FCC-hh: Longitudinal beam dynamics and RF requirements

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Acknowledgements:

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Update since 2018

- RF and beam parameters in CDR are mainly defined by single-bunch stability in longitudinal plane (loss of Landau damping)

Since last FCC meeting, we evaluated:

- Longitudinal coupled-bunch stability and requirements for high order mode (HOM) damping (talk today in the SRF session)
- Transient beam loading and RF power consumption

→ Self-consistent consideration of these effects is needed

Loss of Landau damping (LLD)

Criterion derived in short bunch approximation (F. Sacherer, 1973)

Form factor

RF voltage

Effective impedance for dipole mode

$$\left|\mathrm{Im}Z_{\parallel}/n\right| < \frac{\mathrm{F}_{\mathrm{LLD}} V_{\mathrm{RF}} \cos\phi_{s0}\, \omega_{\mathrm{RF}}^2}{e N_{\mathrm{p}} 8\pi h} \frac{\Delta\omega_s}{\omega_{s0}} \tau_{4\sigma}^3$$

$n = \omega/\omega_0$ frequency normalized by revolution frequency

Single bunch intensity

Harmonic number

Phase of synchronous particle

Synchrotron frequency spread

$$\frac{\Delta\omega_s}{\omega_{s0}} = \frac{\omega_{\mathrm{RF}}^2}{64}\left(1 + \frac{5}{3}\tan^2\phi_{s0}\right)\tau_{4\sigma}^2$$

Equivalent bunch length

$$\tau_{4\sigma} = \frac{2\tau_{\mathrm{FWHM}}}{\sqrt{2\log 2}}$$

Measured quantity

→ What are $\left|\mathrm{Im}Z_{\parallel}/n\right|$ and $\mathrm{F}_{\mathrm{LLD}}$?

Effective impedance

Effective impedance for dipole mode

$$\mathrm{Im}Z_{\parallel}/n = \omega_0 \sum_{p=-\infty}^{\infty} \frac{\mathrm{Im}Z_{\parallel}(\omega_p)}{\omega_p} H(\omega_p)$$

$\omega_p = p\omega_0 + \omega_{s0}$

Normalized spectral density of dipole mode

$|\mathrm{Im}Z_{\parallel}/n| \approx 0.03\ \Omega$ for present FCC-hh impedance model, but calculations for many elements are still missing (see talk of S. Arsenyev today)

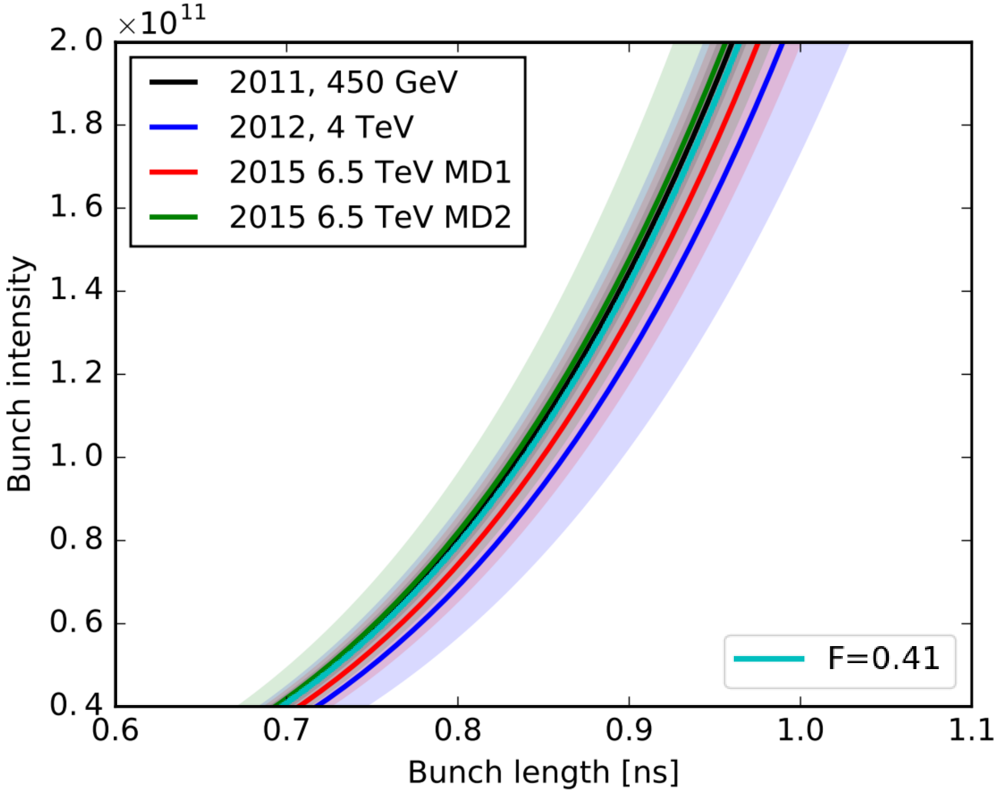
Assuming the same $\mathrm{Im}Z_{\parallel}$ per unit length in LHC and FCC-hh

$$\mathrm{Im}Z_{\parallel\mathrm{FCC}} = \frac{C_{\mathrm{FCC}}}{C_{\mathrm{LHC}}} \mathrm{Im}Z_{\parallel\mathrm{LHC}} \quad \Rightarrow \quad \frac{\mathrm{Im}Z_{\parallel\mathrm{FCC}}}{\omega/\omega_{0\mathrm{FCC}}} = \frac{C_{\mathrm{FCC}}}{C_{\mathrm{LHC}}} \frac{\omega_{0\mathrm{FCC}}}{\omega_{0\mathrm{LHC}}} \frac{\mathrm{Im}Z_{\parallel\mathrm{LHC}}}{\omega/\omega_{0\mathrm{LHC}}} = \frac{\mathrm{Im}Z_{\parallel\mathrm{LHC}}}{\omega/\omega_{0\mathrm{LHC}}}$$

In the LHC measured and calculated $|\mathrm{Im}Z_{\parallel}/n| \approx 0.09\ \Omega$
→ We use $|\mathrm{Im}Z_{\parallel}/n| = 0.2\ \Omega$ as longitudinal impedance budget of FCC-hh (with margin of factor 2)

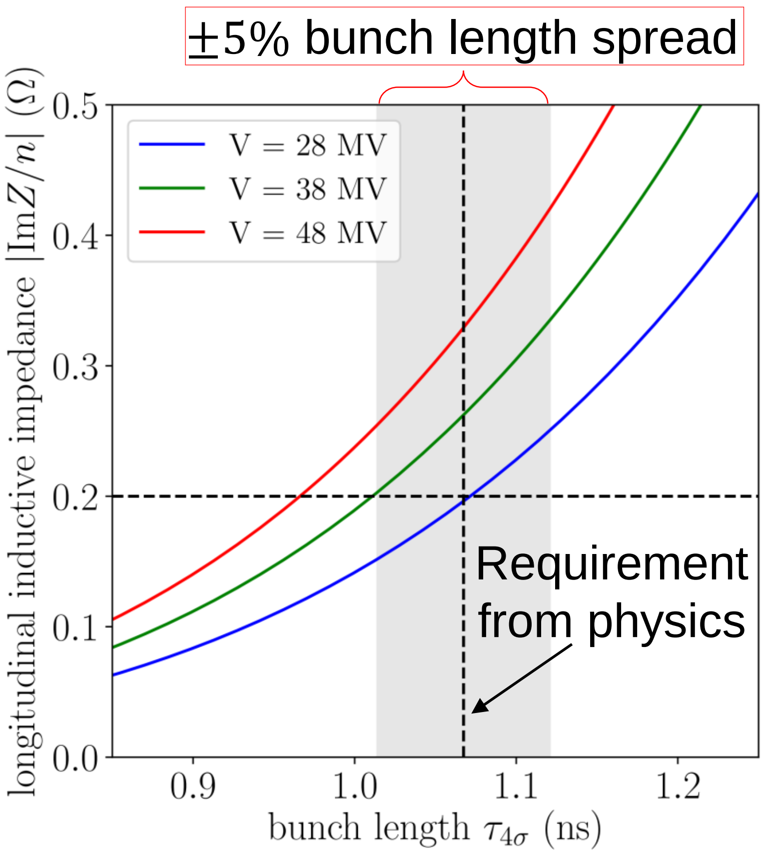
Form factor $\mathrm{F_{LLD}}$

Measured LLD threshold in LHC (J.F. Esteban Müller, PhD thesis, 2016)

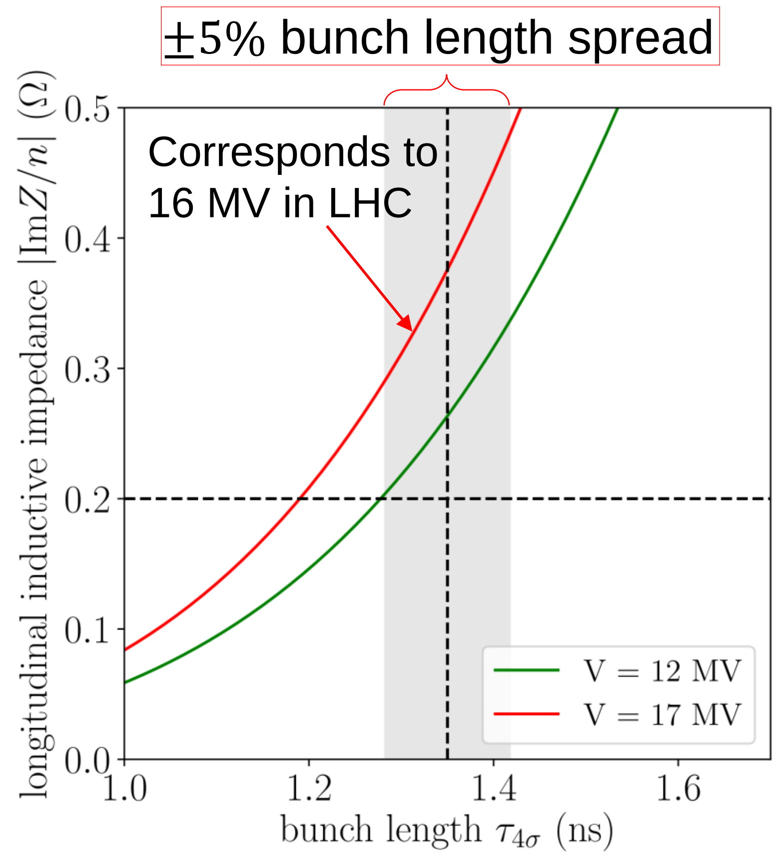


→ Fitting with analytic formula gives $\mathrm{F_{LLD}} \approx 0.41$

RF voltage at 50 TeV and 3.3 TeV



Thresholds at 50 TeV



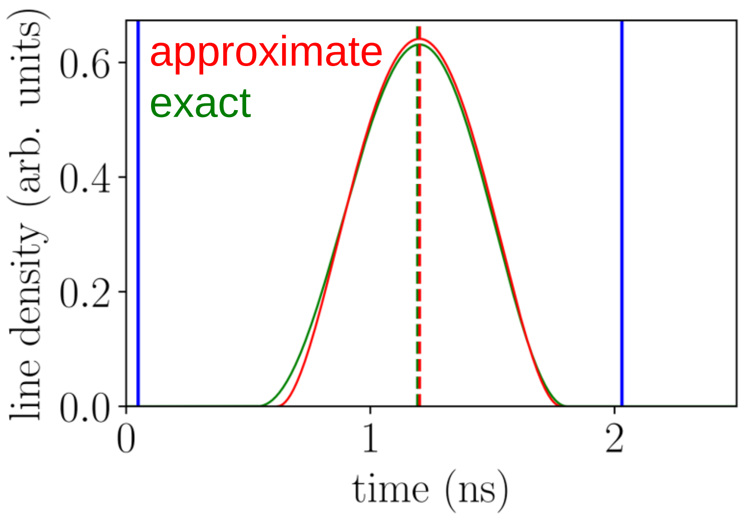
Thresholds at 3.3 TeV

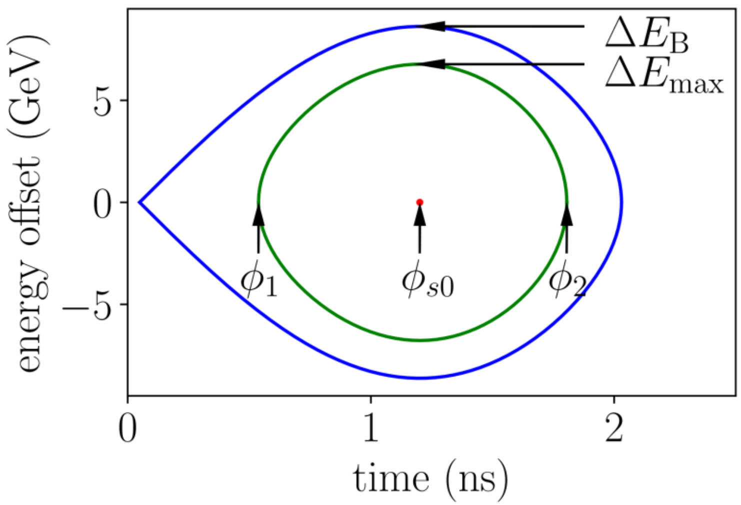
→ RF voltage of 38 MV provides stability for average $\tau_{4\sigma} = 1.07$ ns with $\pm5\%$ bunch length spread at 50 TeV

→ For $V_{\mathrm{RF}} = 12$ MV at 3.3 TeV, 1.35 ns bunch length is required for stability

Bunch parameters

Bunch profile at 50 TeV





- Synchrotron radiation leads to bunch asymmetry (phase of bunch center of mass $\phi_s \neq \phi_{s0}$ phase of synchronous particle
- RF component of the beam current is affected

We assume binomial particle distribution (as a function of Hamiltonian H)

$$\mathcal{F}(H) = \mathcal{F}_0(H_{\max} - H)^{\mu}$$

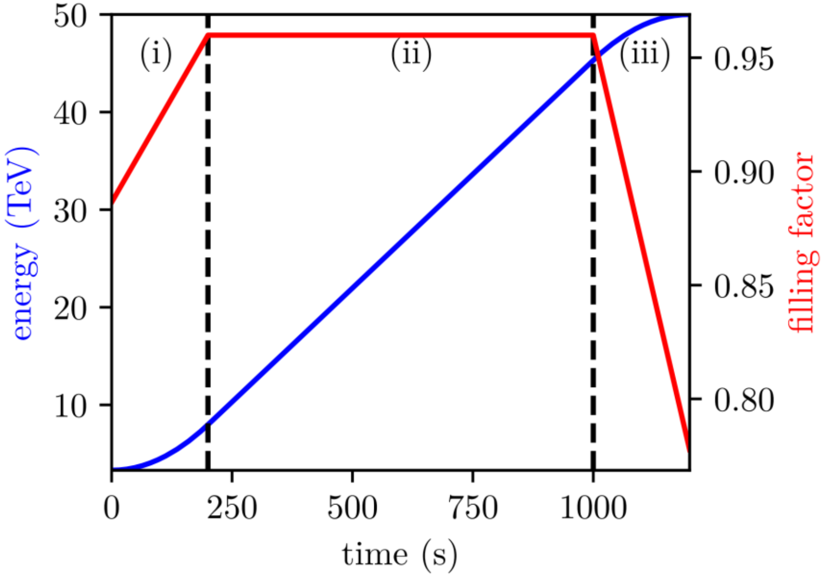
with $\mu = 1.5$ ($\mu = 2$ at LHC flattop and flat bottom)

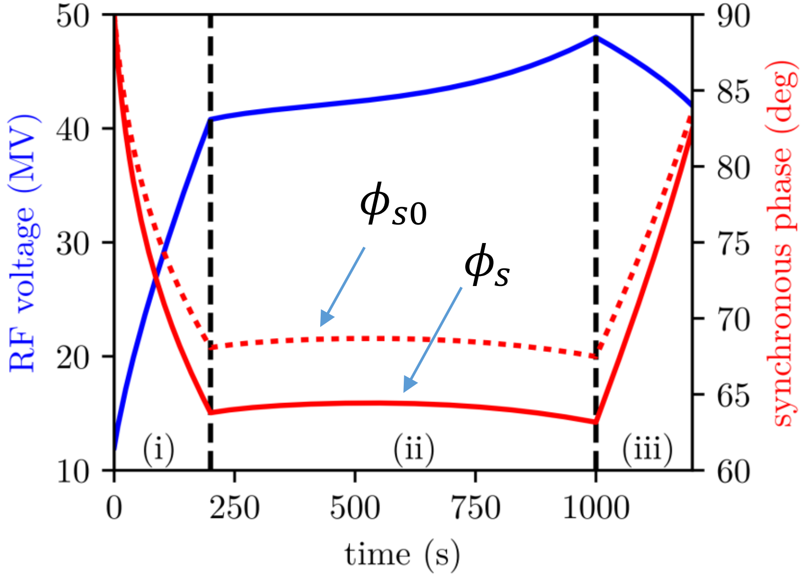
Energy, E	TeV	3.3	50
RF voltage, V_{RF}	MV	12	38
Bunch length, $\tau_{4\sigma}$	ns	1.35	1.07
Full emittance, ϵ	eVs	3.12	13.7
Filling factor, $q_p = \frac{\Delta E_{\max}}{\Delta E_{\mathrm{B}}}$		0.9	0.8

RF voltage program during ramp

Conditions:

- maximum $q_p = 0.94$ to avoid losses
- controlled emittance blow-up $\epsilon \propto \sqrt{E}$ (previously optimized for TMCI)

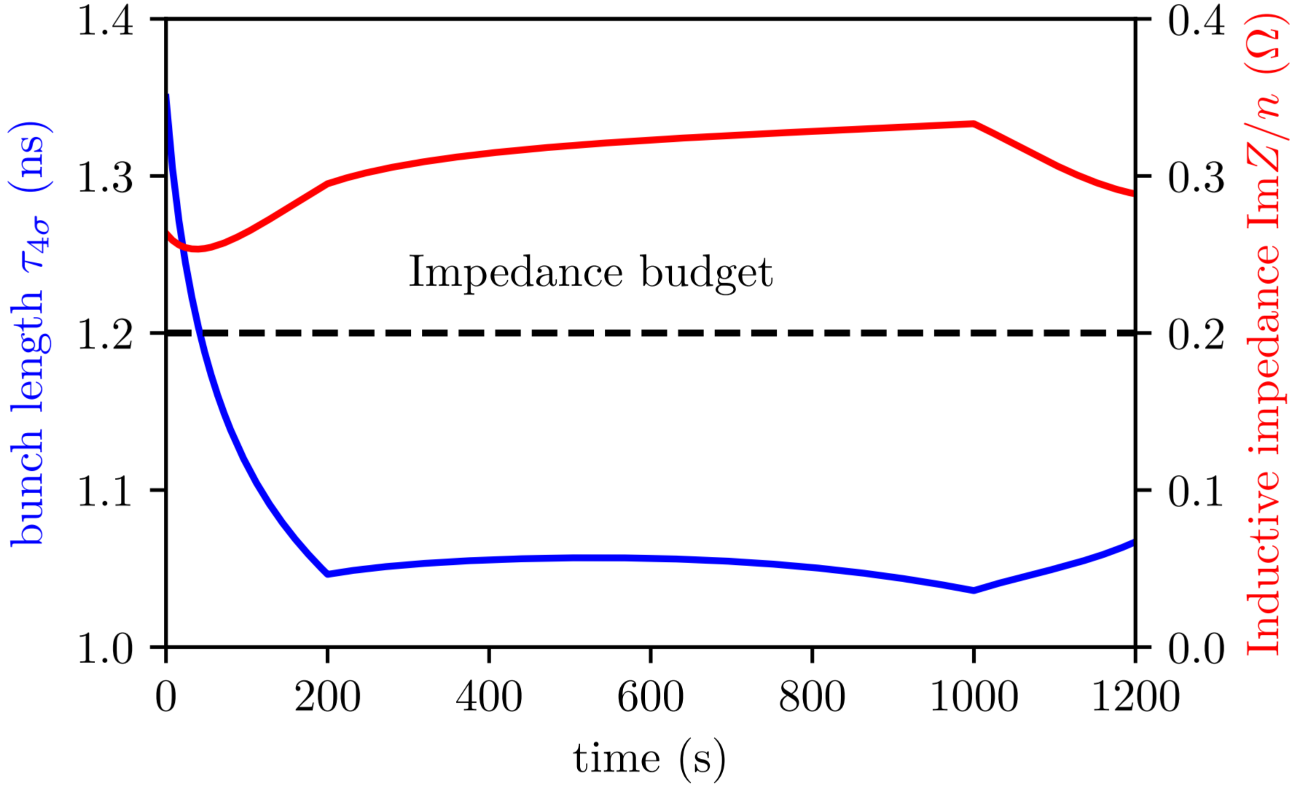




→ Bunch stable phase differs significantly from the phase of the synchronous particle

→ FWHM bunch length calculated from the line density is used for stability evaluation during the ramp

LLD threshold during ramp



→ Sufficient longitudinal single-bunch stability during the cycle

Beam loading and its compensation

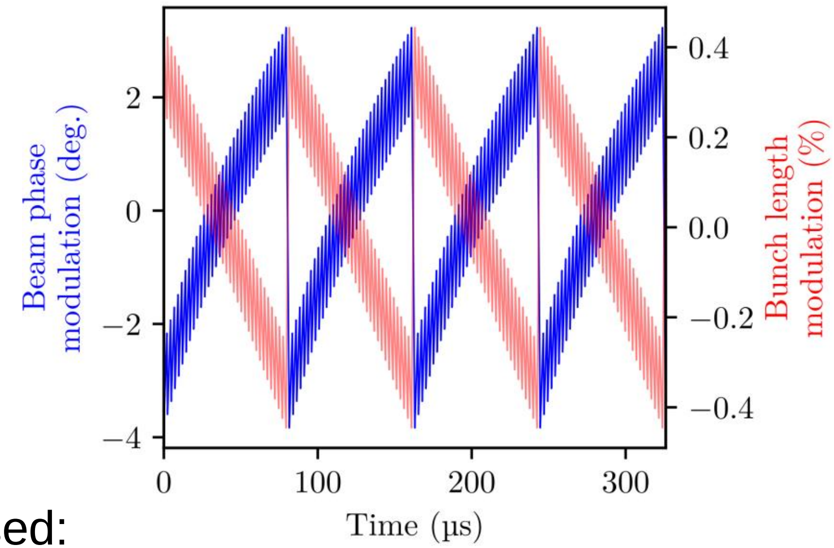
Interaction of the beam with the fundamental cavity impedance results in modulation bunch-by-bunch and cavity parameters

Without transient beam loading compensation the minimum power is

$V_{\mathrm{RF}}/N_{\mathrm{cav}},\ N_{\mathrm{cav}} = 24$

$$P_{\mathrm{opt}} = \frac{V_{\mathrm{cav}} I_{\mathrm{b,DC}} |F_{\mathrm{b}}| \sin \phi_s}{2}$$

Calculations at 50 TeV



It is achieved if the following parameters are used:

Optimal cavity detuning $\Delta\omega_{\mathrm{opt}} = \omega_{\mathrm{r}} - \omega_{\mathrm{RF}} = \omega_{\mathrm{RF}} \frac{(R/Q) I_{\mathrm{b,DC}} |F_{\mathrm{b}}| \cos \phi_s}{2V_{\mathrm{cav}}}$

Optimal quality factor $Q_{\mathrm{L,opt}} = \frac{V_{\mathrm{cav}}}{(R/Q) I_{\mathrm{b,DC}} |F_{\mathrm{b}}| \sin \phi_s}$

Form factor form line density $F_{\mathrm{b}} = 2 \frac{\mathcal{F}[\lambda(t)]_{\omega=\omega_{\mathrm{RF}}}}{\mathcal{F}[\lambda(t)]_{\omega=0}} = |F_{\mathrm{b}}| e^{-i\phi_s}$

→ Evaluation of power consumption for partial and full transient beam loading compensation schemes is need

RF power requirements

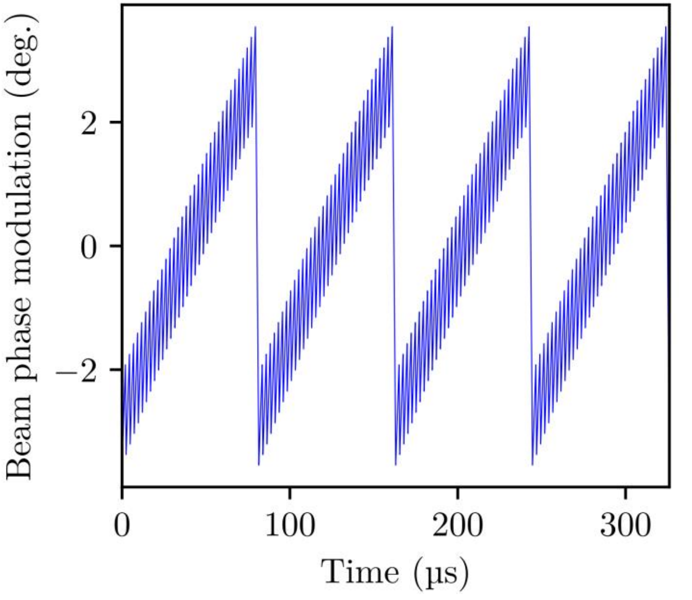
Partial beam loading compensation
(Full-detuning scheme)

$V(t) = V_{\mathrm{cav}} e^{i\phi(t)}$ → no bunch length modulation

$$P_{\mathrm{FD}} = \frac{V_{\mathrm{cav}} I_{\mathrm{b,peak}} |F_{\mathrm{b}}| \sin\phi_s}{2}$$

$I_{\mathrm{b,peak}} = eN_{\mathrm{p}}/t_{\mathrm{bb}}$, t_{bb} - bunch spacing

$$\Delta\omega_{\mathrm{FD}} = \Delta\omega_{\mathrm{opt}} \qquad Q_{\mathrm{L,FD}} = \frac{V_{\mathrm{cav}}}{(R/Q) I_{\mathrm{b,peak}} |F_{\mathrm{b}}| \sin\phi_s}$$



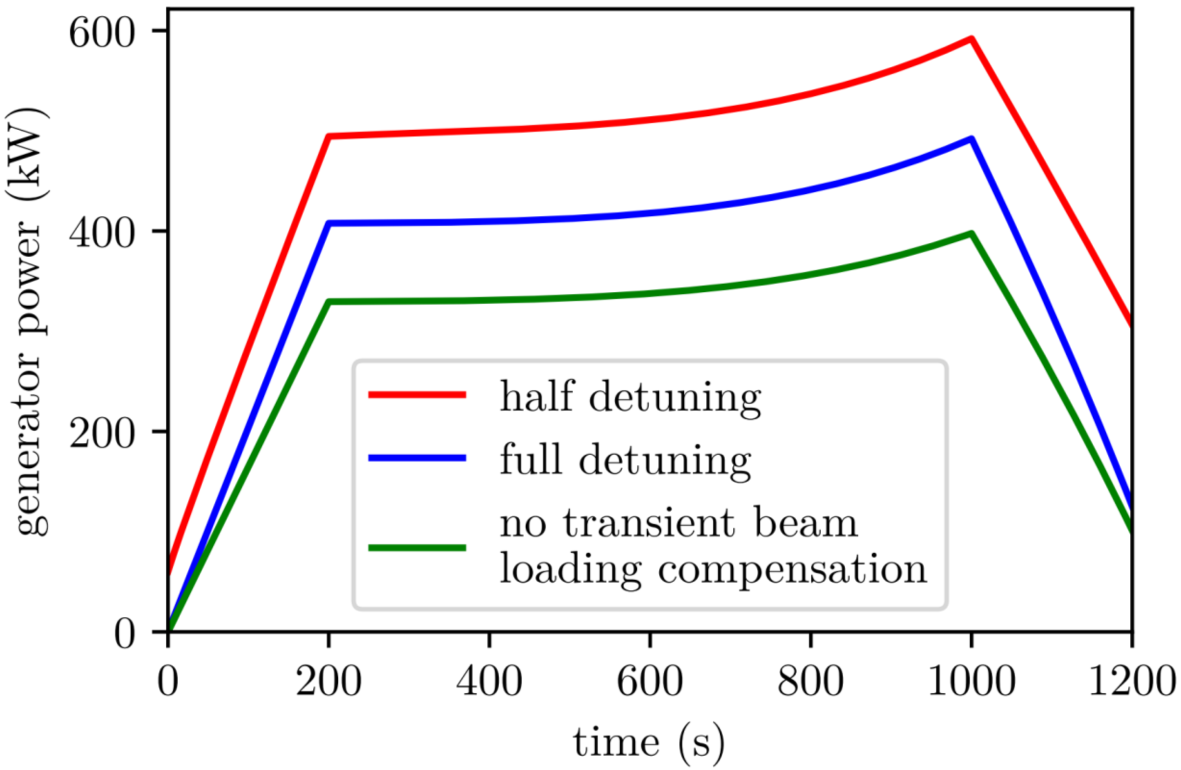
Full transient beam loading compensation (Half-detuning detuning scheme)

Cavity voltage amplitude and phase are constant
→ no modulation of beam parameters

Required power $\quad P_{\mathrm{HD}}(\phi_s) = \dfrac{V_{\mathrm{cav}} I_{\mathrm{b,peak}} |F_{\mathrm{b}}|}{8} \dfrac{1}{|\cos\phi_s + \sin\phi_s|}$

$$\Delta\omega_{\mathrm{HD}} = \omega_{\mathrm{rf}} \frac{\hat{I}_{\mathrm{b,rf}}(R/Q)}{4V_{\mathrm{cav}}} \frac{1}{\cos\phi_s + \sin\phi_s} \qquad Q_{\mathrm{L,HD}} = \frac{\omega_{\mathrm{r}}}{2\Delta\omega_{\mathrm{HD}}}$$

Power consumption during cycle*



→ Keeping constant amplitude and phase of the cavity voltage during the FCC-hh cycle would require about 600 kW peak power (half-detuning scheme).
→ The full-detuning scheme, requires about 25% more power compared with the case without transient beam loading compensation.

*I. Karpov, P. Baudrenghien, submitted to PRAB

Summary

- Evaluation of RF power, single-bunch stability and coupled-bunch stability resulted in update of beam and RF parameters:
  - at flattop V_{RF} = 38 MV,
  - at flat bottom $\tau_{4\sigma}$ = 1.35 ns,
  - emittance blow-up during acceleration $\epsilon \propto \sqrt{E}$
- RF power consumption was calculated for different transient beam loading compensation schemes
  - Full compensation requires 600 kW peak power during acceleration (400 kW without compensation)

Thank you for your attention!