Nuclear Medium Effects in EM and Weak Structure Functions

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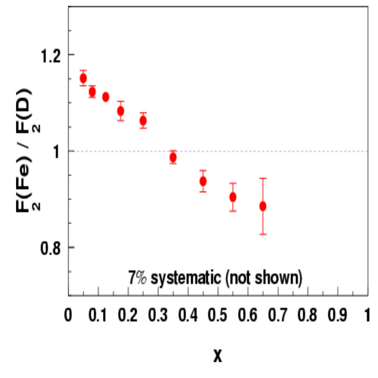
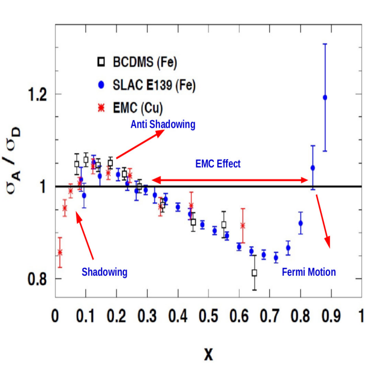
Outline

1. *Introduction*
2. *Nuclear Effects*
   - Phenomenological vs Theoretical approaches
3. *Nuclear Model*
   - Spectral Function
   - Mesonic Contribution
4. *Neutrino-Nucleus Scattering*
5. *Results*

What happens when the interaction takes place with a nucleon bound inside the nucleus?

Aubert et al, Phys. Lett. B123, 275 (1983)





To understand nuclear medium effects, there are two approaches:

- Phenomenological
- Theoretical

Phenomenological Efforts

Phenomenological group	data types used
EKS98	l+A DIS, p+A DY
HKM	l+A DIS
HKN04	l+A DIS, p+A DY
nDS	l+A DIS, p+A DY
EKPS	l+A DIS, p+A DY
HKN07	l+A DIS, p+A DY
EPS08	l+A DIS, p+A DY, $h^{\pm}, \pi^0, \pi^{\pm}$ in d+Au
EPS09	l+A DIS, p+A DY, π^0 in d+Au
nCTEQ	l+A DIS, p+A DY
nCTEQ	l+A and ν+A DIS, p+A DY
DSSZ	l+A and ν+A DIS, p+A DY, $\pi^0, \pi^{\pm}$ in d+Au

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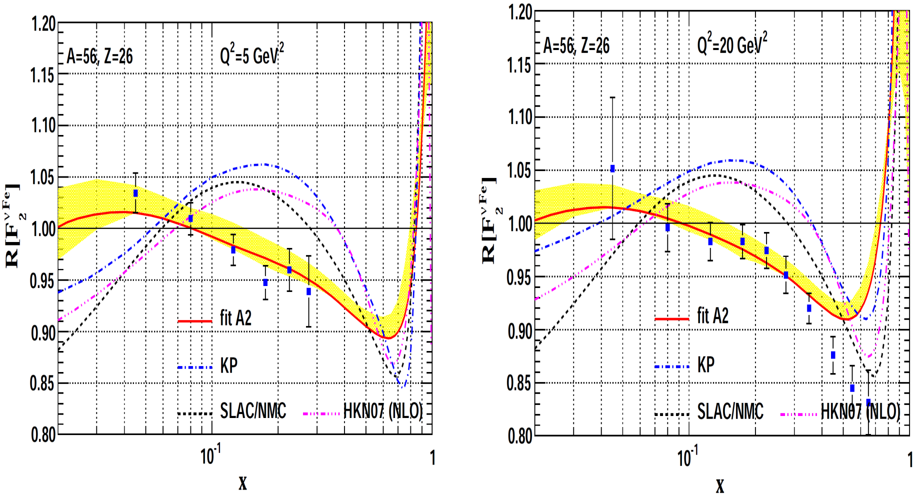
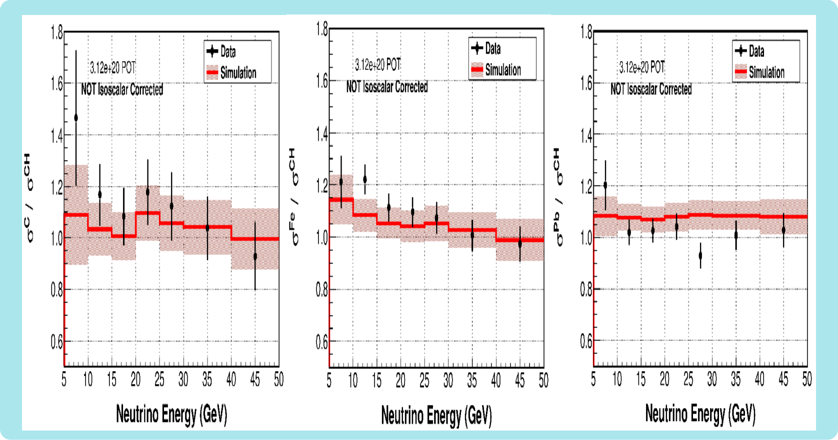
Paukkunen and Salgado:JHEP2010: "find no apparent disagreement with the nuclear effects in neutrino DIS and those in charged lepton DIS."

CTEQ-Grenoble-Karlsruhe collaboration "observed that the nuclear corrections in ν-A DIS are indeed incompatible with the predictions derived from $l^{\pm}$-A DIS and DY data"



J G Morfin J. of Physics: Conf. Ser. 408 (2013) 012054; Kovarik et al. Phys.Rev.Lett. 106 (2011) 122301

MINERvA@Fermilab: Phys. Rev. D93 (2016) 071101



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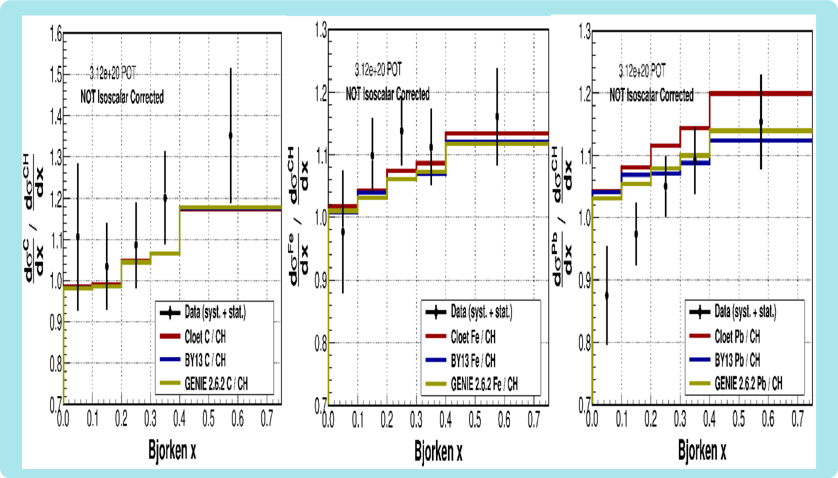
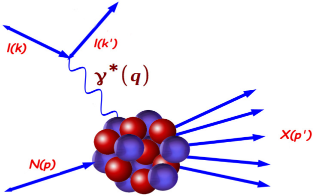
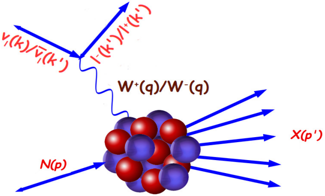


Theoretical approach

- Aligarh-Valencia group
- Kulagin-Petti group





- Fermi motion
- Pauli blocking
- Nucleon correlations
- Mesonic contributions
- Shadowing and Antishadowing

The DCX for $\nu - N$

$$\frac{d^2\sigma^N}{d\Omega dE'} \propto L_{\mu\nu} \, W_N^{\mu\nu}$$

gets modified to

$$\frac{d^2\sigma^A}{d\Omega dE'} \propto L_{\mu\nu} \, W_A^{\mu\nu}$$

for $\nu - A$

$$W_A^{\mu\nu} = \left(\frac{q^\mu q^\nu}{q^2} - g^{\mu\nu}\right) W_{1A} + \left(p_A^\mu - \frac{p_A . q}{q^2} \, q^\mu\right)\left(p_A^\nu - \frac{p_A . q}{q^2} \, q^\nu\right)\frac{W_{2A}}{M_A^2} - i\epsilon^{\mu\nu\rho\sigma}\frac{p_\rho q_\sigma}{2M_A^2} \, W_{3A}$$

$$MW_{1A} = F_{1A} \; ; \; \nu W_{2A} = F_{2A} \; ; \; \nu W_{3A} = F_{3A}$$

Model

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- Nuclear information like Binding energy, Fermi motion, nucleon correlations, is contained in the spectral function.
- The mesonic contribution is incorporated in a many body field theoretical approach.
- For the shadowing and antishadowing effects, Glauber-Gribov multiple scattering model has been used following the works of Kulagin and Petti[PRD76(2007)094033].

Fermi Gas Model

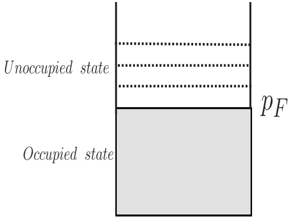
Assumptions

- It is assured that the nucleons in a nucleus (or nuclear matter) occupy one nucleon per unit cell in phase space so that the total number of Nucleons N is given by

$$N = 2V \int \frac{d^3\mathbf{p}}{(2\pi)^3}$$

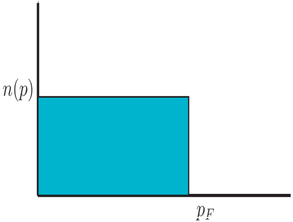
where a factor of two to account spin degree of freedom.

- All states upto a maximum momentum p_F $(p < p_F)$ are filled.
- The momentum states higher than $p > p_F$ are unoccupied.



The occupation number n(p) is defined as:

$$
\begin{aligned}
n(p) &= 1, p < p_F \\
&= 0, p > p_F \\
\implies \rho &= \frac{N}{V} = \frac{p_F^3}{3\pi^2} \\
\therefore p_F &= (3\pi^2\rho)^{\frac{1}{3}}
\end{aligned}
$$



Protons and neutrons are supposed to have different Fermi sphere

$$
p_{F_p} = \left(\frac{3}{2}\pi\rho_p\right)^{\frac{1}{3}} \qquad p_{F_n} = \left(\frac{3}{2}\pi\rho_n\right)^{\frac{1}{3}}
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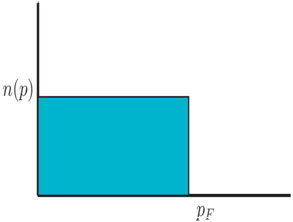

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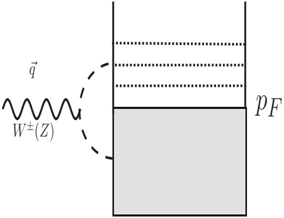
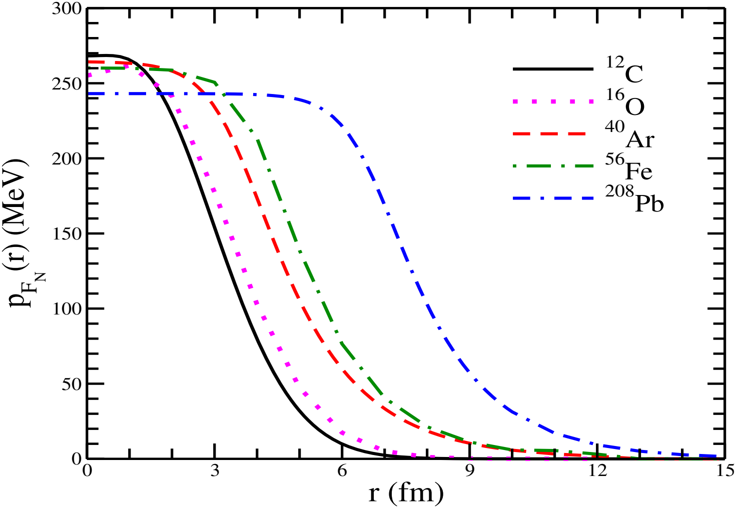
This results a hole in the Fermi sea and a particle above the sea. This is known as 1p1h excitation, with the condition that:

- initial momentum: $|\mathbf{p}| < p_F^i$
- final momentum: $|\mathbf{p}+\mathbf{q}| > p_F^f$

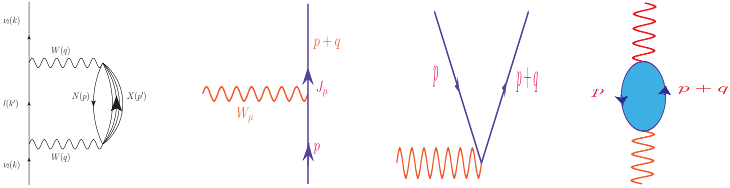


Local Fermi momentum



p-h excitation:

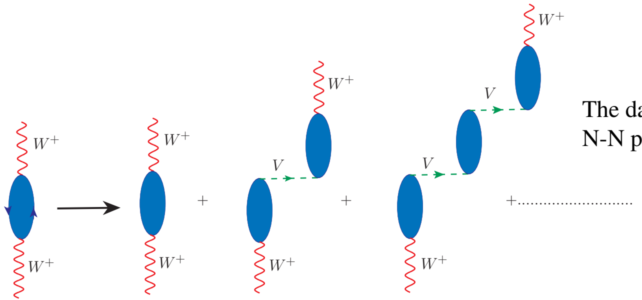
- ν disappears from the elastic flux, by inducing 1p1h, $1\Delta 1h$, etc. excitations.
- Initial nucleon has a momentum (distribution) such that $|\mathbf{p}| < p_F^f$ Fermi momentum of initial nucleon.
- Final nucleon should be outside the Fermi level so $p = |\mathbf{p} + \mathbf{q}| > p_F^f$ Fermi momentum of final nucleon.
- In the interaction (of W or Z) with the nucleon, a hole is created in Fermi sea and excited to a particular state $W + n \longrightarrow p$.
- Creation of 1p1h state: Diagrammatically



ν elastic flux | Feynman Diagram | 1p1h excitation matrix element | 1p1h response

The nucleons in a nucleus interact through two body NN potential (simply modeled with π and ρ exchange.)

Once 1p1h are excited by an external probe, they can interact through the NN–potential(π and ρ exchange) n number of times. In fact in this interaction they can also produce Δ leading to ph–Δh interaction which can be depicted as:



The dashed line shows the N-N potential.

NN potential

$$V(\vec{r}_1,\vec{r}_2) = C_0 \left\{ f_0 + f'_0 \vec{\tau}_1.\vec{\tau}_2 + g_0 \vec{\sigma}_1.\vec{\sigma}_2 + g'_0 \vec{\sigma}_1.\vec{\sigma}_2 \vec{\tau}_1.\vec{\tau}_2 \right\}$$

f_0 & g_0 is strength of the NN-potential in isoscalar spin-independent and spin-dependent channel.

f'_0 & g'_0 is strength of the NN-potential in isovector spin-independent and spin-dependent channel.

σ and τ are Pauli matrices acting on the nucleon spin and isospin spaces.

$$V = V_\pi + V_\rho$$

$$V_\pi = \frac{f_\pi^2}{m_\pi^2} \frac{\vec{\sigma}_1.\vec{q}\vec{\sigma}_2.\vec{q}}{q_0^2 - \vec{q}^2 - m_\pi^2 + i\in}$$

$$V_\rho = \frac{f_\rho^2}{m_\rho^2} \frac{\vec{\sigma}_1 \times \vec{q} \;\; \vec{\sigma}_2 \times \vec{q}}{q_0^2 - \vec{q}^2 - m_\rho^2 + i\in} \vec{\tau}.\vec{\tau}$$

$$V_l(q) = \frac{f^2}{m_\pi^2} \left\{ \left(\frac{\Lambda_\pi^2 - m_\pi^2}{\Lambda_\pi^2 - q^2} \right)^2 \frac{\vec{q}^{\,2}}{q^2 - m_\pi^2} + g' \right\}$$

$$V_t(q) = \frac{f^2}{m_\pi^2} \left\{ C_\rho \left(\frac{\Lambda_\rho^2 - m_\rho^2}{\Lambda_\rho^2 - q^2} \right)^2 \frac{\vec{q}^{\,2}}{q^2 - m_\rho^2} + g' \right\}$$

$\Gamma dtdS$ provides probability times differential of area (dS), which is a contribution to (ν_l, l) cross section

$$d\sigma = \Gamma dtdS = \Gamma \frac{dt}{dl} dSdl = \Gamma \frac{dV}{v} = \Gamma \frac{E(\mathbf{k})}{|\mathbf{k}|} d^3 r$$

$$\begin{aligned} f &= \frac{1}{k^0 - E(\mathbf{k}) + i\frac{\Gamma}{2}} \\ G(k^0, k) &= \sum_r \frac{\bar{u}_r(k')u_r(k)}{k^0 - E(k) - \frac{m_l}{E}\bar{u}_r(k')\Sigma(k)u_r(k)} \\ \Gamma &= -\frac{2m_l}{E(\mathbf{k})} Im\Sigma(k) \end{aligned}$$

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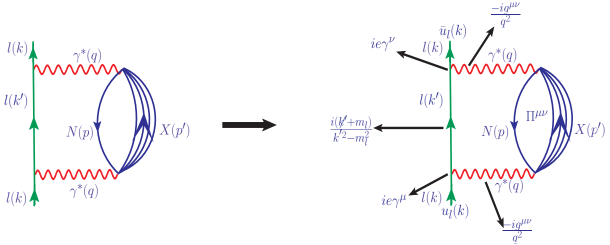


Lepton self energy $\Sigma(k)$:

$$-i\Sigma(k) = \int \frac{d^4q}{(2\pi)^4} \bar{u}_l(\mathbf{k})\, ie\gamma^\mu\, i\frac{\not k' + m}{k'^2 - m^2 + i\varepsilon}\, ie\gamma^\nu u_l(\mathbf{k}) \times \frac{-ig^{\mu\rho}}{q^2} (-i)\, \Pi_{\rho\sigma}(q)\, \frac{-ig^{\sigma\nu}}{q^2}$$

$$Im\Sigma(k) = e^2 \int \frac{d^3q}{(2\pi)^3} \frac{1}{2E} \theta(q^0)\, Im[\Pi^{\mu\nu}(q)] \frac{1}{q^4} \frac{1}{2m_l} L_{\mu\nu} \;,$$

$$\Pi^{\mu\nu}(q) = e^2 \int \frac{d^4p}{(2\pi)^4} G(p) \sum_X \sum_{s_p, s_l} \prod_{i=1}^N \int \frac{d^4p'_i}{(2\pi)^4} \prod_l G_l(p'_l) \prod_j D_j(p'_j)$$

$$< X|J^\mu|H >< X|J^\nu|H >^* (2\pi)^4\, \delta^4(q + p - \sum_{i=1}^N p'_i)$$

Free nucleon propagator

- Let us start with the relativistic Dirac propagator $G^0(p_0, \mathbf{p})$ for a free nucleon:

$$G^0(p^0, \mathbf{p}) = \frac{1}{\not{p} - M + i\varepsilon} = \frac{\not{p} + M}{(p^2 - M^2 + i\varepsilon)}$$

- In terms of positive and negative energy components of the nucleon

$$G^0(p_0, \mathbf{p}) = \frac{M}{E(\mathbf{p})} \left\{ \frac{\sum_r u_r(p)\bar{u}_r(p)}{p^0 - E(\mathbf{p}) + i\varepsilon} + \frac{\sum_r v_r(-p)\bar{v}_r(-p)}{p^0 + E(\mathbf{p}) - i\varepsilon} \right\}$$

- In a non-interacting Fermi sea, it may be written as

$$G^0(p^0, \mathbf{p}) = \frac{M}{E_N(\mathbf{p})} \left\{ \sum_r u_r(\mathbf{p})\bar{u}_r(\mathbf{p}) \left[\frac{1 - n(\mathbf{p})}{p^0 - E_N(\mathbf{p}) + i\varepsilon} + \frac{n(\mathbf{p})}{p^0 - E_N(\mathbf{p}) - i\varepsilon} \right] + \frac{\sum_r v_r(-\mathbf{p})\bar{v}_r(-\mathbf{p})}{p^0 + E_N(\mathbf{p}) - i\varepsilon} \right\}$$

- Relativistic nucleon propagator with positive energy component

$$G^0(p^0,\mathbf{p}) = \frac{M}{E_N(\mathbf{p})}\sum_r u_r(\mathbf{p})\bar{u}_r(\mathbf{p})\left[\underbrace{\frac{1-n(\mathbf{p})}{p^0-E_N(\mathbf{p})+i\varepsilon}}_{p\geq p_F}+\underbrace{\frac{n(\mathbf{p})}{p^0-E_N(\mathbf{p})-i\varepsilon}}_{p<p_F}\right]$$

- Apart from negative energy contribution, which plays no role in our problem, the only difference with a nonrelativistic propagator

$$G^0(p^0,\mathbf{p}) = \frac{1-n(\mathbf{p})}{p^0-\varepsilon(\mathbf{p})+i\,\varepsilon} + \frac{n(\mathbf{p})}{p^0-\varepsilon(\mathbf{p})-i\,\varepsilon}$$

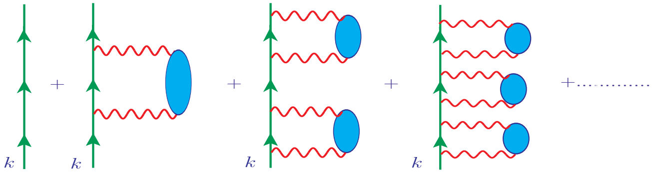
is the presence of

$$\frac{M}{E(\mathbf{p})} \xrightarrow[N.R.limit]{} 1$$

$$\sum_r u_r(\mathbf{p})\bar{u}_r(\mathbf{p}) \xrightarrow[N.R.limit]{} 1$$

Relativistic propagator in the interacting Fermi sea

- We wish to sum the Dyson series of the nucleon self-energy diagrams



- This perturbative expansion is summed in a ladder approximation:

$$\begin{aligned} G(p_0,\mathbf{p}) &= G^0(p_0,\mathbf{p}) + G^0(p_0,\mathbf{p})\Sigma^N(p_0,\mathbf{p})G^0(p_0,\mathbf{p}) + G^0(p_0,\mathbf{p})\Sigma^N(p_0,\mathbf{p})G(p_0,\mathbf{p}) + \ldots\ldots \\ &= \frac{M}{E(\mathbf{p})}\frac{\sum_r u_r(\mathbf{p})\bar{u}_r(\mathbf{p})}{p_0 - E(\mathbf{p})} + \frac{M}{E(\mathbf{p})}\frac{\sum_r u_r(\mathbf{p})\bar{u}_r(\mathbf{p})}{p_0 - E(\mathbf{p})}\Sigma^N(p_0,\mathbf{p})\frac{M}{E(\mathbf{p})}\frac{\sum_s u_s(\mathbf{p})\bar{u}_s(\mathbf{p})}{p_0 - E(p)} + \ldots\ldots \\ &= \frac{M}{E(\mathbf{p})}\sum_r \frac{u_r(\mathbf{p})\bar{u}_r(\mathbf{p})}{p_0 - E(\mathbf{p}) - \bar{u}_r(\mathbf{p})\underbrace{\Sigma^N(p_0,\mathbf{p})}_{self-energy}u_r(\mathbf{p})\frac{M}{\mathbf{E(p)}}} \end{aligned}$$

Nucleon self-energy

- Nucleon self energy $\Sigma^N(p_0, \mathbf{p})$ is complex in nature:

$$\Sigma^N(p^0, \mathbf{p}) = Re\{\Sigma^N(p^0, \mathbf{p})\} \; + \; iIm\{\Sigma^N(p^0, \mathbf{p})\}$$

- We rewrite

$$G(p_0, \mathbf{p}) \quad = \quad \frac{M}{E(\mathbf{p})} \sum_r u_r(\mathbf{p}) \bar{u}_r(\mathbf{p}) \, \frac{p^0 - E(\mathbf{p}) - \frac{M}{E(\mathbf{p})} Re\Sigma + i \frac{M}{E(\mathbf{p})} Im\Sigma}{\left(p^0 - E(\mathbf{p}) - \frac{M}{E(\mathbf{p})} \; Re\Sigma\right)^2 + \left(\frac{M}{E(\mathbf{p})} Im\Sigma\right)^2}$$

- Relativistic nucleon propagator in the nuclear medium:

$$G(p^0, \mathbf{p}) = \frac{M}{E(\mathbf{p})} \sum_r u_r(\mathbf{p}) \bar{u}_r(\mathbf{p}) \left[\int_{-\infty}^{\mu} d\,\omega \frac{S_h(\omega, \mathbf{p})}{p^0 - \omega - i\varepsilon} \; + \int_{\mu}^{\infty} d\,\omega \frac{S_p(\omega, \mathbf{p})}{p^0 - \omega + i\varepsilon} \right]$$

- For $p^0 \leq \mu$

$$S_h(p^0,\mathbf{p}) = \frac{1}{\pi}\frac{\frac{M}{E(\mathbf{p})} Im\Sigma(p^0,\mathbf{p})}{(p^0 - E(\mathbf{p}) - \frac{M}{E(\mathbf{p})} Re\Sigma(p^0,\mathbf{p}))^2 + (\frac{M}{E(\mathbf{p})} Im\Sigma(p^0,\mathbf{p}))^2}$$

- For $p^0 > \mu$

$$S_p(p^0,\mathbf{p}) = -\frac{1}{\pi}\frac{\frac{M}{E(\mathbf{p})} Im\Sigma(p^0,\mathbf{p})}{(p^0 - E(\mathbf{p}) - \frac{M}{E(\mathbf{p})} Re\Sigma(p^0,\mathbf{p}))^2 + (\frac{M}{E(\mathbf{p})} Im\Sigma(p^0,\mathbf{p}))^2}$$

- In an inclusive process only hole spectral function is relevant.
- Through S_h the effects of Fermi motion, Pauli blocking and nucleon correlations are incorporated.

P.Fernandez de Cordoba and E. Oset, PRC 46, 1697(1992)

Behaviour of hole spectral function

- In the absence of interactions, the nucleon energy p_0 is the free relativistic energy $E(\mathbf{p})$
- The dressed propagator $G(p)$ reduces to the free propagator $G^0(p)$ i.e. if $\Sigma^N(p) = 0$ then

$$S_h(p_0, \mathbf{p}) = S_p(p_0, \mathbf{p}) = \delta(p_0 - E(\mathbf{p}))$$

then

$$\int_{-\infty}^{\mu} dp_0\, S_h(p_0, \mathbf{p}) = \int_{-\infty}^{\mu} dp_0\, \delta(p_0 - E(\mathbf{p})) = \begin{cases} 1 & \text{if} \quad \mu > E(\mathbf{p}) \\ 0 & \text{if} \quad \mu < E(\mathbf{p}) \end{cases}$$

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Spectral functions for proton and neutron, are normalized to the total number of protons and neutrons in the nucleus:

$$2\int d^3r \int \frac{d^3p}{(2\pi)^3} \int_{-\infty}^{\mu} S_h^p(\omega, p_p, \rho_p(\mathbf{r}))\, d\omega = Z$$

$$2\int d^3r \int \frac{d^3p}{(2\pi)^3} \int_{-\infty}^{\mu} S_h^n(\omega, p_n, \rho_n(\mathbf{r}))\, d\omega = A - Z$$

We ensure to get correct binding energy for each nucleus and thus there are no free parameters in the model.

- Nuclear hadronic tensor is written as a convolution of nucleonic hadronic tensor with the hole spectral function

$$W_A^{\mu\nu} = 4\int d^3r \int \frac{d^3p}{(2\pi)^3}\, \frac{M}{E_N(\mathbf{p})} \int_{-\infty}^{\mu} dp^0 S_h(p^0, \mathbf{p}, \rho(r)) W_N^{\mu\nu}(p,q)$$

- By choosing the appropriate components for the nucleon and nuclear hadronic tensors,
- Taking **q** along the z direction such that $q = (q_0, 0, 0, |\mathbf{q}|)$

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- Taking **q** along the z direction such that $q = (q_0, 0, 0, |\mathbf{q}|)$

We evaluate dimensionless nuclear structure functions

Nuclear Structure Function

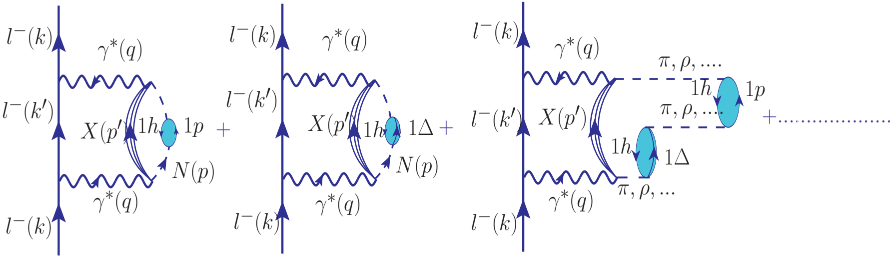
$$F_{1A}(x_A) = 2\sum_{i=p,n} AM \int d^3r \int \frac{d^3p}{(2\pi)^3}\frac{M}{E(\mathbf{p})}\int_{-\infty}^{\mu_i} dp^0 S_h^i(p^0,\mathbf{p},\rho^i(\mathbf{r})) \left[\frac{F_1^N(x_N)}{M} + \frac{1}{M^2}{p_x}^2\frac{F_2^N(x_N)}{\nu}\right]$$

$$F_{2A}(x_A) = 2\sum_{i=p,n}\int d^3r \int \frac{d^3p}{(2\pi)^3}\frac{M}{E(\mathbf{p})}\int_{-\infty}^{\mu_i} dp^0 S_h^i(p^0,\mathbf{p},\rho^i(\mathbf{r}))F_2^N(x_N) \times \left[\frac{Q^2}{q_z^2}\left(\frac{p^2 - p_z^2}{2M^2}\right) + \frac{(p.q)^2}{M^2\nu^2}\left(\frac{p_z\, Q^2}{p.qq_z} + 1\right)^2 \frac{q_0 M}{p_0\, q_0 - p_z\, q_z}\right]$$

$$F_{3A}(x_A) = 2\sum_{i=p,n}\int d^3r \int \frac{d^3p}{(2\pi)^3}\frac{M}{E(\mathbf{p})}\int_{-\infty}^{\mu_i} dp^0 S_h^i(p^0,\mathbf{p},\rho^i(\mathbf{r}))\frac{p^0\gamma - p_z}{(p^0 - p_z\gamma)\gamma}F_3^N(x_N)$$

- We have not used the Bjorken limit.
- Evolution of $F_1(x,Q^2)$ and $F_2(x,Q^2)$ are done independently.
- For the evolution at next-to-the-leading order we use following Refs.:
  - Vermaseren et al., Nucl. Phys. B **724**, 3 (2005)
  - van Neerven et al. Nucl. Phys. B **568**, 263 (2000); ibid **588**, 345 (2000).
  - Moch et al., Phys. Lett. B 606 123, (2005)
  - Moch et al., Nucl. Phys. B **813**, 220 (2009).
- For the nucleon PDFs CTEQ6.6 parameterization is used.

π and ρ mesons contribution to the SF



- Nucleons interact among themselves via the exchange of virtual mesons.

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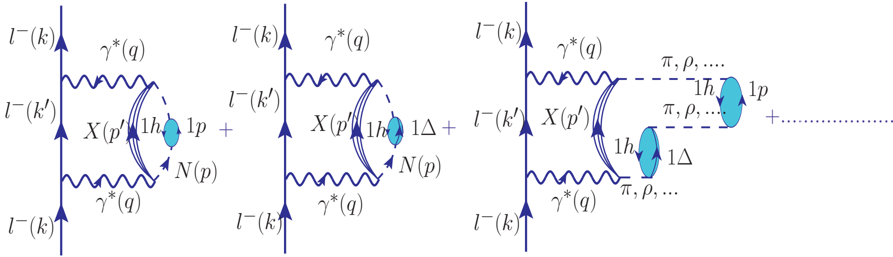

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- $\mathscr{P}^{\gamma^*}_{meson}$ becomes finite along with the $\mathscr{P}^{\gamma^*}_{nucleon}$.

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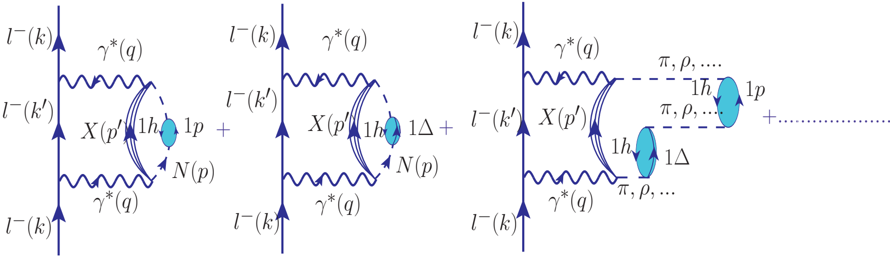

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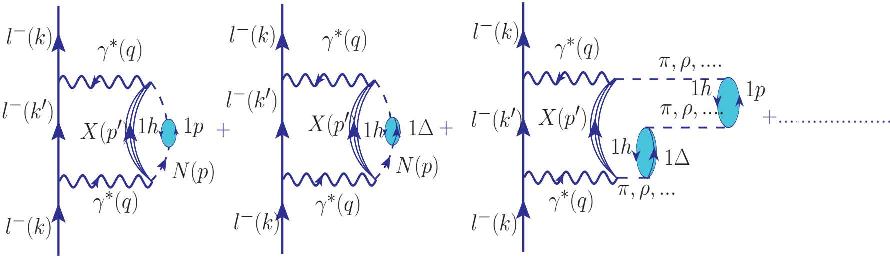

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- Implemented following the many body field theoretical approach.

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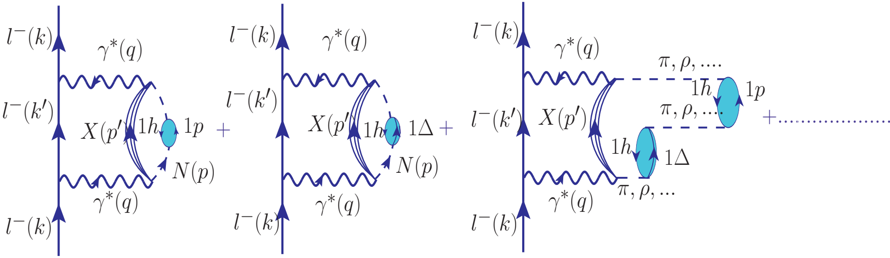


- Nucleons interact among themselves via the exchange of virtual mesons.
- $\mathscr{P}^{\gamma^*}_{meson}$ becomes finite along with the $\mathscr{P}^{\gamma^*}_{nucleon}$.
- Contributions from π and ρ mesons have been incorporated.
- Implemented following the many body field theoretical approach.
- For pion PDFs parameterization by Gluck et al. has been used.

π and ρ mesons contribution to the nuclear structure function

$$F^A_{1,\pi}(x_\pi) = -6AM\int d^3r\int \frac{d^4p}{(2\pi)^4}\theta(p_0)\ \delta ImD(p)\ 2m_\pi\ \times \left[\frac{F_{1\pi}(x_\pi)}{m_\pi} + \frac{|\mathbf{p}|^2 - p_z^2}{2(p_0\,q_0 - p_z q_z)}\frac{F_{2\pi}(x_\pi)}{m_\pi}\right]$$

$$F^A_{1,\rho}(x_\rho) = -12AM\int d^3r\int \frac{d^4p}{(2\pi)^4}\theta(p_0)\ \delta ImD_\rho(p)\ 2m_\rho \times \left[\frac{F_{1\rho}(x_\rho)}{m_\rho} + \frac{|\mathbf{p}|^2 - p_z^2}{2(p_0\,q_0 - p_z\,q_z)}\frac{F_{2\rho}(x_\rho)}{m_\rho}\right]$$

Pion propagator in the nuclear medium

$$D(p) = [{p_0}^2 - \mathbf{p}^2 - m_\pi^2 - \Pi_\pi(p_0,\mathbf{p})]^{-1}$$

with

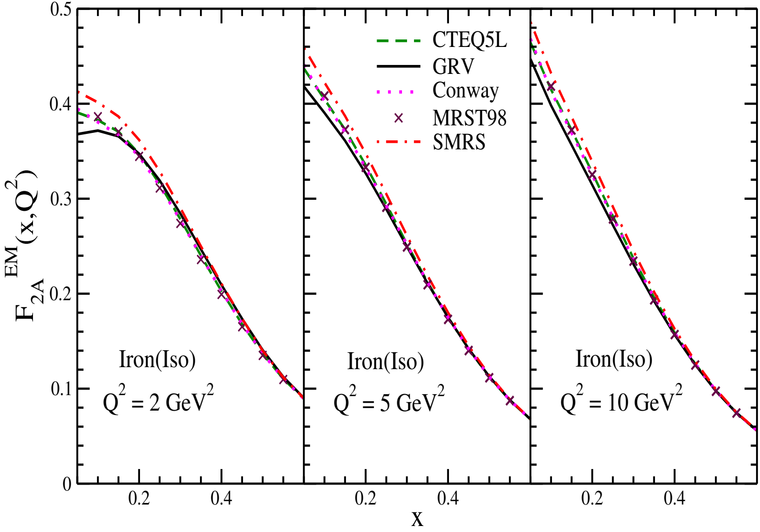
$$\Pi_\pi(p_0,\mathbf{p}) = \frac{f^2/m_\pi^2 F^2(p)\mathbf{p}^2\Pi^*}{1 - f^2/m_\pi^2 V_L'\Pi^*}, \quad \text{and} \quad F(p) = (\Lambda^2 - m_\pi^2)/(\Lambda^2 + p^2)$$

Structure functions for π and ρ mesons without using Callan-Gross relation:

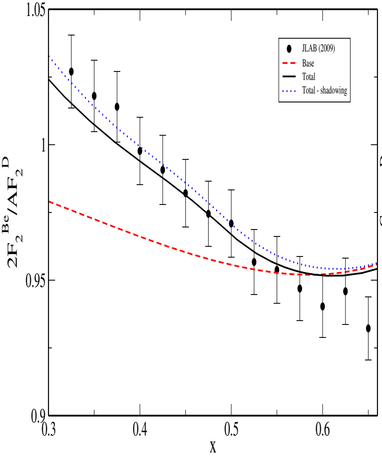
$$F^A_{2,\pi}(x_\pi) = -6\int d^3r \int \frac{d^4p}{(2\pi)^4}\,\theta(p_0)\,\delta ImD(p)\,2m_\pi\,\frac{m_\pi}{p_0 - p_z\,\gamma}\times \left[\frac{Q^2}{q_z^2}\left(\frac{|\mathbf{p}|^2 - p_z^2}{2m_\pi^2}\right) + \frac{(p_0 - p_z\,\gamma)^2}{m_\pi^2}\left(\frac{p_z\,Q^2}{(p_0 - p_z\,\gamma)q_0 q_z} + 1\right)^2\right] F_{2\pi}(x_\pi)$$

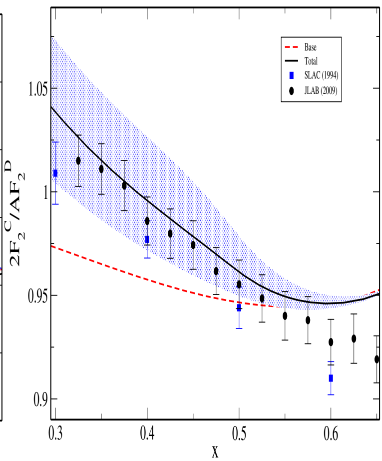
$$F^A_{2,\rho}(x_\rho) = -12\int d^3r \int \frac{d^4p}{(2\pi)^4}\,\theta(p_0)\,\delta ImD_\rho(p)\,2m_\rho\,\frac{m_\rho}{p_0 - p_z\,\gamma}\times \left[\frac{Q^2}{q_z^2}\left(\frac{|\mathbf{p}|^2 - p_z^2}{2m_\rho^2}\right) + \frac{(p_0 - p_z\,\gamma)^2}{m_\rho^2}\left(\frac{p_z\,Q^2}{(p_0 - p_z\,\gamma)q_0 q_z} + 1\right)^2\right] F_{2\rho}(x_\rho)$$

Pion structure function



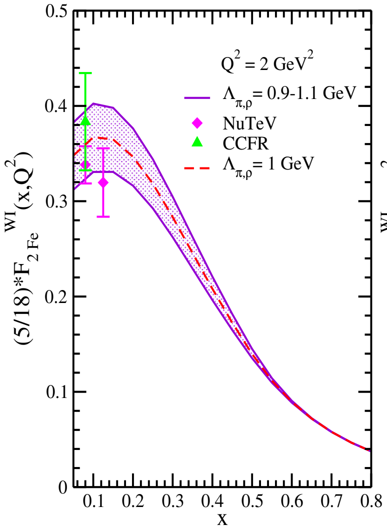
Variation of $\Lambda_{\pi,\rho}$ in $l-A$ scattering

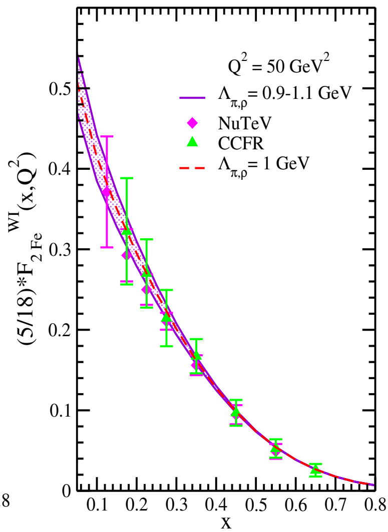


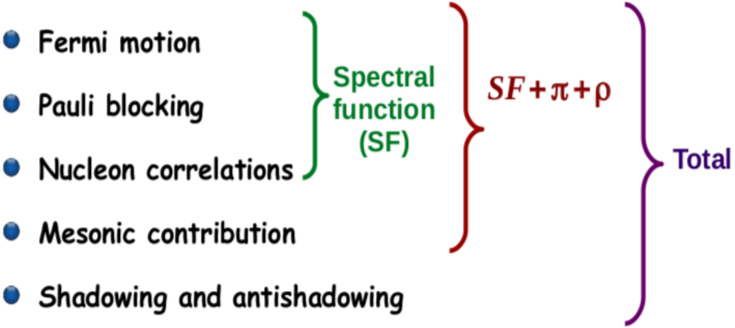


Nucl. Phys. A **857** 29, (2011)

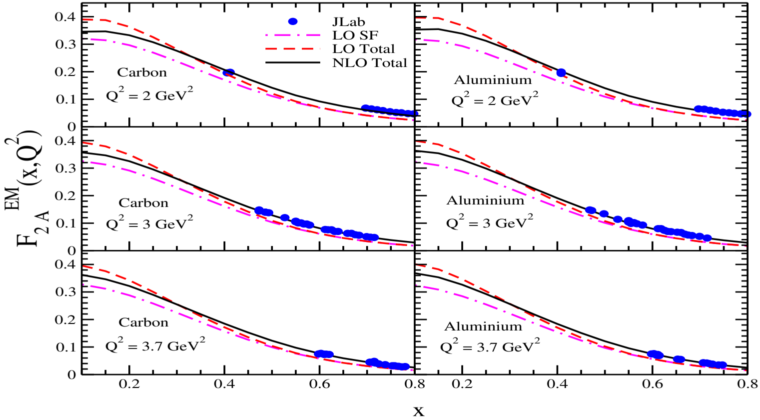
Variation of $\Lambda_{\pi,\rho}$ in $\nu_l - A$ scattering






Fermi motion
Pauli blocking
Nucleon correlations
Mesonic contribution
Shadowing and antishadowing
Spectral function (SF)
SF + π + ρ
Total

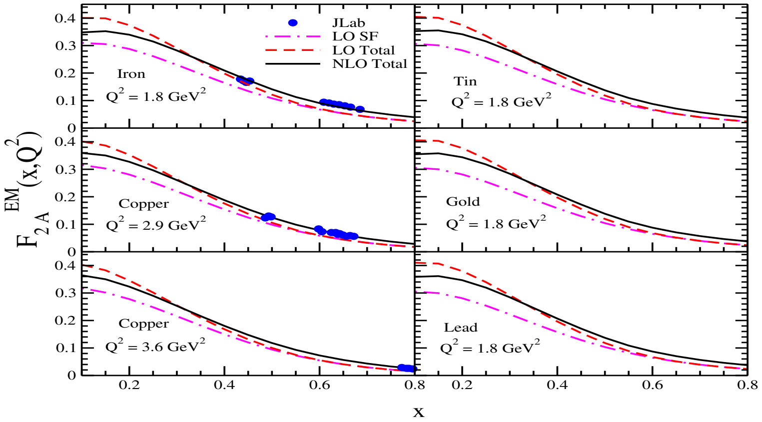
Results for $l^{\pm} - A$: $F_{2A}^{EM}(x,Q^2)$ vs x



- At LO(SF→Full): $\sim 18\%$ increase at low x in ^{12}C, increases with A and negligible at high x.
- At NLO: Results at low x get suppressed while at high x results get enhanced compared to LO results.

Nucl. Phys. A 943 58 (2015) J Lab Data: arXiv: 1202.1457

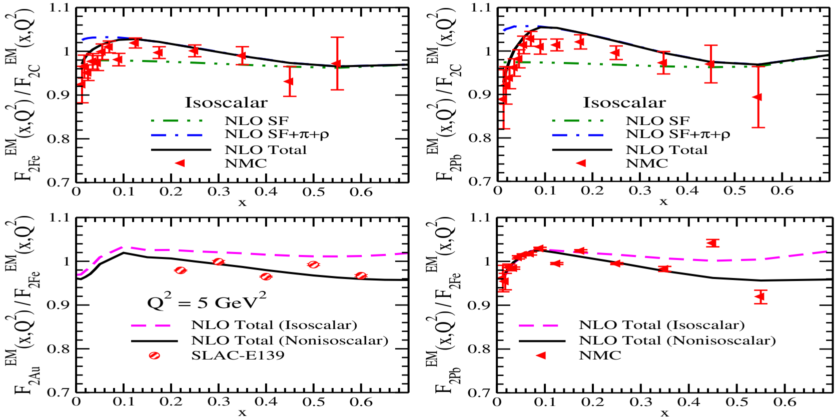
Results for $l^{\pm} - A$: $F_{2A}^{EM}(x,Q^2)$ vs x



- NME increases with the increase in A.

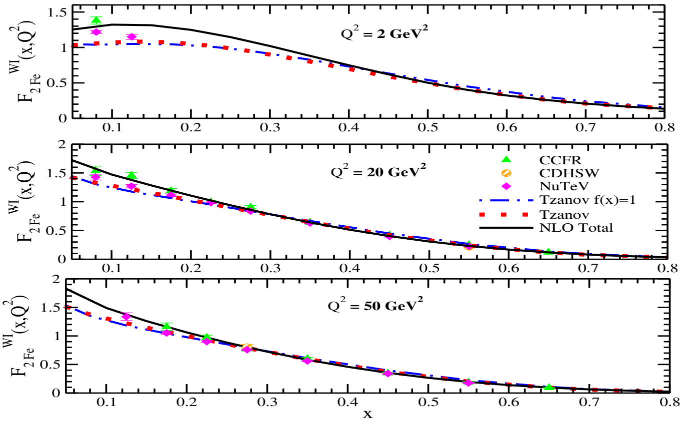
Nucl. Phys. A 943 58 (2015) J Lab Data: arXiv: 1202.1457

Results for $l^{\pm} - A$*:* $F_{2A}^{EM}(x, Q^2)$ *vs x*



- Results are obtained at NLO at $3 < Q^2 < 67$ GeV^2 corresponding to NMC.
- $3-4\%$ deviation from unity due to NME.
- Enhancement due to mesonic contribution(5-8%) & suppression due to shadowing is observed.

Results for $\nu_l - A$: $F_{2A}^{WI}(x, Q^2)$ vs x



- NME are important even for very high Q^2.
- Difference is found be the same as in the case of EM interaction.

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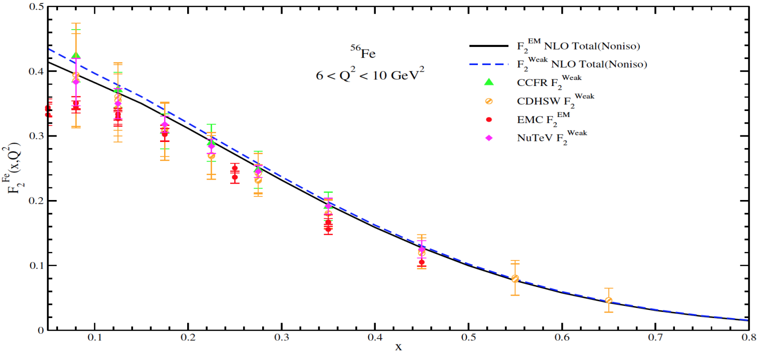
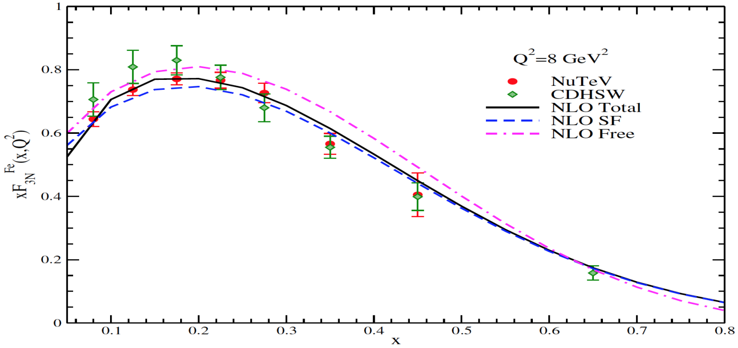
$l^{\pm} - A$ vs $\nu_l - A$ scattering

$F_2^{Fe}(x,Q^2)$ vs x, at $Q^2 = 8Gev^2$

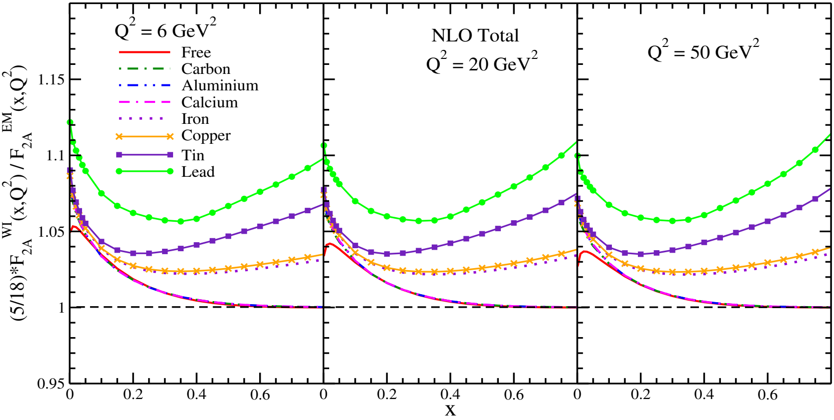


F_3 *vs* x

$F_3^{Fe}(x,Q^2)$ vs x, at $Q^2 = 8Gev^2$



$l^{\pm} - A$ *vs* $\nu_l - A$ *scattering*



Nucl. Phys. A 955 58 (2016)

- Nuclear medium effects in electromagnetic and weak nuclear structure functions are different.
- For the nuclear medium effects, we took into account Fermi motion, nuclear binding, nucleon correlations, effect of meson degrees of freedom, and shadowing effects. The calculations are performed both at LO and NLO.
- Non-isoscalarity corrections are taken properly into account.
- The plan is to perform these calculations with Higher Twist effect.
- We have also performed the calculations for $(\bar{\nu}_\tau)\nu_\tau - N$ DIS processes where F_4 and F_5 are the additional contributions. Also the massive charm quark contributes in $(\bar{\nu}_\tau)\nu_\tau$ induced processes.
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Thank you!