Requirements for longitudinal HOM damping in FCC-hh

Ivan Karpov and Elena Shaposhnikova

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Why damping of HOM is needed?

The FCC-hh is high-current machine with 10400 circulating bunches
→ Interaction of beam with high-order modes (HOM) can result in longitudinal coupled-bunch instability (CBI)

Unlike electron synchrotrons with strong synchrotron radiation, in FCC-hh we have to rely on Landau damping

How to evaluate the threshold? It can be obtained
→ from particle tracking simulations (very difficult for FCC-hh)
→ using semi analytical methods
Method of threshold diagrams

Dispersion relation obtained from Vlasov equation with assumptions (A. N. Lebedev 1968):

- Uniformly filled machine with spacing $t_{bb}$
- $\Delta f_r = \frac{f_r}{2Q} \ll \frac{1}{t_{bb}}$, and $\Delta f_r \ll \left| f_r - \frac{l}{2t_{bb}} \right|

\[
\frac{1}{Z_k(\Omega_c)} = G_{kk}(\Omega_c)
\]

HOM impedance:

\[
\frac{1}{Z_k(\Omega_c)} = \frac{1}{R_{sh}} \left( 1 - iQ \frac{\omega^2 - \omega_r^2}{\omega \omega_r} \right)
\]

$\omega = k\omega_0 + \Omega_c$, $k \approx \omega_r/\omega_0$

→ There is a unique diagram for given resonant frequency $f_r$

→ In practice, it is difficult to use diagrams for threshold evaluation
Approximate threshold

Additional assumptions:
• Single RF system
• Short bunches with binomial distribution

Synchrotron frequency spread
\[
\frac{\Delta \omega_s}{\omega_{s0}} = \frac{\omega_{RF}^2}{64} \left( 1 + \frac{5}{3} \tan^2 \phi_{s0} \right) \tau_b^2
\]

Line density
\[
\lambda(\tau) \propto \left[ 1 - \left( \frac{2\tau}{\tau_b} \right)^2 \right]^{\mu+1/2}
\]

Phase of synchronous particle
\[
\tau_{FWHM} = \tau_b \sqrt{1 - 2^{2\mu+1}}
\]

Threshold shunt impedance
\[
R_{sh} < \frac{t_{bb} \omega_{RF} \tau_b \nu_{RF} |\cos \phi_{s0}|}{4eN_p} \frac{\Delta \omega_s}{\omega_{s0}} G_\mu(f_{r}\tau_b)
\]

\[
G_\mu(x) = \frac{x}{\mu(\mu + 1)} \min_{y\in[0,1]} [(1 - y^2)^{\mu-1} J_1^2(\pi xy)]^{-1}
\]

→ \( R_{sh} \) depends on RF voltage, bunch length, and synchronous phase for constant intensity
Acceleration cycle

Considerations:
- controlled emittance blow-up $\epsilon \propto \sqrt{E}$ for longitudinal single-bunch stability
- Maximum energy filling factor
  $$q_p = \frac{\Delta E_B}{\Delta E_{\text{max}}} = 0.941$$ to avoid losses
Parameters during cycle

→ Threshold of the loss of Landau damping is higher than longitudinal impedance budget
→ Obtained parameters are used for longitudinal CBI threshold calculations
Results at 50 TeV

For the same $\tau_{\text{FWHM}}$:

→ The lowest $R_{\text{sh}}$ is for $\mu = 1$
→ Thresholds are similar for $\mu > 1$
Threshold during cycle

 Obtained from $\tau_{\text{FWHM}}$ bunch length for $\mu = 1$

$\rightarrow$ The lowest value at flat bottom
HOMs in FCC-hh impedance model

Worst case scenario: $f_r$ is the same in all cavities

Crab cavity HOMs:
- FCC-hh Wide Opened Waveguide × 24
- HL-LHC Double Quarter Wave × 12
- HL-LHC RF-Dipole × 12

→ Damping of HOMs has to be revisited for Wide Opened Waveguide crab cavities
Sacherer formalism

Solution of dispersion relation is split in two parts (Sacherer 1973):

• Calculation of complex coherent frequency shift neglecting synchrotron frequency spread
• Removing dependence on $f_r$ from stability diagram using Taylor expansion

→ Sacherer approach underestimates threshold at higher frequencies
→ The minimum of thresholds are similar for small $\mu$
Summary

• The longitudinal coupled-bunch instability thresholds were evaluated for the FCC-hh cycle, which is optimised for longitudinal single-bunch stability.

• For the considered family of the binomial particle distributions, bunches with different $\mu$ (except $\mu = 1$) but the same FWHM bunch length have similar threshold shunt impedances.

• To prevent longitudinal CBI in FCC-hh due to HOMs of WOW crab cavities further damping is required.
Thank you for your attention!