

# FCC-hh: Longitudinal beam dynamics and RF requirements

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Acknowledgements:

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# Update since 2018

- RF and beam parameters in CDR are mainly defined by single-bunch stability in longitudinal plane (loss of Landau damping)

Since last FCC meeting, we evaluated:

- Longitudinal coupled-bunch stability and requirements for high order mode (HOM) damping (talk today in the SRF session)
- Transient beam loading and RF power consumption

→ Self-consistent consideration of these effects is needed

# Loss of Landau damping (LLD)

Criterion derived in short bunch approximation (F. Sacherer, 1973)

$$\begin{array}{c}
 \text{Form factor} \swarrow \\
 \text{Effective impedance for dipole mode} \quad \left| \text{Im} Z_{\parallel} / n \right| < \frac{\text{RF voltage} \downarrow \quad F_{\text{LLD}} V_{\text{RF}} \cos \phi_{s0} \omega_{\text{RF}}^2 \frac{\Delta \omega_s}{\omega_{s0}} \tau_{4\sigma}^3}{\text{Single bunch intensity} \uparrow \quad e N_p 8 \pi h \quad \text{Harmonic number} \swarrow}
 \end{array}$$

$n = \omega / \omega_0$  frequency normalized by revolution frequency

$$\begin{array}{c}
 \text{Synchrotron frequency spread} \quad \frac{\Delta \omega_s}{\omega_{s0}} = \frac{\omega_{\text{RF}}^2}{64} \left( 1 + \frac{5}{3} \tan^2 \phi_{s0} \right) \tau_{4\sigma}^2 \\
 \text{Phase of synchronous particle} \swarrow
 \end{array}$$

$$\begin{array}{c}
 \text{Equivalent bunch length} \quad \tau_{4\sigma} = \frac{2 \tau_{\text{FWHM}}}{\sqrt{2 \log 2}} \quad \text{Measured quantity} \leftarrow
 \end{array}$$

→ What are  $\left| \text{Im} Z_{\parallel} / n \right|$  and  $F_{\text{LLD}}$ ?

# Effective impedance

Effective impedance for dipole mode

$$\text{Im}Z_{\parallel}/n = \omega_0 \sum_{p=-\infty}^{\infty} \frac{\text{Im}Z_{\parallel}(\omega_p)}{\omega_p} H(\omega_p)$$

$\omega_p = p\omega_0 + \omega_{s0}$

Normalized spectral density of dipole mode

$|\text{Im}Z_{\parallel}/n| \approx 0.03 \, \Omega$  for present FCC-hh impedance model, but calculations for many elements are still missing (see talk of S. Arsenyev today)

Assuming the same  $\text{Im}Z_{\parallel}$  per unit length in LHC and FCC-hh

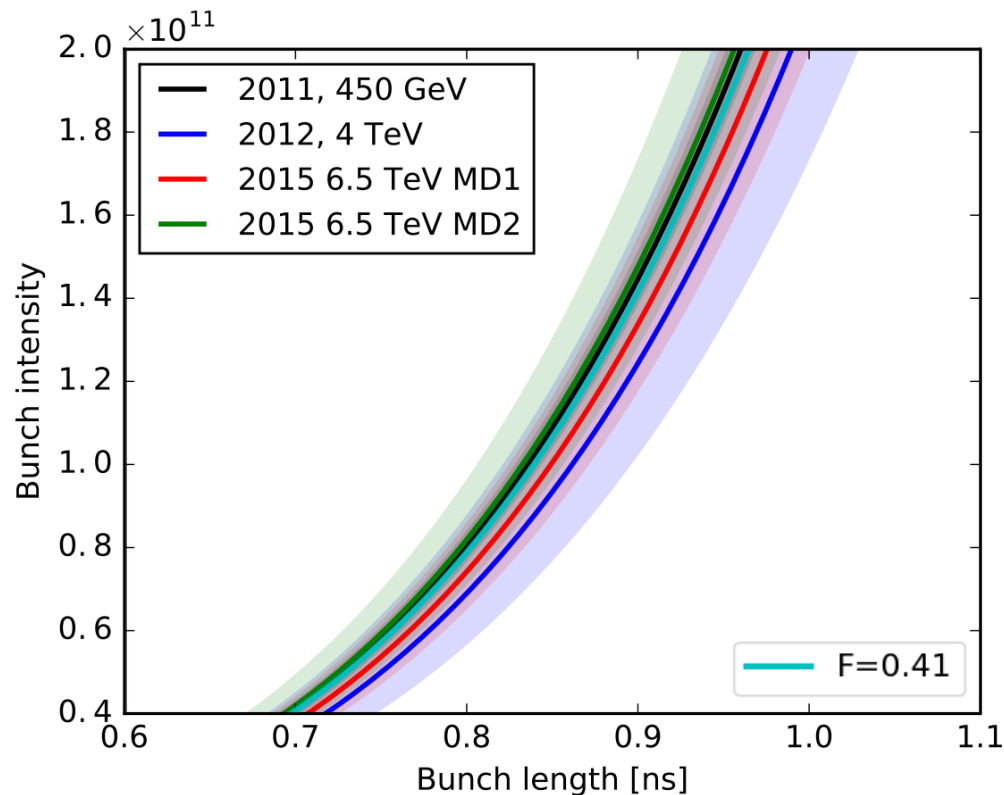
$$\text{Im}Z_{\parallel\text{FCC}} = \frac{C_{\text{FCC}}}{C_{\text{LHC}}} \text{Im}Z_{\parallel\text{LHC}} \quad \Rightarrow \quad \frac{\text{Im}Z_{\parallel\text{FCC}}}{\omega/\omega_{0\text{FCC}}} = \frac{C_{\text{FCC}}}{C_{\text{LHC}}} \frac{\omega_{0\text{FCC}}}{\omega_{0\text{LHC}}} \frac{\text{Im}Z_{\parallel\text{LHC}}}{\omega/\omega_{0\text{LHC}}} = \frac{\text{Im}Z_{\parallel\text{LHC}}}{\omega/\omega_{0\text{LHC}}}$$

In the LHC measured and calculated  $|\text{Im}Z_{\parallel}/n| \approx 0.09 \, \Omega$

→ We use  $|\text{Im}Z_{\parallel}/n| = 0.2 \, \Omega$  as longitudinal impedance budget of FCC-hh (with margin of factor 2)

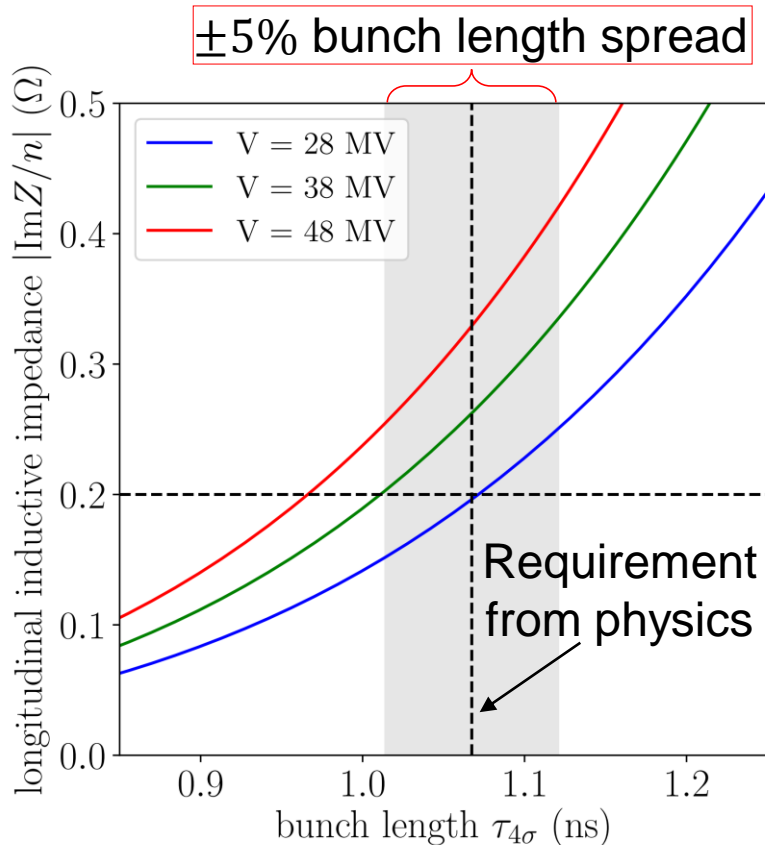
# Form factor $F_{LLD}$

Measured LLD threshold in LHC (J.F. Esteban Müller, PhD thesis, 2016)

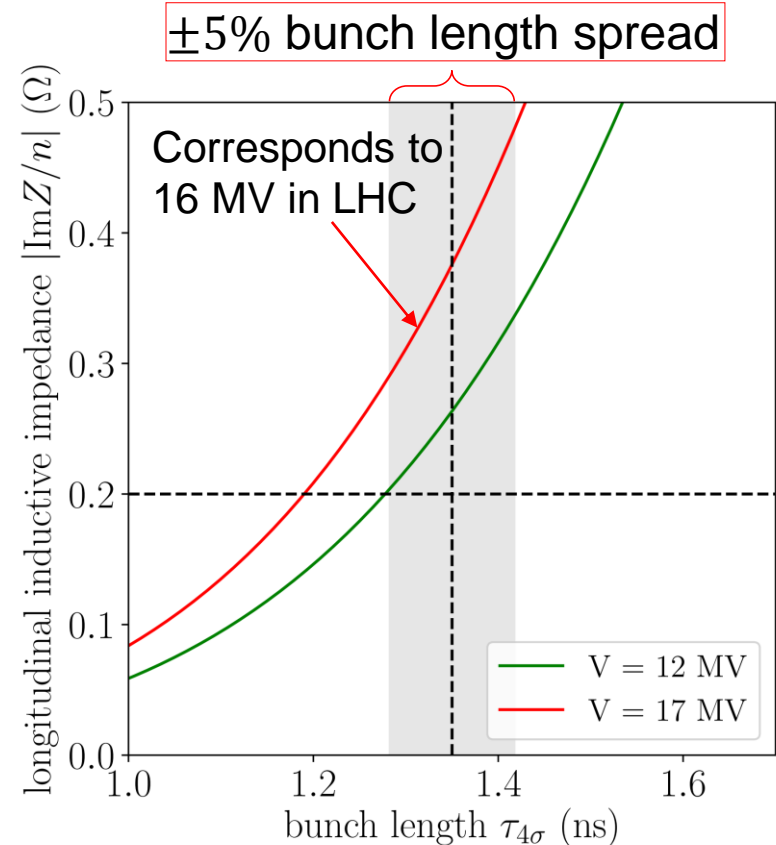


→ Fitting with analytic formula gives  $F_{LLD} \approx 0.41$

# RF voltage at 50 TeV and 3.3 TeV



Thresholds at 50 TeV



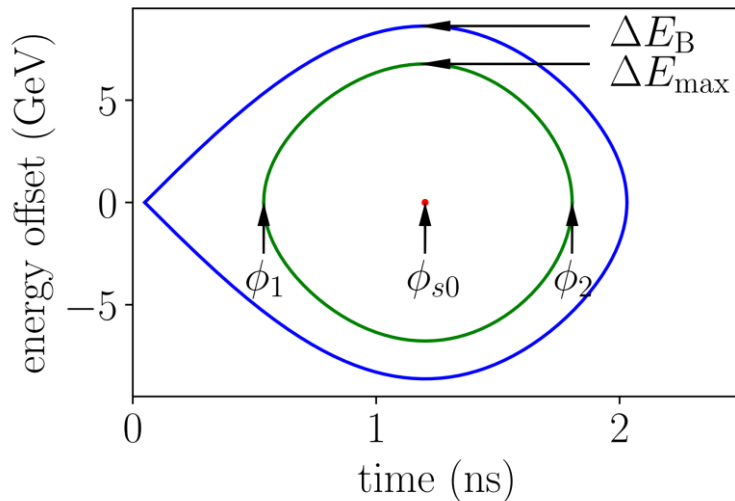
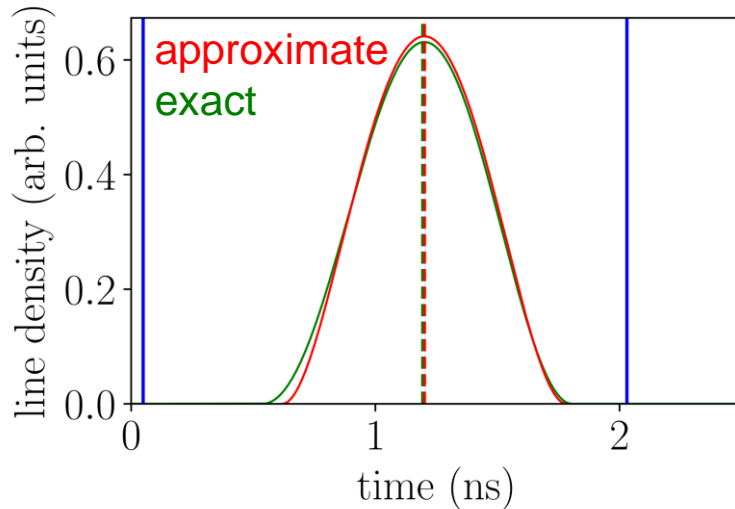
Thresholds at 3.3 TeV

→ RF voltage of 38 MV provides stability for average  $\tau_{4\sigma} = 1.07$  ns with  $\pm 5\%$  bunch length spread at 50 TeV

→ For  $V_{\text{RF}} = 12$  MV at 3.3 TeV, 1.35 ns bunch length is required for stability

# Bunch parameters

Bunch profile at 50 TeV



- Synchrotron radiation leads to bunch asymmetry (phase of bunch center of mass  $\phi_s \neq \phi_{s0}$  phase of synchronous particle)
- RF component of the beam current is affected

We assume binomial particle distribution (as a function of Hamiltonian  $H$ )

$$\mathcal{F}(H) = \mathcal{F}_0 (H_{\max} - H)^\mu$$

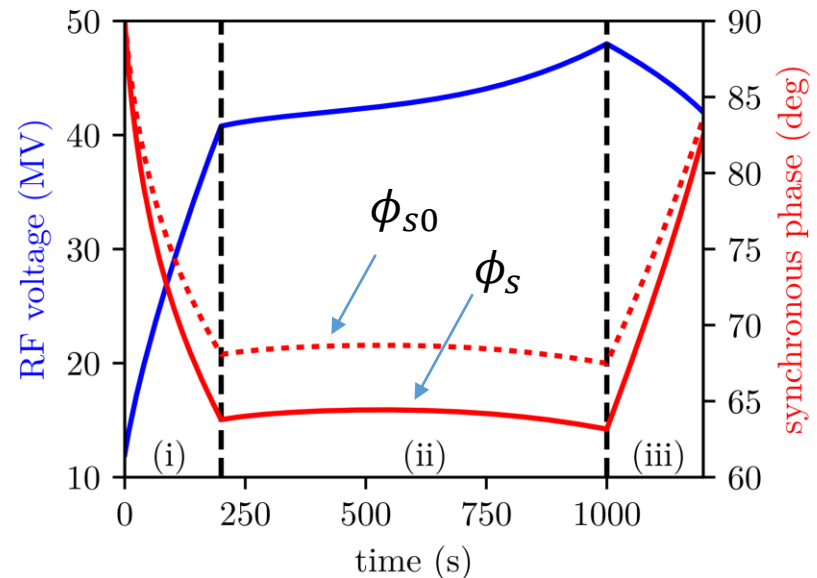
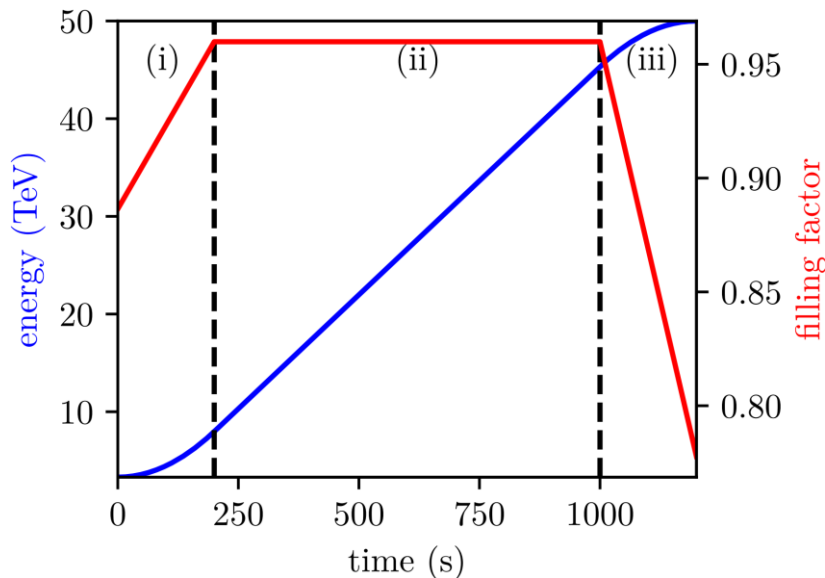
with  $\mu = 1.5$  ( $\mu = 2$  at LHC flattop and flat bottom)

Energy, $E$	TeV	3.3	50
RF voltage, $V_{\text{RF}}$	MV	12	38
Bunch length, $\tau_{4\sigma}$	ns	1.35	1.07
Full emittance, $\epsilon$	eVs	3.12	13.7
Filling factor, $q_p = \frac{\Delta E_{\max}}{\Delta E_B}$		0.9	0.8

# RF voltage program during ramp

Conditions:

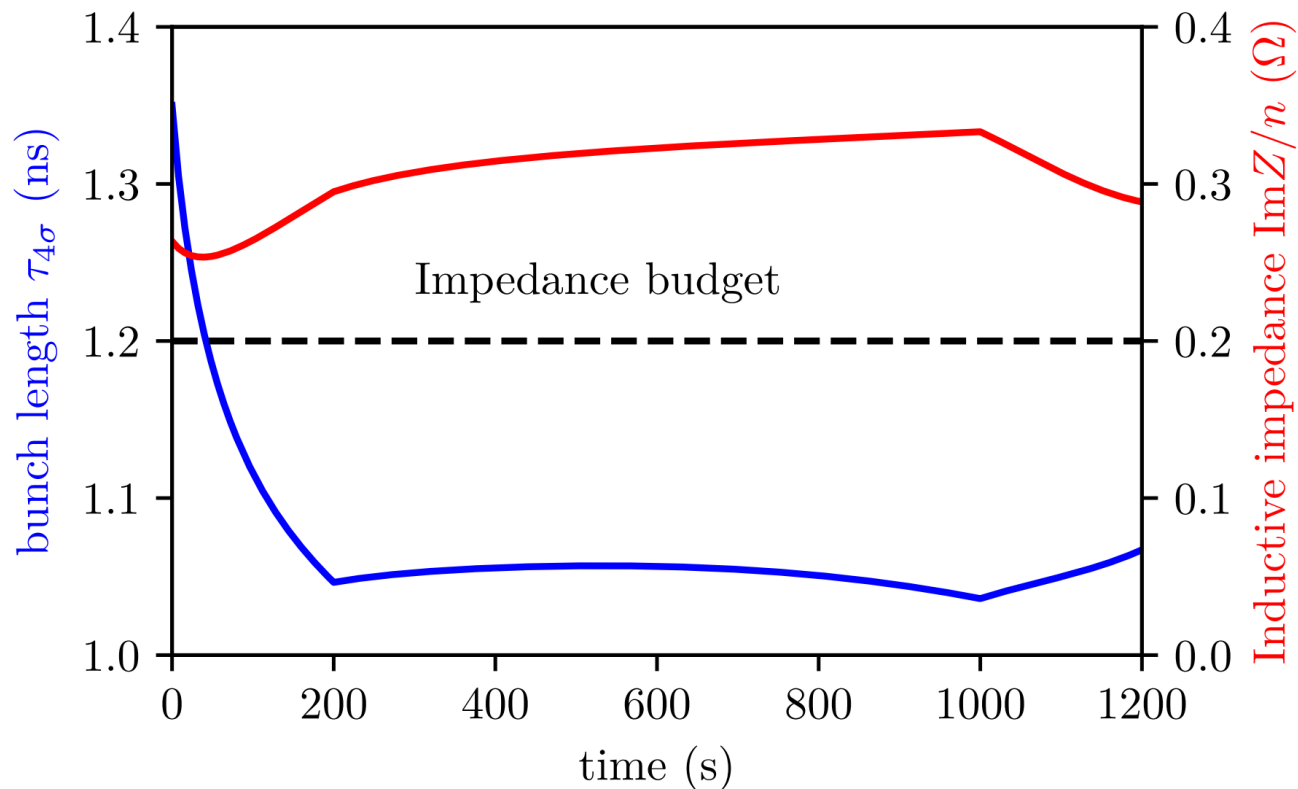
- maximum  $q_p = 0.94$  to avoid losses
- controlled emittance blow-up  $\epsilon \propto \sqrt{E}$  (previously optimized for TMCI)



→ Bunch stable phase differs significantly from the phase of the synchronous particle

→ FWHM bunch length calculated from the line density is used for stability evaluation during the ramp

# LLD threshold during ramp



→ Sufficient longitudinal single-bunch stability during the cycle

# Beam loading and its compensation

Interaction of the beam with the fundamental cavity impedance results in modulation bunch-by-bunch and cavity parameters

Without transient beam loading compensation the minimum power is

$$P_{\text{opt}} = \frac{V_{\text{cav}} I_{\text{b,DC}} |F_{\text{b}}| \sin \phi_s}{2}$$

↙  $V_{\text{RF}}/N_{\text{cav}}, N_{\text{cav}} = 24$

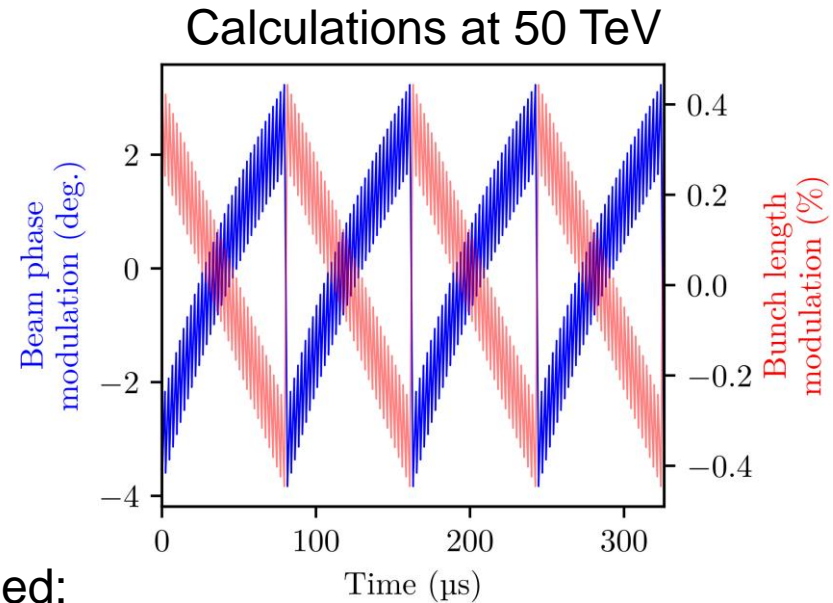
It is achieved if the following parameters are used:

Optimal cavity detuning  $\Delta\omega_{\text{opt}} = \omega_{\text{r}} - \omega_{\text{RF}} = \omega_{\text{RF}} \frac{(R/Q) I_{\text{b,DC}} |F_{\text{b}}| \cos \phi_s}{2V_{\text{cav}}}$

Optimal quality factor  $Q_{\text{L,opt}} = \frac{V_{\text{cav}}}{(R/Q) I_{\text{b,DC}} |F_{\text{b}}| \sin \phi_s}$

Form factor form line density  $F_{\text{b}} = 2 \frac{\mathcal{F}[\lambda(t)]_{\omega=\omega_{\text{RF}}}}{\mathcal{F}[\lambda(t)]_{\omega=0}} = |F_{\text{b}}| e^{-i\phi_s}$

→ Evaluation of power consumption for partial and full transient beam loading compensation schemes is need



# RF power requirements

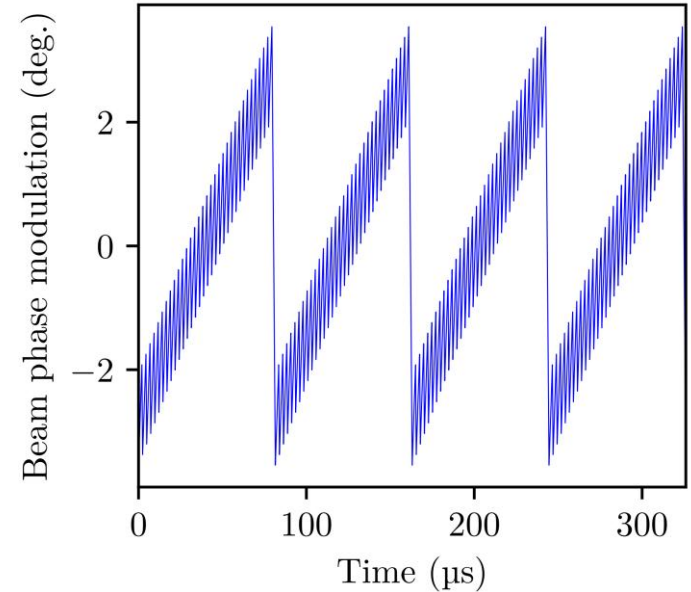
## Partial beam loading compensation (Full-detuning scheme)

$V(t) = V_{\text{cav}} e^{i\phi(t)} \rightarrow$  no bunch length modulation

$$P_{\text{FD}} = \frac{V_{\text{cav}} I_{\text{b,peak}} |F_{\text{b}}| \sin \phi_s}{2}$$

$I_{\text{b,peak}} = eN_{\text{p}}/t_{\text{bb}}$ ,  $t_{\text{bb}}$  - bunch spacing

$$\Delta\omega_{\text{FD}} = \Delta\omega_{\text{opt}} \quad Q_{\text{L,FD}} = \frac{V_{\text{cav}}}{(R/Q) I_{\text{b,peak}} |F_{\text{b}}| \sin \phi_s}$$



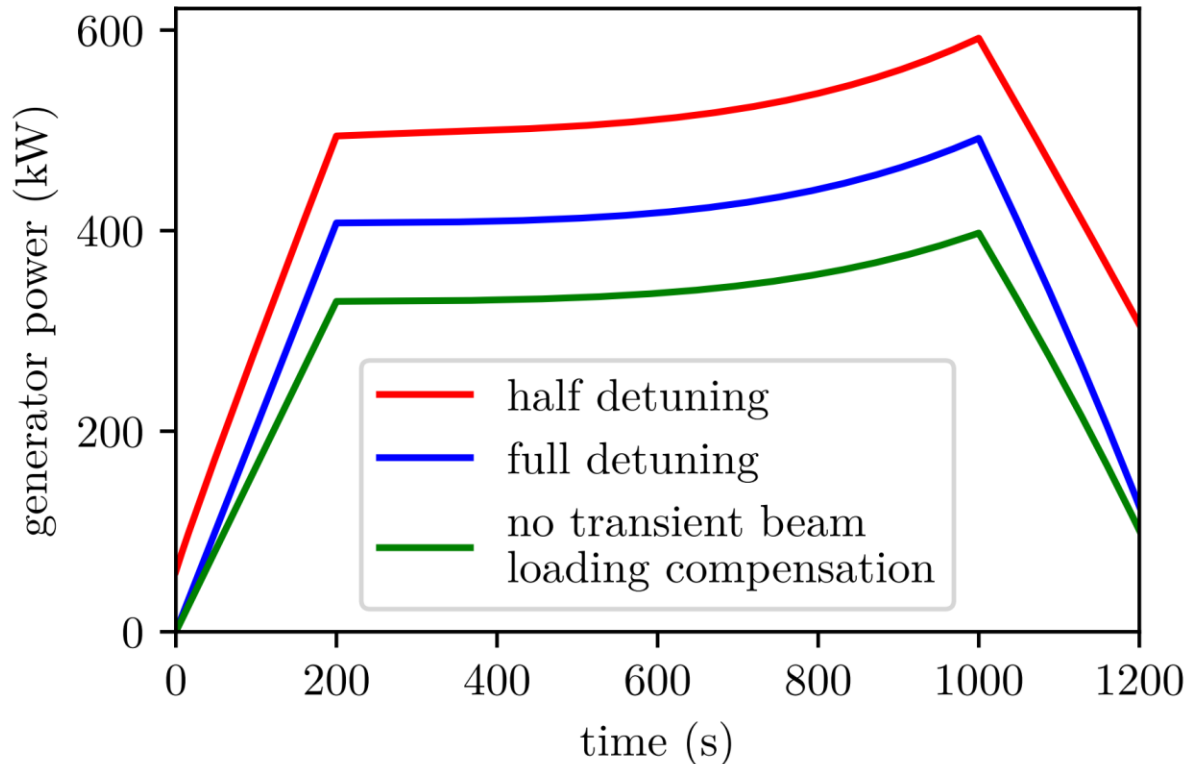
## Full transient beam loading compensation (Half-detuning detuning scheme)

Cavity voltage amplitude and phase are constant  
 $\rightarrow$  no modulation of beam parameters

Required power  $P_{\text{HD}}(\phi_s) = \frac{V_{\text{cav}} I_{\text{b,peak}} |F_{\text{b}}|}{8} \frac{1}{|\cos \phi_s + \sin \phi_s|}$

$$\Delta\omega_{\text{HD}} = \omega_{\text{rf}} \frac{\hat{I}_{\text{b,rf}}(R/Q)}{4V_{\text{cav}}} \frac{1}{\cos \phi_s + \sin \phi_s} \quad Q_{\text{L,HD}} = \frac{\omega_{\text{r}}}{2\Delta\omega_{\text{HD}}}$$

# Power consumption during cycle\*



→ Keeping constant amplitude and phase of the cavity voltage during the FCC-hh cycle would require about 600 kW peak power (half-detuning scheme).

→ The full-detuning scheme, requires about 25% more power compared with the case without transient beam loading compensation.

# Summary

- Evaluation of RF power, single-bunch stability and coupled-bunch stability resulted in update of beam and RF parameters:
  - at flattop  $V_{\text{RF}} = 38 \text{ MV}$ ,
  - at flat bottom  $\tau_{4\sigma} = 1.35 \text{ ns}$ ,
  - emittance blow-up during acceleration  $\epsilon \propto \sqrt{E}$
- RF power consumption was calculated for different transient beam loading compensation schemes
  - Full compensation requires 600 kW peak power during acceleration (400 kW without compensation)

**Thank you for your attention!**