FCC-hh: Longitudinal beam dynamics and RF requirements

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Update since 2018

• RF and beam parameters in CDR are mainly defined by single-bunch stability in longitudinal plane (loss of Landau damping)

Since last FCC meeting, we evaluated:

• Longitudinal coupled-bunch stability and requirements for high order mode (HOM) damping (talk today in the SRF session)

• Transient beam loading and RF power consumption

→ Self-consistent consideration of these effects is needed
Loss of Landau damping (LLD)

Criterion derived in short bunch approximation (F. Sacherer, 1973)

$$|\text{Im} Z_\parallel / n| < \frac{F_{\text{LLD}} V_{\text{RF}} \cos \phi_{s0} \omega_{s0}^2 \Delta \omega_s}{e N_p 8\pi h \omega_{s0}^2} \frac{\tau_{4\sigma}^3}{\tau_{4\sigma}}$$

Effective impedance for dipole mode

$$n = \frac{\omega}{\omega_0} \text{ frequency normalized by revolution frequency}$$

Form factor

RF voltage

Single bunch intensity

Harmonic number

Phase of synchronous particle

Synchrotron frequency spread

$$\frac{\Delta \omega_s}{\omega_{s0}} = \frac{\omega_{\text{RF}}^2}{64} \left( 1 + \frac{5}{3} \tan^2 \phi_{s0} \right) \tau_{4\sigma}^2$$

Equivalent bunch length

$$\tau_{4\sigma} = \frac{2\tau_{\text{FWHM}}}{\sqrt{2 \log 2}} \quad \text{Measured quantity}$$

→ What are $|\text{Im} Z_\parallel / n|$ and $F_{\text{LLD}}$?
Effective impedance

\[ \text{Effective impedance for dipole mode} \]
\[ \frac{\text{Im} Z_{\|}}{n} = \omega_0 \sum_{p=-\infty}^{\infty} \frac{\text{Im} Z_{\|}(\omega_p)}{\omega_p} H(\omega_p) \]
\[ \omega_p = p \omega_0 + \omega_{s0} \]

Normalized spectral density of dipole mode

|\text{Im} Z_{\|}/n| \approx 0.03 \, \Omega \text{ for present FCC-hh impedance model, but calculations for many elements are still missing (see talk of S. Arsenyev today)}

Assuming the same \text{Im} Z_{\|} per unit length in LHC and FCC-hh

\[ \frac{\text{Im} Z_{\|,\text{FCC}}}{C_{\text{FCC}}} = \frac{\text{Im} Z_{\|,\text{LHC}}}{C_{\text{LHC}}} \quad \frac{\text{Im} Z_{\|,\text{FCC}}}{\omega_0^{\text{FCC}}} = \frac{\text{Im} Z_{\|,\text{LHC}}}{\omega_0^{\text{LHC}}} \]

In the LHC measured and calculated |\text{Im} Z_{\|}/n| \approx 0.09 \, \Omega

\[ \rightarrow \text{We use} \, |\text{Im} Z_{\|}/n| = 0.2 \, \Omega \text{ as longitudinal impedance budget of FCC-hh (with margin of factor 2)} \]
Form factor $F_{\text{LLD}}$

Measured LLD threshold in LHC (J.F. Esteban Müller, PhD thesis, 2016)

→ Fitting with analytic formula gives $F_{\text{LLD}} \approx 0.41$
RF voltage at 50 TeV and 3.3 TeV

Thresholds at 50 TeV

→ RF voltage of 38 MV provides stability for average $\tau_{4\sigma} = 1.07$ ns with ±5% bunch length spread at 50 TeV

→ For $V_{RF} = 12$ MV at 3.3 TeV, 1.35 ns bunch length is required for stability
Bunch parameters

- Synchrotron radiation leads to bunch asymmetry (phase of bunch center of mass $\phi_s \neq \phi_{s0}$ phase of synchronous particle)
- RF component of the beam current is affected

We assume binomial particle distribution (as a function of Hamiltonian $H$)

$$\mathcal{F}(H) = \mathcal{F}_0 (H_{\text{max}} - H)^\mu$$

with $\mu = 1.5$ ($\mu = 2$ at LHC flattop and flat bottom)

| Energy, $E$ | TeV | 3.3 | 50 |
| RF voltage, $V_{RF}$ | MV | 12 | 38 |
| Bunch length, $\tau_{4\sigma}$ | ns | 1.35 | 1.07 |
| Full emittance, $\epsilon$ | eVs | 3.12 | 13.7 |
| Filling factor, $q_p = \frac{\Delta E_{\text{max}}}{\Delta E_B}$ | | 0.9 | 0.8 |
RF voltage program during ramp

Conditions:
• maximum $q_p = 0.94$ to avoid losses
• controlled emittance blow-up $\epsilon \propto \sqrt{E}$ (previously optimized for TMCI)

→ Bunch stable phase differs significantly from the phase of the synchronous particle
→ FWHM bunch length calculated from the line density is used for stability evaluation during the ramp
LLD threshold during ramp

→ Sufficient longitudinal single-bunch stability during the cycle
Beam loading and its compensation

Interaction of the beam with the fundamental cavity impedance results in modulation bunch-by-bunch and cavity parameters.

Without transient beam loading compensation the minimum power is

\[ P_{\text{opt}} = \frac{V_{\text{cav}} I_{b,\text{DC}} |F_b| \sin \phi_s}{2} \]

It is achieved if the following parameters are used:

Optimal cavity detuning

\[ \Delta \omega_{\text{opt}} = \omega_r - \omega_{\text{RF}} = \omega_{\text{RF}} \frac{(R/Q) I_{b,\text{DC}} |F_b| \cos \phi_s}{2V_{\text{cav}}} \]

Optimal quality factor

\[ Q_{L,\text{opt}} = \frac{V_{\text{cav}}}{(R/Q) I_{b,\text{DC}} |F_b| \sin \phi_s} \]

Form factor form line density

\[ F_b = 2 \frac{\mathcal{F}[\lambda(t)]_{\omega=\omega_{\text{RF}}}}{\mathcal{F}[\lambda(t)]_{\omega=0}} = |F_b| e^{-i\phi_s} \]

→ Evaluation of power consumption for partial and full transient beam loading compensation schemes is need.
RF power requirements

Partial beam loading compensation
(Full-detuning scheme)

\[ V(t) = V_{\text{cav}}e^{i\phi(t)} \rightarrow \text{no bunch length modulation} \]

\[ P_{\text{FD}} = \frac{V_{\text{cav}}I_{\text{b,peak}}|F_b| \sin \phi_s}{2} \]

\[ I_{\text{b,peak}} = eN_p/t_{\text{bb}}, \ t_{\text{bb}} - \text{bunch spacing} \]

\[ \Delta \omega_{\text{FD}} = \Delta \omega_{\text{opt}} \quad Q_{L,\text{FD}} = \frac{V_{\text{cav}}}{(R/Q)I_{\text{b,peak}}|F_b| \sin \phi_s} \]

Full transient beam loading compensation
(Half-detuning detuning scheme)

Cavity voltage amplitude and phase are constant
\rightarrow \text{no modulation of beam parameters}

Required power

\[ P_{\text{HD}}(\phi_s) = \frac{V_{\text{cav}}I_{\text{b,peak}}|F_b|}{8} \frac{1}{|\cos \phi_s + \sin \phi_s|} \]

\[ \Delta \omega_{\text{HD}} = \omega_{\text{rf}} \frac{\hat{I}_{\text{b,rf}}(R/Q)}{4V_{\text{cav}}} \frac{1}{\cos \phi_s + \sin \phi_s} \quad Q_{L,\text{HD}} = \frac{\omega_r}{2\Delta \omega_{\text{HD}}} \]
Keeping constant amplitude and phase of the cavity voltage during the FCC-hh cycle would require about 600 kW peak power (half-detuning scheme). The full-detuning scheme, requires about 25% more power compared with the case without transient beam loading compensation.

*I. Karpov, P. Baudrenghien, submitted to PRAB
Summary

• Evaluation of RF power, single-bunch stability and coupled-bunch stability resulted in update of beam and RF parameters:
  • at flattop $V_{RF} = 38$ MV,
  • at flat bottom $\tau_{4\sigma} = 1.35$ ns,
  • emittance blow-up during acceleration $\epsilon \propto \sqrt{E}$

• RF power consumption was calculated for different transient beam loading compensation schemes
  • Full compensation requires 600 kW peak power during acceleration (400 kW without compensation)
Thank you for your attention!