Causal thresholds and infrared singularities in the forest

Germán Rodrigo

IFIC INSTITUT DE FÍSICA CORPUSCULAR · EXCELENCIA SEVERO OCHOA · CSIC · VNIVERSITAT DE VALÈNCIA

J.J. Aguilera-Verdugo, F. Driencourt-Mangin, J. Plenter, S. Ramírez-Uribe, G. Rodrigo, G.F.R. Sborlini, W. J. Torres Bobadilla, S. Tracz e-Print: **arXiv:1904.08389**

FCC WEEK 2019

BRUSSELS, BELGIUM

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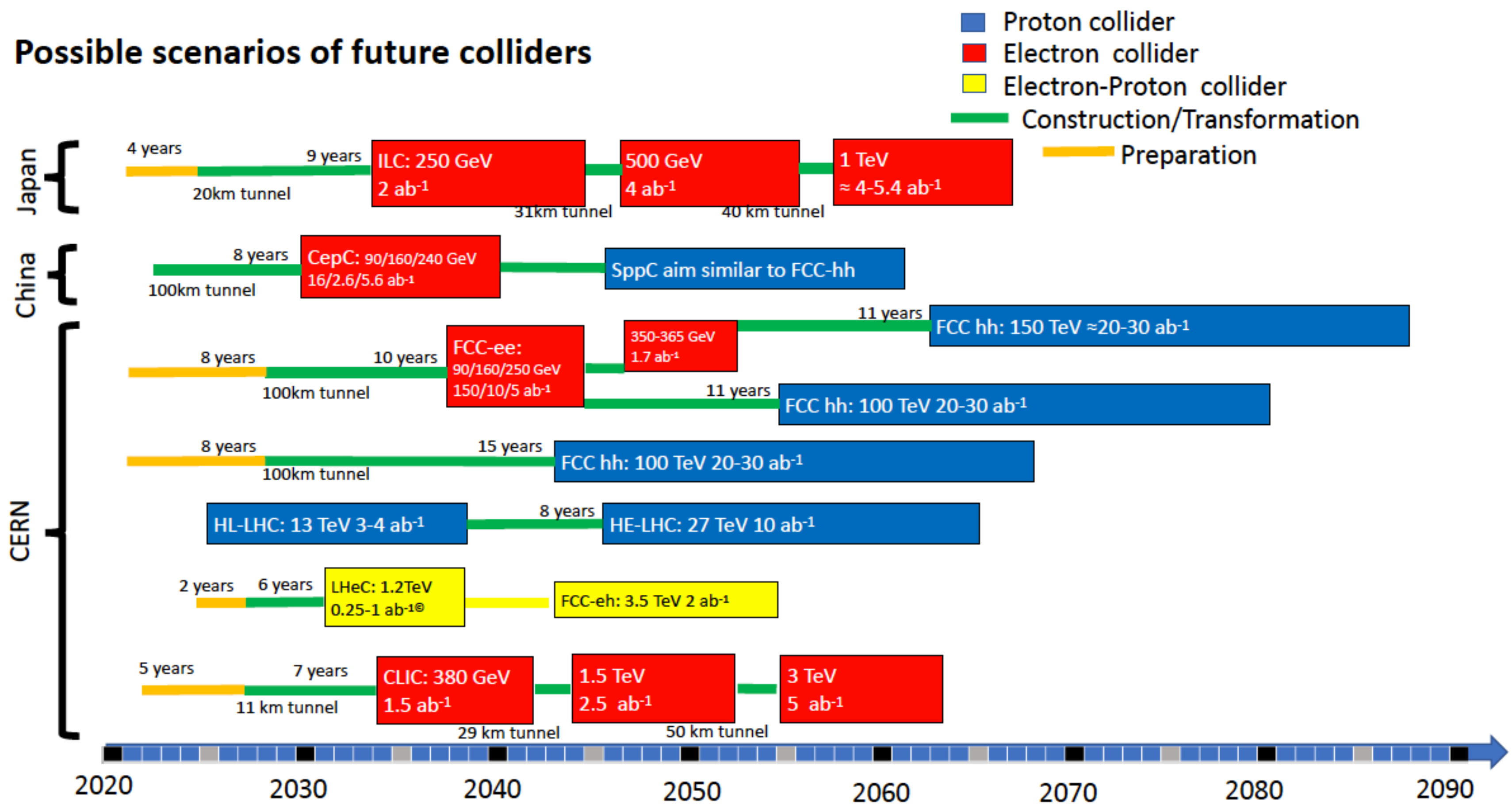
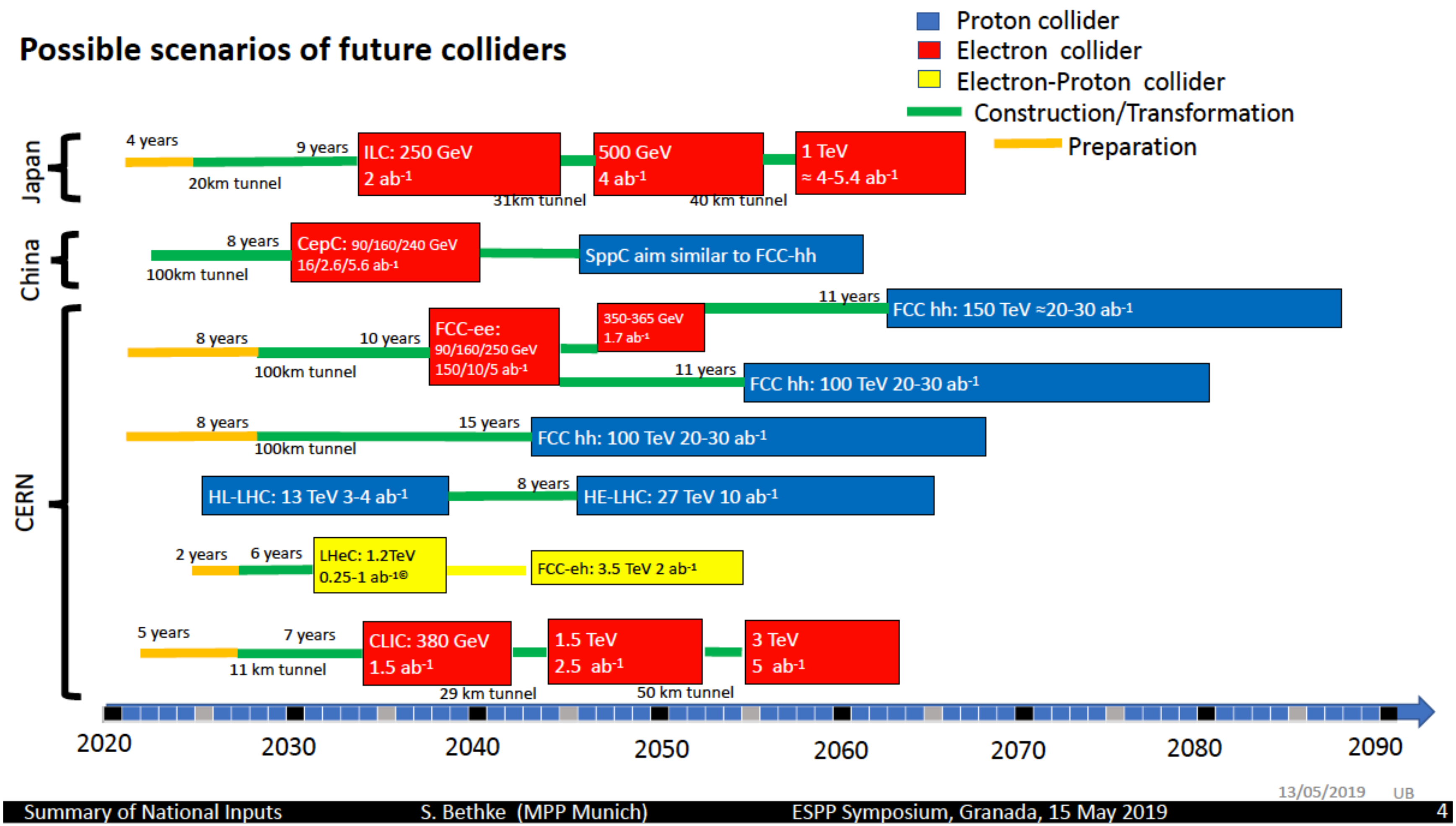
Possible scenarios of future colliders





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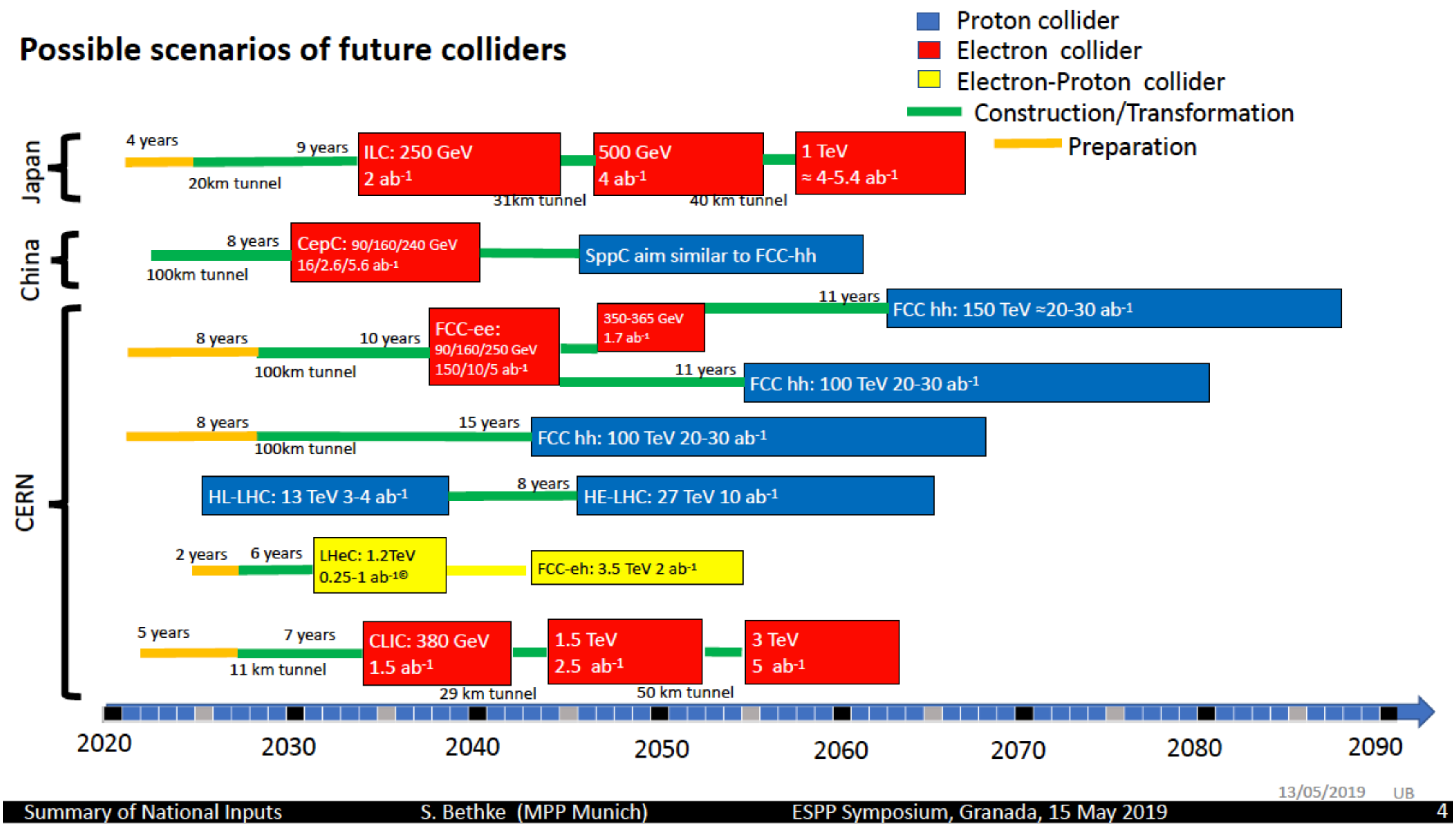

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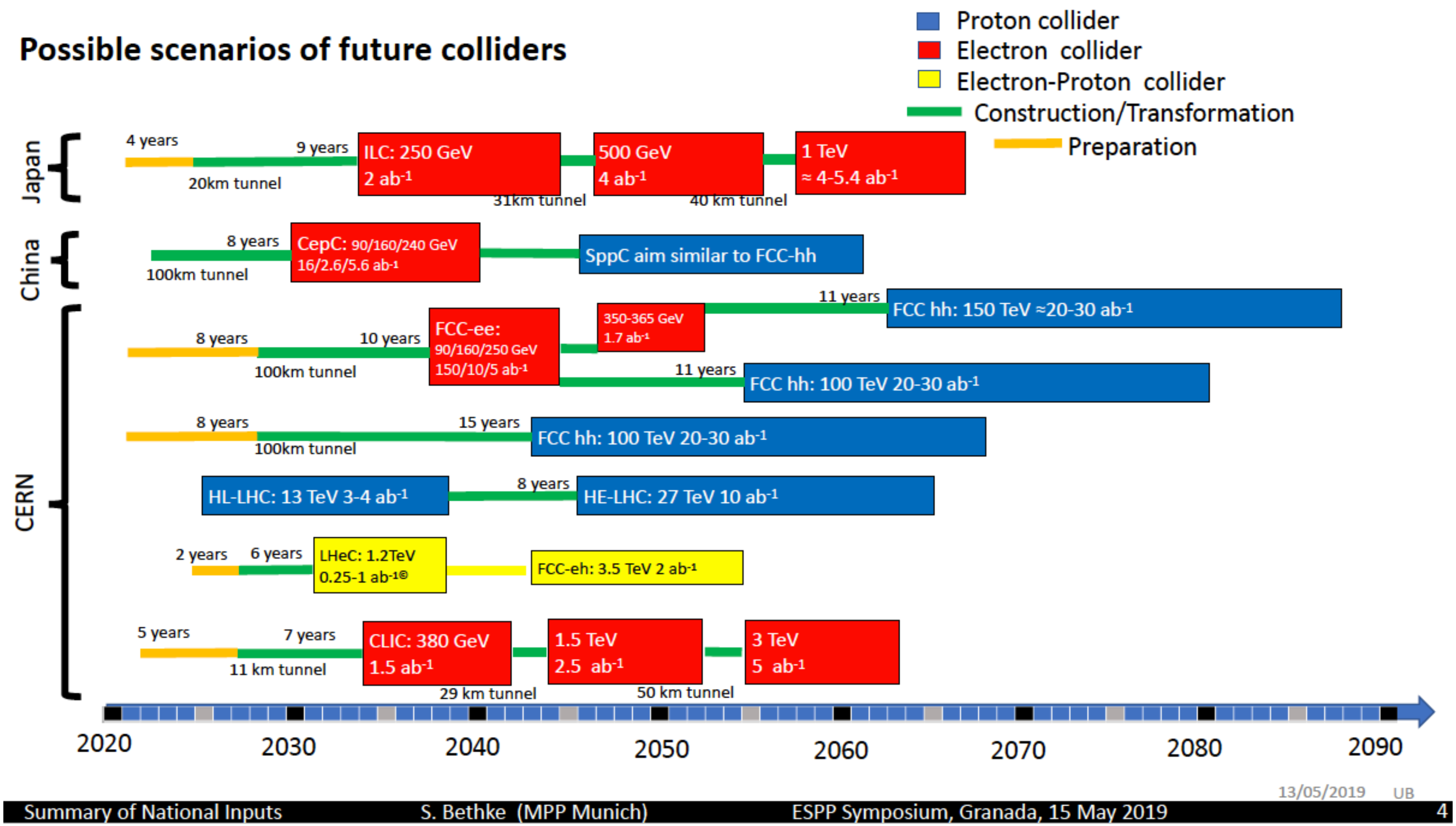

- all options aimed at **attobarn^{-1} physics**
- requires to go **far beyond NNLO for theory**
- Even conservative estimates **not reachable with current techniques**

WEAKNESSES OF QFT

- **SM/BSM** extrapolated to **infinite energy** (**zero distance**) in loop corrections $\gg M_{\text{Plank}}$
- Quantum state with N partons $\neq$ quantum state with **zero energy emission** of extra patrons
- Partons can be emitted in **exactly the same direction**

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in **four** space-time dimensions
$1/\epsilon$ in dimensional regularization

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Ultraviolet singularities (UV)

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soft singularities (IR)

Ultraviolet singularities (UV)

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soft singularities (IR)

collinear singularities (IR)

Ultraviolet singularities (UV)

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soft singularities (IR)

collinear singularities (IR)

Ultraviolet singularities (UV)

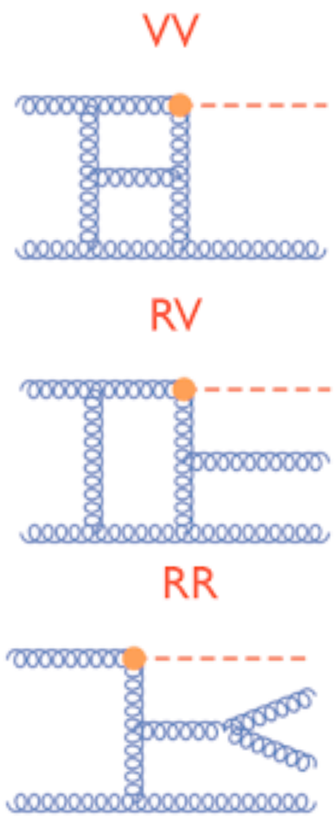
and **threshold** singularities,
integrable but numerically unstable

in **four** space-time dimensions
$1/\epsilon$ in dimensional regularization

G. Rodrigo, FCC Week 2019

SUBTRACTION OF IR SINGULARITIES & DREG

- Subtraction of IR singularities at NLO is solved: efficient algorithms applicable to any process for which matrix elements are known
- At NNLO several working algorithms, successfully applied to "simple" processes with up to four legs. Heavy computational costs

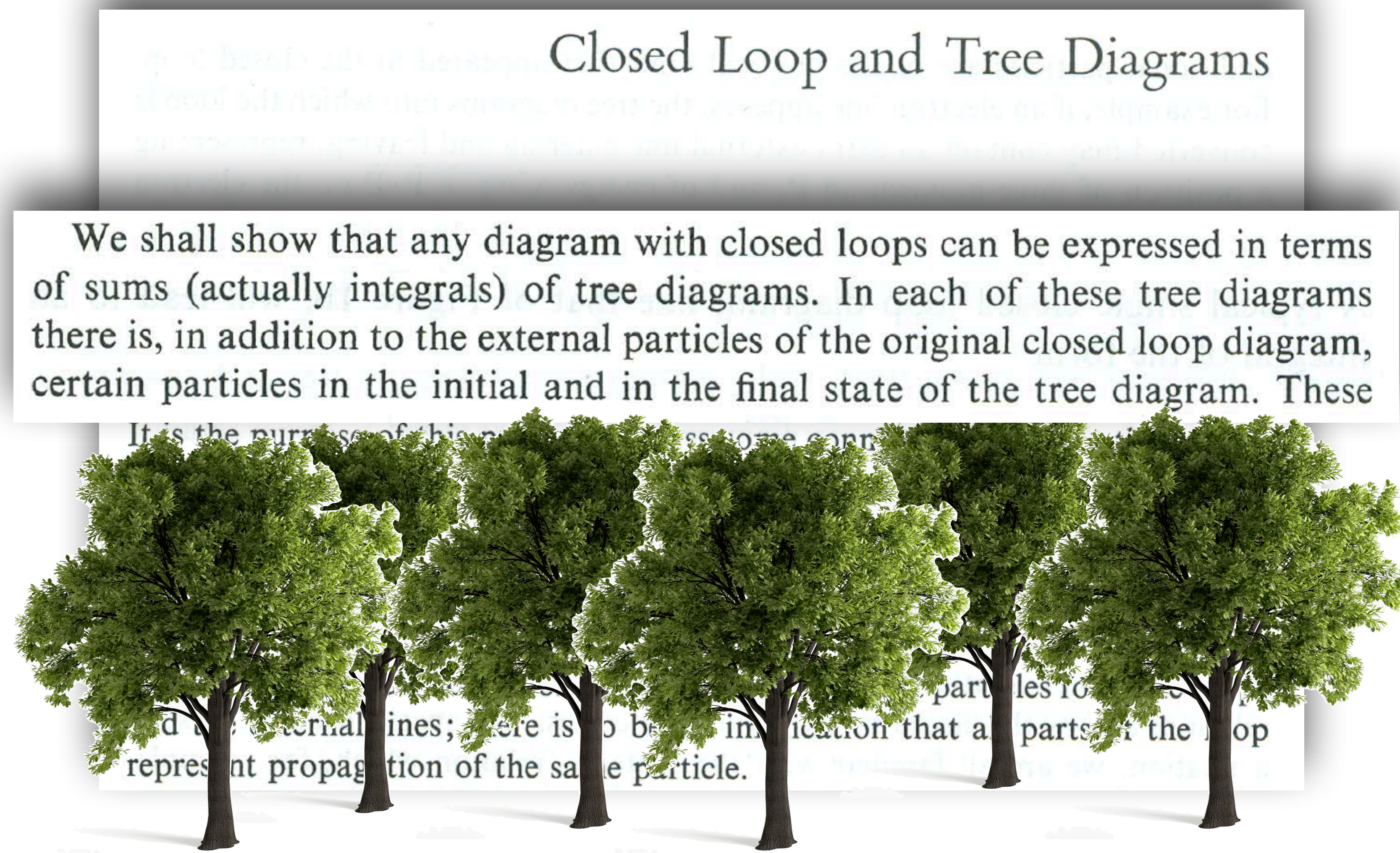


- Antennae Subtraction [Gehrmann et al.]
- Stripper [Czacon et al.]
- Nested Soft-Collinear Subtraction [Caola et al.]
- Colourful Subtraction [Del Duca et al.]
- N-Jettiness [Boughezal, Petriello et al., Gaunt et al.]
- q_T Substraction [Catani, Grazzini et al.]
- Projection to Born [Bonciani et al.]
- Geometric Substraction [Herzog]

R. P. Feynman, Closed Loop And Tree Diagrams. (talk),

Magic Without Magic: J. A. Wheeler, A Collection of Essays in Honor of his Sixtieth Birthday. Edited by John R. Klauder, 1972, p.355

In *Brown, L.M. (ed.): Selected papers of Richard Feynman* 867-887



Closed Loop and Tree Diagrams

We shall show that any diagram with closed loops can be expressed in terms of sums (actually integrals) of tree diagrams. In each of these tree diagrams there is, in addition to the external particles of the original closed loop diagram, certain particles in the initial and in the final state of the tree diagram. These

Gnendiger et al., To d, or not to d: Recent developments and comparisons of regularization schemes, EPJ C77 (2017) 471

LOCAL SUBTRACTION IN THE UV

LOCAL UNSUBTRACTION IN THE IR

- **LTD:** open loops to trees
- **FDU:** mapping of V ▶R kinematics

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- **V+R simultaneous:**
  - More efficient event generators

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- **V+R simultaneous:**
  - More efficient event generators
- LTD suitable for **amplitudes**, FDU aimed at **physical observ.**

PROPAGATORS

$$G_F(q_i) = \frac{1}{q_i^2 - m_i^2 + i0}$$

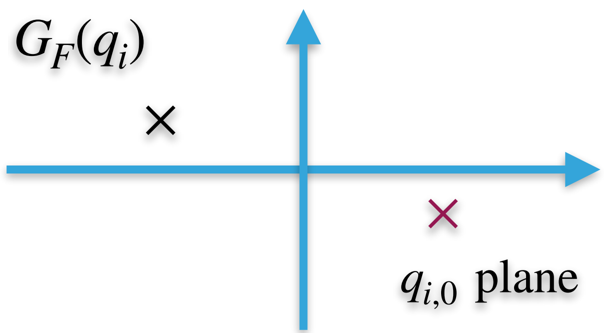
- MATH: the +i0 is a small quantity usually ignored, assuming that the **analytical continuation** to the physical kinematics is well defined
- PHYS: the +i0 encodes **CAUSALITY | positive frequencies are propagated forward in time, and negative backward**

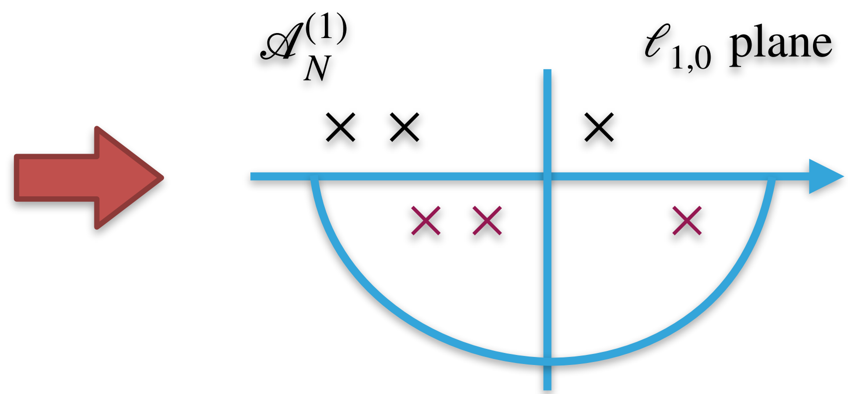
 [Catani et al. JHEP **0809**, 065]

THE LOOP-TREE DUALITY (LTD)

Cauchy residue theorem

in the loop energy complex plane





Feynman Propagator +i0:

positive frequencies are propagated forward in time, and negative backward

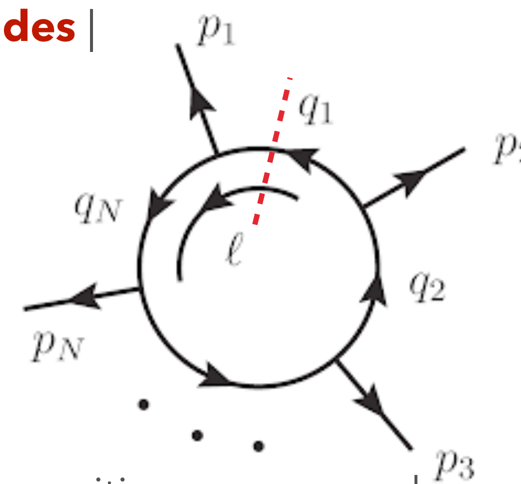
$$G_F(q_i) = \frac{1}{q_i^2 - m_i^2 + i0}$$

selects residues with definite **positive energy and negative imaginary part** (indeed in any other coordinate system)

[Catani et al. JHEP **0809**, 065]

THE LOOP-TREE DUALITY (LTD)

One-loop amplitudes in any relativistic, local and unitary QFT represented as a linear combination of *N* **single-cut phase-space/dual amplitudes** | **no disjoint trees** (at higher orders: number of cuts equal to the number of loops)



$$\int_{\ell_1} \mathcal{N}(\ell_1) \prod G_F(q_i) = - \int_{\ell_1} \mathcal{N}(\ell_1) \otimes \sum \tilde{\delta}(q_i) \prod_{i \neq j} G_D(q_i; q_j)$$

- $\tilde{\delta}(q_i) = i\, 2\pi\, \theta(q_{i,0})\, \delta(q_i^2 - m_i^2)$ sets internal line on-shell, positive energy mode
- $G_D(q_i; q_j) = \dfrac{1}{q_j^2 - m_j^2 - i0\,\eta\, k_{ji}}$ **dual propagator** $k_{ji} = q_j - q_i$

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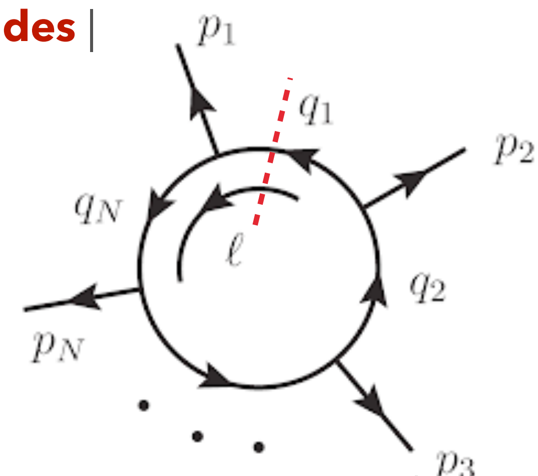

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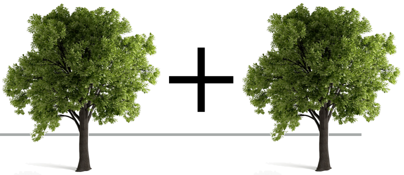
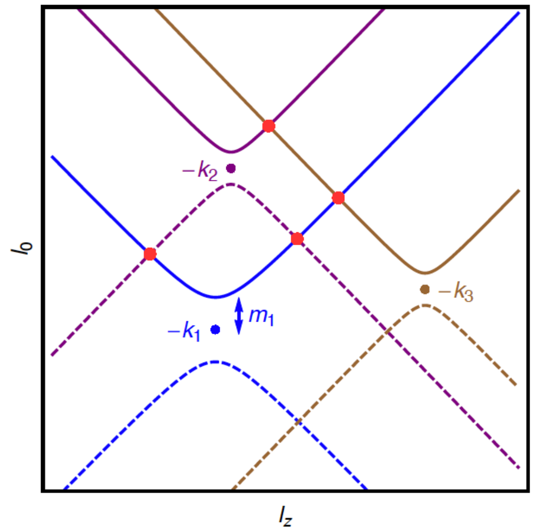
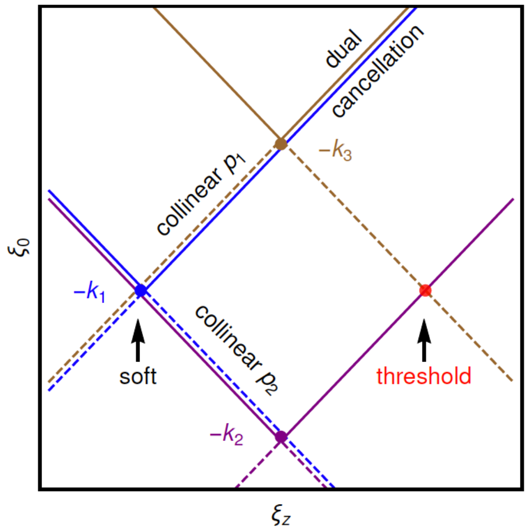
- LTD realised by **modifying the customary +i0 prescription** of the Feynman propagators (only the sign matters), it encodes in a compact way the effect of **multiple-cut** contributions that appear in the **Feynman's Tree Theorem**
- best choice $\eta^\mu = (1, \mathbf{0})$: energy component integrated out, remaining integration in **Euclidean space**

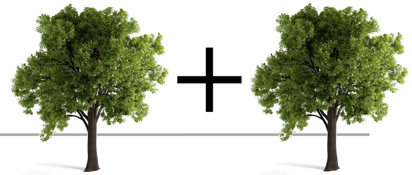


THE DUAL FOREST | CAUSALITY





- **LTD** is equivalent to integrate along the forward on-shell hyperboloids (light-cones for massless) or positive energy modes



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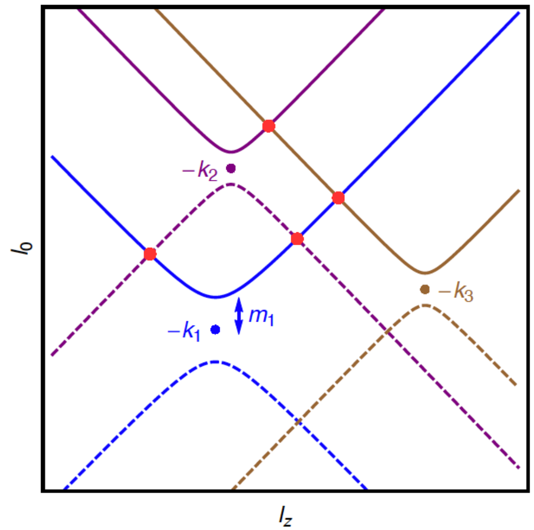

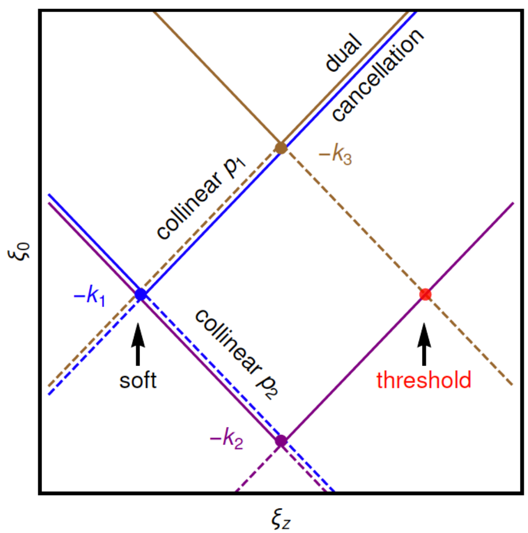

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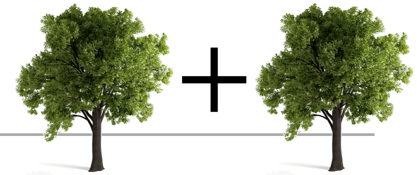
- The dual integrand becomes singular when a second propagator gets eventually on-shell



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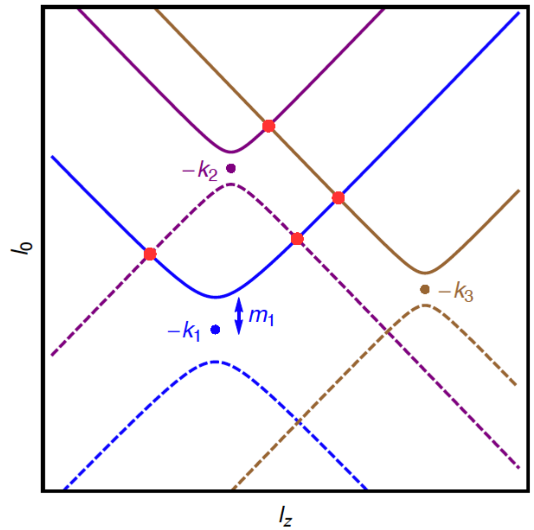

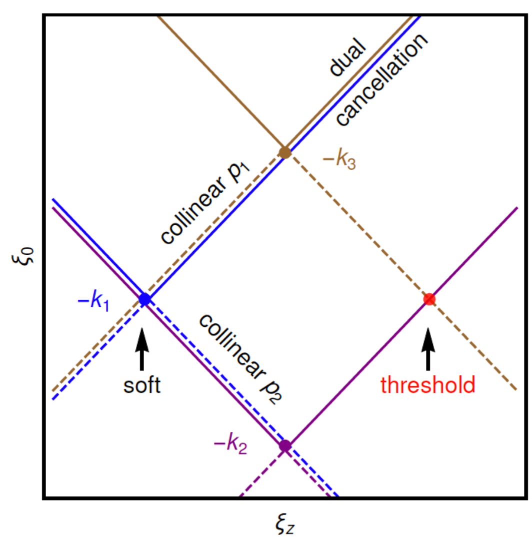

- **LTD** is equivalent to integrate along the forward on-shell hyperboloids (light-cones for massless) or positive energy modes
- The dual integrand becomes singular when a second propagator gets eventually on-shell
- The location of singularities is determined by a linear identity in the on-shell energies

$$\lambda_{ij}^{\pm\pm} = \pm\, q_{i,0}^{(+)} \pm q_{j,0}^{(+)} + k_{ji,0} \to 0$$

where

$$q_{r,0}^{(+)} = \sqrt{\boldsymbol{q}_r^2 + m_r^2} \qquad\qquad k_{ji} = q_j - q_i$$

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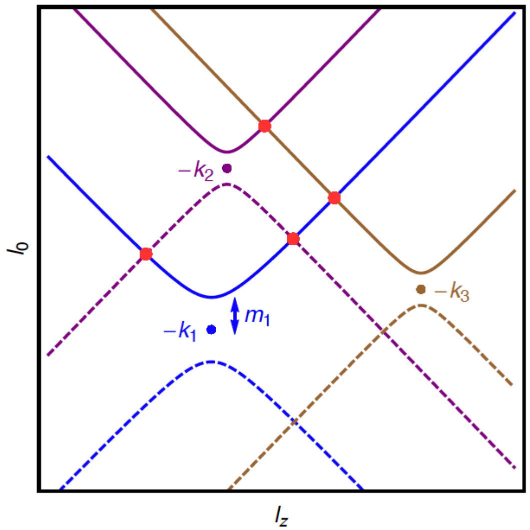
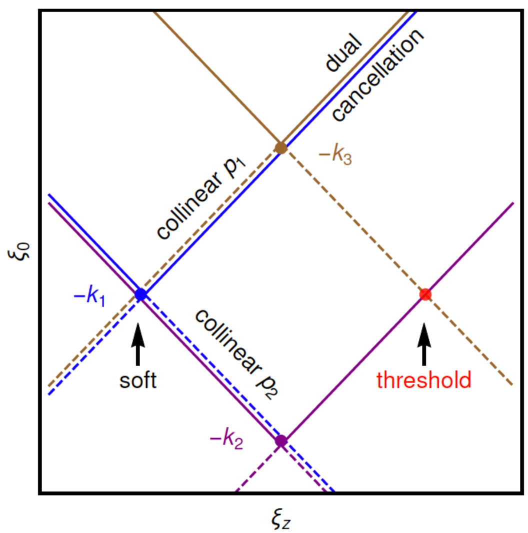
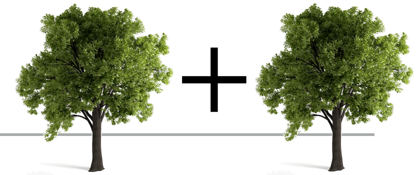


LTD: $\mathcal{S}_{ij}^{(1)} = (2\pi\imath)^{-1}\, G_D(q_i; q_j)\, \tilde{\delta}(q_i) + (i \leftrightarrow j)$





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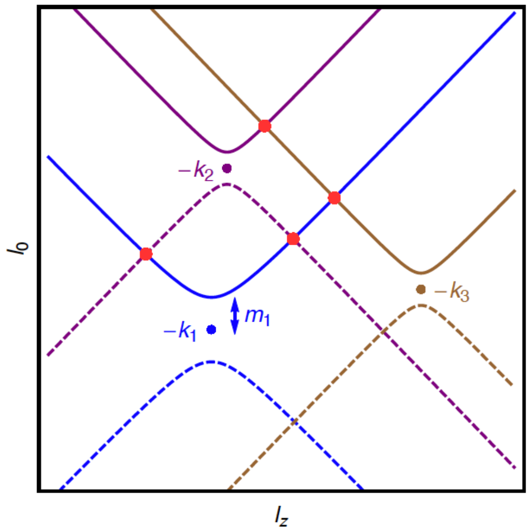
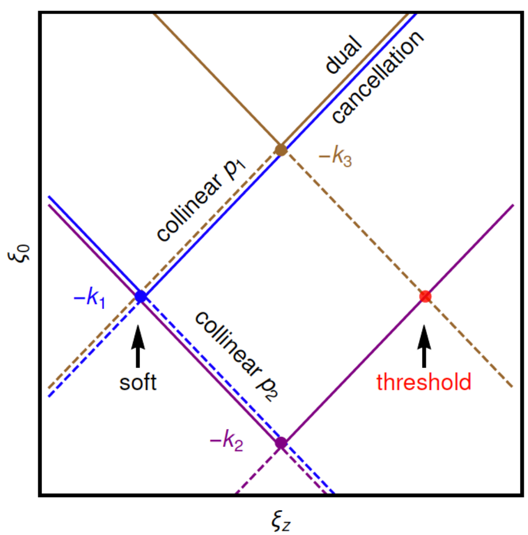




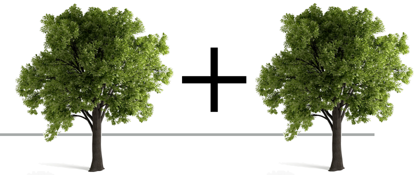
LTD: $$\mathcal{S}^{(1)}_{ij} = (2\pi\imath)^{-1}\, G_D(q_i;q_j)\, \tilde\delta(q_i) + (i \leftrightarrow j)$$

- **Time-like distance (causally connected):** generates physical threshold singularities: always **+i0**

$$\lim_{\lambda^{++}_{ij}\to 0} \mathcal{S}^{(1)}_{ij} = \frac{\theta(-k_{ji,0})\,\theta(k_{ji}^2-(m_i+m_j)^2)}{x_{ij}\,(-\lambda^{++}_{ij} - \imath 0\, k_{ji,0})} + \mathcal{O}\left((\lambda^{++}_{ij})^0\right)$$

$$x_{ij} = 4\, q^{(+)}_{i,0} q^{(+)}_{j,0}$$

$+i0$



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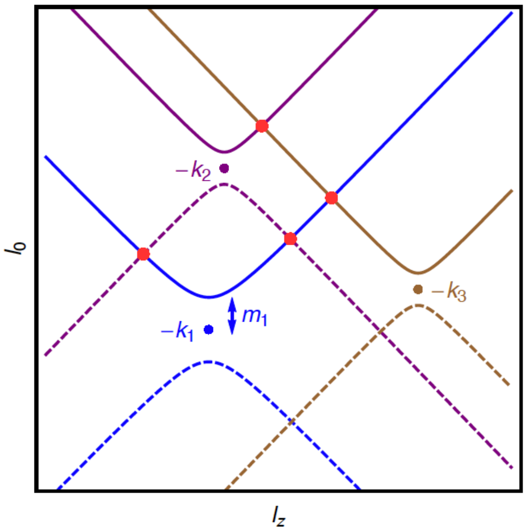

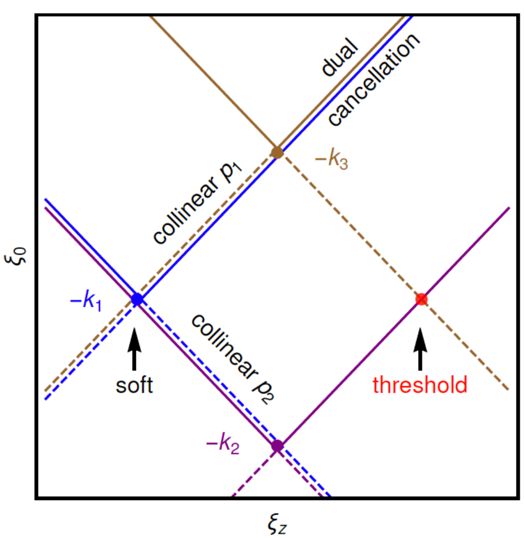

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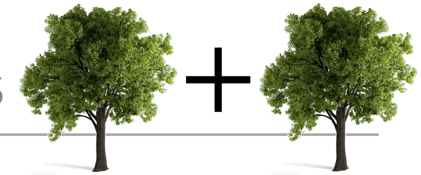

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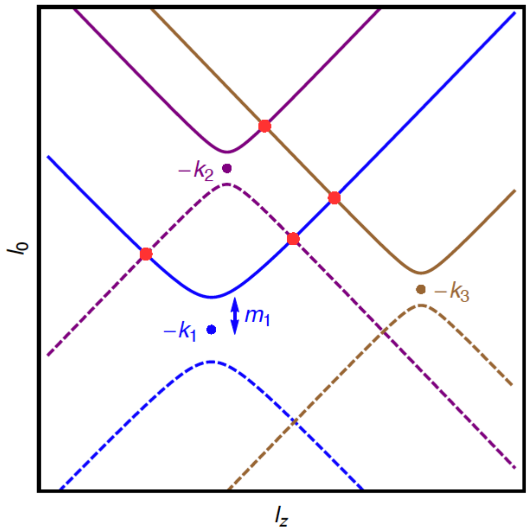

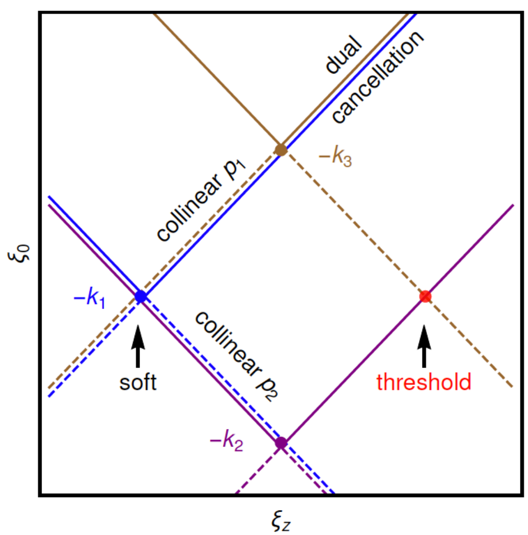

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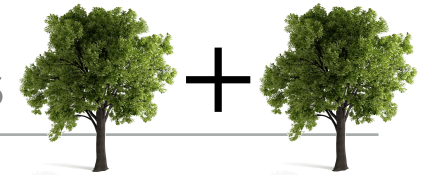
- **Light-like distance:** both singular configurations, partial cancellation, IR singularities remain in a compact region



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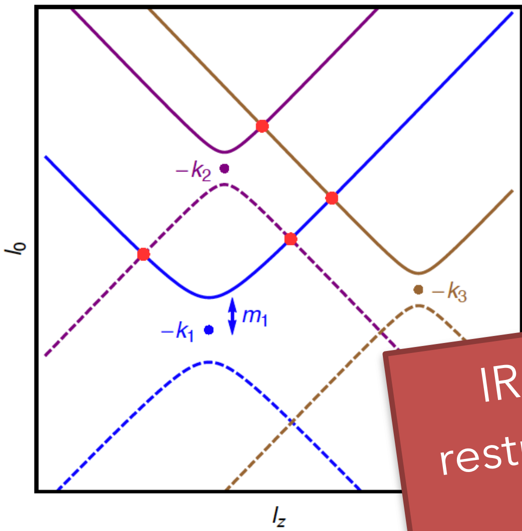

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$$\ldots{}_i + m_j)^2) \quad + \mathcal{O}\left((\lambda_{ij}^{++})^0\right)$$
$$\ldots k_{ji,0})$$

$+i0$

IR and threshold singularities are restricted to a **compact region** of the loop three-momentum

Essential feature for FDU

[…] there is a perfect cancellation of singularities, due to the dual **+i0** prescription

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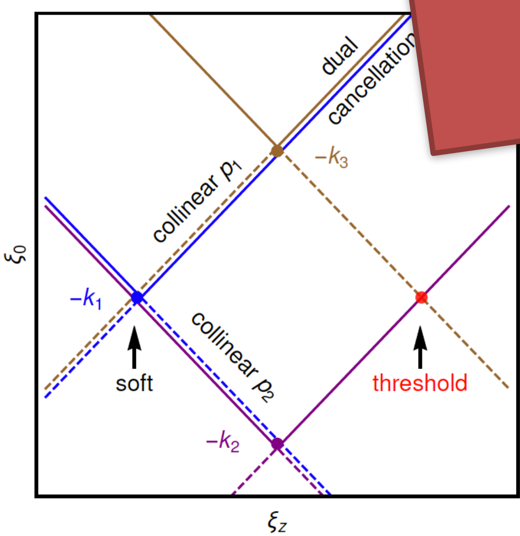

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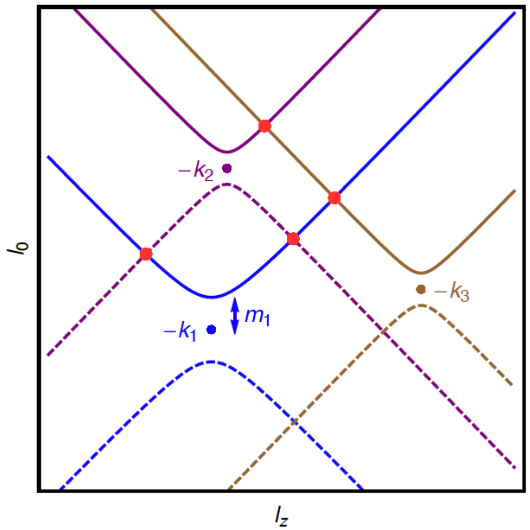
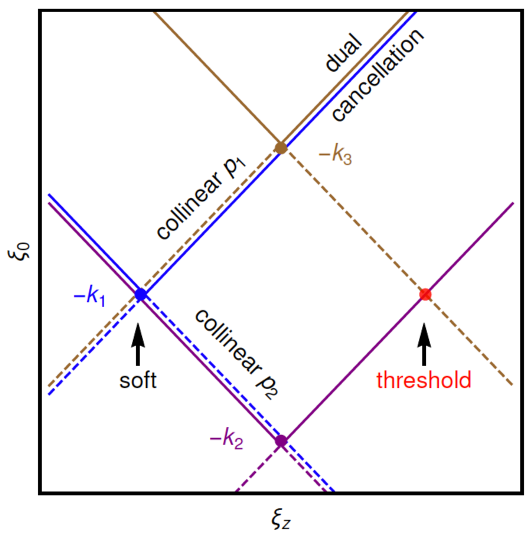
THE FEYNMAN'S FOREST | CAUSALITY





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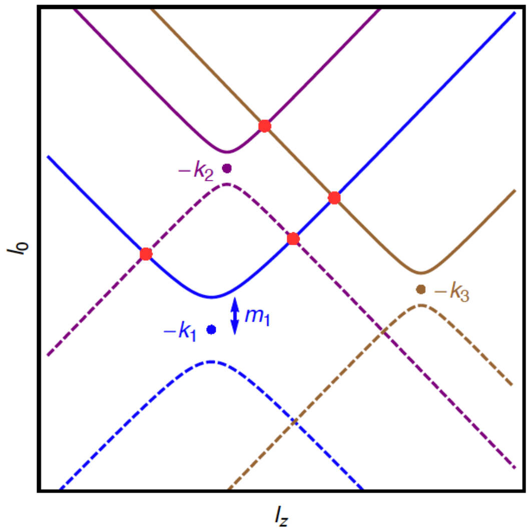
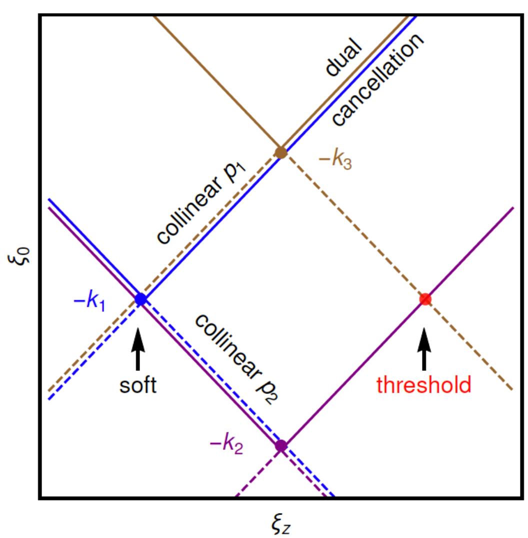


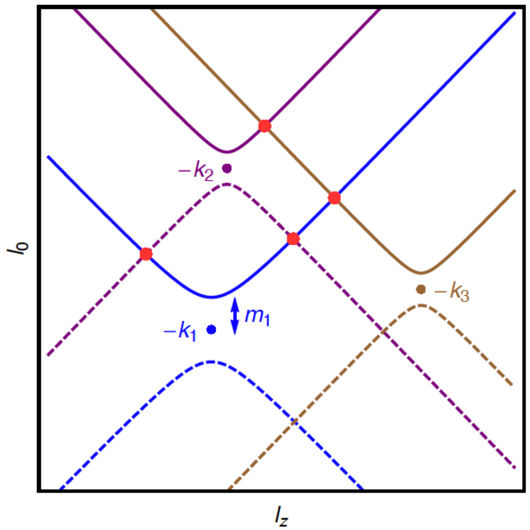


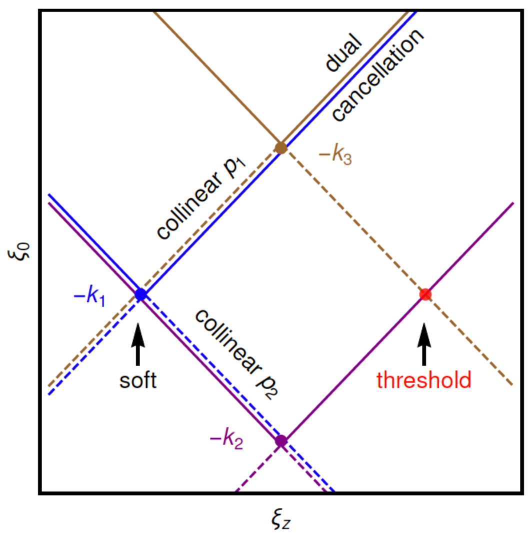
FTT: $$\mathcal{F}^{(1)}_{ij} = (2\pi\imath)^{-1}\, G_F(q_j)\, \tilde{\delta}(q_i) + (i \leftrightarrow j)$$

- **Time-like distance (causally connected):** physics does not depend on the FTT or LTD representation

$$\lim_{\lambda_{ij}^{++}\to 0} \mathcal{F}^{(1)}_{ij} = \frac{\theta(-k_{ji,0})\,\theta(k_{ji}^2 - (m_i + m_j)^2)}{x_{ij}\,(-\lambda_{ij}^{++} + \imath 0)} + \mathcal{O}\left((\lambda_{ij}^{++})^0\right)$$

THE FEYNMAN'S FOREST | CAUSALITY





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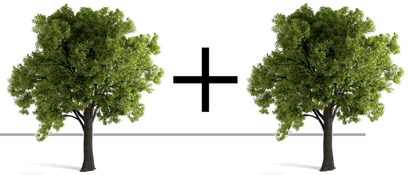

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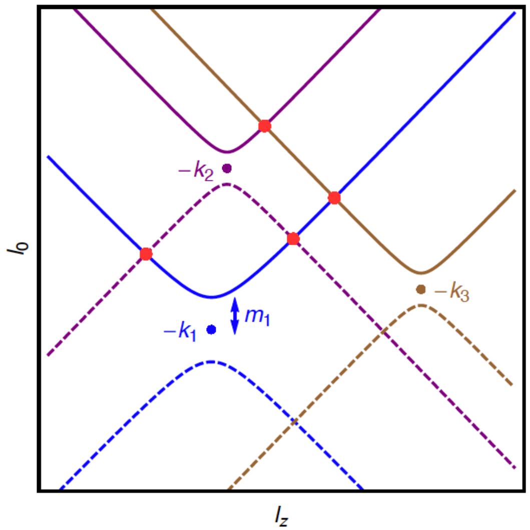

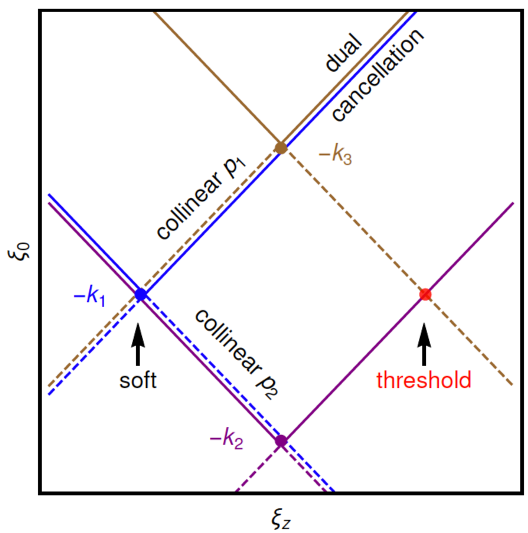

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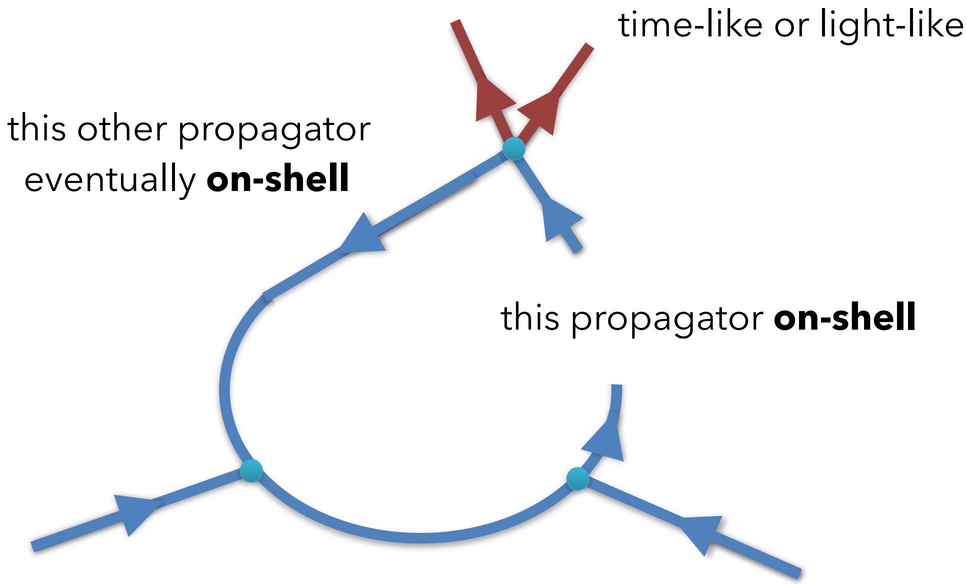
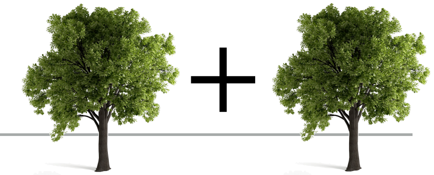
- needs to be compensated by the contribution from **multiple cuts**

WHEN A BRANCHES GET BROKEN

energy of the **on-shell** propagator smaller than the energy of the emitted particles





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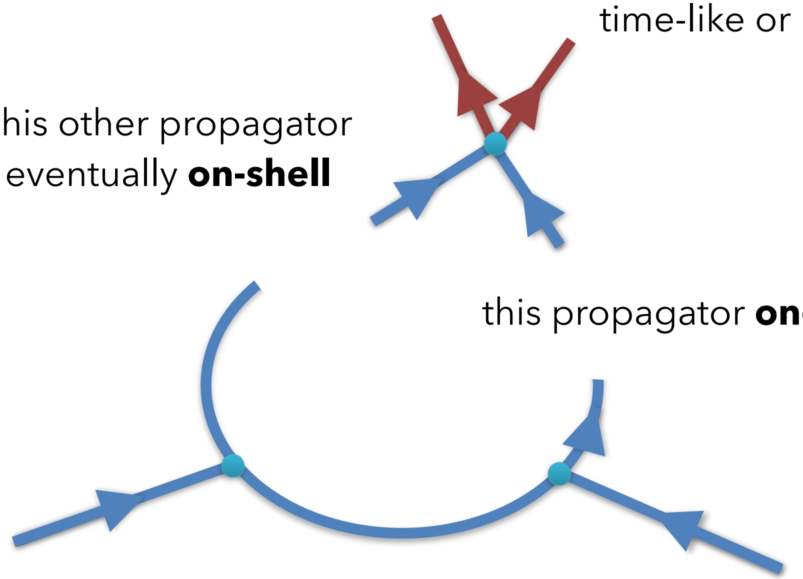
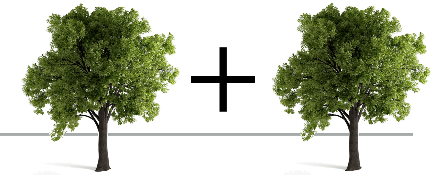


- **Causally connected**

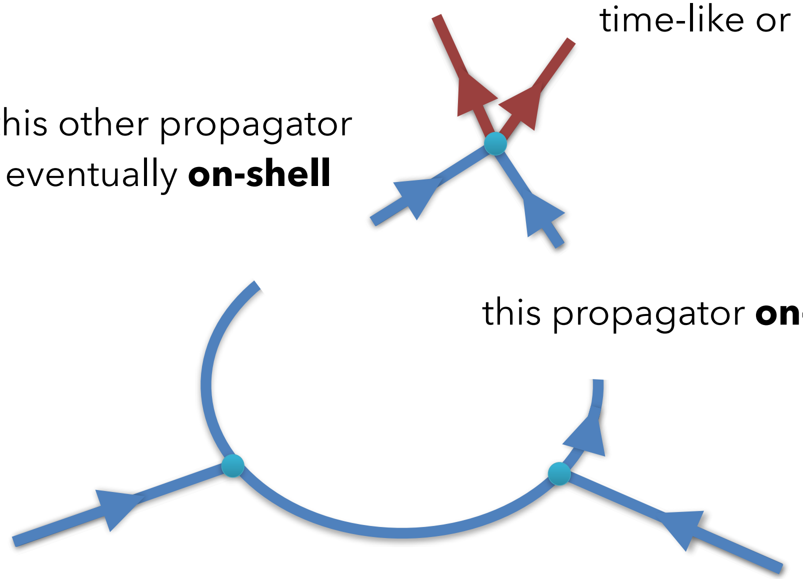


WHEN A BRANCHES GET BROKEN

energy of the **on-shell** propagator smaller than the energy of the emitted particles

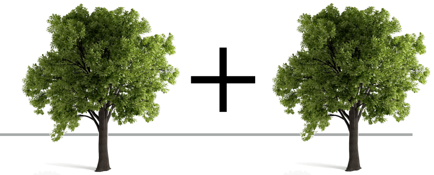
time-like or light-like

this other propagator eventually **on-shell**



this propagator **on-shell**

- **Causally connected**
- **Threshold** singularities occur when a second propagator gets on-shell: consistent with **Cutkosky**



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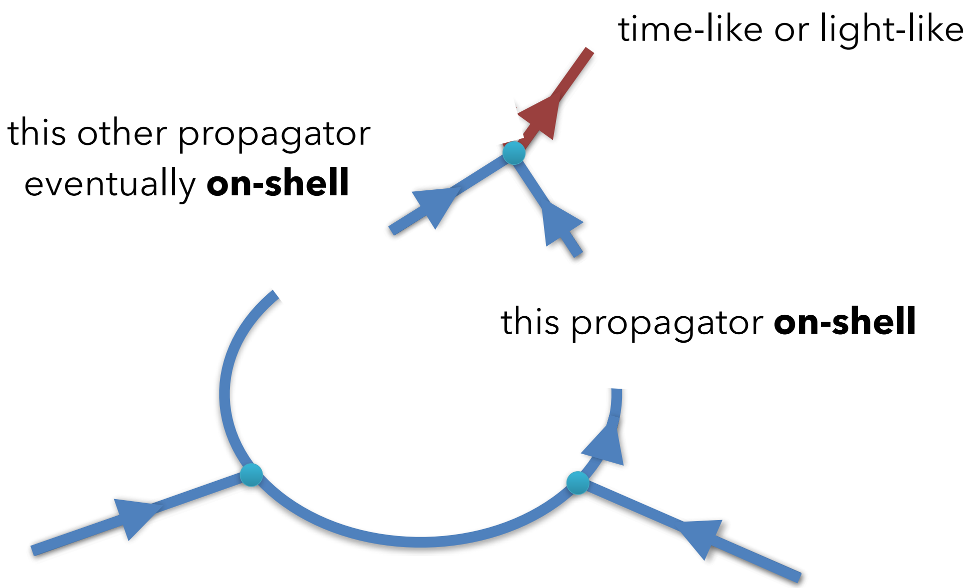
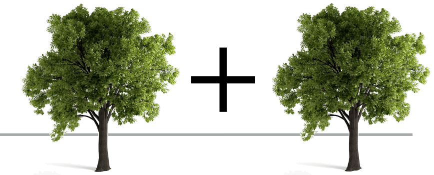


- **Causally connected**
- **Threshold** singularities occur when a second propagator gets on-shell: consistent with **Cutkosky**
- It becomes **collinear (soft)** when a single massless particle is emitted



SINGULARITIES OF SINGLE-CUT TREES AND THE FOREST

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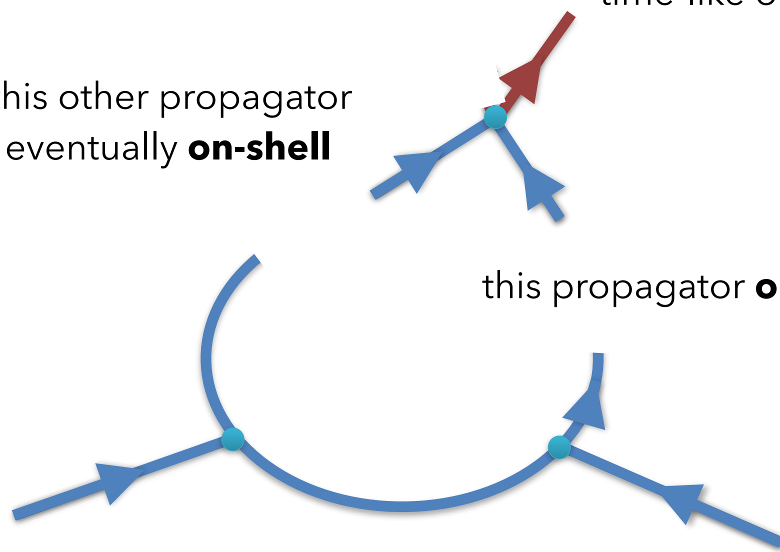
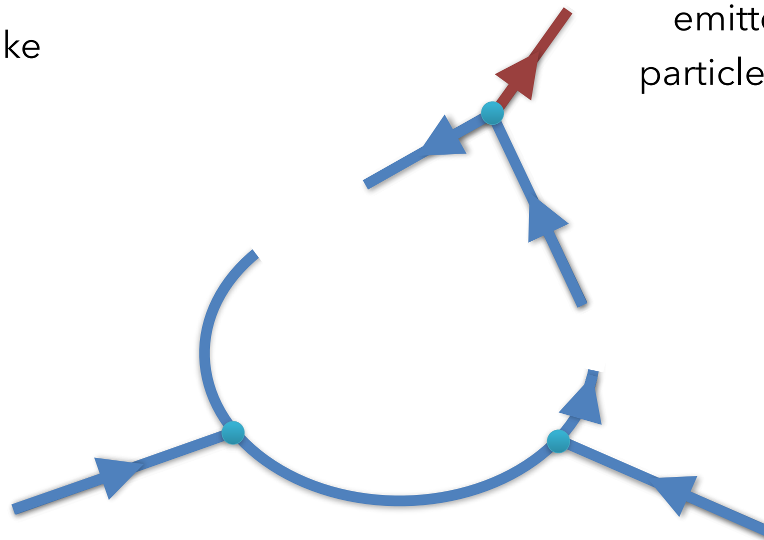
time-like or light-like

this other propagator eventually **on-shell**

this propagator **on-shell**



Space-like or light-like at energy of the **on-shell** propagator larger than the energy of the emitted particle(s)



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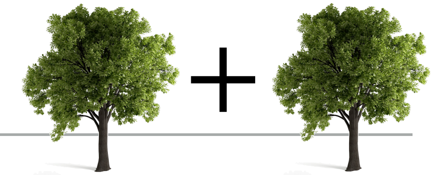

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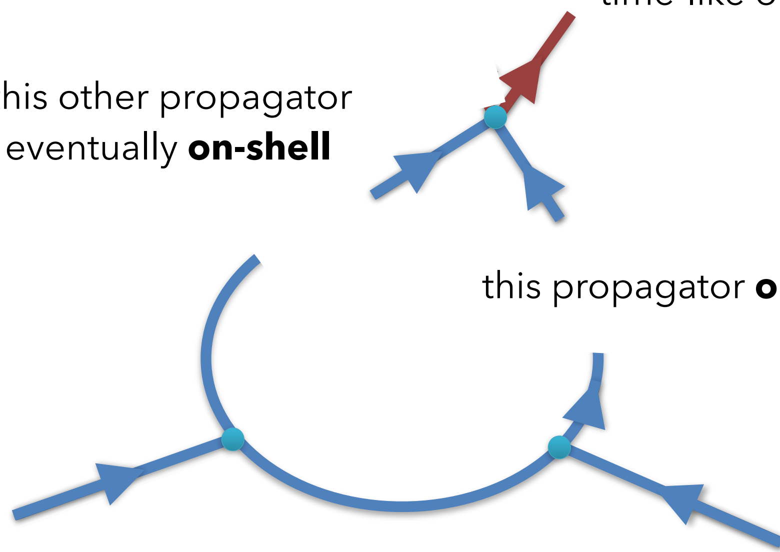

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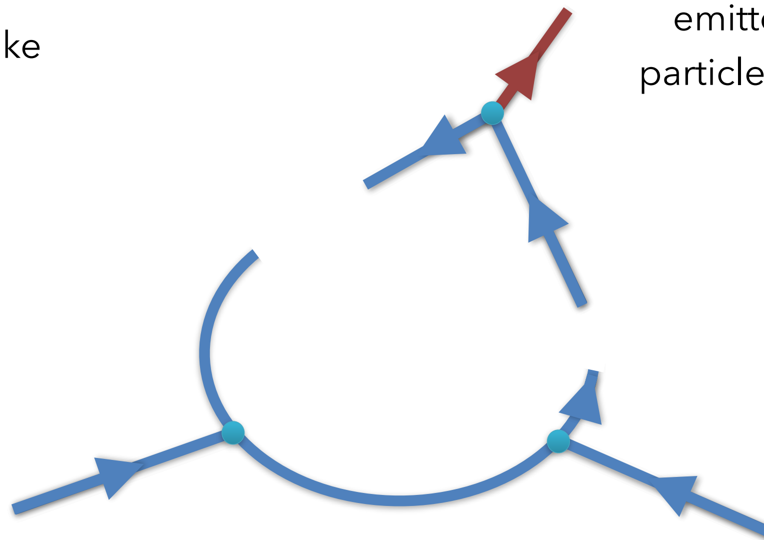

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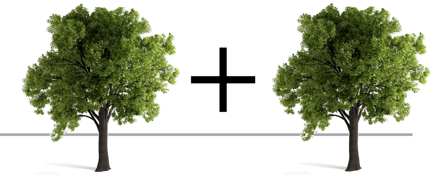

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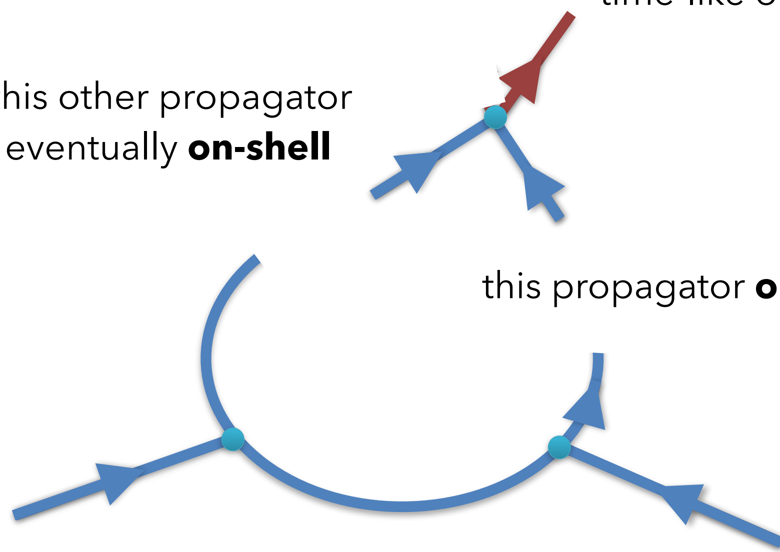

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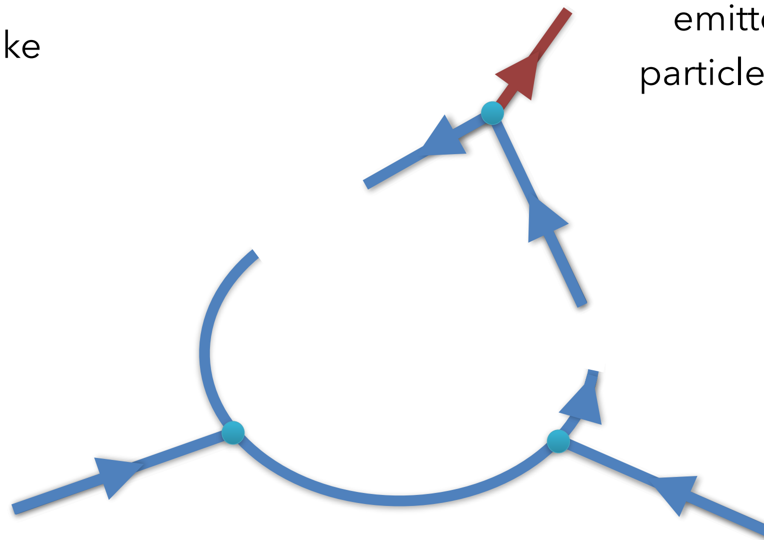

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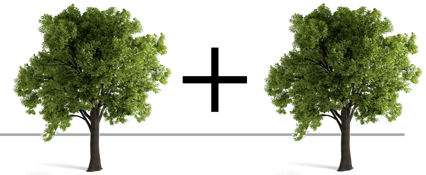

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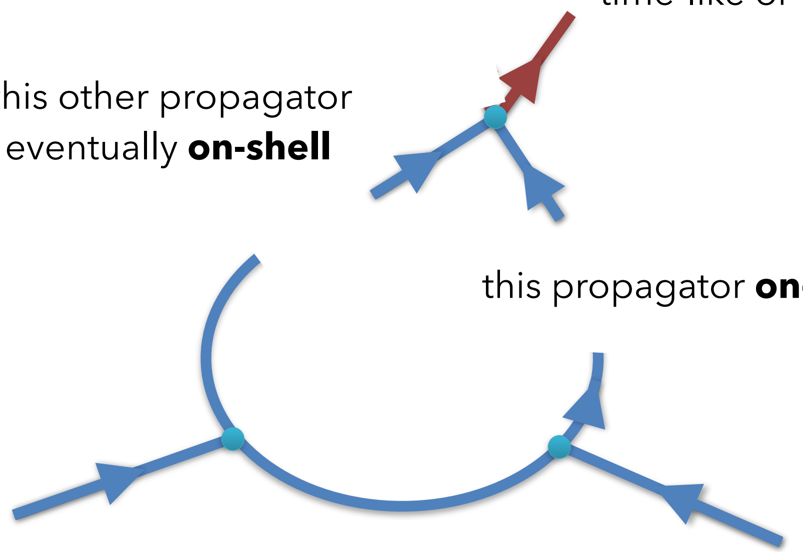

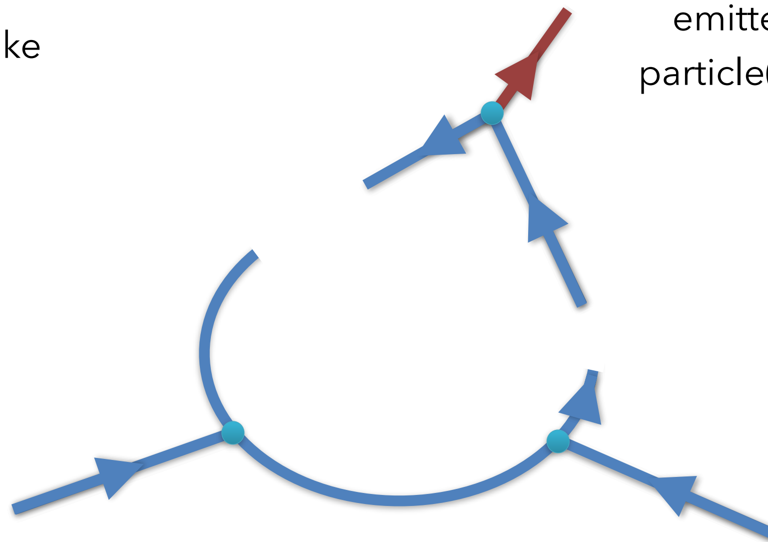

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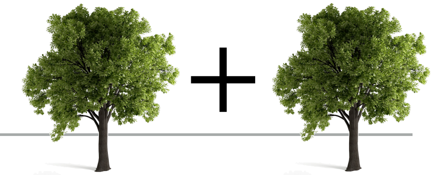

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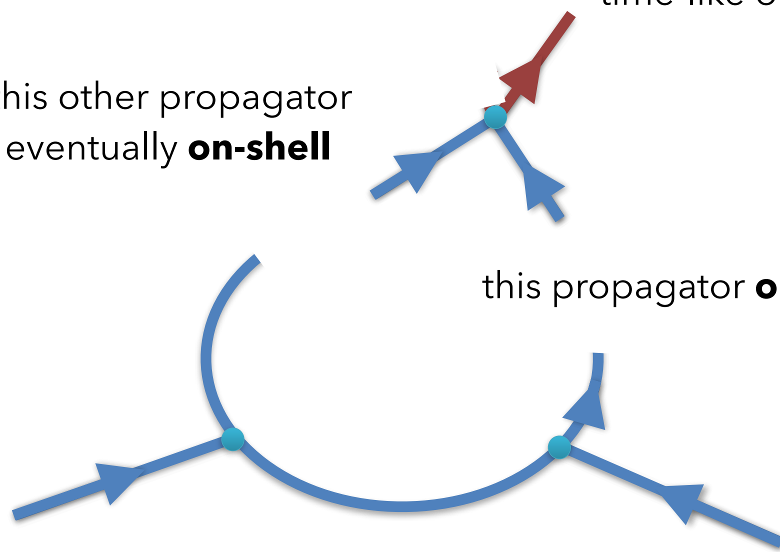
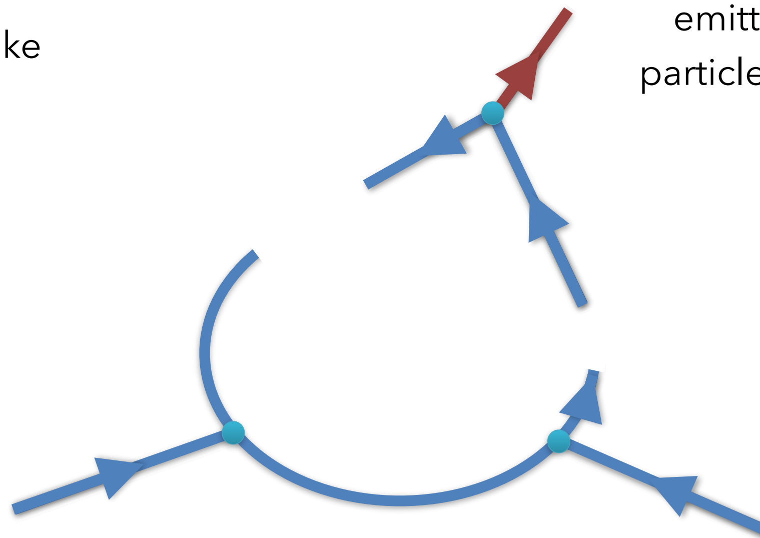


this propagator **on-shell**



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- Potential **threshold and IR singularities cancel** in the sum of single-cut trees
- Non-singular configurations at very large energies (**UV**) expected to be **suppressed.** If not sufficiently suppressed, **renormalise**
- **The bulk of the physics** is in the **"low" energy** region of the loop momentum

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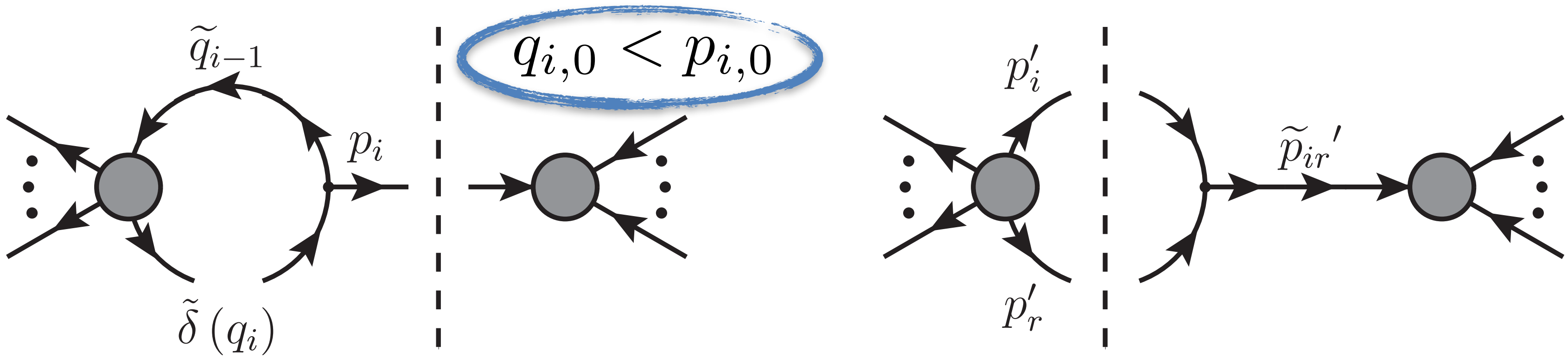
MOMENTUM MAPPING: MULTI-LEG



- Motivated by the **factorisation properties of QCD**: assuming q_i^μ on-shell, and close to collinear with p_i^μ ,we define the momentum mapping

$$
\begin{aligned}
p_r'^\mu &= q_i^\mu \ , \\
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- All the primed momenta (real process) on-shell and momentum conservation: p_i^μ is the **emitter**, p_j^μ the **spectator** needed to absorb momentum recoil

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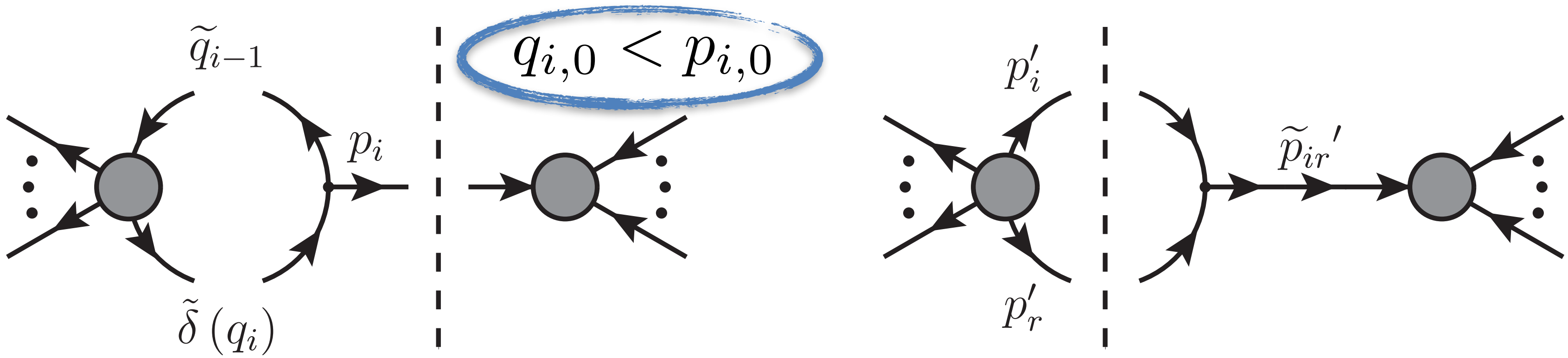

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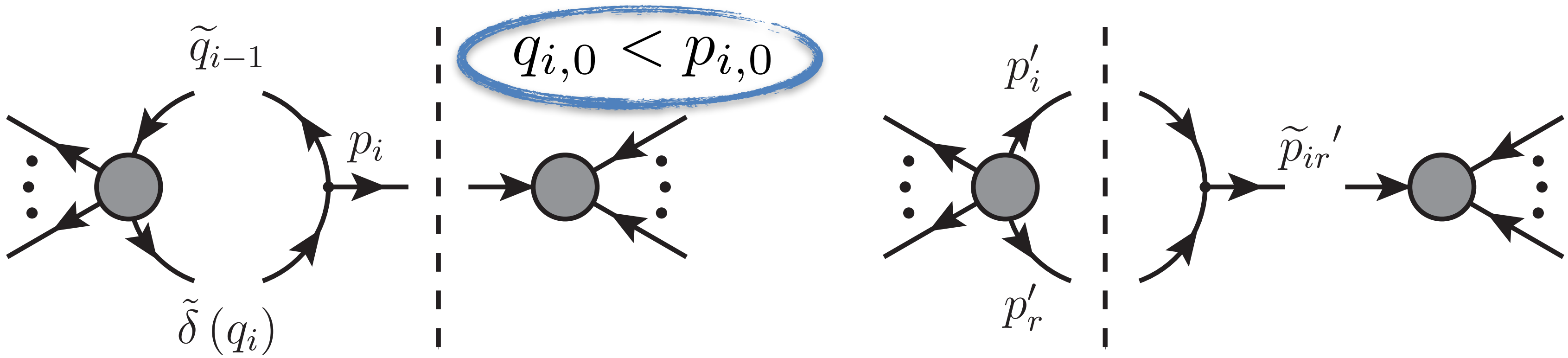

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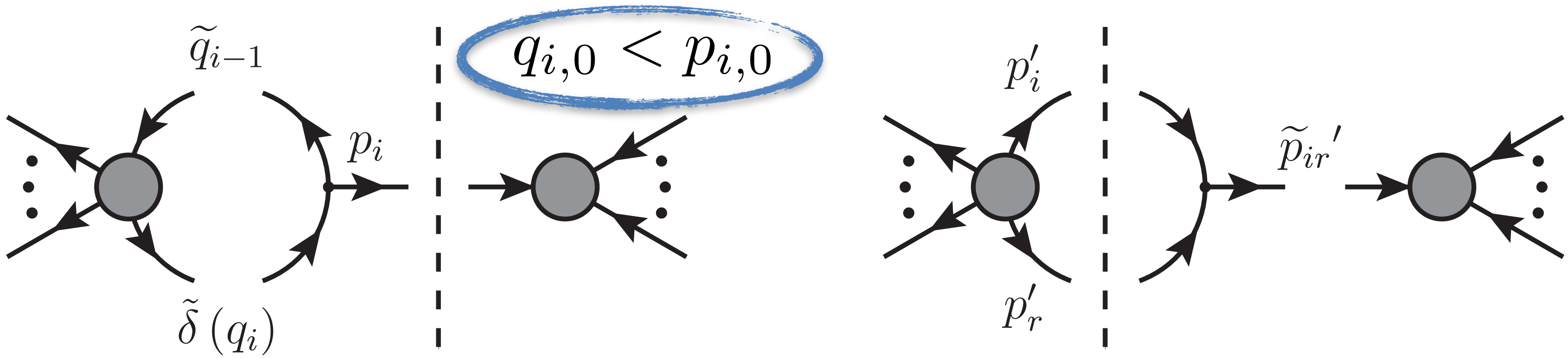

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- All the primed momenta (real process) on-shell and momentum conservation: p_i^μ is the **emitter**, p_j^μ the **spectator** needed to absorb momentum recoil
- **Quasi-collinear configurations** can also be conveniently mapped such that the **massless limit is smooth** [Sborlini, Driencourt-Mangin, GR, JHEP **1610**, 162]

FOUR-DIMENSIONAL UNSUBTRATION (FDU) @ NLO

- The **LTD representation** of the renormalised loop cross-section: one single integral in the loop three-momentum

$$\int_N d\sigma_{\rm V}^{(1,{\rm R})} = \int_N \int_{\vec\ell_1} 2\,{\rm Re}\,\langle \mathcal{M}_N^{(0)} | \left(\sum_i \mathcal{M}_N^{(1)}(\tilde\delta(q_i)) \right) - \mathcal{M}_{\rm UV}^{(1)}(\tilde\delta(q_{\rm UV})) \rangle$$

- A **partition** of the real phase-space

$$\sum_i \mathcal{R}_i(\{p'_j\}_{N+1}) = 1$$

- The real contribution **mapped** to the Born kinematics + loop three-momentum

$$\int_{N+1} d\sigma_{\rm R}^{(1)} = \int_N \int_{\vec\ell_1} \sum_i \mathcal{J}_i(q_i)\, \mathcal{R}_i(\{p'_j\})\, |\mathcal{M}_{N+1}^{(0)}(\{p'_j\})|^2 \Big|_{\{p'_j\}_{N+1} \to (q_i, \{p_k\}_N)}$$

ANOMALOUS THRESHOLDS

$$\mathcal{S}^{(1)}_{ijk} = (2\pi\imath)^{-1}\, G_D(q_i; q_k)\, G_D(q_i; q_j)\, \tilde{\delta}(q_i) + \text{perm}\,.$$

ANOMALOUS THRESHOLDS

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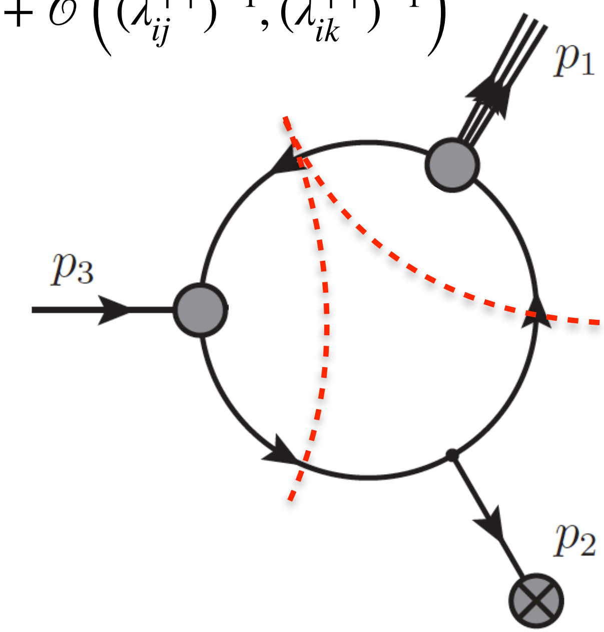
- The singularities that appear at the intersection of three (four) forward on-shell hyperboloids (light cones) cancel
- **Anomalous thresholds:** causal thresholds involving more than two propagators

$$\lim_{\lambda^{++}_{ij},\lambda^{++}_{ik}\to 0} \mathcal{S}^{(1)}_{ijk} = \frac{1}{x_{ijk}} \prod_{r=j,k} \frac{\theta(-k_{ri,0})\,\theta(k_{ri}^2-(m_i+m_r)^2)}{-\lambda^{++}_{ir} - \imath 0\, k_{ri,0}} + \mathcal{O}\left((\lambda^{++}_{ij})^{-1},(\lambda^{++}_{ik})^{-1}\right)$$

$$x_{ijk} = 8\, q^{(+)}_{i,0} q^{(+)}_{j,0} q^{(+)}_{k,0}$$

$$\lambda^{++}_{ij}, \lambda^{++}_{ik} \to 0 \qquad \lambda^{-+}_{jk} = \lambda^{++}_{ik} - \lambda^{++}_{ij}$$



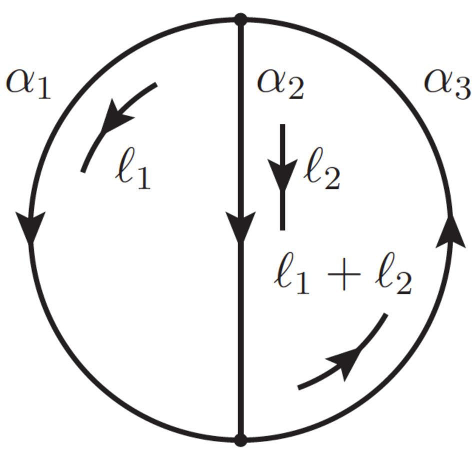
absence of singularity in $\lambda^{-+}_{jk} \to 0$

LTD AT TWO-LOOPS (AND BEYOND)

- At two-loops (LTD representation):

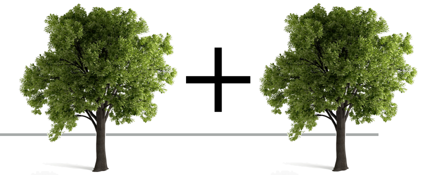
$$\int_{\ell_1}\int_{\ell_2} \mathcal{N}(\ell_1,\ell_2)\, G_F(\alpha_1 \cup \alpha_2 \cup \alpha_3) = \int_{\ell_1}\int_{\ell_2} \mathcal{N}(\ell_1,\ell_2) \otimes \Big\{ G_D(\alpha_1)\, G_D(\alpha_2 \cup \alpha_3)$$

$$+\, G_D(-\alpha_2 \cup \alpha_1)\, G_D(\alpha_3) \;-\; G_D(\alpha_1)\, G_F(\alpha_2)\, G_D(\alpha_3) \Big\}$$



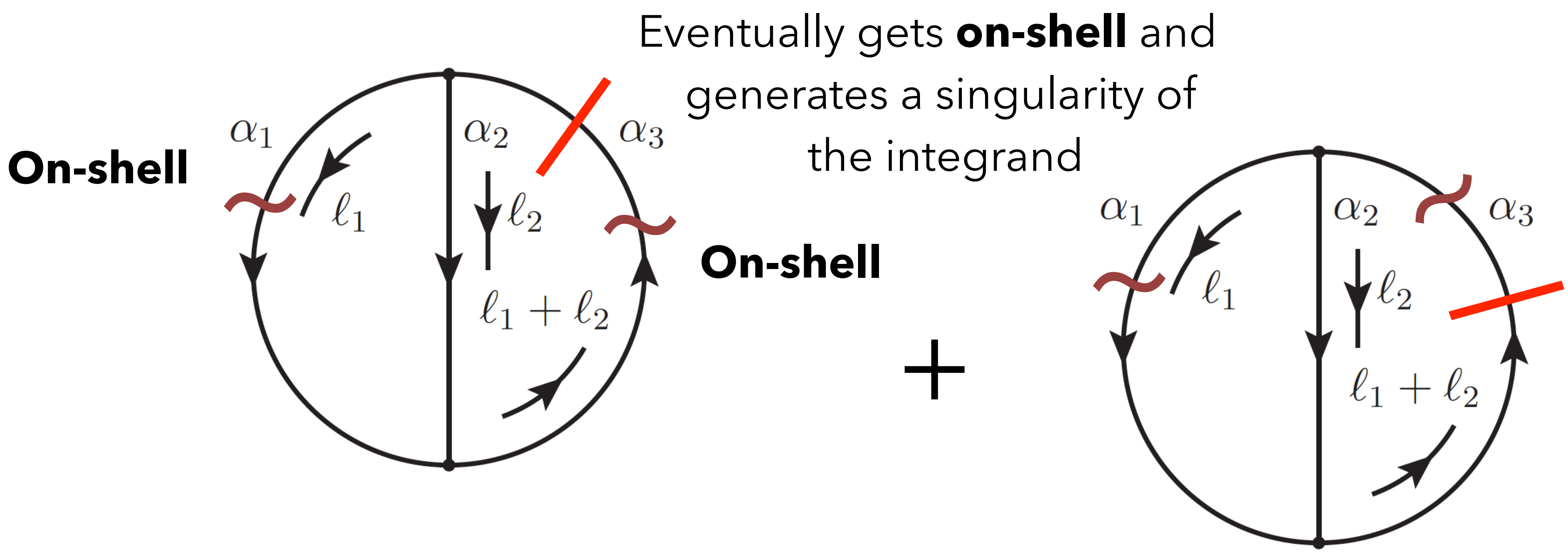
With a **number of cuts equal to the number of loops** the loop amplitude opens to a **non-disjoint**  level like object

$$G_F(\alpha_k) = \prod_{i\in\alpha_k} G_F(q_i) \qquad G_D(\alpha_k) = \sum_{i\in\alpha_k} \tilde\delta(q_i) \prod_{j\neq i} G_D(q_i;q_j)$$



THE TWO-LOOP FOREST

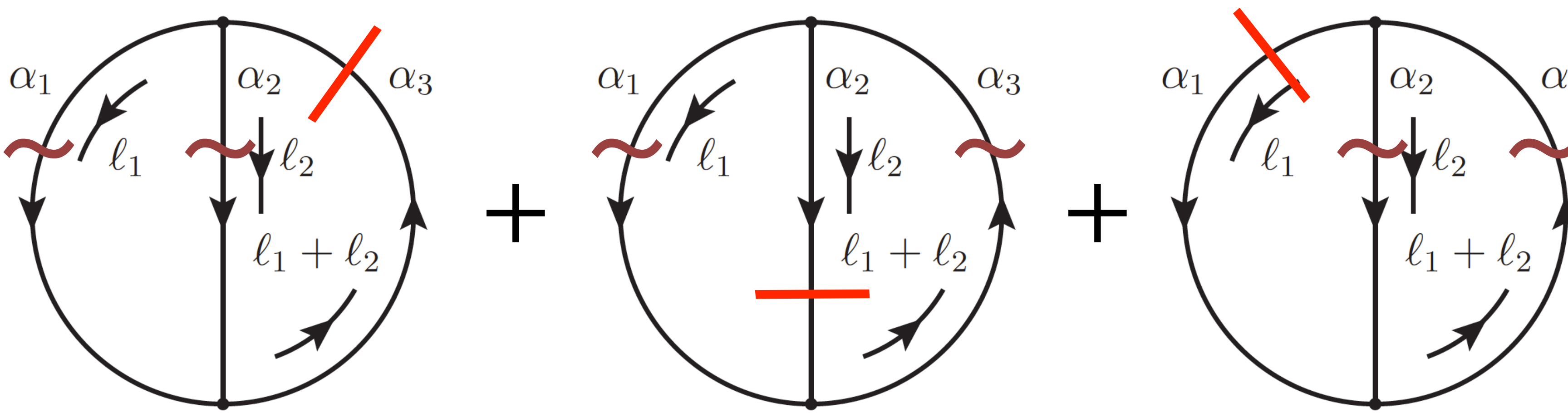
- One propagator gets eventually on-shell in the same line where there is a cut propagator: equivalent to the **one-loop** case



- There are **two potential singular configurations**: one of them cancels in the sum of cuts, the other leads to IR/thresholds

THE TWO-LOOP FOREST

- The genuine two-loop case occurs when the singularity is generated in another loop line



- There are **four singular configurations**: two of them are non-causal and cancel in the sum, the other two lead to potential IR/thresholds (two-loop)

Driencourt-Mangin, Sborlini, GR, Torres Bobadilla, JHEP **1902,**143
and J.J. Aguilera-Verdugo, F. Driencourt-Mangin, J. Plenter, S. Ramírez-Uribe, G. Rodrigo, G.F.R. Sborlini, W. J. Torres Bobadilla, S. Tracz e-Print: **arXiv:1904.08389**

UNITARITY THRESHOLDS AT TWO LOOPS

$$\mathcal{S}^{(2)}_{ijk} = (2\pi\imath)^{-2} \left[G_D(q_j; q_k)\, \tilde{\delta}(q_i, q_j) + G_D(-q_j; q_i)\, \tilde{\delta}(-q_j, q_k) + \left[G_D(q_k; q_j) + G_D(q_i; -q_j) - G_F(q_j) \right] \tilde{\delta}(q_i, q_k) \right]$$

- The location of singularities is determined by a **linear identity*** in the on-shell energies

$$\lambda^{\pm\pm\pm}_{ijk} = \pm\, q^{(+)}_{i,0} \pm q^{(+)}_{j,0} \pm q^{(+)}_{k,0} + k_{k(ij),0} \to 0 \qquad\qquad k_{k(ij),0} = q_k - q_i - q_j$$

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- The four independent singularities are

$$\lim_{\lambda^{+++}_{ijk} \to 0} \mathcal{S}^{(2)}_{ijk} = \frac{\theta(-k_{k(ij),0})\, \theta(k^2_{k(ij)} - (m_i + m_j + m_k)^2)}{x_{ijk}(-\lambda^{+++}_{ijk} - \imath 0\, k_{kj,0})} + \mathcal{O}\left((\lambda^{+++}_{ijk})^0 \right) \qquad x_{ijk} = 8\, q^{(+)}_{i,0} q^{(+)}_{j,0} q^{(+)}_{k,0}$$

$$+i0$$

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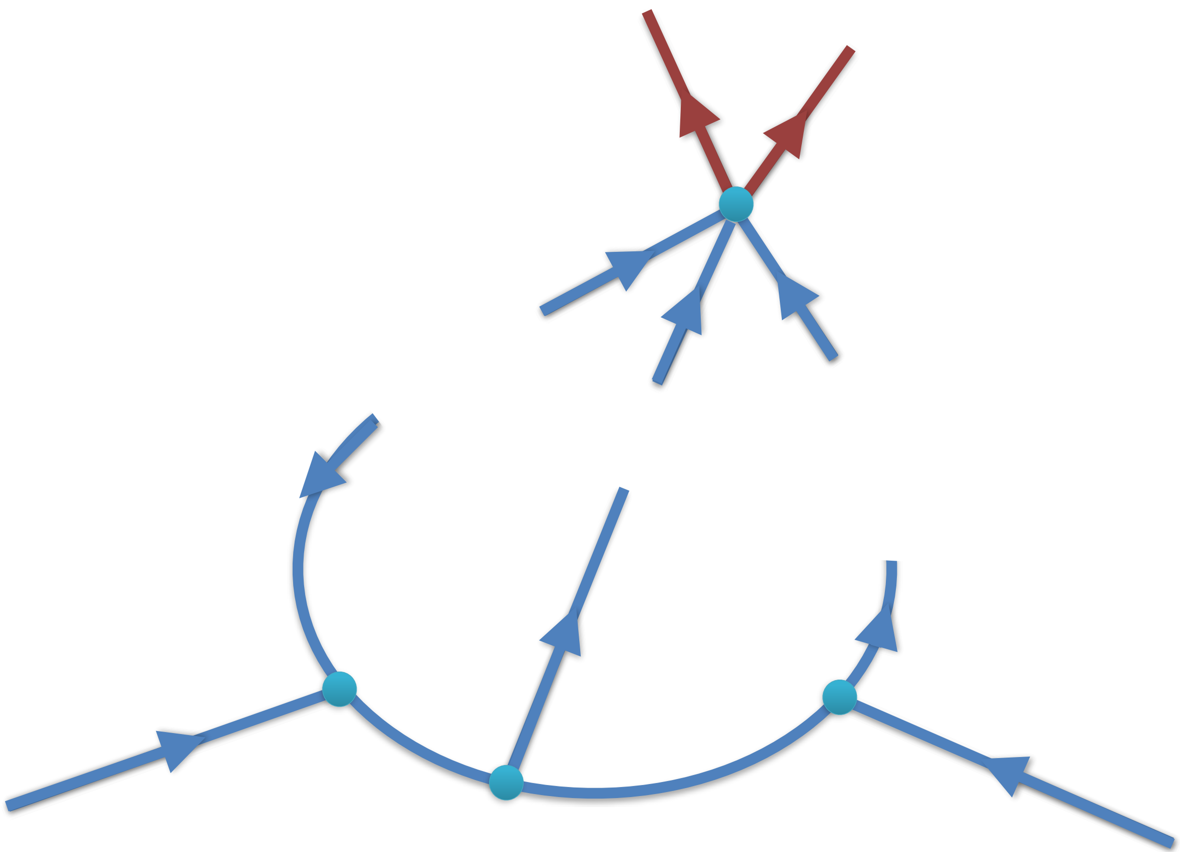
- * Non-linear dual prescriptions as proposed in Runkel, Ször, Vesga, Weinzierl, arXiv:1902.02135 incompatible with this identity
- Dual cancellations tested numerically in Capatti, Hirschi, Kermanschah, Ruijl, arXiv:1906.06138

UNITARITY THRESHOLD / TRIPLE COLLINEAR



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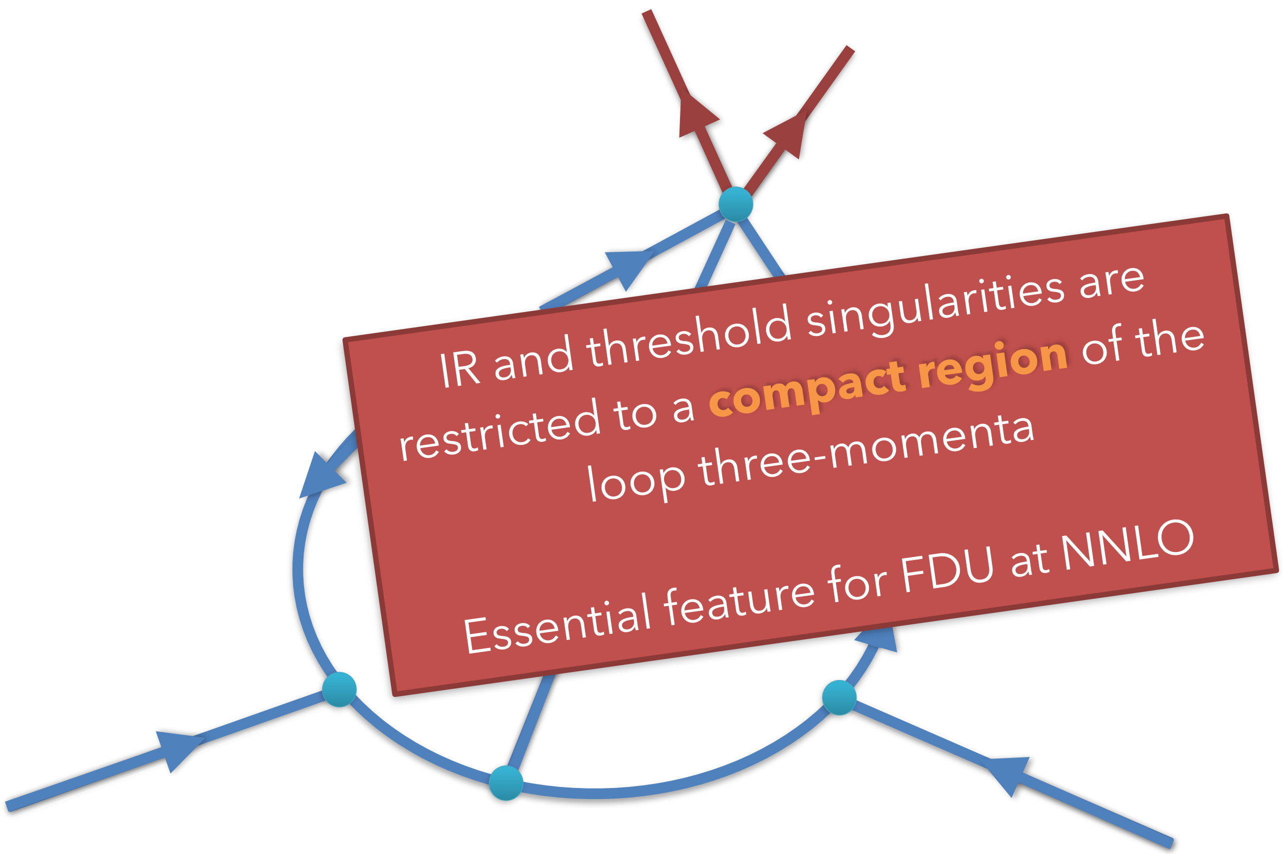
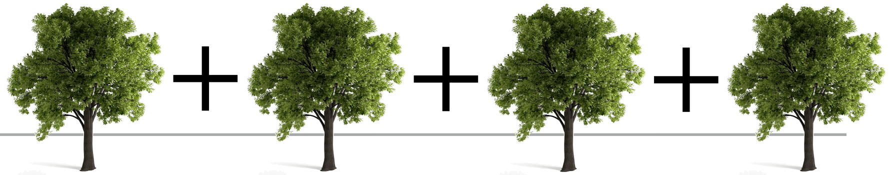
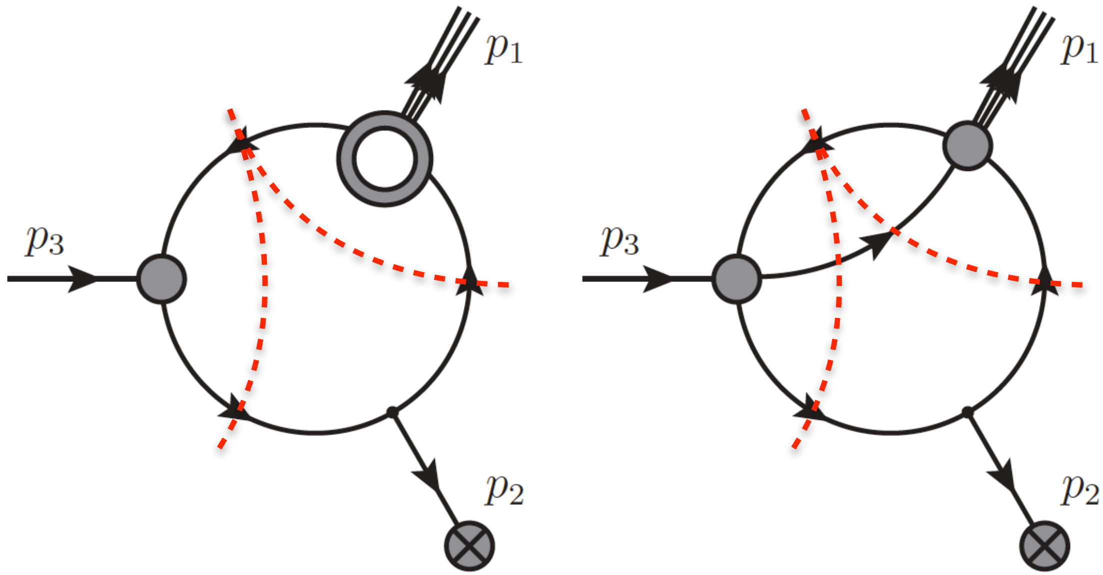


IR and threshold singularities are restricted to a **compact region** of the loop three-momenta

Essential feature for FDU at NNLO



ANOMALOUS THRESHOLDS AT TWO LOOPS



$$\lim_{\lambda^{+++}_{i_1jk},\lambda^{+++}_{i_2jk}\to 0} \mathcal{S}^{(2)}_{i_1i_2jk} = \frac{1}{x_{i_1i_2jk}} \prod_{i=i_1i_2} \frac{\theta(-k_{k(ij),0})\,\theta(k^2_{k(ij)} - (m_i+m_j+m_k)^2)}{-\lambda^{+++}_{ijk} - \imath 0\, k_{kj,0}}$$
$$+\mathcal{O}\left((\lambda^{+++}_{i_1jk})^{-1}, (\lambda^{+++}_{i_2jk})^{-1}\right)$$

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UV RENORMALISATION: LOCAL SUBTRACTION

- Expand propagators and numerators around a UV propagator [Weinzierl, Pittau]

$$G_F(q_i) = \frac{1}{q_{\rm UV}^2 - \mu_{\rm UV}^2 + i0}\left[1 - \frac{2q_{\rm UV}\cdot k_i + k_i^2 - m_i^2 + \mu_{\rm UV}^2}{q_{\rm UV}^2 - \mu_{\rm UV}^2 + i0} + \frac{(2q_{\rm UV}\cdot k_i)^2}{(q_{\rm UV}^2 - \mu_{\rm UV}^2 + i0)^2}\right] + \ldots$$

$$q_{\rm UV} = \ell + k_{\rm UV} \qquad k_i = q_i - q_{\rm UV}$$

- and adjust **subleading** terms to subtract only the pole ($\overline{MS}$ **scheme**), or to define any other renormalisation scheme. For the scalar two point function

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Hernández-Pinto, Sborlini, GR, JHEP **1602,** 044

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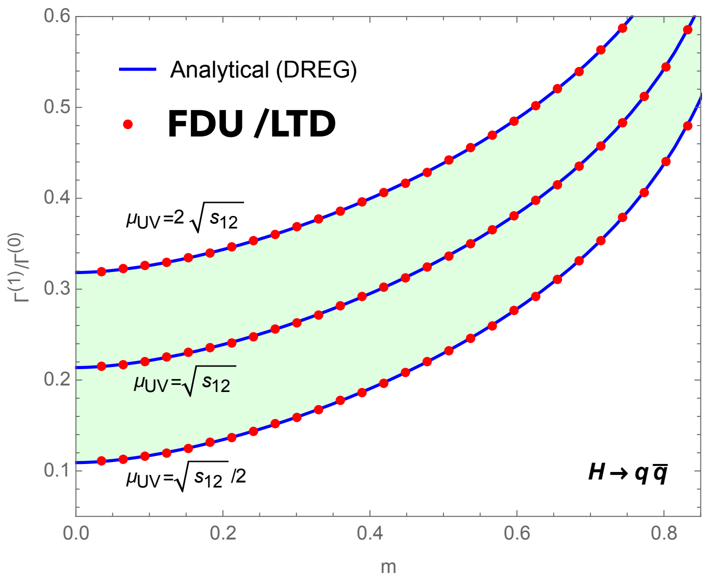
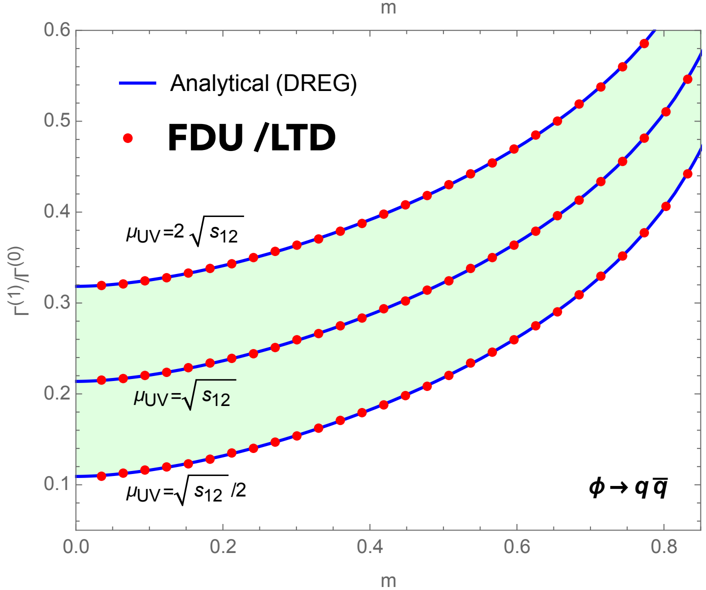
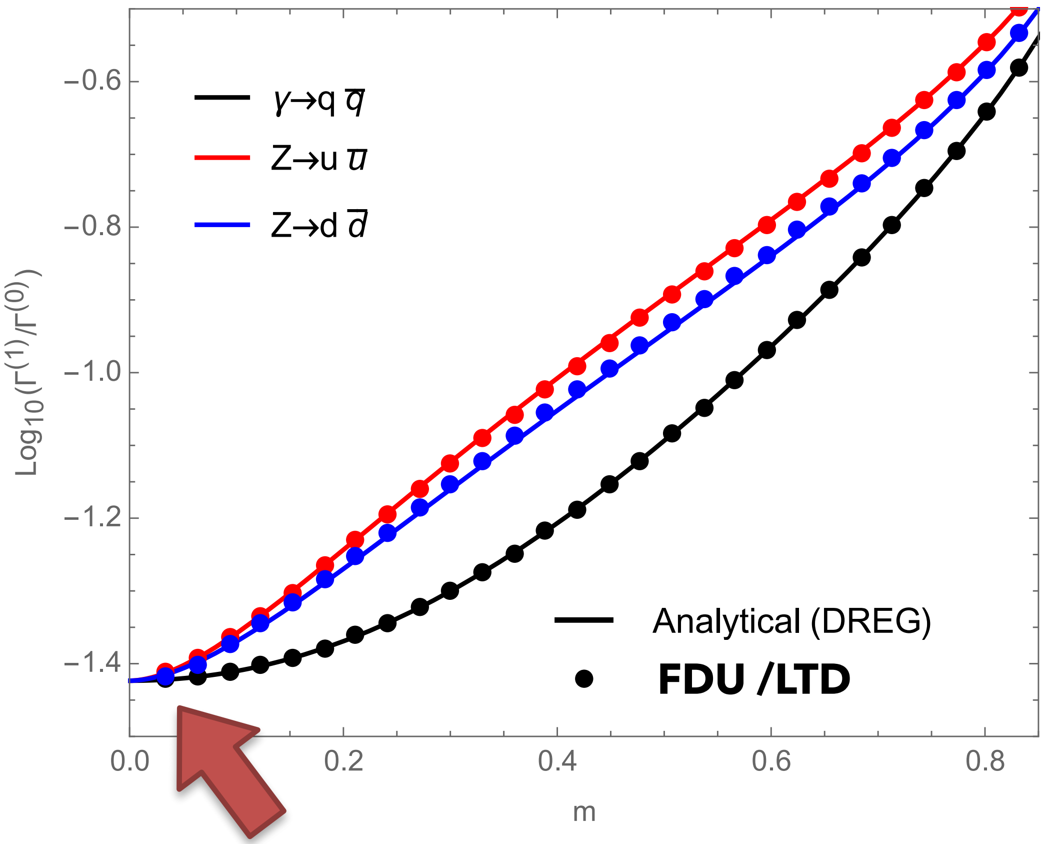
- Integration on the UV on-shell hyperboloid: loop three-momentum unconstrained, but **loop contributions suppressed** for loop energies larger than $\mu_{\rm UV}$

Benchmark application: $A^* \to q\bar{q}(g)$

Sborlini, Driencourt-Mangin, GR, JHEP **1610**, 162



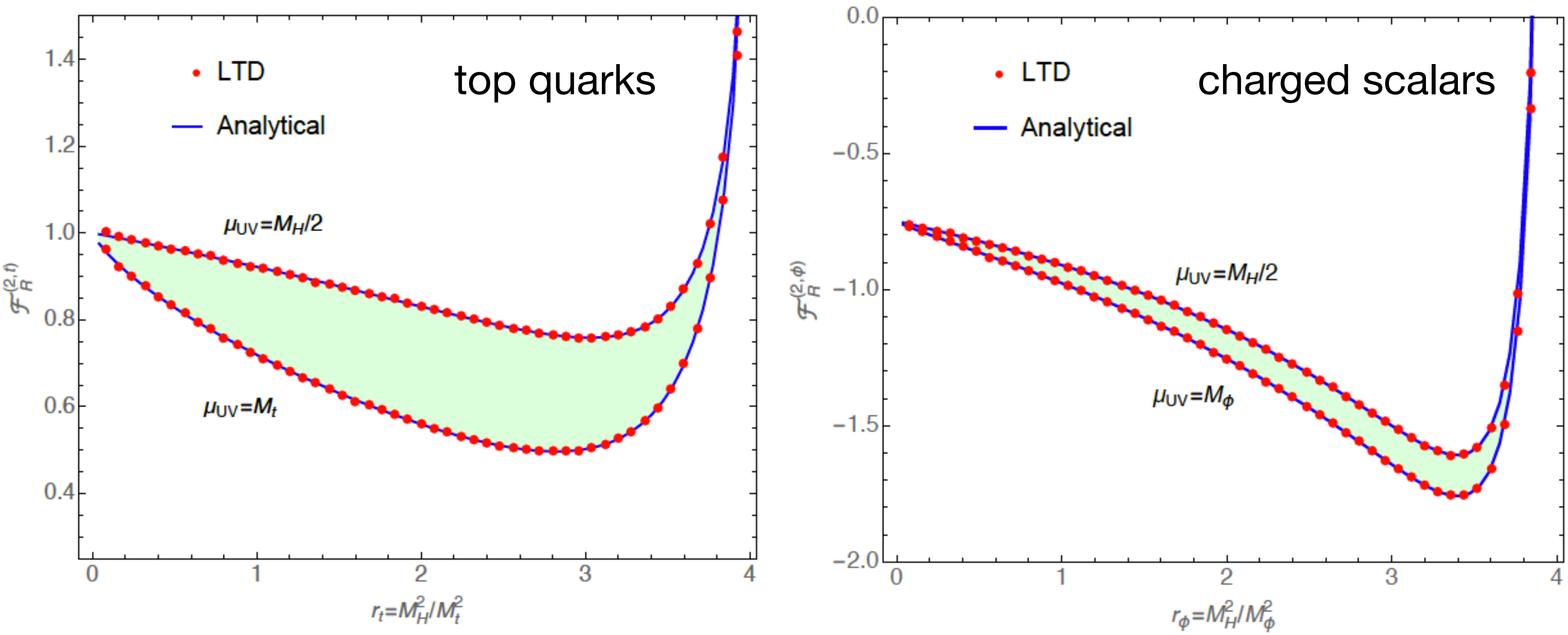




- Excellent agreement with analytic DREG
- Efficient numerical implementation
- **Smooth massless limit**

Driencourt-Mangin, Sborlini, GR, Torres Bobadilla, JHEP **1902,**143

DUAL AMPLITUDE FOR $H \to \gamma\gamma$ AT TWO-LOOPS



Analytic expressions from Aglietti, Bonciani, Degrassi, Vicini, JHEP **0701** (2007) 021

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