? [hep-ph].
The effects of the nuclear environement on the parton densities can be shown as modified parton densities or nuclear correction factors (for example for lead as shown below)



BACKUP SLIDES

nPDFs from neutrino DIS data [PRD 80 (2009) 094004]

Ratio of iron and deuteron structure functions depending on the used deuteron.

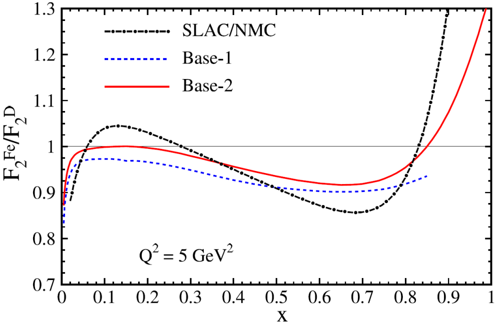
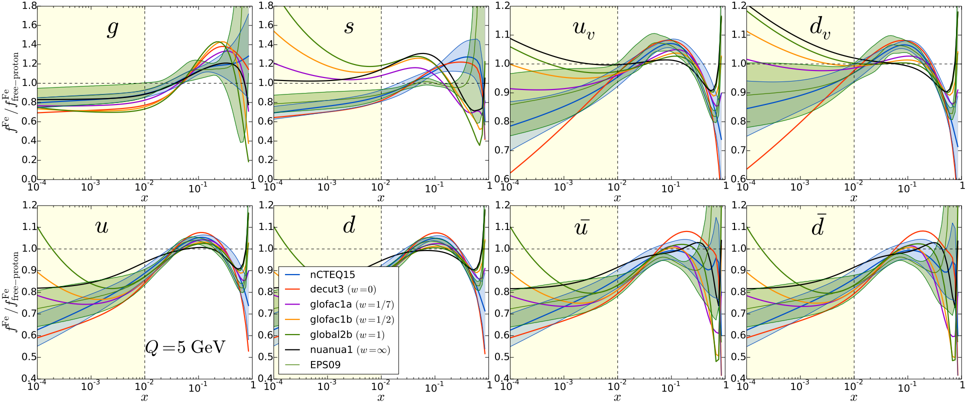


FIG. 10: Predictions (solid and dashed line) for the structure function ratio F_2^{Fe}/F_2^D using the iron PDFs extracted from fits to NuTeV neutrino and anti-neutrino data (fit 'A2'). The SLAC/NMC parameterization is shown with the dot-dashed line. The structure function F_2^D in the denominator has been computed using either the Base-2 (solid line) or the Base-1 (dashed line) PDFs. A nuclear correction factor for deuterium has been included in the Base-1 calculation [10].

How does it look at the level of nPDFs?

$$f_i^{\mathrm{Fe}}(x,Q) = \frac{26}{56} f_i^{p/\mathrm{Fe}}(x,Q) + \frac{56-26}{56} f_i^{n/\mathrm{Fe}}(x,Q)$$



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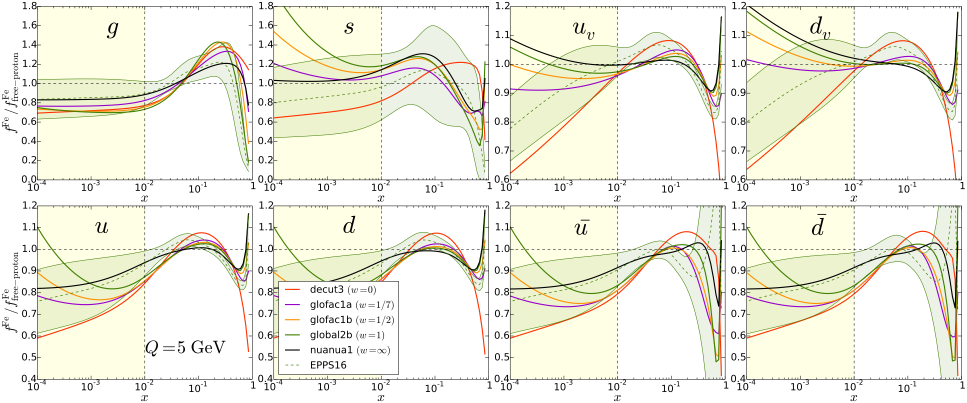


The framework of the present analysis

[H. Paukkunen, DIS2013, 22-26 April 2013, Marseille, France]

- **The main elements of our calculations**

  - NLO pQCD & SACOT prescription for the heavy quarks
  - Target mass correction (Qiu et.al. *JHEP 0807 (2008) 090*)

$$\int_x^1 \frac{dz}{z}\omega_{ik}(z)\, f_k^A\left(\frac{x}{z}\right) \to \int_x^1 \frac{dz}{z}\omega_{ik}(z)\, f_k^A\left(\frac{\xi}{z}\right) \qquad \xi \equiv 2x/(1+\sqrt{1+4x^2M^2/Q^2})$$

  - Electroweak radiation (Bardin et.al *JHEP 0506 (2005) 078*)

$$F_i^A = \sum_k \left[\omega_{ik}^{\mathrm{LO}}\left(1+\Delta_k^{\mathrm{radiative}}\right) + \omega_{ik}^{\mathrm{NLO}}\right] \otimes f_k^A$$

  - Use CTEQ6.6 free proton PDFs & EPS09 nuclear modifications

- **Plot the data as a weighted average**

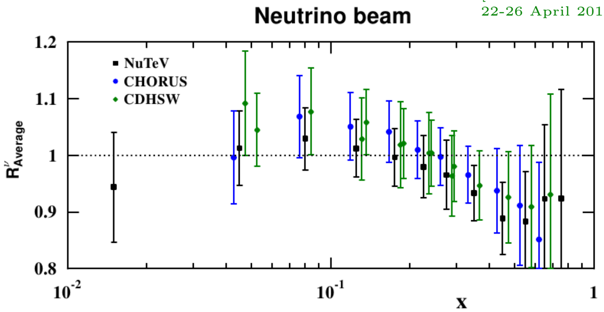
$$R_{\mathrm{Average}}^{\mathrm{CTEQ6.6}} \equiv \left(\sum_{i\in \mathrm{fixed}\, x}^{N} \frac{R_i^{\mathrm{CTEQ6.6}}}{\sigma_i}\right)\left(\sum_{i\in \mathrm{fixed}\, x}^{N} \frac{1}{\sigma_i}\right)^{-1} \pm N \times \left(\sum_{i\in \mathrm{fixed}\, x}^{N} \frac{1}{\sigma_i}\right)^{-1}$$

$$R^{\mathrm{CTEQ6.6}} \equiv \frac{\sigma^{\nu,\overline{\nu}}\,(\mathrm{Experimental})}{\sigma^{\nu,\overline{\nu}}\,(\mathrm{CTEQ6.6})}$$

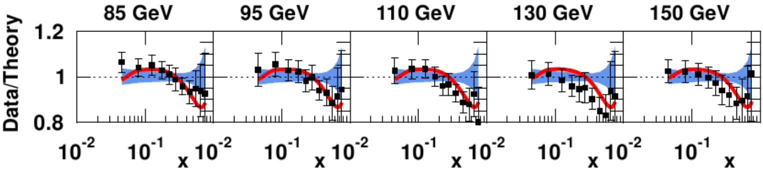
← *practically independent of Q^2*

Inconsistecies in the absolute normalization

[H. Paukkunen, DIS2013,
22-26 April 2013, Marseille, France]



- The NuTeV data few percents below the rest
- Not a big surprise as it has been shown (JHEP 1007 (2010) 032), that the NuTeV data shows different normalization from a neutrino energy to antoher. For example:



Normalize the data "by itself"

[H. Paukkunen, DIS2013,
22-26 April 2013, Marseille, France]

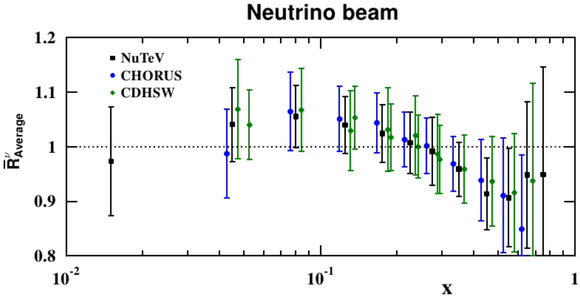
- **Try to account for the differences in the absolute normalization. Define**

$$I^{\nu}_{\rm exp}(E) \equiv \sum_{i \in {\rm fixed}\, E} \sigma_{{\rm exp},i}(x,y,E) \times B_i(x,y)$$

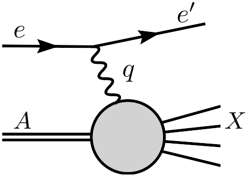
Size of the experimental bin

- **Instead of "bare" cross-section ratios, consider ratios of *normalized* cross-sections**

$$\overline{R}^{\nu}(x,y,E) \equiv \frac{\sigma^{\nu}_{\rm exp}(x,y,E)/I^{\nu}_{\rm exp}(E)}{\sigma^{\nu}_{\rm CTEQ6.6}(x,y,E)/I^{\nu}_{\rm CTEQ6.6}(E)}.$$



Variables: DIS of nuclear target $eA \to e'X$



- DIS variables in case on nucleons

  in nucleus $\begin{cases} Q^2 \equiv -q^2 \\ x_A \equiv \frac{Q^2}{2\, p_A \cdot q} \end{cases}$

  - p^A – nucleus momentum
  - $x_A \in (0, 1)$ – analog of Bjorken variable (fraction of the nucleus momentum carried by a nucleon)

- Analogue variables for partons:

  - $p_N = \frac{p_A}{A}$ – *average* nucleon momentum
  - $x \equiv \frac{Q^2}{2\, p_N \cdot q} = A\, x_A$ – parton momentum fraction with respect to the average nucleon momentum p_N
  - $x \in (0, A)$ – parton can carry more than the average nucleon momentum p_N.

Available nuclear PDFs

- Multiplicative nuclear correction factors

$$f_i^{p/A}(x_N, \mu_0) = R_i(x_N, \mu_0, A)\ f_i^{\text{free proton}}(x_N, \mu_0)$$

  - **HKN**: Hirai, Kumano, Nagai [PRC 76, 065207 (2007)]
  - **DSSZ**: de Florian, Sassot, Stratmann, Zurita [PRD 85, 074028 (2012)]
  - **EPS09**: Eskola, Paukkunen, Salgado [JHEP 04 (2009) 065]
  - **EPPS16**: Eskola, Paakkinen, Paukkunen, Salgado [EPJC 77 (2017) 163]
  - **KT16**: Khanpour, Tehrani [PRD 93, 014026 (2016)]

- Convolution relation

$$f_i^{p/A}(x_N, Q_0^2) = \int_{x_N}^{A} \frac{dy}{y} W_i(y, A, Z)\ f_i^{\text{free proton}}\left(\frac{x_N}{y}, Q_0^2\right)$$

  - **DS04**: de Florian, Sassot [PRD 69, 074028 (2004)]

- Native nuclear PDFs

$$f_i^{p/A}(x_N, \mu_0) = f_i(x_N, A, \mu_0)$$
$$f_i(x_N, A = 1, \mu_0) \equiv f_i^{\text{free proton}}(x_N, \mu_0)$$

  - **nCTEQ15**: Kovarik, Kusina, Jezo, Clark, Keppel, Lyonnet, Morfin, Olness, Owens, Schienbein, Yu [PRD 93, 085037 (2016), `arXiv:1509.00792`]

nPDF framework

- **Parametrization**
  - PDF of nucleus (A - mass, Z - charge)

$$f_i^{(A,Z)}(x,Q) = \frac{Z}{A} f_i^{p/A}(x,Q) + \frac{A-Z}{A} f_i^{n/A}(x,Q)$$

  - bound neutron PDFs, $f_i^{n/A}$, constructed assuming iso-spin symmetry
    (Note that $f_i^{p/A}$, $f_i^{n/A}$ are not physical objects just convenient way of parameterizing f_i^A)
  - bound proton PDFs parametrized:

nPDF framework

- **Parametrization**
  - PDF of nucleus (A - mass, Z - charge)

$$f_i^{(A,Z)}(x,Q) = \frac{Z}{A} f_i^{p/A}(x,Q) + \frac{A-Z}{A} f_i^{n/A}(x,Q)$$

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    (Note that $f_i^{p/A}$, $f_i^{n/A}$ are not physical objects just convenient way of parameterizing f_i^A)
  - bound proton PDFs parametrized:

nCTEQ15 [arXiv:1509.00792]

$$x f_i^{p/A}(x,Q_0) = x^{c_1}(1-x)^{c_2} e^{c_3 x}(1+e^{c_4}x)^{c_5}$$

$$c_k \to c_k(A) \equiv c_{k,0} + c_{k,1}\left(1 - A^{-c_{k,2}}\right)$$

nPDF framework

- **Parametrization**
  - PDF of nucleus (A - mass, Z - charge)

$$f_i^{(A,Z)}(x,Q) = \frac{Z}{A} f_i^{p/A}(x,Q) + \frac{A-Z}{A} f_i^{n/A}(x,Q)$$

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$$c_k \to c_k(A) \equiv c_{k,0} + c_{k,1}\left(1 - A^{-c_{k,2}}\right)$$

EPPS16 [arXiv:1612.05741]

$$f_i^{p/A}(x,Q) = R_i^A(x,Q)\, f_i^p(x,Q),$$

$$R_i^A(x,Q_0) = \begin{cases} a_0 + a_1(x-x_a)^2 & x \le x_a \\ b_0 + b_1 x^\alpha + b_2 x^{2\alpha} + b_3 x^{3\alpha} & x_a \le x \le \\ c_0 + (c_1 - c_2 x)(1-x)^{-\beta} & x_e \le x \le \end{cases}$$

$$d_i \to d_i(A) = d_i(A_{\rm ref})\left(\frac{A}{A_{\rm ref}}\right)^{\gamma_i[d_i(A_{\rm ref})-1]},$$

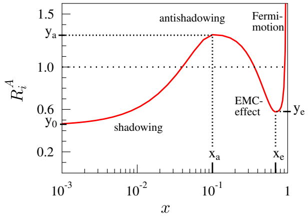
with $d_i = a_i, b_i, \ldots$ and $A_{\rm ref} = 12$

nPDF framework

- **Parametrization**
  - PDF of nucleus (A - mass, Z - charge)

$$f_i^{(A,Z)}(x,Q) = \frac{Z}{A} f_i^{p/A}(x,Q) + \frac{A-Z}{A} f_i^{n/A}(x,Q)$$

  - bound neutron PDFs, $f_i^{n/A}$, constructed assuming iso-spin symmetry
    (Note that $f_i^{p/A}$, $f_i^{n/A}$ are not physical objects just convenient way of parameterizing f_i^A)
  - bound proton PDFs parametrized:



EPPS16 [arXiv:1612.05741]

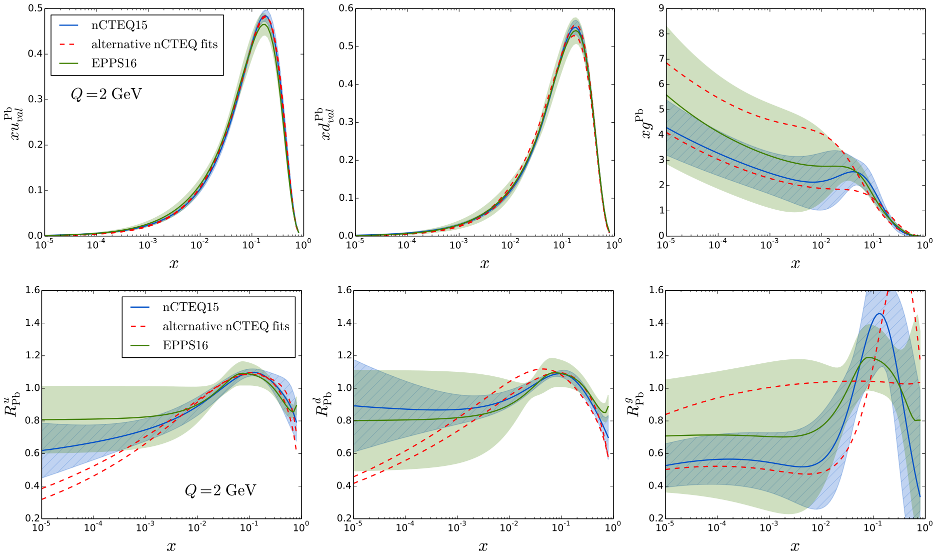
$$f_i^{p/A}(x,Q) = R_i^A(x,Q)\, f_i^p(x,Q),$$

$$R_i^A(x,Q_0) = \begin{cases} a_0 + a_1(x-x_a)^2 & x \le x_a \\ b_0 + b_1 x^{\alpha} + b_2 x^{2\alpha} + b_3 x^{3\alpha} & x_a \le x \le \\ c_0 + (c_1 - c_2 x)(1-x)^{-\beta} & x_e \le x \le \end{cases}$$

$$d_i \to d_i(A) = d_i(A_{\rm ref}) \left(\frac{A}{A_{\rm ref}}\right)^{\gamma_i [d_i(A_{\rm ref})-1]},$$

with $d_i = a_i, b_i, \ldots$ and $A_{\rm ref} = 12$

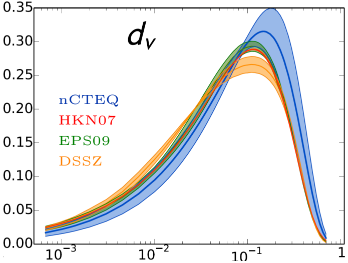
Current nPDFs



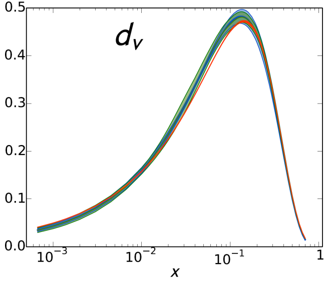
Note

- Bound proton PDFs, $f_i^{p/A}$, are only **effective** means of parameterizing the full nPDFs f_i^A

$d_v^{p/\mathrm{Pb}}$



$d_v^{\mathrm{Pb}} = \frac{82}{208} d_v^{p/\mathrm{Pb}} + \frac{208-82}{208} d_v^{n/\mathrm{Pb}}$



Comparison of available nPDFs

	EPPS16	EPS09	nCTEQ15	DSSZ12	HKN07	KA15
FT e-DIS	✓	✓	✓	✓	✓	✓
FT ν-DIS	✓	✗	✗#	✓	✗	✗
FT Drell-Yan	✓	✓	✓	✓	✓	✓
RHIC π^0	✓	✓	✓	✗	✗	✗
LHC W/Z	✓	✗	✗*	✗	✗	✗
LHC dijet	✓	✗	✗	✗	✗	✗
QCD order	NLO	LO & NLO	NLO	NLO	LO & NLO	NNLO
Kinematic cuts	$Q > 1.3$GeV	$Q > 1.3$GeV	$Q > 2$GeV $W > 3.5$GeV	$Q > 1$GeV	$Q > 1$GeV	$Q > 1$GeV
No data points	1811	929	740	1579	1241	1479
No free param.	20	15	16	25	12	16
χ^2/dof	1.00	0.79	0.81	0.99	1.21	1.15
Error analysis	Hessian	Hessian	Hessian	Hessian	Hessian	Hessian
Tolerance $\Delta\chi^2$	52	50	35	30	13.7	1?
Proton baseline	CT14NLO	CTEQ6.1	CTEQ6.1-like	MSTW2008	MRST1998	JR09
Heavy-quark eff.	✓	✗	✓	✓	✗	✗
Flavour sep.	✓	✗	✓ (val.)	✗	✗	✗
Reference	[1612.05741]	[0902.4154]	[1509.00792]	[1112.6324]	[0709.3038]	[1601.00939]

In a separate dedicated analysis [PRL106, 122301, (2011), 1012.0286; PRD80, 094004, (2009), 0907.2357]
* See a reweighting study [EPJC77, 488 (2017), 1610.02925]

LHeC

EPS09:

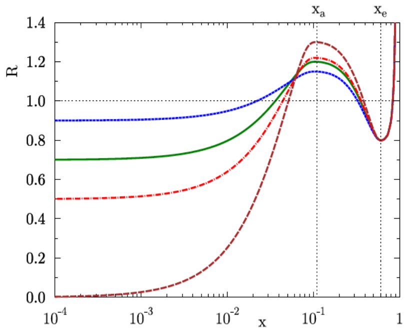
[N. Armesto, 2015 LHeC Workshop, Chavannes-de-Bogis, June 26th 2015]

- Very little freedom at small x.

The fit function in EPS09:

$$R^{\mathrm{EPS09}}(x) = \begin{cases} a_0 + (a_1 + a_2 x)\left(e^{-x} - e^{-x_a}\right) & x \leq x_a \\ b_0 + b_1 x + b_2 x^2 + b_3 x^3 & x_a \leq x \leq x_e \\ c_0 + (c_1 - c_2 x)(1-x)^{-\beta} & x_e \leq x \leq 1 \end{cases}$$

(power-law parametrization of A-dependence at x_a, x_e, and $x \to 0$)

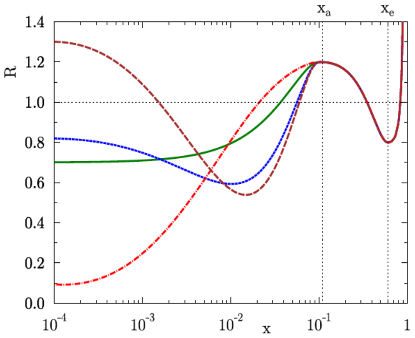


New ICs:

[N. Armesto, 2015 LHeC Workshop, Chavannes-de-Bogis, June 26th 2015]

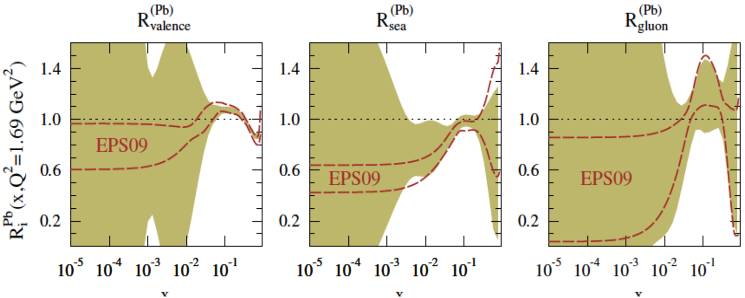
- Use a far more flexible form to reduce the bias at small x:

$$R(x \leq x_a) = a_0 + a_1(x - x_a)^2 + \sqrt{x}(x_a - x)\left[a_2 \log\left(\frac{x}{x_a}\right) + a_3 \log^2\left(\frac{x}{x_a}\right) + a_4 \log^3\left(\frac{x}{x_a}\right)\right]$$



New fit framework:

The baseline fit using the new fit functions: no control over small x!



The lower bound restricted here by $F_L(Q^2 = 2\,\mathrm{GeV}^2, x > 10^{-5}) > 0$

Maybe against "physical intuition" (small-x theory predicts shadowing, $R_i < 1$), but consistent with the data.

E.g. in EPS09, small-x shadowing was essentially built in

[N. Armesto, 2015 LHeC Workshop, Chavannes-de-Bogis, June 26th 2015]

Uncertainties in global analysis

- **Experimental errors** (included in PDFs error analysis)
- **Theoretical uncertainties** (e.g. HF schemes; not included)
- **"Details"** of Global Fits
  (e.g. parametrization; not included)

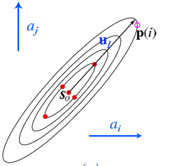
Propagating experimental errors to PDFs:

- Hessian Method
  - Eigenvector PDFs
  - Quadratic approximation
  - Simple computation of correlations
- Lagrange Multipliers
- Monte Carlo Methods
  - generate N data samples by varying data within errors;
  - perform N fits to the samples $\rightarrow$ PDF replicas
  - estimate uncertainty by calculating moments of PDF replicas

Hessian method

- **Expand** χ^2 function around minimum, $\{a_i^0\}$,

$$\chi^2 = \chi_0^2 + \sum_{ij} \frac{1}{2}(a_i - a_i^0)(a_j - a_j^0)\left(\frac{\partial^2 \chi^2}{\partial a_i \partial a_j}\right)_0$$

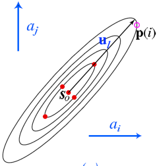


(a)
Original parameter basis

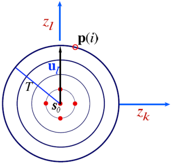
Hessian method

- **Expand** χ^2 function around minimum, $\{a_i^0\}$, and **diagonalize**

$$\chi^2 = \chi_0^2 + \sum_{ij} \frac{1}{2}(a_i - a_i^0)(a_j - a_j^0)\left(\frac{\partial^2 \chi^2}{\partial a_i \partial a_j}\right)_0 = \chi_0^2 + \sum_i \lambda_i z_i^2$$



(a)
Original parameter basis



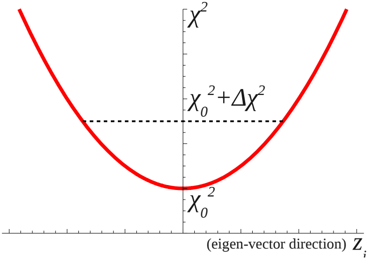
(b)
Orthonormal eigenvector basis

Hessian method

- **Expand** χ^2 function around minimum, $\{a_i^0\}$, and **diagonalize**

$$\chi^2 = \chi_0^2 + \sum_{ij} \frac{1}{2}(a_i - a_i^0)(a_j - a_j^0)\left(\frac{\partial^2\chi^2}{\partial a_i \partial a_j}\right)_0 = \chi_0^2 + \sum_i \lambda_i z_i^2$$

- Choose tolerance criteria $\Delta\chi^2 = \chi^2 - \chi_0^2$ value (defining 1-σ uncertainty),
  - ideal case $\Delta\chi^2 = 1$
  - realistic global analysis $\Delta\chi^2 \sim 1 - 100$



Hessian method

- **Expand** χ^2 function around minimum, $\{a_i^0\}$, and **diagonalize**

$$\chi^2 = \chi_0^2 + \sum_{ij} \frac{1}{2}(a_i - a_i^0)(a_j - a_j^0)\left(\frac{\partial^2 \chi^2}{\partial a_i \partial a_j}\right)_0 = \chi_0^2 + \sum_i z_i^2$$

- Choose tolerance criteria $\Delta\chi^2 = \chi^2 - \chi_0^2$ value (defining 1-σ uncertainty),
  - ideal case $\Delta\chi^2 = 1$
  - realistic global analysis $\Delta\chi^2 \sim 1 - 100$
- Construct error PDFs corresponding to each eigenvector direction:

$$f_i^{\pm} = f(\{z_i\}) = f(0, \ldots, z_i = \pm\sqrt{\Delta\chi^2}, \ldots, 0)$$
$$z_i = \pm\sqrt{\Delta\chi^2}$$

- Calculate errors of observable X:

$$\Delta X = \sqrt{\sum_i \left(\frac{\partial X}{\partial z_i} \times \delta z_i\right)^2} \simeq \frac{1}{2}\sqrt{\sum_i \left[X(f_i^+) - X(f_i^-)\right]^2}$$