Luminosity with $e^+e^- \to \gamma\gamma$: theory perspective

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FCC Week 2019

Crowne Plaza, Brussels, June 24 - 28, 2019

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based on `arXiv:1906.08056`

Outline

- Introduction
- $e^+e^- \to \gamma\gamma$ at large angle at FCC-ee for luminometry
  - Why $e^+e^- \to \gamma\gamma$? Advantages and disadvantages (theoretical point of view)
- The **BabaYaga** event generator
  - Sketch of its theoretical formulation
- Phenomenology of QED/EWK radiative corrections (NLO and higher orders)
- Considerations about the achievable theoretical accuracy
- Conclusions & Outlook

Reference processes for luminosity

- Instead of getting the luminosity from machine parameters, it's more effective and precise to exploit the relation

$$\sigma = \frac{N}{L} \quad \to \quad L = \frac{N_{\text{ref}}}{\sigma_{\text{theory}}} \qquad \frac{\delta L}{L} = \frac{\delta N_{\text{ref}}}{N_{\text{ref}}} \oplus \frac{\delta \sigma_{\text{theory}}}{\sigma_{\text{theory}}}$$

- Reference *(normalization)* processes are required to have a clean topology, high statistics and be calculable with high theoretical accuracy

- QED processes are golden processes to push theo. accuracy at the [sub-]permill level

  ⋆ At LEP: small-angle $e^+e^- \to e^+e^-$ (Bhabha)
  (mainly t-channel γ exchange, tiny Z "contamination")

  ⋆ At flavour factories: large-angle QED processes
  Bhabha, $e^+e^- \to \gamma\gamma$, $e^+e^- \to \mu^+\mu^-$

- At FCC-ee, Bhabha will still be the reference process, but $e^+e^- \to \gamma\gamma$ is worth being studied

S. Jadach *et al.*, PLB 790 (2019) 314 and A. Blondel *et al.*, `arXiv:1809.01830 [hep-ph]`

↪ Inclusion of Radiative Corrections is mandatory (in particular QED RC)

↪ Fully-fledged Monte Carlo event generators needed

The event generator BabaYaga/BabaYaga@NLO

- ↦ Fully-exclusive generator developed for QED processes at flavour factories
- ↦ It simulates **Bhabha**, $e^+e^- \to \mu^+\mu^-$, $e^+e^- \to \gamma\gamma$ at large angles
- ↦ Theoretical accuracy at 0.1% (or slightly better) for integrated cross sections for luminosity monitoring
- ↦ Based on an *in-house* implementation of a **QED Parton-Shower**, ***consistently matched with exact QED NLO RCs***
  - ↪ An arbitrary number of (extra) photons can be generated
- ↦ The same QED PS & NLO matching framework successfully applied also to Drell-Yan processes (`HORACE`) and $H \to 4\ell$ (`Hto4l`)

  CMCC *et al.*, JHEP 0710 (2007) 109; CMCC *et al.*, JHEP 0612 (2006) 016; S. Boselli *et al.*, JHEP 1506 (2015) 023

- ⋆ One of the few generators to implement $e^+e^- \to \gamma\gamma$, with exact QED NLO & resummation (to the best of my knowledge)

  see also S. Eidelman *et al.*, EPJC **71** (2011) 1597 (`MCGPJ` generator)

Relevant references for BabaYaga

⋆ Webpage

http://www.pv.infn.it/hepcomplex/babayaga.html

or better ask the authors!

⋆ **BabaYaga** core references:

- Barzè *et al.*, Eur. Phys. J. C **71** (2011) 1680 — BabaYaga with a dark photon
- Balossini *et al.*, Phys. Lett. **663** (2008) 209 — BabaYaga@NLO for $e^+e^- \to \gamma\gamma$
- Balossini *et al.*, Nucl. Phys. **B758** (2006) 227 — BabaYaga@NLO for Bhabha
- CMCC *et al.*, Nucl. Phys. Proc. Suppl. **131** (2004) 48 — BabaYaga@NLO
- CMCC, Phys. Lett. B **520** (2001) 16 — improved PS BabaYaga
- CMCC *et al.*, Nucl. Phys. B **584** (2000) 459 — BabaYaga

⋆ Related work:

- S. Actis *et al.*
  "Quest for precision in hadronic cross sections at low energy: Monte Carlo tools vs. experimental data", Eur. Phys. J. C **66** (2010) 585
  Report of the Working Group on Radiative Corrections and Monte Carlo Generators for Low Energies
- CMCC *et al.*, JHEP **1107** (2011) 126
  NNLO massive pair corrections

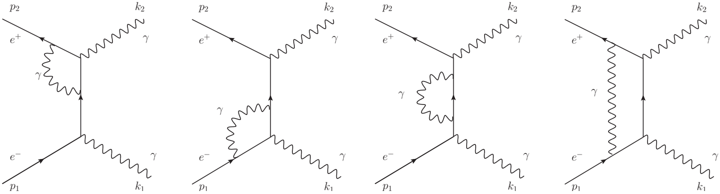
Why $e^+e^- \to \gamma\gamma$ at FCC-ee? Pros and cons

see also M. Dam's talk at FCC-ee week, Rome, April 2016
and at 10th FCC-ee physics workshop, CERN, February 2016
P. Janot's presentation at FCC-ee Joint Accelerator-Physics meeting, June 2015
my talks at 11th FCC-ee workshop: Theory and Experiment, CERN, January 2019
FCC (TLEP) workshop (TLEP9), Pisa, February 2015

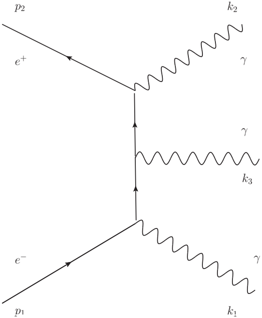
- ✓ at LO, purely QED process, ***at any energy***
- ✓ at NLO, weak corrections (loops with Z & $W^\pm$), but not fermionic loops yet (in particular, ***no hadronic loops***)
- ✓ hadronic vacuum polarization (**and its uncertainty**) enters only at NNLO (2-loops, order α^2)

- ✗ Large Bhabha background: at Z pole huge, much better at higher energies *[see later]*
- ✗ At NNLO less explored than Bhabha, but modern 2-loop techniques can be straightforwardly used
- ✗ Lack of independent MC codes for cross-checks/validation

NLO QED diagrams



$+\ \{k_1 \leftrightarrow k_2\}$
$+$ counterterms



$$+\ \wp\,[k_1, k_2, k_3]$$

virtual (+soft) RC from F.A. Berends & R. Kleiss, NPB 186 (1981) 22
real RC calculated with the help of Vermaseren's `FORM`

Simulation setup & cross sections

$\mapsto$ 4 "standard" cms energy points: $\sqrt{s} = 91,\ 160,\ 240,\ 365$ GeV

$\mapsto$ Only QED corrections (NLO & higher orders) *[weak & hadronic RC discussed later]*

- [1] Full phase space, i.e. no cuts
- [2] (Acceptance) cuts:

  at least two γ's with: $20^\circ < \theta_\gamma < 160^\circ \quad \wedge \quad E_\gamma \geq 0.25 \times \sqrt{s}$

- [1] without cuts

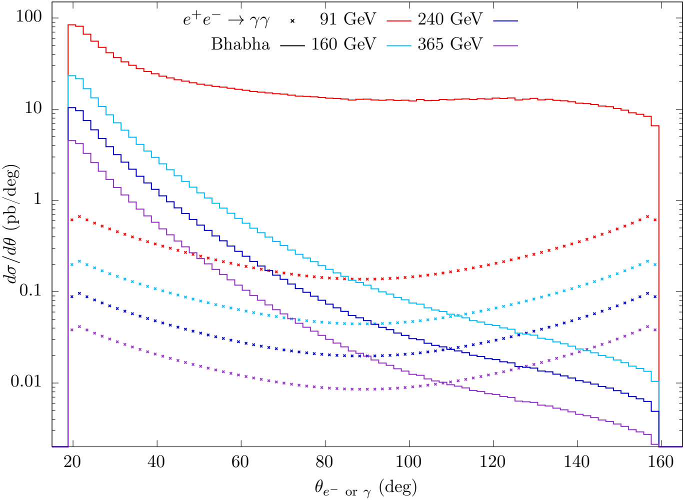
$\sqrt{s}$ (GeV)	LO (pb)	NLO (pb)	w h.o. (pb)	stat. acc.*
91	364.68	447.27 [+23%]	445.6(9) [−0.46%]	$3.9 \cdot 10^{-6}$
160	123.71	154.37 [+25%]	153.2(2) [−0.95%]	$2.3 \cdot 10^{-5}$
240	56.816	71.809 [+26%]	71.07(6) [−1.30%]	$5.3 \cdot 10^{-5}$
365	25.385	32.515 [+28%]	32.09(2) [−1.67%]	$1.4 \cdot 10^{-4}$

- [2] with cuts

$\sqrt{s}$ (GeV)	LO (pb)	NLO (pb)	w h.o. (pb)	Bhabha LO (pb)
91	39.821	41.043 [+3.07%]	40.870(4) [−0.43%]	2625.9 [$66 \times \sigma^{\gamma\gamma}$]
160	12.881	13.291 [+3.18%]	13.228(1) [−0.49%]	259.98 [$20 \times \sigma^{\gamma\gamma}$]
240	5.7250	5.9120 [+3.26%]	5.8812(6) [−0.54%]	115.77 [$20 \times \sigma^{\gamma\gamma}$]
365	2.4752	2.5581 [+3.35%]	2.5438(3) [−0.58%]	50.373 [$20 \times \sigma^{\gamma\gamma}$]

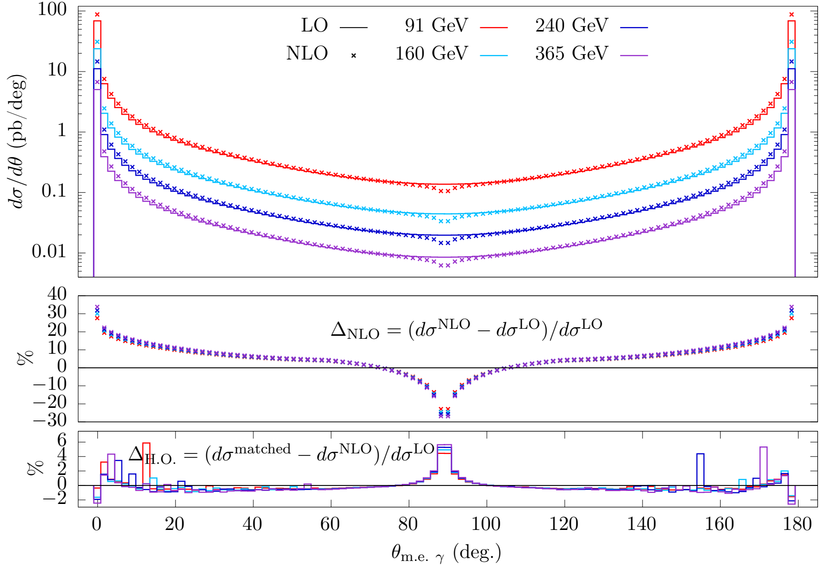
*Assuming integrated luminosities as in Tab. 1 of A. Blondel *et al.*, arXiv:1809.01830 [hep-ph]

$e^+e^- \to \gamma\gamma$ vs Bhabha (at LO, with acceptance cuts)

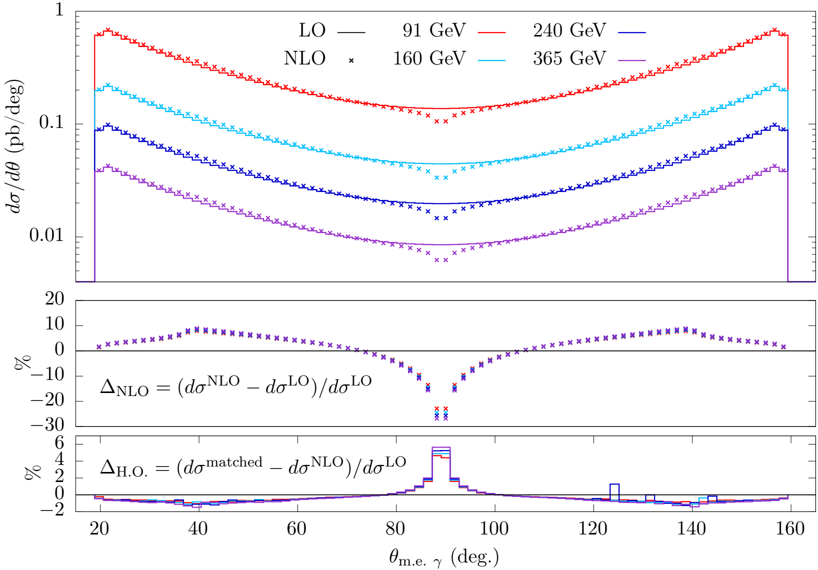


- in the backward region $d\sigma_{e^+e^-\to\gamma\gamma}/d\theta_\gamma \geq d\sigma_{\text{Bhabha}}/d\theta_{e^-}$ (except at 91 GeV)

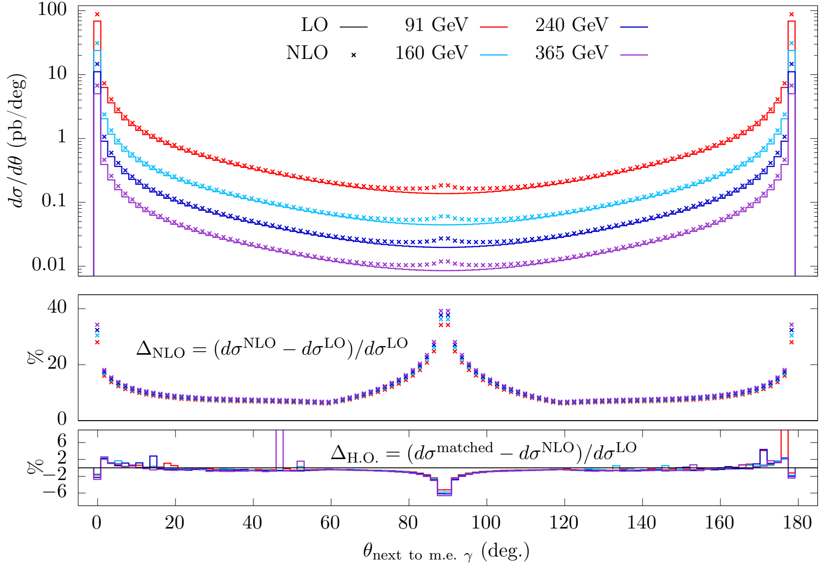
Most energetic γ angle (without cuts)



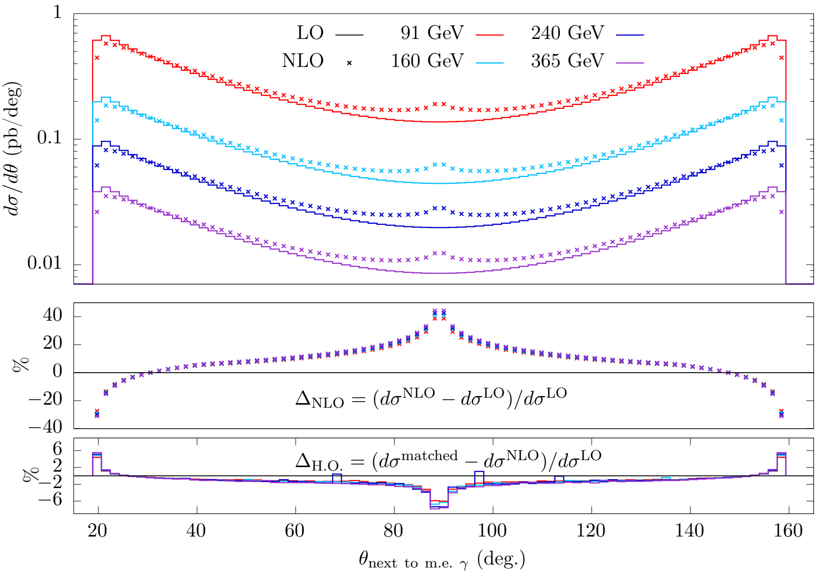
Most energetic γ angle (with acceptance cuts)



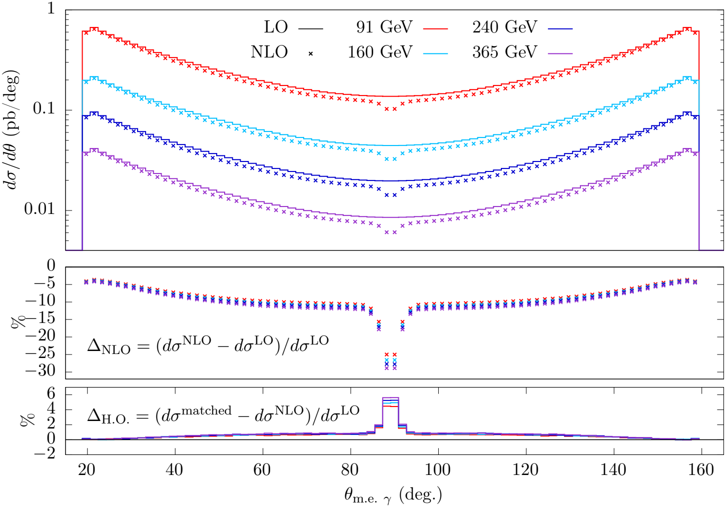
Next to most energetic γ angle (without cuts)



Next to most energetic γ angle (with acceptance cuts)

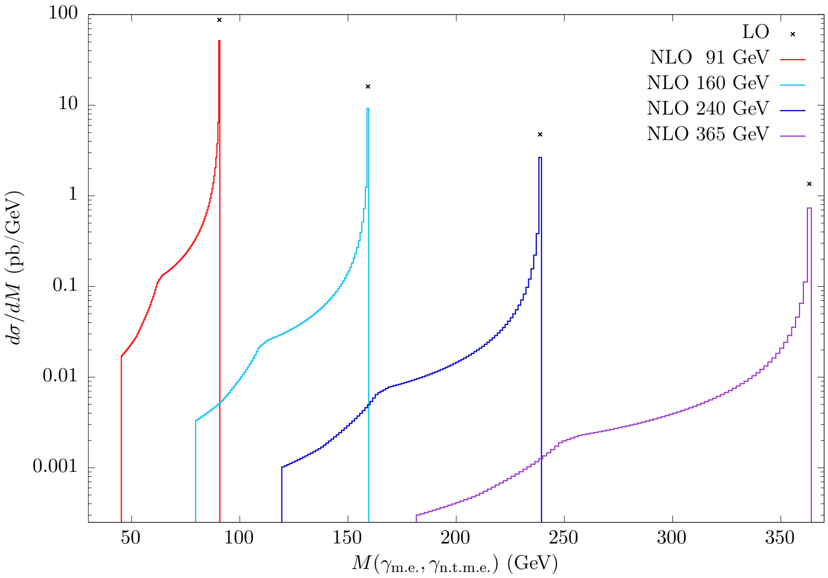


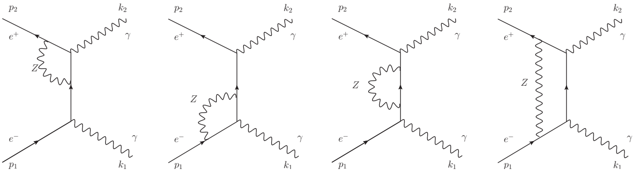
Most energetic γ angle (with acceptance cuts + **acollinearity cut**)

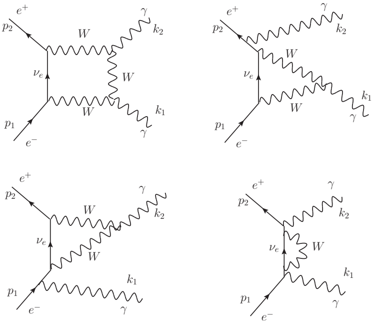


- Including an elasticity cut (acollinearity $\leq 10^\circ$) changes the shape of the RCs

Invariant mass distribution at NLO (with cuts)







$+\ \{k_1 \leftrightarrow k_2\}$

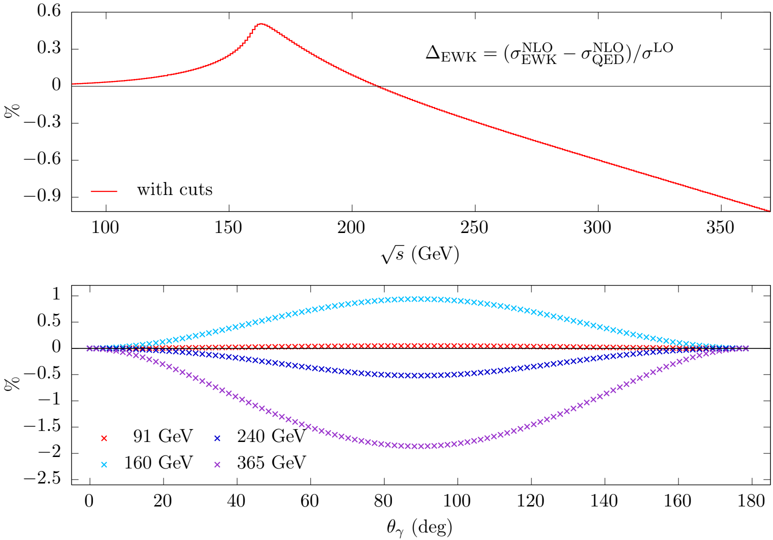
+ counterterms

$\mapsto$ Calculated in the on-shell (complex mass) scheme with the help of `Recola-1.4.0`

S. Actis *et al.*, JHEP 04:037, 2013

S. Actis *et al.*, CPC 214:140–173, 2017

Pure weak corrections



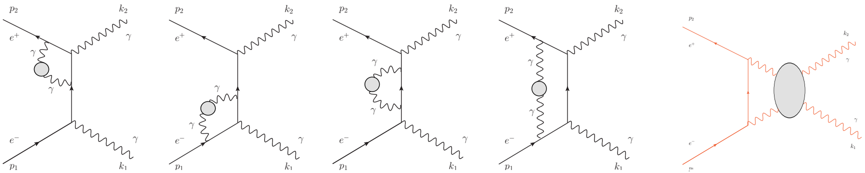
- at higher energies, weak RCs get of the same order of QED h.o.

Rough estimate of (NNLO) VP hadronic corrections (and uncertainties)

- for Bhabha, $\Delta\alpha_{had}$ uncertainty affects *[today]* the theoretical accuracy at $\mathcal{O}\left(10^{-4}\right)$, entering at NLO

  see Tables 2 & 3 of S. Jadach *et al.*, PLB 790 (2019) 314 and A. Blondel *et al.*, `arXiv:1809.01830 [hep-ph]`

- for $e^+e^- \to \gamma\gamma$ it enters only at NNLO ***[and also light-by-light graphs contribute!]***



$$\sigma^{\rm NNLO}_{\Delta\alpha_{had}} \pm \delta\sigma \overset{\text{very naive!}}{\approx} \left(\sigma^{\rm NLO}_{\rm QED} - \sigma^{\rm LO}\right) \times \left[\Delta\alpha_{had}(s) \pm \delta\Delta\alpha_{had}\right]$$

$\sqrt{s}$ (GeV)	$\Delta\alpha_{had}(s)^\dagger$	$\delta\sigma/\sigma_{LO}$ [1]	$\delta\sigma/\sigma_{LO}$ [2]
91	$(276.7 \pm 1.2)\cdot 10^{-4}$	$2.8\cdot 10^{-5}$	$3.7\cdot 10^{-6}$
160	$(309.1 \pm 1.2)\cdot 10^{-4}$	$3.0\cdot 10^{-5}$	$3.8\cdot 10^{-6}$
240	$(333.2 \pm 1.2)\cdot 10^{-4}$	$3.1\cdot 10^{-5}$	$3.9\cdot 10^{-6}$
365	$(358.5 \pm 1.2)\cdot 10^{-4}$	$3.4\cdot 10^{-5}$	$4.0\cdot 10^{-6}$

†from F. Jegerlehner's recent `hadr5n16.f`

Conclusions & Outlook

- $\mapsto$ The process $e^+e^- \to \gamma\gamma$ at large-angle is worth being studied as monitor for luminosity at FCC-ee
- $\mapsto$ On the theory side:
  - $\rightsquigarrow$ NLO QED RCs affect differential cross sections at the 10 - 20% level
  - $\rightsquigarrow$ QED higher-order RCs lie in the $\%$ range
  - $\rightsquigarrow$ EWK RCs grow with energy, and lie in the $\%$ range
  - $\rightsquigarrow$ Hadronic VP enters only at NNLO,
    its uncertainty is (likely) negligible [$\sim \mathcal{O}(10^{-6})$]
- $\mapsto$ As of today, the theoretical error on $e^+e^- \to \gamma\gamma$ is of $\mathcal{O}(0.1\%)$
  (if supplemented with NLO EWK RCs for FCC-ee energies)
- $\mapsto$ **My educated guess:**
  with complete NNLO at hand, matched with QED h.o. resummation, $e^+e^- \to \gamma\gamma$ theoretical accuracy can be controlled at the 10^{-4} level.

  Perhaps better than small-angle Bhabha?

SPARES

Theory of QED corrections into Monte Carlo generators

- ⋆ The most precise MC generators include exact $\mathcal{O}(\alpha)$ (NLO) photonic corrections matched with higher-order leading logarithmic contributions [multiple photon corrections]

  [+ vacuum polarization, using a data driven routine for the calculation of the non-perturbative $\Delta\alpha^{(5)}_{\rm had}(q^2)$ hadronic contribution]

- ⋆ Common methods used to account for multiple photon corrections are the analytical collinear QED Structure Functions (SF), YFS exponentiation and QED Parton Shower (PS)

- The QED PS [implemented in BabaYaga/BabaYaga@NLO] is an exact MC solution of the QED DGLAP equation for the non-singlet electron SF $D(x,Q^2)$

$$Q^2\frac{\partial}{\partial Q^2}D(x,Q^2) = \frac{\alpha}{2\pi}\int_x^1 \frac{dt}{t}P_+(t)D(\frac{x}{t},Q^2)$$

- The PS solution can be cast into the form

$$D(x,Q^2) \ = \ \Pi(Q^2)\sum_{n=0}^{\infty} \ \int \frac{\delta(x-x_1\cdots x_n)}{n!} \ \prod_{i=0}^{n}\left[\frac{\alpha}{2\pi}P(x_i) \ L \ dx_i\right]$$

  → $\Pi(Q^2) \equiv e^{-\frac{\alpha}{2\pi}LI_+}$ Sudakov form factor, $I_+ \equiv \int_0^{1-\epsilon}P(x)dx$, $L \equiv \ln Q^2/m^2$ collinear log, ϵ soft–hard separator and Q^2 virtuality scale

  → the kinematics of the photon emissions can be recovered → exclusive photons generation

- The accuracy is improved by matching exact NLO with higher-order leading log corrections
  - ⋆ theoretical error starts at $\mathcal{O}(\alpha^2)$ (NNLO) QED corrections, for all QED channels [Bhabha, $\gamma\gamma$ and $\mu^+\mu^-$]

Exact $\mathcal{O}(\alpha)$ (NLO) soft+virtual (SV) corrections and hard-bremsstrahlung (H) matrix elements can be combined with QED PS *via* a matching procedure

- $d\sigma^{\infty}_{PS} = \Pi(Q^2,\varepsilon)\ \sum_{n=0}^{\infty} \frac{1}{n!}\ |\mathcal{M}_{n,PS}|^2\ d\Phi_n$
- $d\sigma^{\alpha}_{PS} = [1 + C_{\alpha,PS}]\,|\mathcal{M}_0|^2 d\Phi_2 + |\mathcal{M}_{1,PS}|^2 d\Phi_3 \equiv d\sigma^{SV}_{PS}(\varepsilon) + d\sigma^{H}_{PS}(\varepsilon)$
- $d\sigma^{\alpha}_{\rm NLO} = [1 + C_{\alpha}]\,|\mathcal{M}_0|^2 d\Phi_2 + |\mathcal{M}_1|^2 d\Phi_3 \equiv d\sigma^{SV}_{\rm NLO}(\varepsilon) + d\sigma^{H}_{\rm NLO}(\varepsilon)$
- $F_{SV} = 1 + (C_\alpha - C_{\alpha,PS}) \qquad F_H = 1 + \frac{|\mathcal{M}_1|^2 - |\mathcal{M}_{1,PS}|^2}{|\mathcal{M}_{1,PS}|^2}$

$$d\sigma^{\infty}_{\rm matched} = F_{SV}\ \Pi(Q^2,\varepsilon)\ \sum_{n=0}^{\infty} \frac{1}{n!}\ \left(\prod_{i=0}^{n} F_{H,i}\right)\ |\mathcal{M}_{n,PS}|^2\ d\Phi_n$$

$d\Phi_n$ is the exact phase space for n final-state particles
(2 fermions + an arbitrary number of photons)

Matching NLO and PS in BabaYaga@NLO

- F_{SV} and $F_{H,i}$ are infrared/collinear safe and account for missing $\mathcal{O}(\alpha)$ non-logs, avoiding double counting of leading-logs
- $\left[\sigma^{\infty}_{matched}\right]_{\mathcal{O}(\alpha)} = \sigma^{\alpha}_{\rm NLO}$
- resummation of higher orders LL (PS) contributions is preserved
- the cross section is still fully differential in the momenta of the final state particles (e^+, e^- and $n\gamma$)
  (F's correction factors are applied on an event-by-event basis)
- as a by-product, part of photonic $\alpha^2 L$ included by means of terms of the type $F_{SV\ |\ H,i} \otimes$ [leading-logs]

G. Montagna et al., **PLB** 385 (1996)

- the theoretical error is shifted to $\mathcal{O}(\alpha^2)$ (NNLO, 2 loop) not infrared, singly collinear terms: very naively and roughly (for photonic corrections)

$$\frac{1}{2}\alpha^2 L \equiv \frac{1}{2}\alpha^2 \log\frac{s}{m_e^2} \sim 5\times 10^{-4}$$

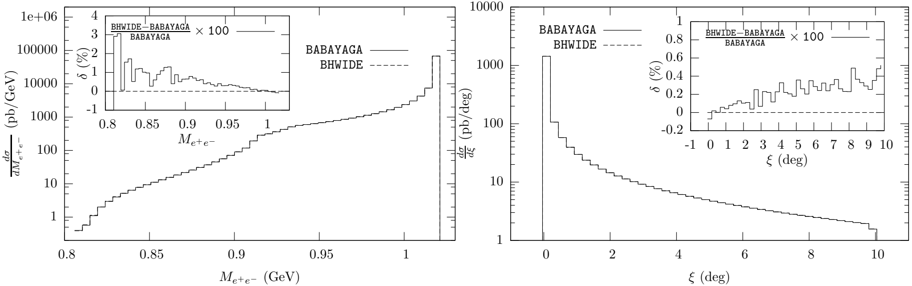
Estimating the theoretical accuracy

S. Actis et al. Eur. Phys. J. C **66** (2010) 585

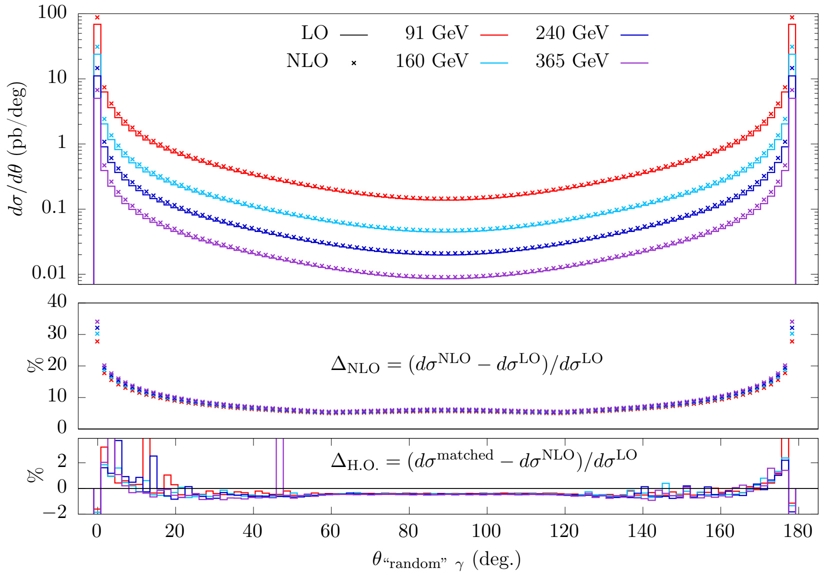
"Quest for precision in hadronic cross sections at low energy: Monte Carlo tools vs. experimental data"

- It is extremely important to compare independent calculations/implementations/codes, in order to
  - $\mapsto$ asses the technical precision, spot bugs (with the same th. ingredients)
  - $\mapsto$ estimate the theoretical error when including partial/incomplete higher-order corrections
- E.g. comparison **`BabaYaga@NLO`** vs. **`Bhwide`** at KLOE

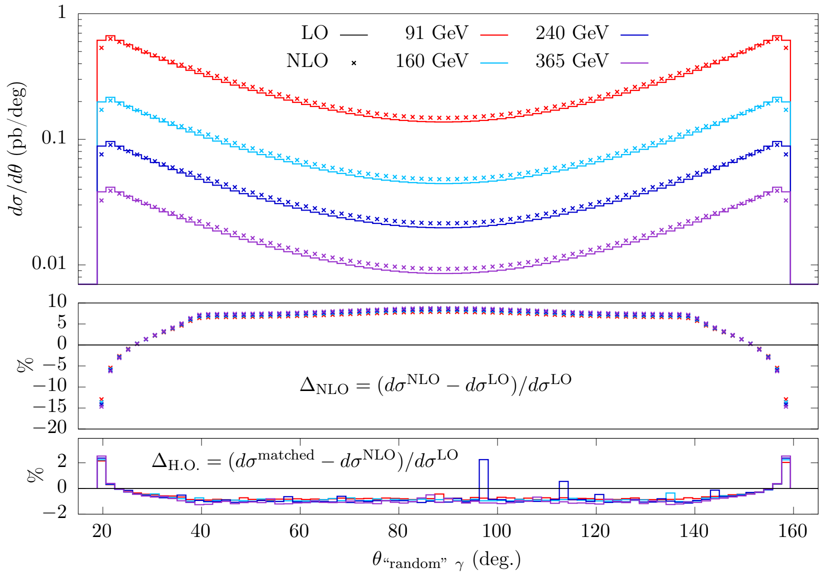
S. Jadach *et al.* PLB 390 (1997) 298



"Randomized" γ angle (without cuts)



“Randomized” γ angle (with acceptance cuts)



Acollinearity distribution at NLO