Requirements for longitudinal HOM damping in FCC-hh

Ivan Karpov and Elena Shaposhnikova

Acknowledgements:
Rama Calaga, Erk Jensen, and James A. Mitchell

Why damping of HOM is needed?

The FCC-hh is high-current machine with 10400 circulating bunches

→ Interaction of beam with high-order modes (HOM) can result in longitudinal coupled-bunch instability (CBI)

Unlike electron synchrotrons with strong synchrotron radiation, in FCC-hh we have to rely on Landau damping

How to evaluate the threshold? It can be obtained

→ from particle tracking simulations (very difficult for FCC-hh)

→ using semi analytical methods

Method of threshold diagrams

Dispersion relation obtained from Vlasov equation with assumptions (A. N. Lebedev 1968):

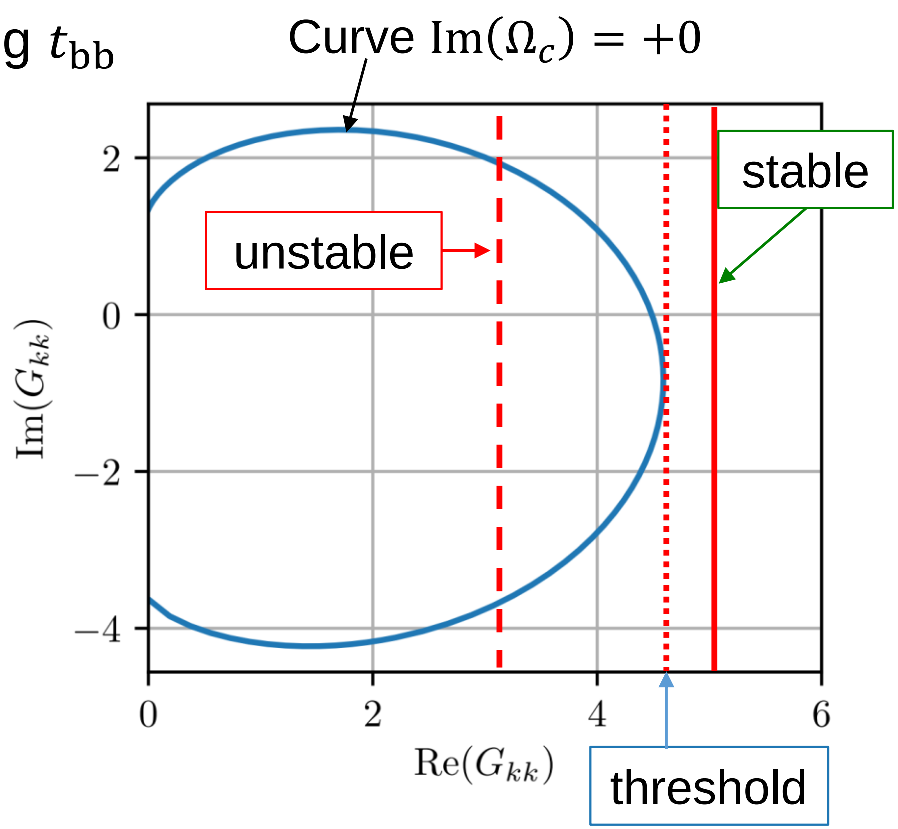
- Uniformly filled machine with spacing t_{bb}
- $\Delta f_{\mathrm{r}} = \frac{f_{\mathrm{r}}}{2Q} \ll \frac{1}{t_{\mathrm{bb}}}$, and $\Delta f_{\mathrm{r}} \ll \left| f_{\mathrm{r}} - \frac{l}{2t_{\mathrm{bb}}} \right|$

$$\frac{1}{Z_k(\Omega_c)} = G_{kk}(\Omega_c)$$

HOM impedance:

$$\frac{1}{Z_k(\Omega_c)} = \frac{1}{R_{\mathrm{sh}}}\left(1 - iQ\frac{\omega^2 - \omega_{\mathrm{r}}^2}{\omega\omega_{\mathrm{r}}}\right)$$

$$\omega = k\omega_0 + \Omega_c,\ k \approx \omega_{\mathrm{r}}/\omega_0$$



→ There is a unique diagram for given resonant frequency f_{r}

→ In practice, it is difficult to use diagrams for threshold evaluation

Approximate threshold

Additional assumptions:

- Single RF system
- Short bunches with binomial distribution

Phase of synchronous particle

Synchrotron frequency spread $$\frac{\Delta\omega_s}{\omega_{s0}} = \frac{\omega_{\mathrm{RF}}^2}{64}\left(1 + \frac{5}{3}\tan^2\phi_{s0}\right)\tau_{\mathrm{b}}^2$$

Line density $$\lambda(\tau) \propto \left[1 - \left(\frac{2\tau}{\tau_{\mathrm{b}}}\right)^2\right]^{\mu+1/2}$$

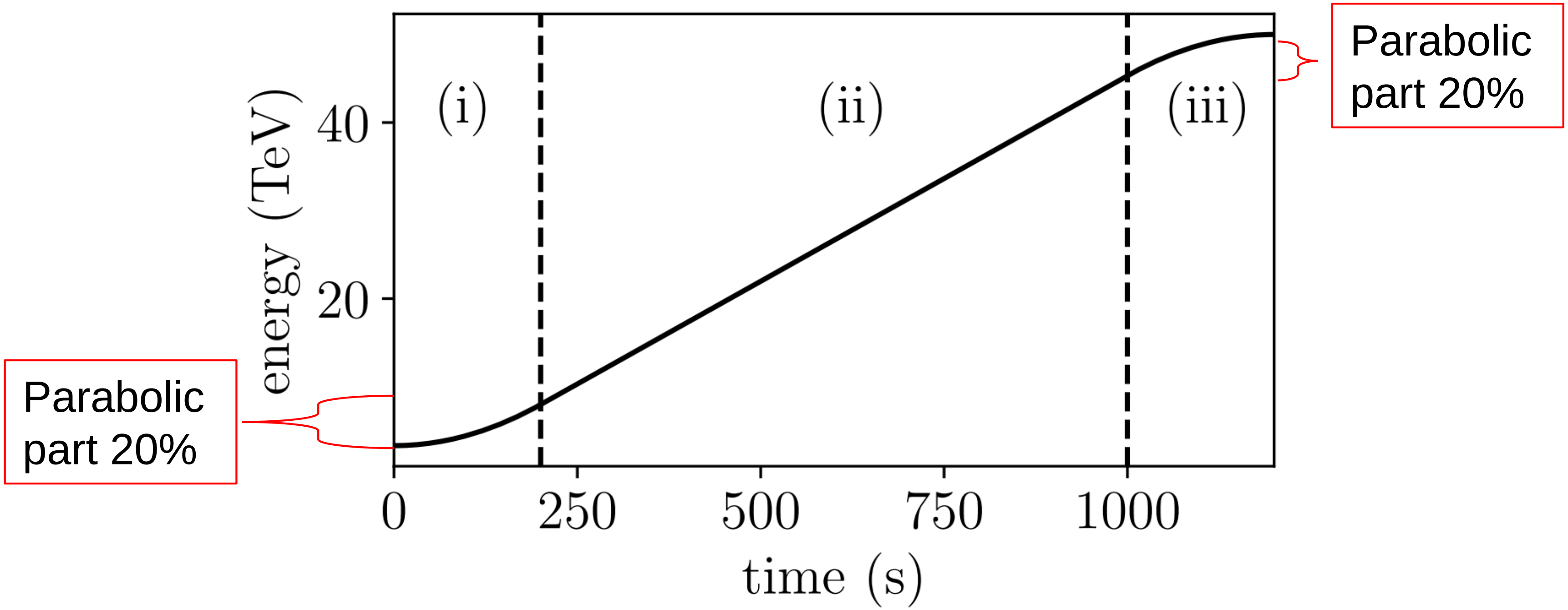
$$\tau_{\mathrm{FWHM}} = \tau_{\mathrm{b}}\sqrt{1 - 2^{\frac{2}{2\mu+1}}}$$

Threshold shunt impedance $$R_{\mathrm{sh}} < \frac{t_{\mathrm{bb}}\omega_{\mathrm{RF}}\tau_{\mathrm{b}}V_{\mathrm{RF}}\,|\cos\phi_{s0}|}{4eN_{\mathrm{p}}}\frac{\Delta\omega_s}{\omega_{s0}}G_\mu(f_{\mathrm{r}}\tau_{\mathrm{b}})$$

$$G_\mu(x) = \frac{x}{\mu(\mu+1)}\min_{y\in[0,1]}\left[(1-y^2)^{\mu-1}J_1^2(\pi x y)\right]^{-1}$$

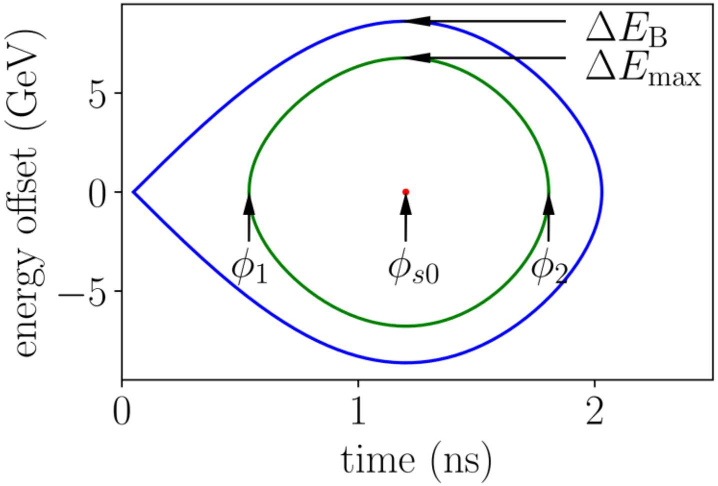
→ R_{sh} depends on RF voltage, bunch length, and synchronous phase for constant intensity

Acceleration cycle

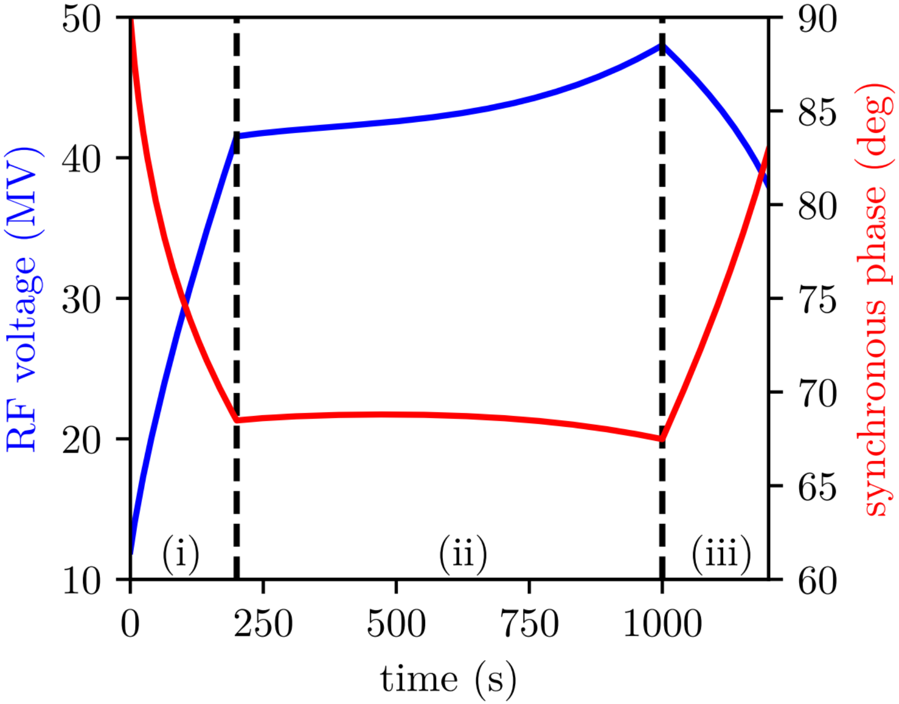


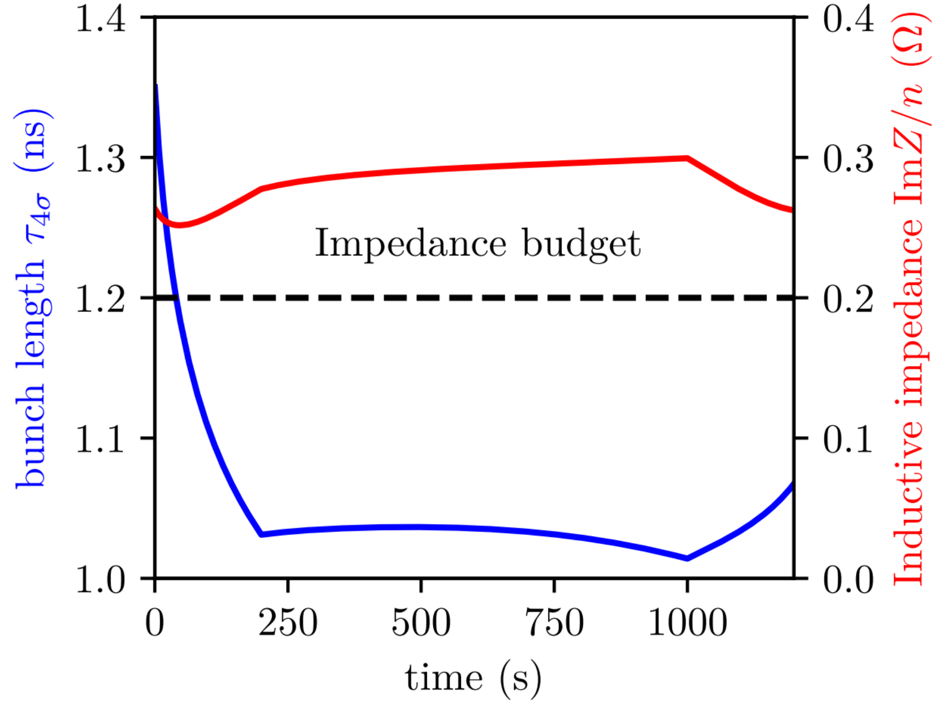
Considerations:

- controlled emittance blow-up $\epsilon \propto \sqrt{E}$ for longitudinal single-bunch stability
- Maximum energy filling factor $q_p = \frac{\Delta E_B}{\Delta E_{max}} = 0.941$ to avoid losses



Parameters during cycle

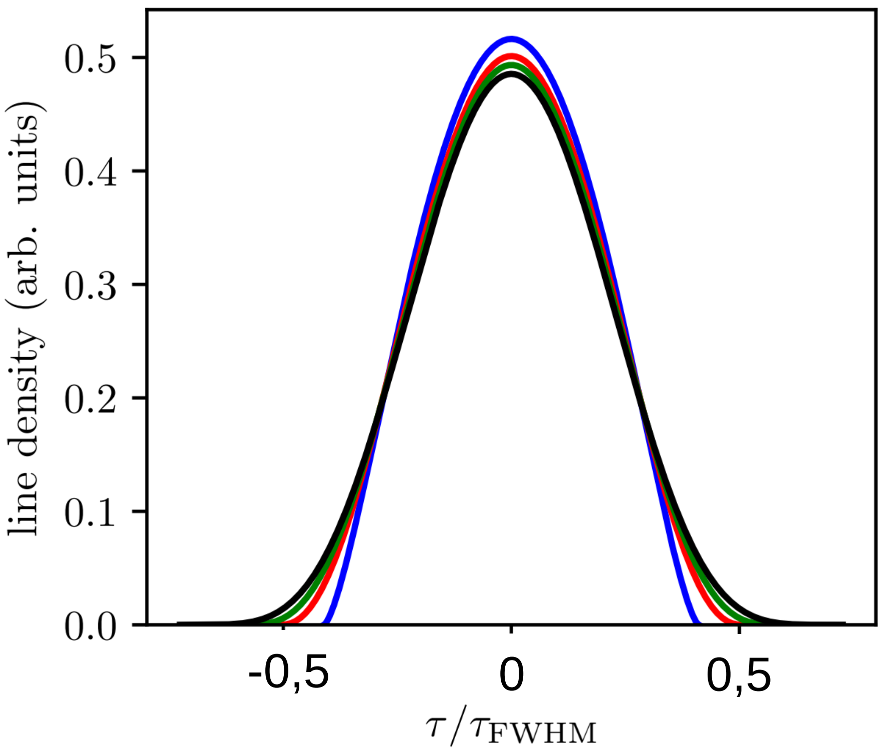


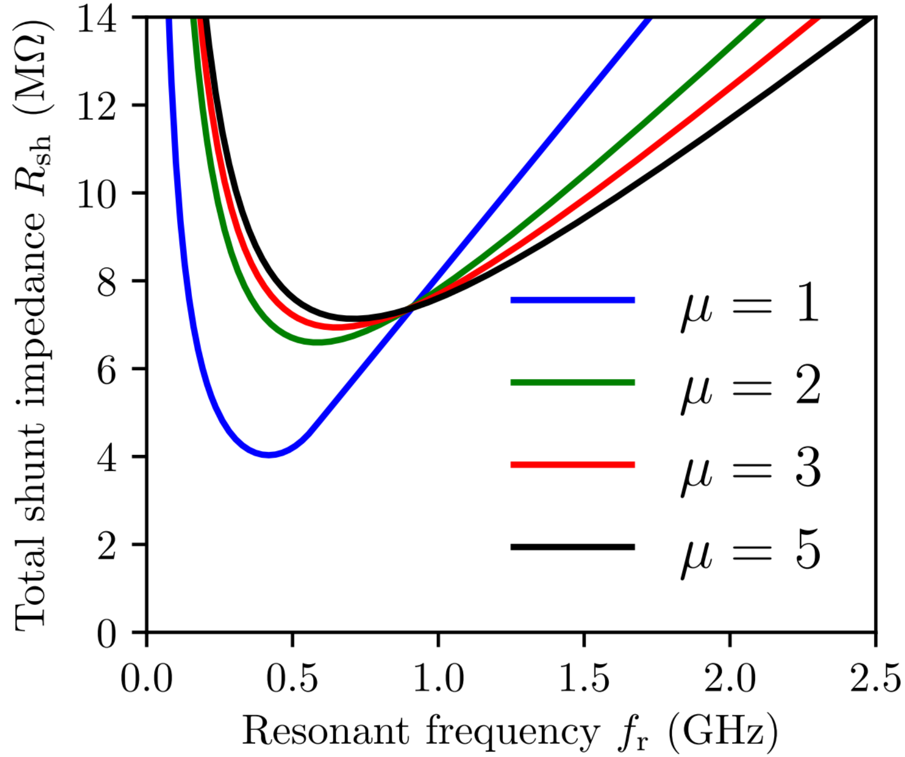


→ Threshold of the loss of Landau damping is higher then longitudinal impedance budget

→ Obtained parameters are used for longitudinal CBI threshold calculations

Results at 50 TeV





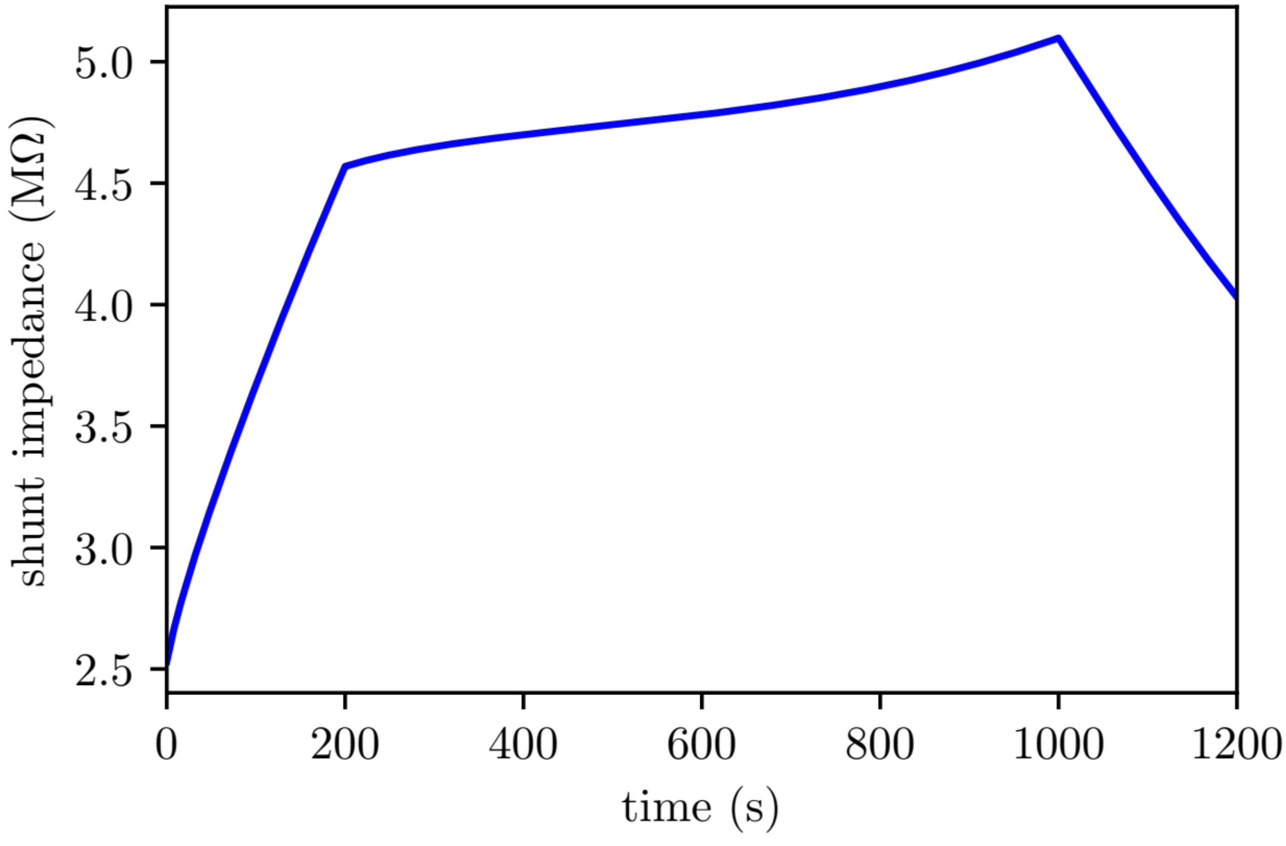
For the same τ_{FWHM}:

→ The lowest R_{sh} is for $\mu = 1$

→ Thresholds are similar for $\mu > 1$

Threshold during cycle

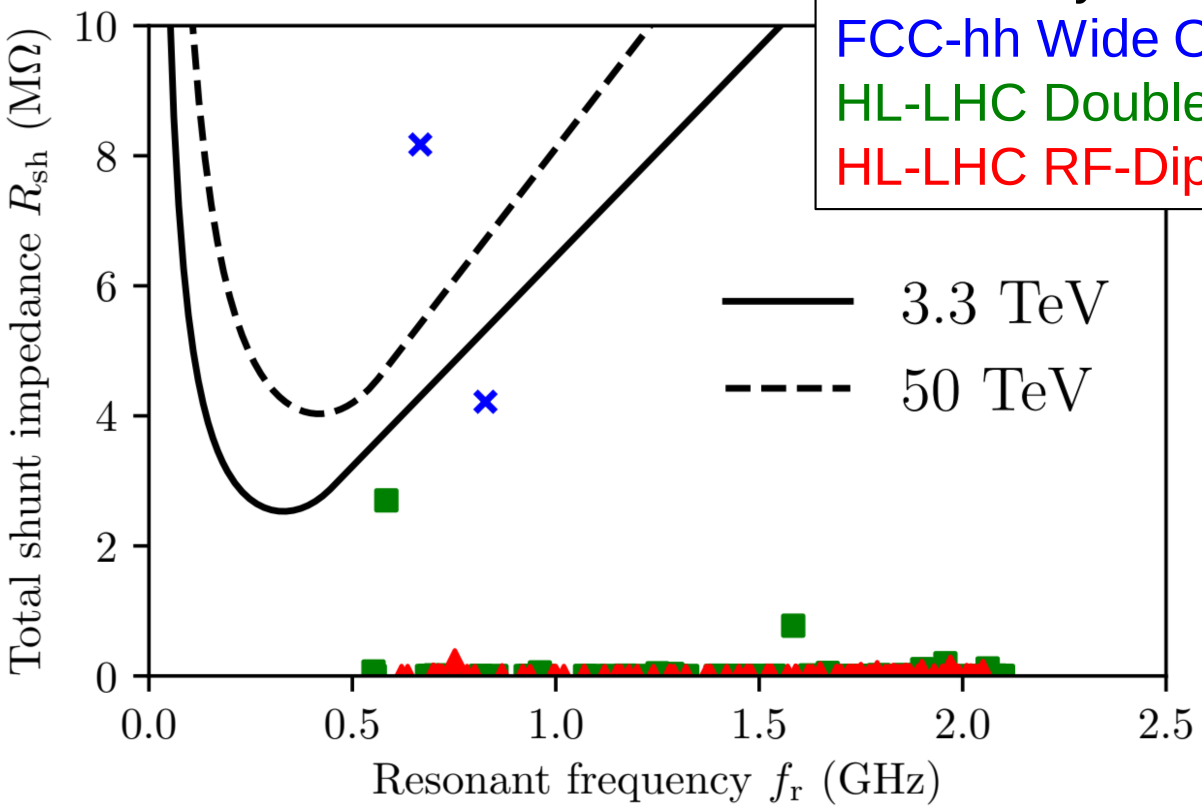
Obtained from τ_{FWHM} bunch length for μ = 1



→ The lowest value at flat bottom

HOMs in FCC-hh impedance model

Worst case scenario: f_r is the same in all cavities



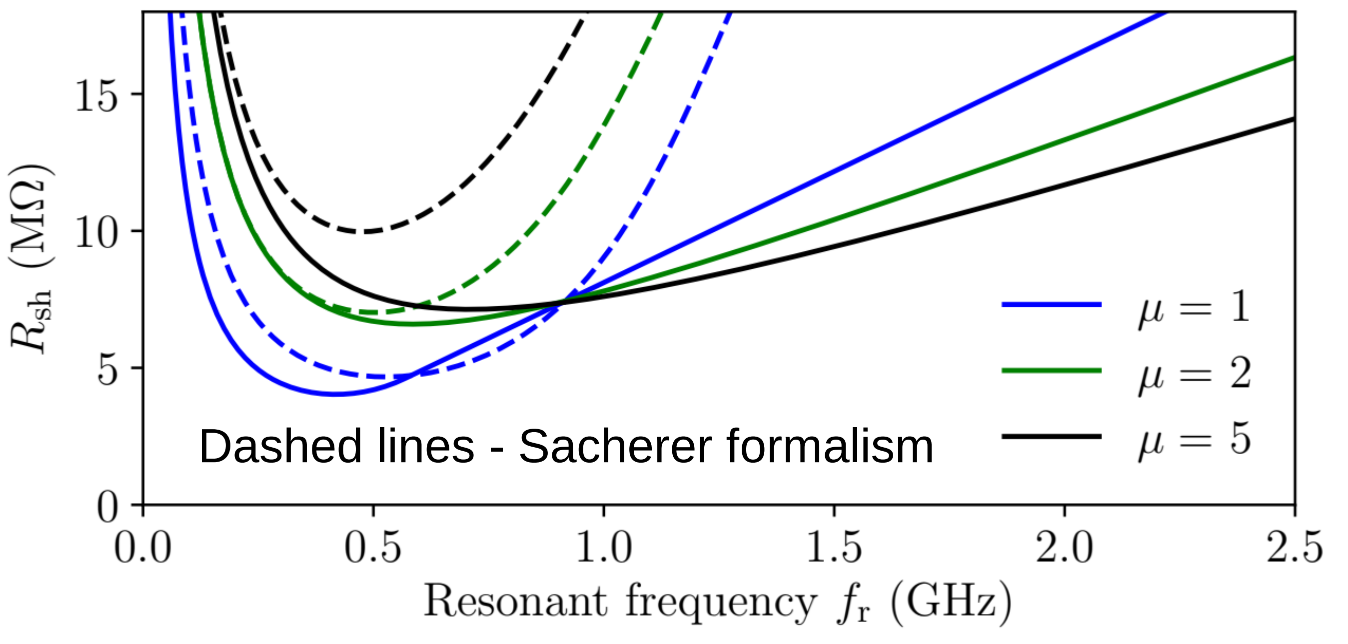
Crab cavity HOMs:
FCC-hh Wide Opened Waveguide × 24
HL-LHC Double Quarter Wave ×12
HL-LHC RF-Dipole ×12

→ Damping of HOMs has to be revisited for Wide Opened Waveguide crab cavities

Sacherer formalism

Solution of dispersion relation is split in two parts (Sacherer 1973):

- Calculation of complex coherent frequency shift neglecting synchrotron frequency spread
- Removing dependence on f_r from stability diagram using Taylor expansion



→ Sacherer approach underestimates threshold at higher frequencies
→ The minimum of thresholds are similar for small μ

Summary

- The longitudinal coupled-bunch instability thresholds were evaluated for the FCC-hh cycle, which is optimised for longitudinal single-bunch stability.
- For the considered family of the binomial particle distributions, bunches with different μ (except $\mu = 1$) but the same FWHM bunch length have similar threshold shunt impedances.
- To prevent longitudinal CBI in FCC-hh due to HOMs of WOW crab cavities further damping is required.

Thank you for your attention!