# *Challenges for EW b physics measurements*
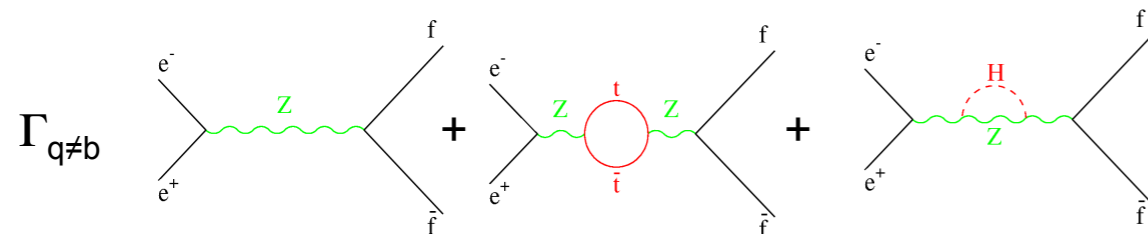
Future Circular Collider Conference

Fabrizio Palla
INFN Sezione di Pisa

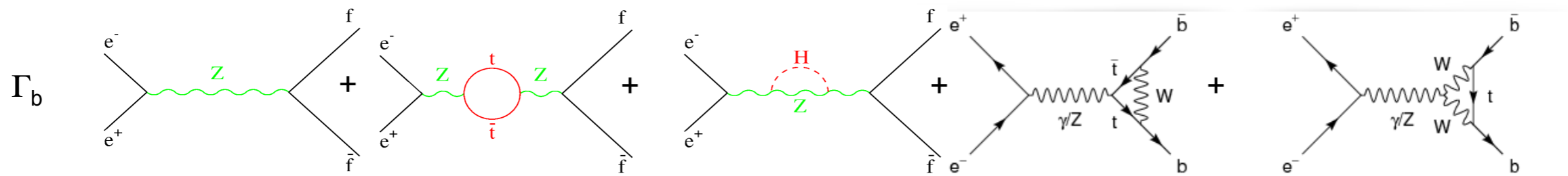FCC Week 2019
Bruxelles, June 25 2019

# Introduction

- Couplings of b-quarks to Z bosons through partial widths and forward-backward asymmetry sensitive to specific radiative corrections and possibly new physics



$\Gamma_{q \neq b}$

$$\Gamma(Z^0 \to q\bar{q}) = \frac{G_\mu M_Z^3}{8\pi\sqrt{2}}\left(v_q^2 + a_q^2\right)$$ **Born**

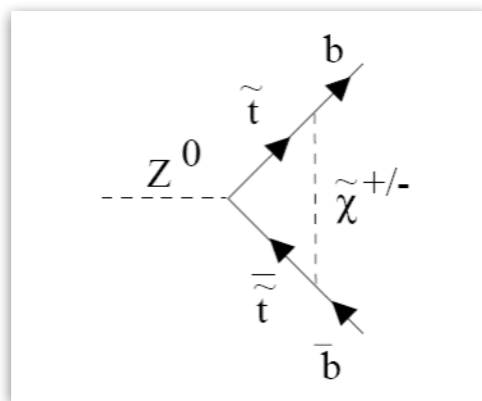$$v_q = (1 + \delta\rho)\left(-1 + 4Q_q \sin^2\theta_{\text{eff}}^q\right)$$

$$a_q = -(1 + \delta\rho)$$ *radiative corrections*



$\Gamma_b$

*b-quark specific*

$$v_b \to v_b\left(1 + \frac{4}{3}\delta\rho\right)$$

$$a_b \to a_b\left(1 + \frac{4}{3}\delta\rho\right)$$

# Introduction

|  | Measured | Theory prediction | Pull |
|---|---|---|---|
| $R_b$ | $0.21629 \pm 0.00066$ | $0.21582 \pm 0.00002$ | $0.7$ |
| $R_c$ | $0.1721 \pm 0.0030$ | $0.17221 \pm 0.00003$ | $0.0$ |
| $A_{FB}^{(0,e)}$ | $0.0145 \pm 0.0025$ | $0.01618 \pm 0.00006$ | $-0.7$ |
| $A_{FB}^{(0,\mu)}$ | $0.0169 \pm 0.0013$ |  | $0.6$ |
| $A_{FB}^{(0,\tau)}$ | $0.0188 \pm 0.0017$ |  | $1.5$ |
| $A_{FB}^{(0,b)}$ | $0.0992 \pm 0.0016$ | $0.1030 \pm 0.0002$ | $-2.3$ |

$$R_b = \frac{\Gamma_b}{\Gamma_{Z \to hadrons}} = R_d \left[ 1 - \frac{20}{13} \frac{\alpha}{\pi} \left( \frac{m_t^2}{M_Z^2} + \frac{13}{6} \log \frac{m_t^2}{M_Z^2} \right) \right] \sim R_d \left( 1 - 0.02 \right)$$

# Statistics

- LEP experiments (ALEPH, DELPHI, L3, OPAL) and SLD at SLC ~15 +0.4 Million hadronic Z decays

- Expected statistics at FCC-ee ~ 3 Tera hadronic Z decays FCC-ee

LEP

| Year | $Z \to q\bar{q}$ | | | | |
|------|------|------|------|------|------|
|      | A | D | L | O | LEP |
| 1990/1991 | 433 | 357 | 416 | 454 | 1660 |
| 1992 | 633 | 697 | 678 | 733 | 2741 |
| 1993 | 630 | 682 | 646 | 649 | 2607 |
| 1994 | 1640 | 1310 | 1359 | 1601 | 5910 |
| 1995 | 735 | 659 | 526 | 659 | 2579 |
| Total | 4071 | 3705 | 3625 | 4096 | 15497 |



FCC-ee

| working point | luminosity/IP [$10^{34}$ cm$^{-2}$s$^{-1}$] | total luminosity (2 IPs)/ yr | physics goal | run time [years] |
|------|------|------|------|------|
| Z first 2 years | 100 | 26 ab$^{-1}$/year | **150 ab$^{-1}$** | **4** |
| Z later | 200 | 52 ab$^{-1}$/year | | |
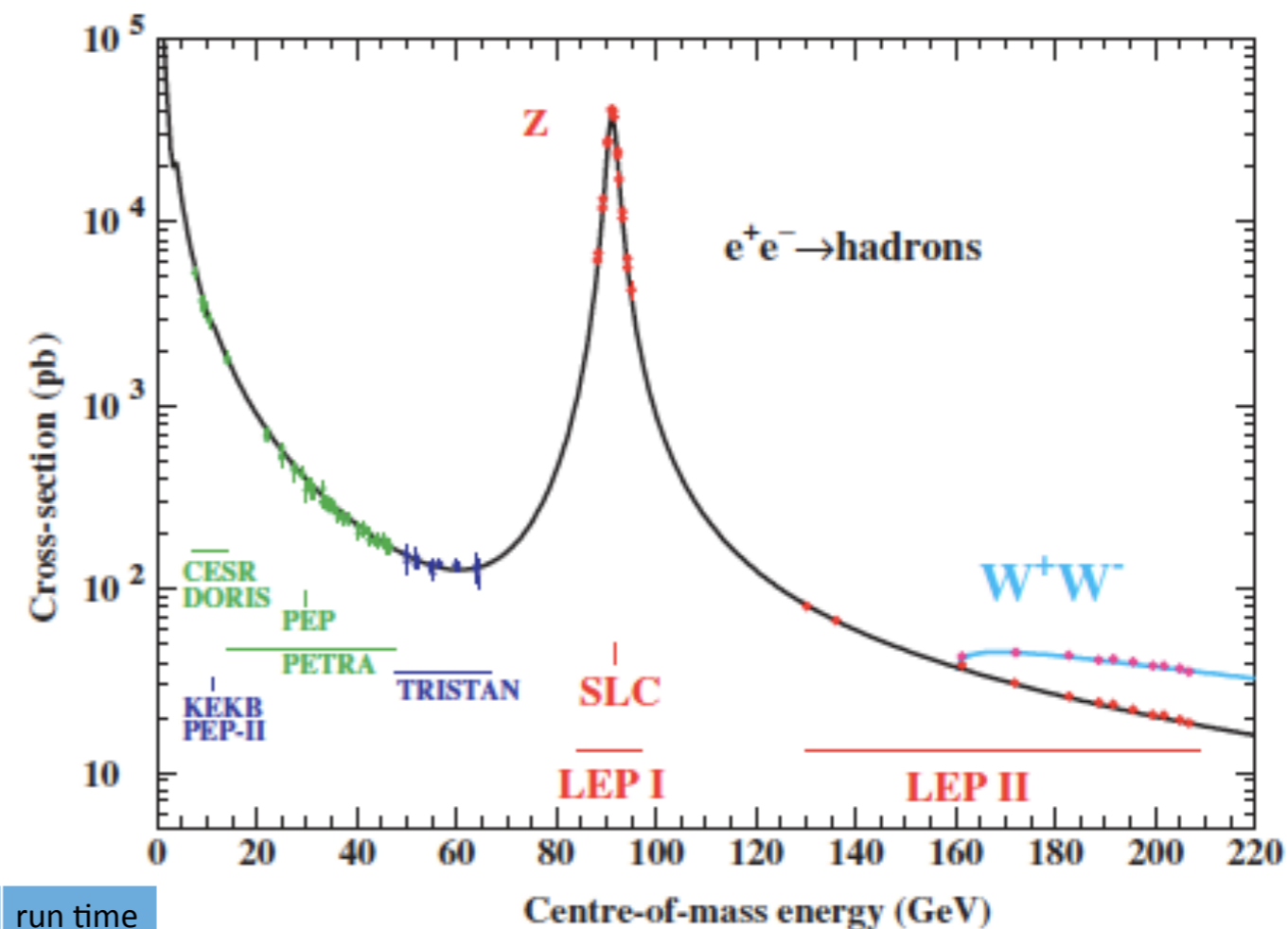
# R$_b$: tagging methods

- Leptons: B(b→µ/e)~10%

- Lifetime (τ$_B$ ~1.6 ps~500 µm)
  - ◆ impact parameter
  - ◆ secondary vertex displacement

- Mass (m$_B$~ 5.3 GeV)
  - ◆ invariant mass of secondary vertex
  - ◆ event shapes

- Double tagging techniques
  - ◆ N tags:
    - • N(N+1)/2 double tag fractions
    - • N single tag fractions
    - • 3N efficiencies
    - • N(N+1)/2 correlations

  - • multivariate analysis could benefit reducing the impact of correlations, and measure a few of them
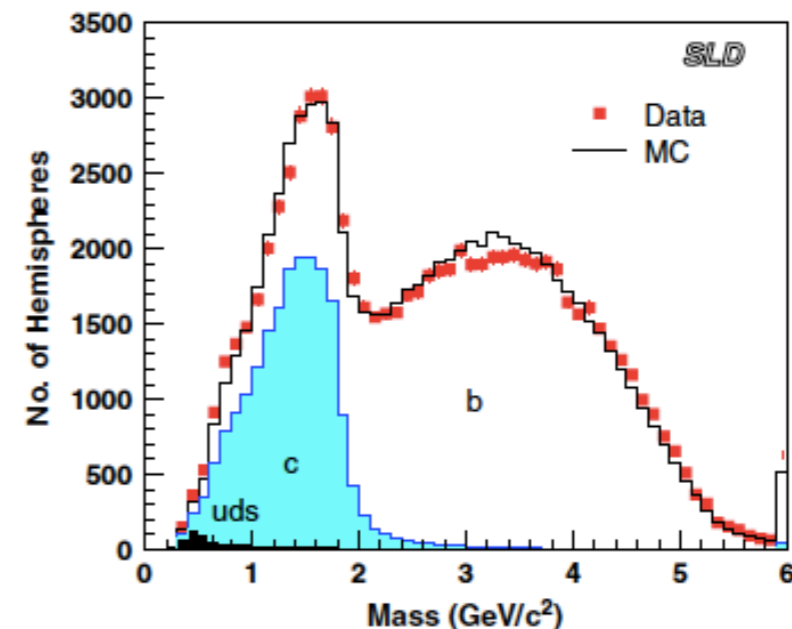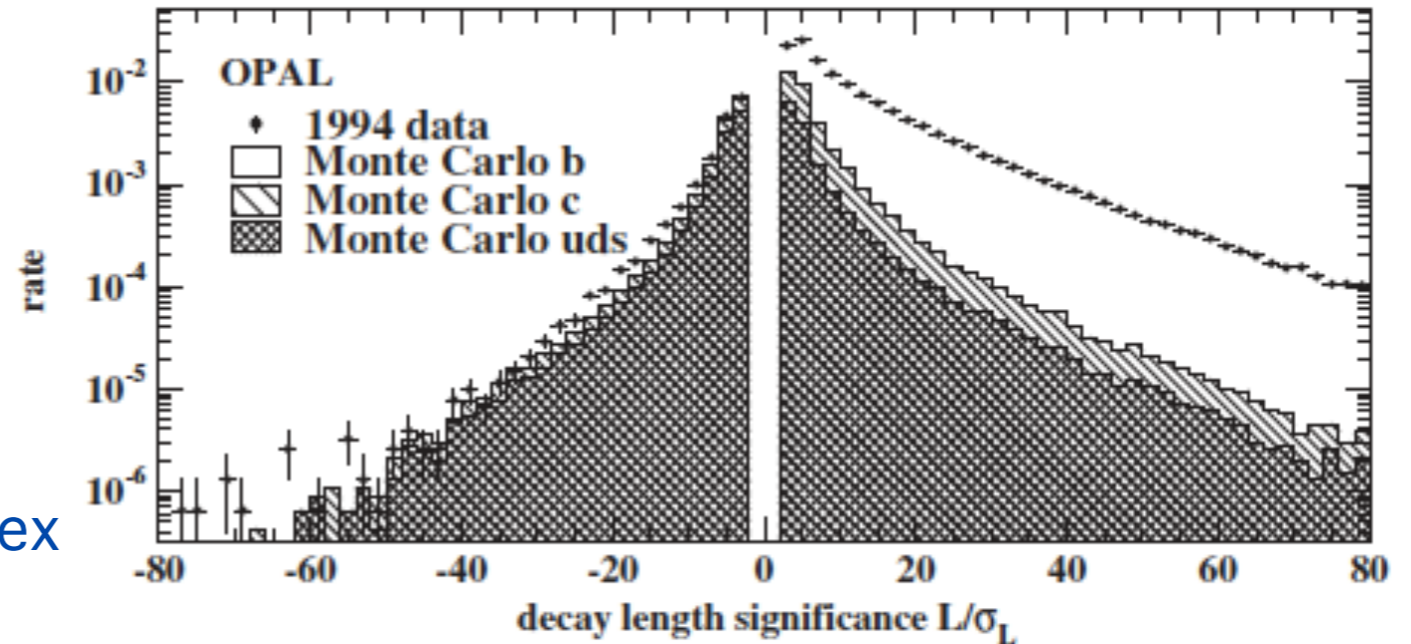
# Detectors and b-tag performances

- Tracking resolutions (best from SLD)
  - impact parameter $r\phi$ 7.7 $\mu m \oplus 33\mu m /(p \sin\vartheta^{3/2})$
  - impact parameter z 9.6 $\mu m$
  - beam 3$\mu m$ x 700 $\mu m$

- FCC-ee
  - impact parameter $r\phi$ 3 $\mu m \oplus 15 \mu m /(p \sin\vartheta^{3/2})$
  - impact parameter z 9.6 $\mu m$
  - beam 6 $\mu m$ x 420 $\mu m$

- Impact parameter resolution a factor 2 better
  - SLD PV z resolution 17 $\mu m$ for b, 10 $\mu m$ for charm and uds
  - FCC-ee PV resolution ~ 9 $\mu m$ and 5 $\mu m$ respectively

- Lifetime-mass tagging efficiencies for SLD (should be better at FCC-ee)
  - 60% (b), 1% (c), 0.1% (uds)

Vertex detector characteristics and experimental resolutions

| | ALEPH | DELPHI | L3 | OPAL | SLD |
|---|---|---|---|---|---|
| Number of layers | 2 | 3 | 2 | 2 | 3 |
| Radius of layers (cm) | 6.5/11.3 | 6.3/9/11 | 6.2/7.7 | 6.1/7.5 | 2.7–4.8 |
| $R\phi$ imp. par. res. ($\mu$m) | 25[a] | 20 | 30 | 16 | 8 |
| $z$ imp. par. res. ($\mu$m) | | 30 | 100 | 35 | 10 |
| Primary vertex res. | $58 \times 10$ | $60 \times 10$ | $77 \times 10$ | $80 \times 12$ | $4 \times 4$ |
| $x \times y \times z$ ($\mu$m) | $\times 60$ | $\times 70$ | $\times 100$ | $\times 85$ | $\times 17$ |

# Uncertainties

$$N_{single}^b = 2N_Z \left[ R_b \varepsilon_b + R_c \varepsilon_c + (1 - R_b - R_c) \varepsilon_{uds} \right]$$

$$N_{double}^b \sim N_Z \left[ R_b \varepsilon_b^2 (1 + \rho_b) + R_c \varepsilon_c^2 + (1 - R_b - R_c) \varepsilon_{uds}^2 \right]$$

$$\frac{\Delta R_b}{R_b} \sim \Delta \rho_b$$

$$\frac{\Delta R_b}{R_b} \sim -2 \frac{\Delta \varepsilon_c}{\varepsilon_b} \frac{R_c}{R_b}$$

| Source | $R_b^0$ [$10^{-3}$] | $A_{FB}^{0,b}$ [$10^{-3}$] |
|---|---|---|
| Statistics | 0.44 | 1.5 |
| Internal systematics | 0.28 | 0.6 |
| QCD effects | 0.18 | 0.4 |
| $B(D \to neut.)$ | 0.14 | 0 |
| D decay multiplicity | 0.13 | 0 |
| B decay multiplicity | 0.11 | 0 |
| $B(D^+ \to K^- \pi^+ \pi^+)$ | 0.09 | 0 |
| $B(D_s \to \phi \pi^+)$ | 0.02 | 0 |
| $B(\Lambda_c \to p K^- \pi^+)$ | 0.05 | 0 |
| D lifetimes | 0.07 | 0 |
| B decays | 0 | 0.1 |
| Decay models | 0 | 0.1 |
| Non-incl. mixing | 0 | 0.1 |
| Gluon splitting | 0.23 | 0.1 |
| c Fragmentation | 0.11 | 0.1 |
| Light quarks | 0.07 | 0 |
| Beam polarisation | 0 | 0 |
| Total correlated | 0.42 | 0.4 |
| Total error | 0.66 | 1.6 |

Statistical error scales $\sim 1/\varepsilon_b^2$

Correlation (taken from MC) due to:
- *detector inhomogeneities [checked with data]*
- *common primary vertex in case of i.p. based taggers, not important for SV*
- *kinematic correlations*
  - *momentum dependent efficiency*
  - *(hard) gluon radiation*

Large b-tagging efficiencies reduce correlations

Uncertainty due to B (and C) physics affect correlation
- *lifetimes, decay multiplicity, fractions, fragmentation*

Charm and uds tag efficiencies from MC
*Uncertainties due to physics and modelling (see next)*
*Impact proportional to the charm and uds tag efficiencies*

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Table 5.4
The most important external parameters used in the heavy flavour analyses

**Already now a factor 2-3 better**

| Error source | Used range | |
|---|---|---|
| $\langle x_E \rangle_b$ | $0.702 \pm 0.008$ | |
| $\langle x_E \rangle_c$ | $0.484 \pm 0.008$ | |
| Choice of b fragmentation function | See Section 5.6.1 | |
| Choice of c fragmentation function | See Section 5.6.1 | |
| $B(b \to \bar{c} \to \ell^-)$ | $(1.62^{+0.44}_{-0.36})\%$ | Already better now, |
| $B(b \to \tau^- \to \ell^-)$ | $(0.419 \pm 0.055)\%$ | Will improve with BESIII and Belle2 |
| $B(b \to (J/\psi, \psi') \to \ell\ell)$ | $(0.072 \pm 0.006)\%$ | |
| Semilept. model $b \to \ell^-$ | ACCMM $(^{+ISGW}_{-ISGW**})$ (Section 5.6.6) | |
| Semilept. model $c \to \ell^+$ | ACCMM1 $(^{+ACCMM2}_{-ACCMM3})$ (Section 5.6.6) | |
| $B \to D$ model | Peterson $\epsilon = 0.42 \pm 0.07$ | |
| $D^0$ lifetime | $0.415 \pm 0.004$ ps | $\pm 0.0015$ ps |
| $D^+$ lifetime | $1.057 \pm 0.015$ ps | $\pm 0.007$ ps |
| $D_s$ lifetime | $0.467 \pm 0.017$ ps | $\pm 0.004$ ps |
| $\Lambda_c^+$ lifetime | $0.206 \pm 0.012$ ps | $\pm 0.006$ ps |
| B lifetime | $1.576 \pm 0.016$ ps | $\pm 0.0029$ ps |
| $B(D^0 \to K^- \pi^+)$ | $0.0385 \pm 0.0009$ | $\pm 0.00031$ |
| $B(D^+ \to K^- \pi^+ \pi^+)$ | $0.090 \pm 0.006$ | $\pm 0.0016$ |
| $B(D_s^+ \to \phi\pi^+)$ | $0.036 \pm 0.009$ | $\pm 0.004$ |
| $\frac{B(D_s^+ \to \bar{K}^{*0}K^+)}{B(D_s^+ \to \phi\pi^+)}$ | $0.92 \pm 0.09$ | Belle |
| $B(\Lambda_c \to pK^- \pi^+)$ | $0.050 \pm 0.013$ | $\pm 0.0032$ |
| B charged decay multiplicity | $4.955 \pm 0.062$ | |
| D charged decay multiplicity | See Section 5.6.3 | Can improve with BESIII and Belle2 |
| D neutral decay multiplicity | See Section 5.6.3 | |
| $g \to c\bar{c}$ per multi-hadron | $(2.96 \pm 0.38)\%$ | Can be better measured at FCC-ee |
| $g \to b\bar{b}$ per multi-hadron | $(0.254 \pm 0.051)\%$ | |
| Rate of long-lived light hadrons | Tuned JETSET $\pm 10\%$ (Section 5.6.8) | |
| Light quark fragmentation | See Section 5.6.8 | |
| QCD hemisphere correlations | See Section 5.6.7 | |

$$\frac{\Delta R_b}{R_b} \sim -2 \frac{\Delta \epsilon_c}{\epsilon_b} \frac{R_c}{R_b}$$

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$$\frac{\Delta R_b}{R_b} \sim -2 \frac{\Delta \epsilon_c}{\epsilon_b} \frac{R_c}{R_b}$$

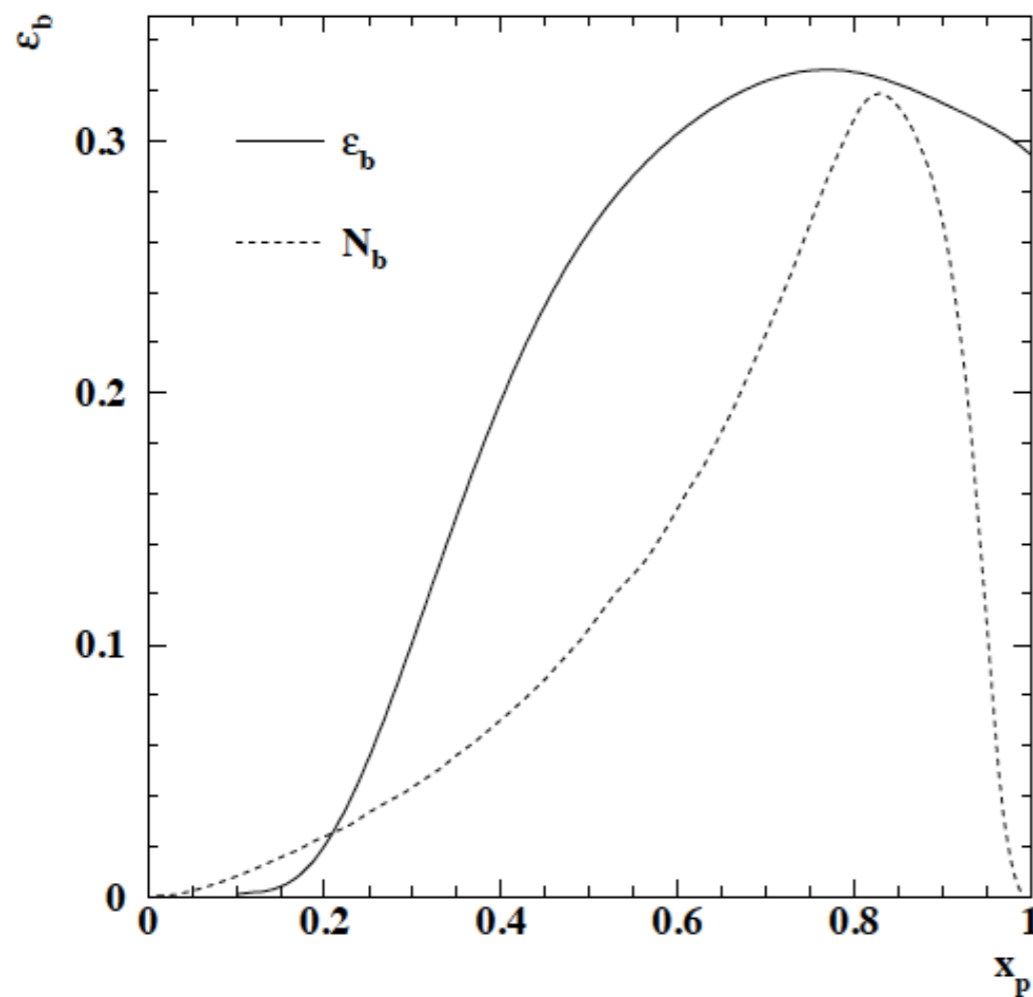$$\frac{\Delta R_b}{R_b} \sim -2 \frac{\Delta \epsilon_{uds}}{\epsilon_b} \frac{R_{uds}}{R_b}$$

# "Measuring" the correlation

- Compare "components" of correlations in data and MC

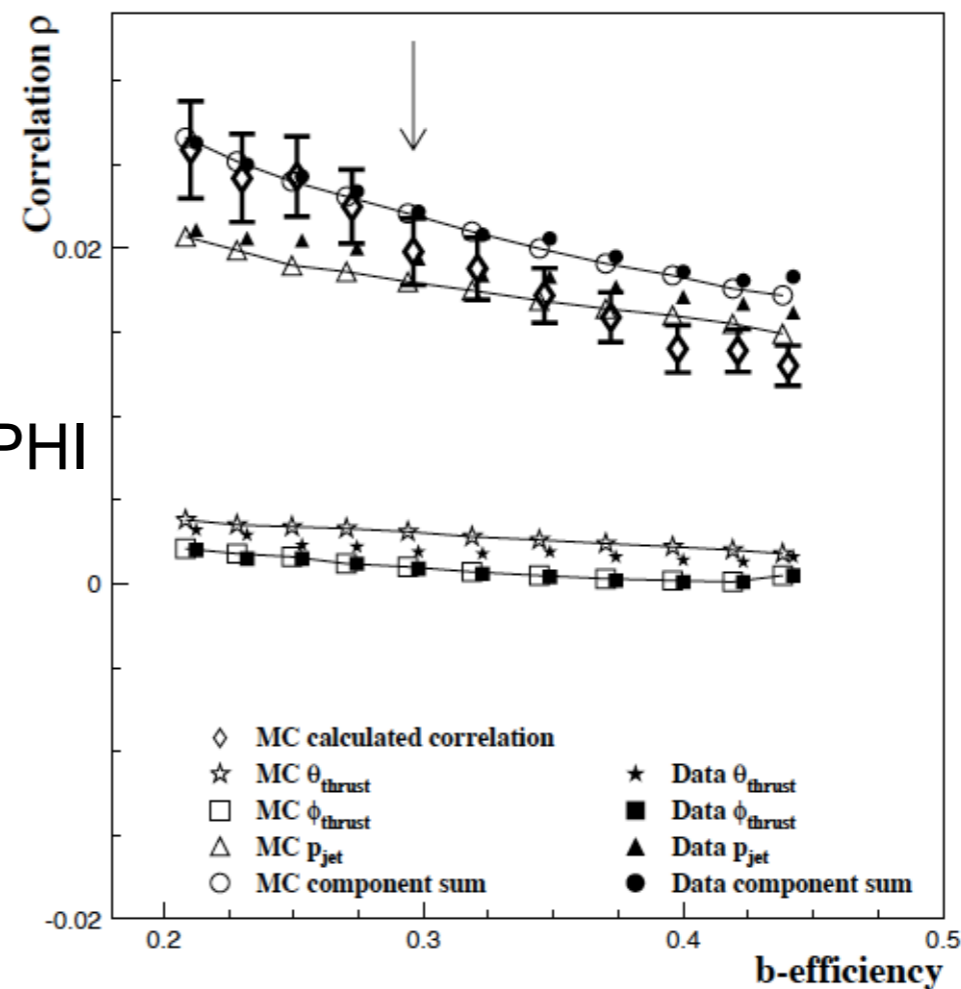  - usually the correlation decreases if efficiency is large or flat in the variable

$$\epsilon_\nu \equiv \int E(\nu) \cdot F(\nu) \, d\nu \qquad \rho_\nu = \frac{\epsilon_\nu - \epsilon_b}{\epsilon_b}$$

$E(\nu)$ = Efficiency to tag vs variable $\nu$

$F(\nu)$ = Biased distribution of variable $\nu$ after tagging the opposite hemisphere



DELPHI

- Require hermetic detector coverage

- Design a tagger using (using several tag variables in a NN) to flatten out the efficiency vs momentum

# Role of b-tagging

- Thanks to the better performance of the detector one could achieve a better rejection of light-quark and charm background with an efficiency of ~60%.

- Statistical error ~0.3 10$^{-6}$ with 60% b-tagging efficiency
  - Going down to ~30% one could extrapolate a 5 times smaller charm efficiency, for a total of ~99.5% purity, and a stat error ~0.8 10$^{-6}$ thanks to a charm impact 5 times smaller.

    - However the correlation is larger for smaller efficiency
    - Incidentally $\frac{\Delta R_b}{R_b} \sim -2 \frac{\varepsilon_c}{\varepsilon_b} \frac{\Delta R_c}{R_b}$ hence reducing charm efficiency is beneficial
    - Must find a trade off between statistical and systematic error

- Extrapolating from the current sensitivity, one could go to 50-100 x 10$^{-6}$ (10 times better than now!)
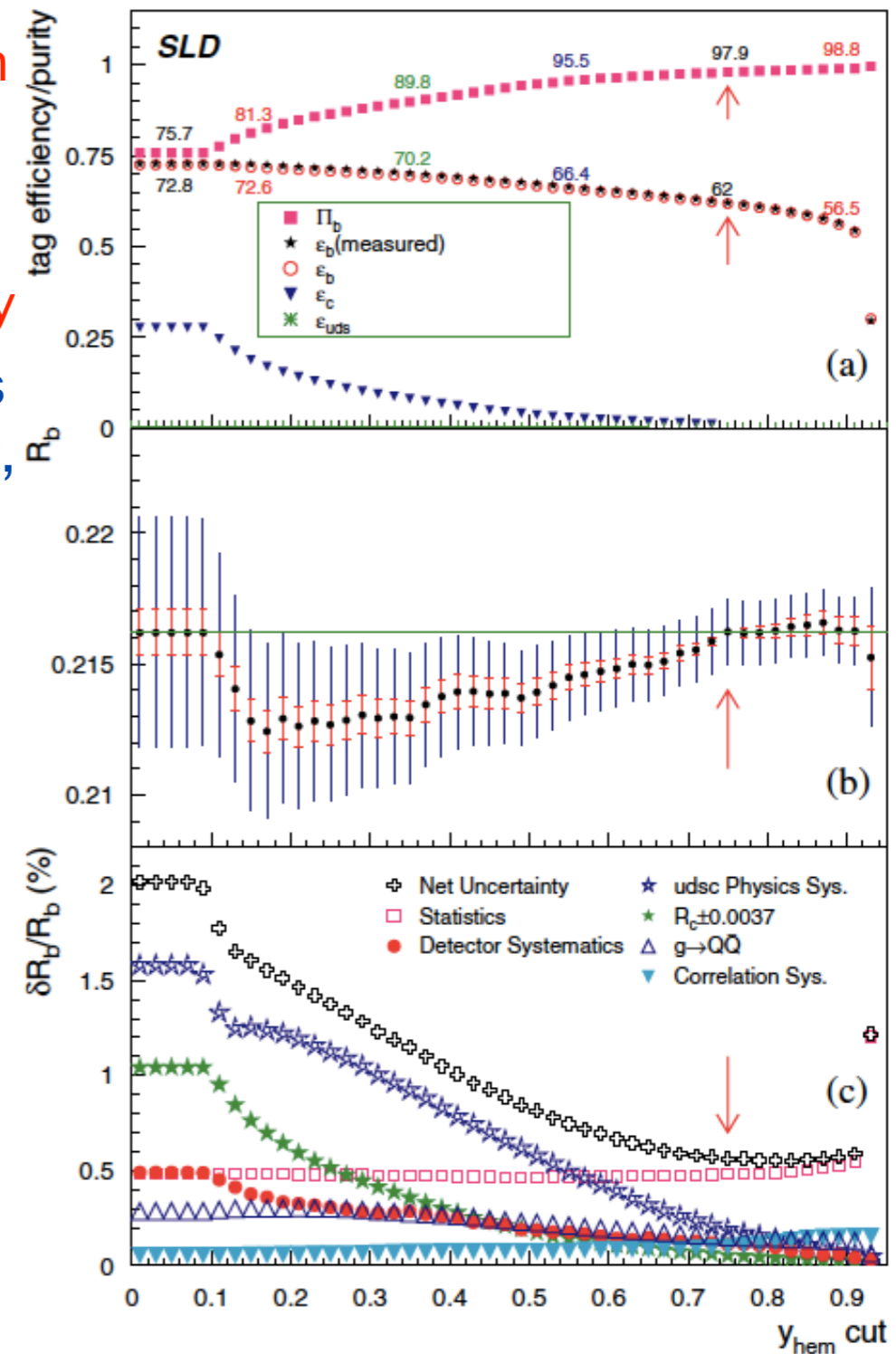
# What about Theory?

Central EW precision (pseudo-)observables at the Z pole

FCC-ee: update of Blondel et al., 1901.02648 (in prep.);    ILC: Moortgat-Pick et al., 1504.01726

| | experimental accuracy | | | intrinsic theory uncertainty | | | parametric unc. | |
|---|---|---|---|---|---|---|---|---|
| | current | ILC | FCC-ee | current | current source | prospect | prospect | source |
| $\Delta M_{\rm Z}$[MeV] | 2.1 | – | 0.1 | | | | | |
| $\Delta \Gamma_{\rm Z}$[MeV] | 2.3 | 1 | 0.1 | 0.4 | $\alpha^3, \alpha^2\alpha_{\rm s}, \alpha\alpha_{\rm s}^2$ | 0.15 | 0.1 | $\alpha_{\rm s}$ |
| $\Delta \sin^2\theta_{\rm eff}^\ell$ $[10^{-5}]$ | 23 | 1.3 | 0.6 | 4.5 | $\alpha^3, \alpha^2\alpha_{\rm s}$ | 1.5 | 2(1) | $\Delta\alpha_{\rm had}$ |
| $\Delta R_{\rm b}$ $[10^{-5}]$ | 66 | 14 | 6 | 11 | $\alpha^3, \alpha^2\alpha_{\rm s}$ | 5 | 1 | $\alpha_{\rm s}$ |
| $\Delta R_\ell$ $[10^{-3}]$ | 25 | 3 | 1 | 6 | $\alpha^3, \alpha^2\alpha_{\rm s}$ | 1.5 | 1.3 | $\alpha_{\rm s}$ |

Theory requirements for Z-pole pseudo-observables:

- needed:    ◇ EW and QCD–EW 3-loop calculations

    ◇ $1 \rightarrow 2$ decays, fully inclusive

- problems:    ◇ technical: massive multi-loop integrals, $\gamma_5$

    ◇ conceptual: pseudo-obs. on the complex Z-pole

↪ Enormous challenge, but feasible (anticipating progress + support!)

Physikalisches Institut

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Stefan Dittmaier, *Precision Electroweak Calculations*   Symposium on the European Strategy, Granada, May 2019 – 8

# Forward-Backward asymmetry A<sub>FB</sub>(b)

- **Fit of F-B asymmetry as a function of the scattering angle**

$$A_{FB}^{q\bar{q}} = \frac{\sigma_F^q - \sigma_B^q}{\sigma_F^q + \sigma_B^q},$$

$$\frac{d\sigma^q}{d\cos\theta} = \sigma_{tot}^q \left[ \frac{3}{8}(1 + \cos^2\theta) + A_{FB}^{q\bar{q}} \cos\theta \right]$$

$$A_{FB}^{q\bar{q}}(\cos\theta) = \frac{8}{3} A_{FB}^{q\bar{q}} \frac{\cos\theta}{1 + \cos^2\theta} = \mathscr{A}_e \mathscr{A}_q \frac{2\cos\theta}{1 + \cos^2\theta}$$

  - At SLD, given the polarised beam they measured directly $\mathscr{A}_q$

- **Ingredients**
  - Tag quark-flavour (b-quark)
    - b-tagging as for $R_b$
  - Identification of quark vs anti-quark
    - charge of the leptons, jet-charge, vertex-charge, kaons
  - Determine the quark direction ($\vartheta$)
    - Use the "thrust" axis
      - ➡ sensitive to QCD effects

# quark vs anti-quark tagging

- Leptons
    - direct (b→l) versus cascade (b→c→l).
        - sensitive to effective B mixing.
        - uncertainties on sample composition from mode decays are large.
- Jet and secondary vertex charge

$$Q_h = \frac{\sum_i q_i \, p_{\parallel i}^{\kappa}}{\sum_i p_{\parallel i}^{\kappa}}$$

- typical values of k 0.3 to 1
- At SLD thanks to its superior tracking performance use secondary vertex charge
- Use double tagging techniques in a pure sample of b-quarks to estimate charge tagging mistake (otherwise limited by fragmentations and B-decays if taken from MC)
    - fraction of same sign double tags: 2 w*(1-w)



L3

- Data
- uds
- c
- fake l
- b→l
- b→c→l

Number of muons
Muon transverse momentum (GeV/c)
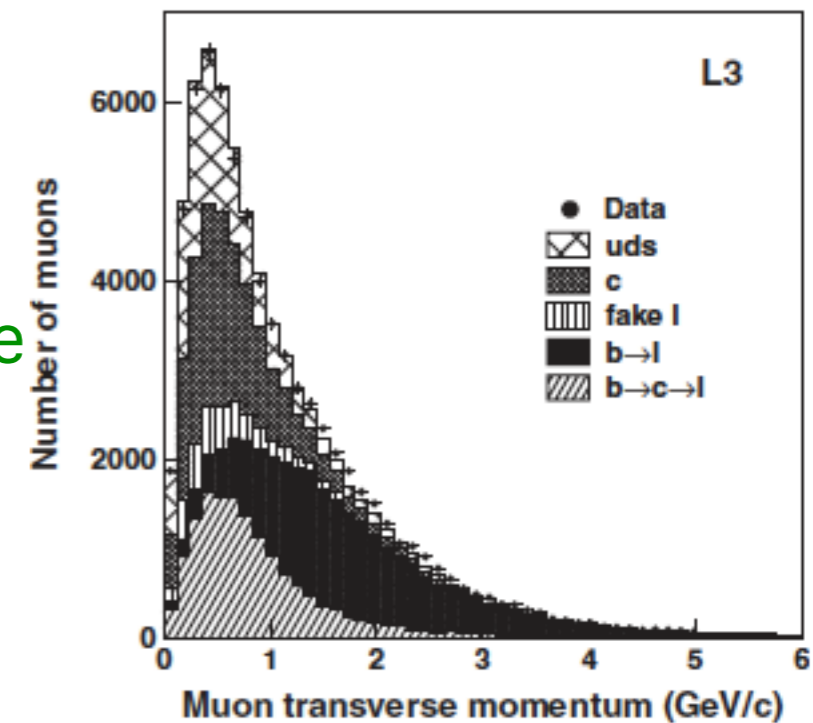
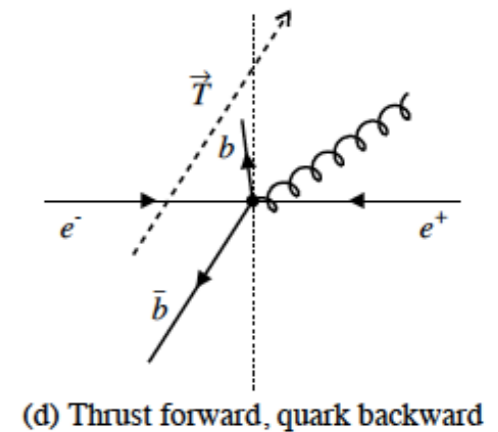# A$_{FB}$(b) precision
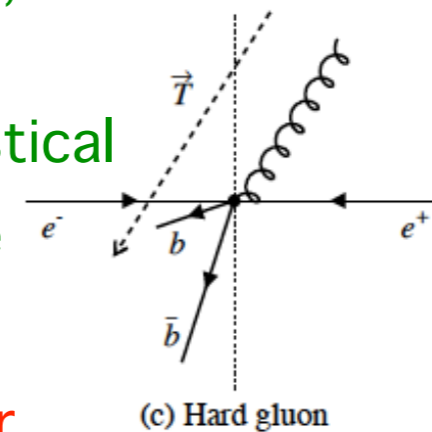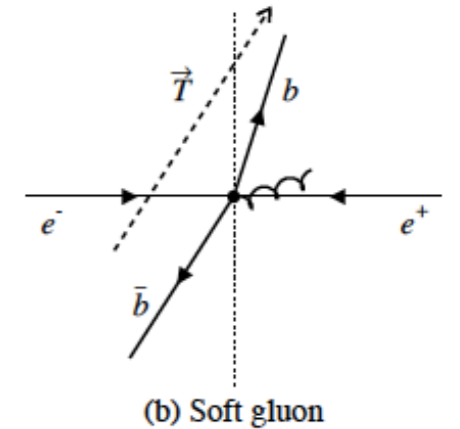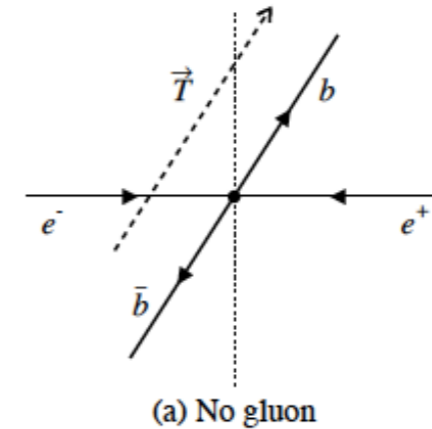
Current precision is limited by statistics

$$\Delta A = \sqrt{\frac{1 - A^2}{N}}$$

- At LEP+SLD 1.5x10-3

  - Statistical error at FCC-ee will be ~1000 times smaller, hence 1.5 $10^{-6}$
  - internal systematics (detector) 0.6x$10^{-3}$ mostly statistical
    - ➤Could be reduced by at least a factor ~2 at FCC-ee
  - QCD effects 0.4x$10^{-3}$
    - ➤Mostly theoretical (mass effects and missing higher orders)
    - ➤Experimental correction mainly due to selection and analysis methods, introducing bias, corrected using MC.
    - ➤Can be reduced by ~2 at FCC-ee ?

At FCC-ee: $\Delta A_{FB}(b) \sim \pm O(10^{-4})$ (systematic dominated)



(a) No gluon     (b) Soft gluon

(c) Hard gluon     (d) Thrust forward, quark backward

$$(A_{FB}^{q\bar{q}})_{meas} = (1 - C_{QCD})(A_{FB}^{q\bar{q}})_{no\ QCD}$$

| Error on $C_{QCD}^{had,T}$ | $b\bar{b}$ |
|---|---|
| Higher orders [192] | 0.0025 |
| Mass effects [140] | 0.0015 |
| Higher order mass [192] | 0.005 |
| $\alpha_s = 0.119 \pm 0.003$ | 0.0012 |

# Other methods to measure $A_{FB}(b)$

- **Could use exclusive B decays**
  - Likewise we did for charm using D*
  - Some BR:
    - BR $(B^+ \rightarrow D^o(K\pi)\ \pi^+) \sim 16 \times 10^{-5}$
    - BR $(B^+ \rightarrow D^o(K^-\pi^+\pi^o)\ \pi^+) \sim 56 \times 10^{-5}$
    - BR $(B^+ \rightarrow D^o(K^-2\pi^+\pi^-)\ \pi^+) \sim 32 \times 10^{-5}$
    - BR $(B^+ \rightarrow K^{*o}(892)\pi^+) \sim 10^{-5}$
    - BR $(B^0 \rightarrow K^+\pi^-) \sim 2 \times 10^{-5}$
    - BR $(B^o \rightarrow D^-(K^+2\pi^-)\pi^+) \sim 23 \times 10^{-5}$
    - BR $(B^o \rightarrow D^-(K^+2\pi^-\pi^o)\pi^+) \sim 15 \times 10^{-5}$
    - Plus many more decay modes

270 Bs candidates/4M Z



| Channel | Efficiency (%) |
|---|---|
| $B_s^0 \rightarrow D_s^- \pi^+ (\phi\pi^-)$ | 19 |
| $B_s^0 \rightarrow D_s^- \pi^+ (K^{\bullet 0} K^-)$ | 14 |
| $B_s^0 \rightarrow D_s^- \pi^+ (K_S^0 K^-)$ | 11 |
| $B_s^0 \rightarrow D_s^- a_1^+ (\phi\pi^-)$ | 10 |
| $B_s^0 \rightarrow D_s^- a_1^+ (K^{\bullet 0} K^-)$ | 5 |
| $B_s^0 \rightarrow D_s^- a_1^+ (K_S^0 K^-)$ | 7 |

- With $10^{12}$ B$^+$ (and B$^0$) and assuming a conservative 10% efficiency one could have few $10^8$ reconstructed events
  - stat error $<10^{-4}$
  - potential smaller systematics
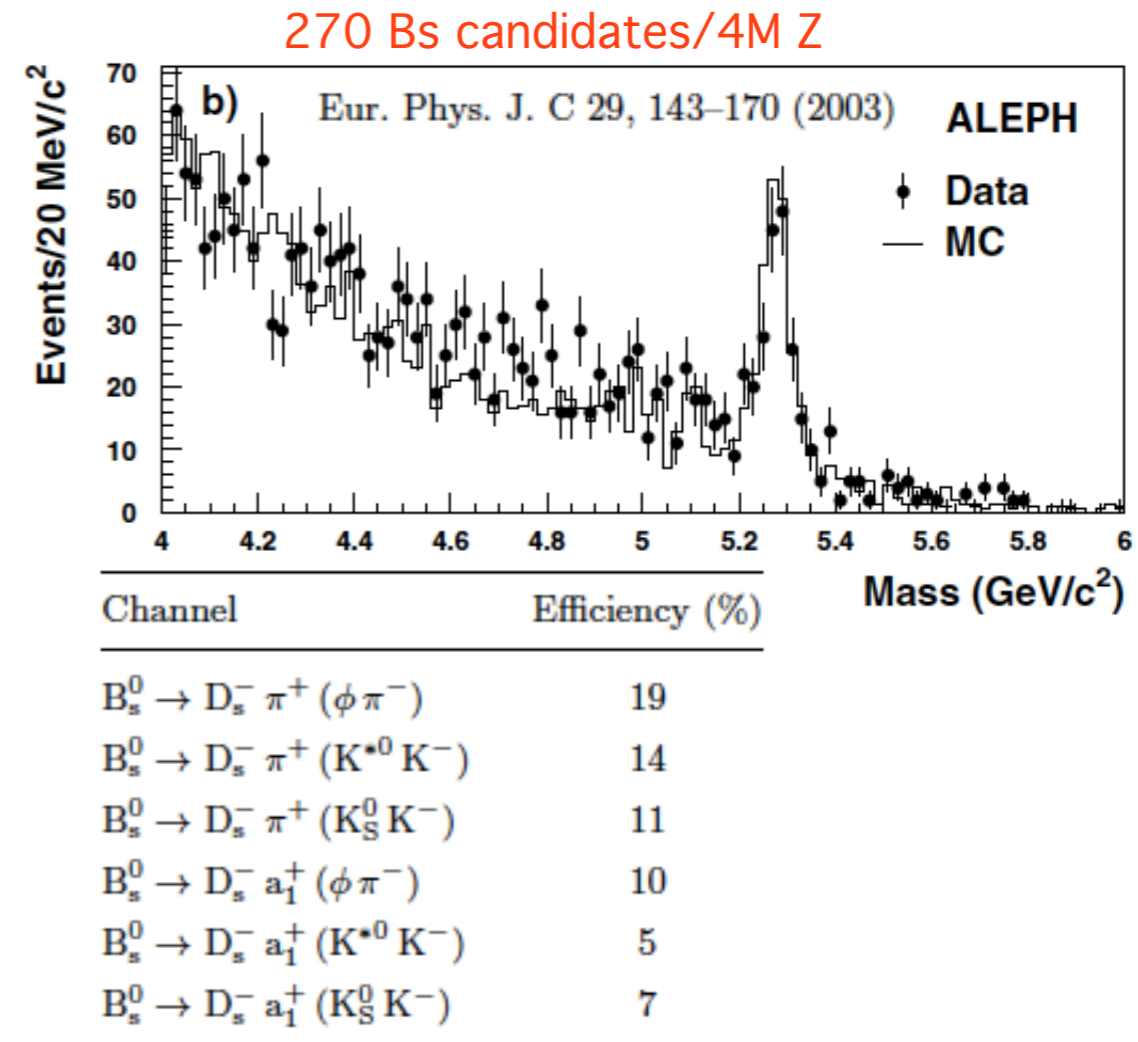
# Conclusions

- The incredible statistics foreseen to be collected at FCC-ee will allow unprecedented precision in measuring the electroweak b-physics parameters by at least one order of magnitude better than the current ones
  - Both $R_b$ and $A_{FB}(b)$ will be limited by systematics
    - $\Delta R_b$ (x $10^{-6}$) ~$\pm 0.3$ (stat) $\pm 60$ (syst)
    - $\Delta A_{FB}(b)$ (x $10^{-6}$) ~$\pm 1.5$ (stat) $\pm 100$ (syst)
  - Theory error for $R_b$ is expected to reach $50$ x $10^{-6}$

- Any hint of new physics could emerge already from early FCC-ee operations!

Backup

$$\Gamma(Z^0 \rightarrow q\bar{q}) = \frac{G_\mu M_Z^3}{8\pi\sqrt{2}}\left(v_q^2 + a_q^2\right)$$ *Born*

$$v_q = \left(1 + \delta\rho\right)\left(-1 + 4Q_q \sin^2\theta_{\text{eff}}^q\right)$$ *radiative corrections*

$$a_q = -\left(1 + \delta\rho\right)$$

$$v_b \rightarrow v_b\left(1 + \frac{4}{3}\delta\rho\right)$$

**b-quark specific**

$$a_b \rightarrow a_b\left(1 + \frac{4}{3}\delta\rho\right)$$
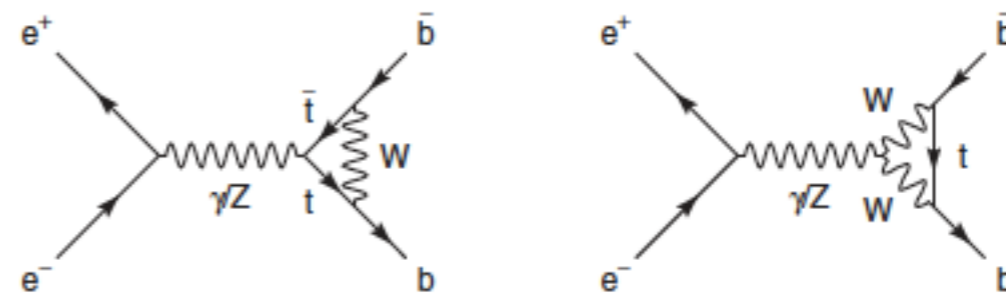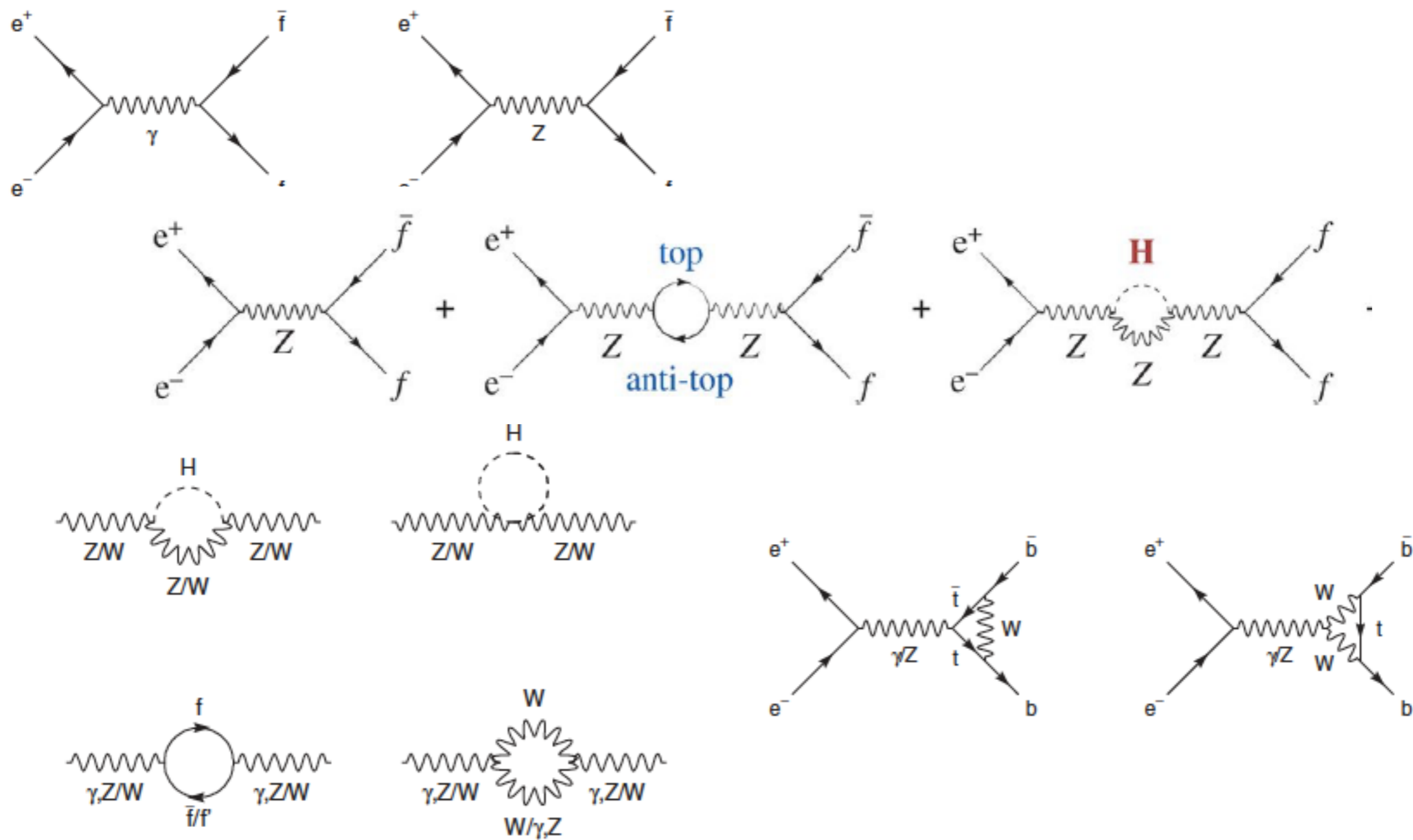


$$R_b = \frac{\Gamma_b}{\Gamma_{Z \rightarrow hadrons}} = R_d\left[1 - \frac{20}{13}\frac{\alpha}{\pi}\left(\frac{m_t^2}{M_Z^2} + \frac{13}{6}\log\frac{m_t^2}{M_Z^2}\right)\right] \sim R_d\left(1 - 0.02\right)$$

$$A_{\text{FB}}(b) = \frac{3}{4}A_e A_b$$

$$A_q = 2\frac{v_b a_b}{v_b^2 + a_b^2}$$

| | Measured | Theory prediction | Pull |
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| $R_b$ | $0.21629 \pm 0.00066$ | $0.21582 \pm 0.00002$ | $0.7$ |
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| $A_{FB}^{(0,b)}$ | $0.0992 \pm 0.0016$ | $0.1030 \pm 0.0002$ | $-2.3$ |

# How the measurement is effectively done …

$$\langle Q_{FB} \rangle = \langle Q_F - Q_B \rangle = \delta_q A_{FB}^{q\bar{q}},$$

$$\delta_q = \langle Q_q - Q_{\bar{q}} \rangle, \tag{5.16}$$

for a pure sample of $q\bar{q}$-events. The "charge separation" $\delta_q$ can be measured from data using:[19]

$$\left(\frac{\delta_q}{2}\right)^2 = \frac{\langle Q_F \cdot Q_B \rangle + \rho_{q\bar{q}}\sigma(Q)^2 + \mu(Q)^2}{1 + \rho_{q\bar{q}}}, \tag{5.17}$$

where $\mu(Q)$ is the mean value of $Q$ for all hemispheres and $\sigma(Q)$ is its variance. $\mu(Q)$ is slightly positive due to an excess of positive particles in secondary hadronic interactions. The hemisphere correlations, $\rho_{q\bar{q}}$, arise from charge conservation, hard gluon radiation and some other small effects and have to be taken from simulation.
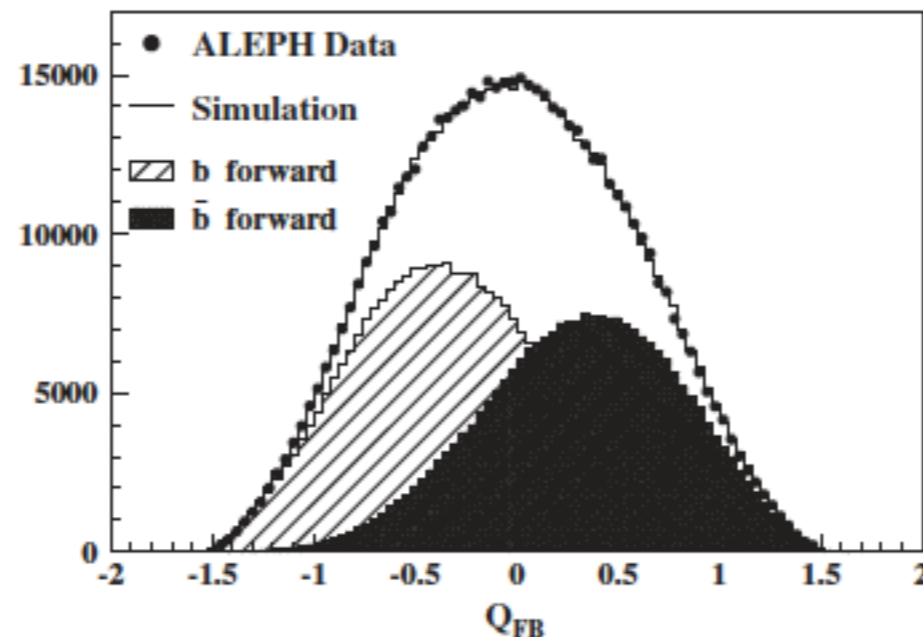
# DELPHI

**Table 14.** Detailed error breakdown for the measurement of $R_b$ from the multivariate analysis for the combined result

| Source of error | Range | $\Delta R_b \times 10^4$ |
|---|---|---|
| Data statistics | | $\pm 6.7$ |
| Simulation statistics | | $\pm 3.3$ |
| Event selection | | $\pm 0.9$ |
| Tracking | | $\pm 1.3$ |
| $K^0$, $\Lambda^0$, photons, etc. | see text | $\pm 0.4$ |
| Gluon splitting $g \to c\bar{c}$ | $(2.33 \pm 0.50)\%$ | $\pm 0.8$ |
| Gluon splitting $g \to b\bar{b}$ | $(0.269 \pm 0.067)\%$ | $\pm 2.7$ |
| $D^+$ fraction in $c\bar{c}$ events | $0.233 \pm 0.027$ | $\pm 1.2$ |
| $D_s$ fraction in $c\bar{c}$ events | $0.103 \pm 0.029$ | $\pm 0.3$ |
| $c$-baryon fraction in $c\bar{c}$ events | $0.063 \pm 0.028$ | $\pm 1.2$ |
| $BR(D^0 \to \text{no neutrals})$ | $(14.1 \pm 1.1)\%$ | $\pm 0.6$ |
| $BR(D^0 \to 1\,\text{neut.}, \geq 2\,\text{charged})$ | $(37.7 \pm 1.7)\%$ | $\pm 0.3$ |
| $BR(D^+ \to \text{no neutrals})$ | $(11.2 \pm 0.6)\%$ | $\pm 0.5$ |
| $BR(D^+ \to 1\,\text{neut.}, \geq 2\,\text{charged})$ | $(26.1 \pm 2.3)\%$ | $\pm 0.2$ |
| $BR(D_s \to K^0 X)$ | $(33 \pm 18)\%$ | $\pm 1.2$ |
| $D^0$ lifetime | $0.415 \pm 0.004$ ps | $\pm 0.3$ |
| $D^+$ lifetime | $1.057 \pm 0.015$ ps | $\pm 0.3$ |
| $D_s$ lifetime | $0.447 \pm 0.017$ ps | $\pm 0.3$ |
| $\Lambda_c$ lifetime | $0.206 \pm 0.012$ ps | $\pm 0.0$ |
| D decay multiplicity | see [18] | $\pm 0.8$ |
| $\langle x_E(c) \rangle$ | $0.484 \pm 0.008$ | $\pm 0.5$ |
| Two $b$'s same hemisphere | $\pm 30\%$ | $\pm 0.5$ |
| $\langle x_E(b) \rangle$ | $0.702 \pm 0.008$ | $\pm 1.2$ |
| B decay multiplicity | $4.97 \pm 0.07$ | $\pm 0.9$ |
| Average B lifetime | $1.55 \pm 0.05$ ps | $\pm 0.0$ |
| Angular effects | see text | $\pm 0.9$ |
| Gluon radiation | see text | $\pm 2.2$ |
| Total systematic error | | $\pm 6.0$ |

# SLD

| | $\delta R_b(10^{-5})$ | $\delta R_c(10^{-5})$ |
|---|---|---|
| MC statistics | 13 | 91 |
| $g \to b\bar{b}$ $0.254 \pm 0.051\%$ | $-24$ | 9 |
| $g \to c\bar{c}$ $2.96 \pm 0.38\%$ | $-23$ | $-101$ |
| long lived light hadron prod. $\pm 10\%$ | $-1$ | $-1$ |
| $D^+$ production $0.233 \pm 0.028$ | $-10$ | $-6$ |
| $D_s$ production $0.102 \pm 0.037$ | $-11$ | $-15$ |
| $c$-baryon production $0.065 \pm 0.029$ | $-11$ | 22 |
| charm fragmentation | $-18$ | 18 |
| $D^0$ lifetime $0.415 \pm 0.004$ ps | $-3$ | 8 |
| $D^+$ lifetime $1.057 \pm 0.015$ ps | $-2$ | 5 |
| $D_s$ lifetime $0.467 \pm 0.017$ ps | $-3$ | $-3$ |
| $\Lambda_c$ lifetime $0.206 \pm 0.012$ ps | $-1$ | $-91$ |
| D decay multiplicity | $-27$ | 60 |
| D decay $K^0$ | 19 | 56 |
| D decay no-$\pi^0$ | $-9$ | 12 |
| B lifetime $\pm 0.05$ ps | 0 | 5 |
| B decay $\langle N_{ch} \rangle = 5.73 \pm 0.35$ | $-20$ | 3 |
| $b$ fragmentation | 4 | 26 |
| $\Lambda_b$ production fraction $0.074 \pm 0.030$ | 5 | $-2$ |
| QCD hemisphere correlation | 6 | 22 |
| hard gluon radiation | $-2$ | 26 |
| tag geometry dependency | 9 | 17 |
| tag time dependency | 1 | 1 |
| component correlation | 14 | 45 |
| tracking resolution | 27 | 22 |
| tracking efficiency | 13 | 3 |
| $\langle IP \rangle_{xy}$ tail | 2 | 0 |
| event selection bias | 17 | 20 |
| 4 jet rate in $b$ events | 15 | 0 |
| $R_c = 0.1723 \pm 0.0037$ | $-12$ | |
| $R_b = 0.2157 \pm 0.0010$ | | $-62$ |
| Total (excl. $R_{b/c}$) | 73 | 200 |

TABLE IV.    Hemisphere correlation component check results

| Component | $(C_{b-\text{tag}} - 1) \times 10^5$ | |
| --- | --- | --- |
| | 97–98 | 96 |
| Primary vertex | +46 | +13 |
| Geometrical correlation $\theta$ | +49 | +60 |
| Geometrical correlation $\phi$ | −4 | +212 |
| Time dependence | 11 | +434 |
| $B/D$ momentum and thrust angle | +107 | +95 |
| Hard gluon radiation | −37 | −23 |
| Component sum | +170 | +670 |
| MC overall correlation | +42 | +891 |
| MC statistical error | ±47 | ±113 |
| discrepancy | +128 | −121 |

SLD