QED at the Z pole: Challenges

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Explaining Master Table of
https://arxiv.org/abs/1903.09895

What are PSEUDO-OBSERVABLEs (POs)?

What is QED-induced uncertainty in PO?

Desired improvement factor for QED!

<table>
<thead>
<tr>
<th>Observable</th>
<th>Where from</th>
<th>Present (LEP)</th>
<th>FCC stat.</th>
<th>FCC syst</th>
<th>Now FCC</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_Z$ [MeV]</td>
<td>$Z$ linesh.</td>
<td>$91187.5 \pm 2.1{0.3}$</td>
<td>0.005</td>
<td>0.1</td>
<td>3</td>
</tr>
<tr>
<td>$\Gamma_Z$ [MeV]</td>
<td>$Z$ linesh.</td>
<td>$2495.2 \pm 2.1{0.2}$</td>
<td>0.008</td>
<td>0.1</td>
<td>2</td>
</tr>
<tr>
<td>$R_l^Z = \Gamma_h/\Gamma_l$</td>
<td>$\sigma(M_Z)$</td>
<td>$20.767 \pm 0.025{0.012}$</td>
<td>$6 \cdot 10^{-5}$</td>
<td>$1 \cdot 10^{-3}$</td>
<td>12</td>
</tr>
<tr>
<td>$\sigma_{\text{had}}^0$ [nb]</td>
<td>$\sigma_{\text{had}}^0$</td>
<td>$41.541 \pm 0.037{0.025}$</td>
<td>$0.1 \cdot 10^{-3}$</td>
<td>$4 \cdot 10^{-3}$</td>
<td>6</td>
</tr>
<tr>
<td>$N_{\nu}$</td>
<td>$\sigma(M_Z)$</td>
<td>$2.984 \pm 0.003{0.006}$</td>
<td>$5 \cdot 10^{-6}$</td>
<td>$1 \cdot 10^{-3}$</td>
<td>6</td>
</tr>
<tr>
<td>$N_{\nu}$</td>
<td>$Z\gamma$</td>
<td>$2.69 \pm 0.15{0.06}$</td>
<td>$0.8 \cdot 10^{-3}$</td>
<td>$&lt; 10^{-3}$</td>
<td>60</td>
</tr>
<tr>
<td>$\sin^2 \theta_W^{eff} \times 10^5$</td>
<td>$A_{FB}^{\text{lept.}}$</td>
<td>$23099 \pm 53{28}$</td>
<td>0.3</td>
<td>0.5</td>
<td>55</td>
</tr>
<tr>
<td>$\sin^2 \theta_W^{eff} \times 10^5$</td>
<td>$\langle P_{\tau}\rangle, A_{FB}^{\text{pol,\tau}}$</td>
<td>$23159 \pm 41{12}$</td>
<td>0.6</td>
<td>$&lt; 0.6$</td>
<td>20</td>
</tr>
<tr>
<td>$M_W$ [MeV]</td>
<td>ADLO</td>
<td>$80376 \pm 33{6}$</td>
<td>0.5</td>
<td>0.3</td>
<td>12</td>
</tr>
<tr>
<td>$A_{FB,\mu}^{M_Z \pm 3.5 \text{GeV}}$</td>
<td>$\frac{d\sigma}{d\cos \theta}$</td>
<td>$\pm 0.020{0.001}$</td>
<td>$1.0 \cdot 10^{-5}$</td>
<td>$0.3 \cdot 10^{-5}$</td>
<td>100</td>
</tr>
</tbody>
</table>

How LEP and FCC-ee exp. precisions do compare?
What are EW pseudo-observables (EWPOs)?

Example of EWPO: $\sigma^0_{\text{had}}$

Experimental $\sigma_{\text{had}}(s_i)$ measured at 7 energies are fit using 1-D convolution formula

$$\sigma(s) = \int_0^1 dz \sigma^{\text{Born}}(zs) \rho_{\text{QED}}(z)$$

and $\sigma^0_{\text{had}} = \sigma^0_{\text{had}}(M_Z)$ is calculated afterwards! $Z$ Mass and width from the same fit.

Induced QED uncertainty (next slide) enters through $\rho_{\text{QED}}$

### Table 2.4: Individual results on Z parameters and their correlation coefficients from the four experiments. Systematic errors are included here except those summarised in Table 2.9.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Correlation Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_Z$ [GeV]</td>
<td>91.1858 ± 0.0030</td>
<td>1.000</td>
</tr>
<tr>
<td>$\Gamma_Z$ [GeV]</td>
<td>2.4948 ± 0.0041</td>
<td>0.049</td>
</tr>
<tr>
<td>$\sigma^0_{\text{had}}$</td>
<td>41.501 ± 0.055</td>
<td>0.031</td>
</tr>
<tr>
<td>$R^{\gamma}$</td>
<td>20.901 ± 0.084</td>
<td>0.108</td>
</tr>
<tr>
<td>$R^{\rho}$</td>
<td>20.811 ± 0.058</td>
<td>0.001</td>
</tr>
<tr>
<td>$R^{\mu}$</td>
<td>20.832 ± 0.091</td>
<td>0.001</td>
</tr>
<tr>
<td>$A_{\text{FB}}^0$</td>
<td>0.0089 ± 0.0045</td>
<td>-0.053</td>
</tr>
<tr>
<td>$A_{\text{FB}}^\mu$</td>
<td>0.0159 ± 0.0023</td>
<td>0.077</td>
</tr>
<tr>
<td>$A_{\text{FB}}^\tau$</td>
<td>0.0145 ± 0.0030</td>
<td>0.059</td>
</tr>
</tbody>
</table>

Where is QED-induced uncertainty of PO in the landscape of theory and exp. errors?

**EXPERIMENT**

- Total Error of the experimental pseudo-observable
  - Experimental error
    - Statistical
    - Systematics: detector, backgrounds, accelerator...
  - Systematics: QED, EW, QCD (perturbative h.o., progr. bugs in MCs)

**THEORY**

- Total Error of the SM prediction
  - Parametric: Error due to input parameters
  - Intrinsic: Theory uncertainties due to perturbative higher orders
  - Error due to input parameters
  - Statistical systematic

**CONFRONT**

- HERE!
Induced QED error in LEP pseudo-observables?

In LEP experiments QED uncertainty was safely below pure experimental errors
What are EW pseudo-observables (EWPOs)?

Example of charge asymmetry is more complicated:

\[ A_{FB}^{\mu,0} = \frac{\int_F d\sigma^{Born} - \int_B d\sigma^{Born}}{\int_F d\sigma^{Born} + \int_B d\sigma^{Born}} \bigg|_{s=M_Z^2} \]

**calculated** using \[ \frac{d\sigma^{Born}(s)}{d \cos \theta} [g_{V}^{\mu}, g_{A}^{\mu}] \]

Eff. Born is central in EWPO construction!

\[ \frac{2s}{\pi N_c^2} \frac{d\sigma_{ew}}{d \cos \theta} (e^+e^- \rightarrow ff) = \]
\[ \frac{|\alpha(s)Q_i|^2 (1 + \cos^2 \theta)}{\sigma^+} \]
\[ -8R \left\{ \alpha(s)Q_i \chi(s) \left[ G_{Ve} G_{Vi} (1 + \cos^2 \theta) + 2G_{Ve} G_{Ai} \cos \theta \right] \right\} \]
\[ \gamma-Z \text{ interference} \]
\[ +16|\chi(s)|^2 \left[ |G_{Ve}|^2 + |G_{Ve} A_i|^2 \right] (1 + \cos^2 \theta) \]
\[ +8|G_{Ve} G_{Ai} | \Re \left\{ G_{Ve} G_{Ai}^* \cos \theta \right\} \]
\[ \sigma^Z \]

with:
\[ \chi(s) = \frac{G_F m_Z^2}{8\pi\sqrt{2}} \frac{s}{s - m_Z^2 + i\sigma_Z/m_Z}, \]

\[ g_{V,A}^{f} = R(G_{Vf,Af}) \]

\[ A_{FB}^{\mu}(s_i), \sigma(s_i) \] are fit to \[ A_{FB}^{\mu}(s_i), \sigma(s_i) \] at several \( s_i \) using convolution formula

\[ \frac{d\sigma^{\mu}}{d \cos \theta^*}(s, \theta^*) = \text{CONV} \left\{ \frac{d\sigma^{Born}(s)}{d \cos \theta}, \rho_{QED} \right\}, \quad \theta^* \neq \theta \]
What are EW pseudo-observables (EWPOs)?

From experimental DATA to EWPO — effective Born is central object!

Two key points:

1. The convolution formula approximates QED, including (at LEP) \(\mathcal{O}(\alpha^1), \mathcal{O}(L_e^2\alpha^2), \mathcal{O}(L_e^3\alpha^3), \mathcal{O}(L_e^2\alpha^1)\), etc. (It may include 1-st order IFI.)
   
   Most likely will be replaced by the Monte Carlo to attain FCC-ee precision.

2. The role of the effective Born is to encapsulate/represent data within exp. precision in the (SM) Model independent way. At FCC-ee precision it may necessarily include more of h.o. SM (EW boxes?), then just only imaginary parts of \(g_V, g_A\) !!!
Basic circular test (B)->(C)->(D)->(B) will be at FCC-ee the same as in LEP

Main difference with LEP is Monte Carlo use in steps (B)->(C) and (B)->(D) instead of progs like ZFITTER/TOPAZ0

For LEP version see:
At the FCC-ee exp. precisions present QED uncertainty is unacceptable!

Current QED precision vs. FCCee exp. error

Anticipated FCC-ee experimental precision
Needed improvement for QED precision at FCCee

Depending on the observable factor 6-200 improvements needed!
The same but with difficulty rating and planning what to be done?

<table>
<thead>
<tr>
<th>Observable</th>
<th>Source</th>
<th>Err. ${QED}$ LEP</th>
<th>Stat[Syst] FCC-ee</th>
<th>LEP FCC-ee</th>
<th>main development to be done</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_Z$ [MeV]</td>
<td>$Z$ linesh.</td>
<td>2.1{0.3}</td>
<td>0.005[0.1]</td>
<td>$3 \times 3^*$</td>
<td>light fermion pairs</td>
</tr>
<tr>
<td>$\Gamma_Z$ [MeV]</td>
<td>$Z$ linesh.</td>
<td>2.1{0.2}</td>
<td>0.008[0.1]</td>
<td>$2 \times 3^*$</td>
<td>fermion pairs</td>
</tr>
<tr>
<td>$R^Z_l \times 10^3$</td>
<td>$\sigma(M_Z)$</td>
<td>25{12}</td>
<td>0.06[1.0]</td>
<td>$12 \times 3^{**}$</td>
<td>better FSR</td>
</tr>
<tr>
<td>$\sigma^0_{\text{had}}$ [pb]</td>
<td>$\sigma^0_{\text{had}}$</td>
<td>37{25}</td>
<td>0.1[4.0]</td>
<td>$6 \times 3^*$</td>
<td>better lumi MC</td>
</tr>
<tr>
<td>$N_\nu \times 10^3$</td>
<td>$\sigma(M_Z)$</td>
<td>8{6}</td>
<td>0.005[1.0]</td>
<td>$6 \times 3^*$</td>
<td>CEEX in lumi MC</td>
</tr>
<tr>
<td>$\sin^2 \theta_W \times 10^5$</td>
<td>$Z\gamma$</td>
<td>150{60}</td>
<td>0.8[&lt; 1]</td>
<td>$60 \times 3^{**}$</td>
<td>$\mathcal{O}(\alpha^2)$ for $Z\gamma$</td>
</tr>
<tr>
<td>$\sin^2 \theta_W \times 10^5$</td>
<td>$A^\text{lept.}_{FB}$</td>
<td>53{28}</td>
<td>0.3[0.5]</td>
<td>$55 \times 3^{**}$</td>
<td>h.o. and EWPOs</td>
</tr>
<tr>
<td>$\langle P_{\tau}, A_{FB}^\text{pol,}\tau \rangle$</td>
<td>$A_{FB}$</td>
<td>41{12}</td>
<td>0.6[&lt; 0.6]</td>
<td>$20 \times 3^{**}$</td>
<td>better $\tau$ decay MC</td>
</tr>
<tr>
<td>$M_W$ [MeV]</td>
<td>mass rec.</td>
<td>33{6}</td>
<td>0.5[0.3]</td>
<td>$12 \times 3^{**}$</td>
<td>QED at threshold</td>
</tr>
<tr>
<td>$A_{FB,\mu}$ $^{M_Z \pm 3.5}\text{GeV} \times 10^5$</td>
<td>$\frac{d\sigma}{d\cos\theta}$</td>
<td>2000{100}</td>
<td>1.0[0.3]</td>
<td>$100 \times 3^{***}$</td>
<td>improved IFI</td>
</tr>
</tbody>
</table>

Table 2: Comparing experimental and theoretical errors at LEP and FCC-ee as in Table 1, 3rd column shows LEP experimental error together with uncertainty induced by QED and 4th column shows anticipated FCC-ee experimental statistical [systematic] errors. Additional factor $\times 3$ in the 5-th column (4th in Table 1) reflects what is needed for QED effects to be subdominant. Rating from $^*$ to $^{**}$ marks whether the needed improvement is relatively straightforward, difficult or very difficult to achieve.

More details for selected observables
Present (LEP)

**No cut-offs** (except on $\sum E_T$)

QED err. according to ADLO 2005: $\delta M_Z, \delta \Gamma_Z \simeq 0.2 - 0.3$ MeV

$\sigma_{\text{had}}$ ISR: $\mathcal{O}(\alpha^1 L_1, \alpha^1, \alpha^2 L_2^2, \alpha^2 L_1^1, \alpha^3 L_3^3)_\gamma$ $\mathcal{O}(\alpha^2 L_2^2, \alpha^2 L_1^1, \alpha^3 L_3^3)_\text{pairs}$

$\sigma_{\text{lept}}$ ISR+FSR

Non-MC implementation, 1-d or 2-d convolution

Initial-final interference (IFI) neglected

**Simplified idealised cut-offs**

ZFITTER and TOPAZ0 non-MC programs

AND

MC event generators: KORALZ, KKMC, BHWIDE

**Arbitrary realistic cut-offs**

MC event generators: KORALZ, KKMC, BHWIDE

For luminosity uncertainty see next…

FCC-ee

**No cut-offs**

exp. $\delta M_Z, \delta \Gamma_Z \leq 0.1$ MeV, QED $\leq 0.03$ MeV

Factor $\sim 10$ improvement in QED is needed!

LEP simplistic convolution may survive only for $\sigma_{\text{had}}$ provided pairs improved, $\mathcal{O}(\alpha^2 L_1^0, \alpha^3 L_2^2, \alpha^4 L_3^4)_\gamma$ are added and mixed QCD-QED corrections are improved.

For leptons MC will *take over* due to IFI and pairs

**Simplified idealised cut-offs**

Only MC event generators of the KKMC class or better will be able to match FCC-ee precision

**Arbitrary realistic cut-offs**

Only MC event generators of the KKMC class or better:

Upgrades of the matrix element:

$\mathcal{O}(\alpha^2 L_1^1)$ penta-boxes, $\mathcal{O}(\alpha^3 L_2^3)$ in CEEX m.e.

Inventing new MC approach for light fermion pairs.

Provisions for SM parameter fitting and extracting new EWPOs from data
Charge and spin asymmetries at mZ

Present (LEP)

**Charge asymmetry**

QED err. at LEP: \( \delta A^\mu_{FB}(M_Z) \simeq 50 \cdot 10^{-5} \)

translates into \( \delta \sin^2 \theta_{\text{eff}} \simeq 28 \cdot 10^{-5} \)

[Conservative estimate based on comparisons of KKMC, ZFITTER, KORALZ, Phys. Ref. D63 (2001) 113009]

However, the effects due to ISR, IFI, EW boxes, imaginary parts of Z couplings, gamma exch. background are genuinely of order \( \delta A^\mu_{FB}(M_Z) \simeq 10 \cdot 10^{-5} \)

**FCC-ee**

FCC-ee exp. error \( \delta A^\mu_{FB}(M_Z) \simeq 1 \cdot 10^{-5} \)

\( \delta \sin^2 \theta_{\text{eff}} \simeq 0.5 \cdot 10^{-5} \)

Factor \( \sim 50-150 \) improvement in QED is needed!

Once they are mastered with 10\% precision, the way to \( \delta A^\mu_{FB}(M_Z) \simeq 1 \cdot 10^{-5} \) is open!

KKMC with complete \( \mathcal{O}(\alpha^2) \) matrix element, soft photon resummation including IFI, EW corrections is already there. One needs the same for Bhabha!

The biggest challenge is, may be, the consistent definition of \( \sin^2 \theta_{\text{eff}} \) at the FCC-ee precision!

**Spin asymmetries**

\( \langle \mathcal{P}_\tau \rangle \) and \( A^\text{pol,}\tau_{FB} \) at LEP were worth \( \delta \sin^2 \theta_{\text{eff}} \simeq 41 \cdot 10^{-5} \)

including QED induced uncertainty due to photon emissions in tau decays \( \delta \sin^2 \theta_{\text{eff}} \simeq 12 \cdot 10^{-5} \)

QED err. is small due to weak dependence on CMS energy.

Expected FCC-ee exp. error \( \delta \sin^2 \theta_{\text{eff}} \simeq 0.6 \cdot 10^{-5} \)

Factor \( \sim 20-60 \) improvement in QED is needed!

To be studied:
- polarimeter biases due to decay channel cross-talk and photon emission in tau decays
- QED effects in tau-pair production
- exploiting super-Belle tau decay data in order to calibrate tau decay MC simulation
\( \alpha_{QED}(M_Z) \) from \( A_{FB}(M_Z \pm 3.5 GeV) \)

- Determination of \( \alpha_{QED}(M_Z) = \alpha(0)/(1 - \Delta \alpha) \) with precision \( \sim 3 \times 10^{-5} \) critical for SM fits.
- Table of parametric uncertainty with
  \[ \delta M_Z \approx 0.1 MeV, \quad \delta m_t \approx 50 MeV \]
  \[ \delta \alpha_s \approx 2 \cdot 10^{-4}, \quad \delta(\Delta \alpha) \approx 5 \cdot 10^{-5} \]

\[
\begin{array}{|c|c|c|c|c|}
\hline
\text{EWPO} & \text{Exp. direct error} & \text{Param. error} & \text{Main source} & \text{Theory uncert.} \\
\hline
\Gamma_Z [\text{MeV}] & 0.1 & 0.1 & \delta \alpha_s & 0.07 \\
R_b [10^{-5}] & 6 & 1 & \delta \alpha_s & 3 \\
R_t [10^{-3}] & 1 & 1.3 & \delta \alpha_s & 0.7 \\
\sin^2 \theta^f_{\ell} [10^{-5}] & 0.5 & 1 & \delta(\Delta \alpha) & 0.7 \\
M_W [\text{MeV}] & 0.5 & 0.6 & \delta(\Delta \alpha) & 0.3 \\
\hline
\end{array}
\]

Table 3: Estimated experimental precision for the direct measurement of several important EWPOs at FCC-ee [2] (column two) and experimental parametric error (column three), with the main source shown in the forth column. Important input parameter errors are \( \delta(\Delta \alpha) = 3 \cdot 10^{-5}, \delta \alpha_s = 0.00015 \) see FCC CDR, vol. 2 [1]. Last column shows anticipated theory uncertainties at start of FCC-ee.

- Measuring \( A_{FB}(M_Z \pm 3.5 GeV) \) with precision \( 3 \times 10^{-5} \), factor 200 more precisely than at LEP was proposed in order to get \( \alpha_{QED}(M_Z) \) with the needed precision \( \sim 10^{-5} \).
- QED Initial-Final state interference IFI is the main obstacle!
- IFI cancels partly in the difference \( \tilde{A}_{FB}(M_Z \pm 3.5 GeV) \), but \( \sim 1\% \) effect remains.
  Can one control IFI in \( A_{FB} \) with the precision \( 3 \times 10^{-5} \) ???
- In arXiv:1801.08611 Phys. Rev. D (S.J. and S.Yost) it was shown that using KKMC and new KKfoam programs one may get precision \( \leq 10^{-4} \)
• **LEP legacy, lumi TH error budget**

<table>
<thead>
<tr>
<th>Type of correction/error</th>
<th>LEP1 1996</th>
<th>LEP2 1996</th>
<th>LEP1 1999</th>
<th>LEP2 1999</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Missing photonic $O(\alpha^2)$ [4,5]</td>
<td>0.10%</td>
<td>0.20%</td>
<td>0.027%</td>
<td>0.04%</td>
</tr>
<tr>
<td>(b) Missing photonic $O(\alpha^2 L^3)$ [6]</td>
<td>0.015%</td>
<td>0.03%</td>
<td>0.015%</td>
<td>0.03%</td>
</tr>
<tr>
<td>(c) Vacuum polarization [7,8]</td>
<td>0.04%</td>
<td>0.10%</td>
<td>0.04%</td>
<td>0.10%</td>
</tr>
<tr>
<td>(d) Light pairs [9,10]</td>
<td>0.03%</td>
<td>0.05%</td>
<td>0.03%</td>
<td>0.05%</td>
</tr>
<tr>
<td>(e) Z-exchange [11,12]</td>
<td>0.015%</td>
<td>0.0%</td>
<td>0.015%</td>
<td>0.0%</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>0.11% [12]</td>
<td>0.25% [12]</td>
<td>0.061% [13]</td>
<td>0.12% [13]</td>
</tr>
</tbody>
</table>

*Table 1: Summary of the total (physical+technical) theoretical uncertainty for a typical calorimeteric detector. For LEP1, the above estimate is valid for a generic angular range within $1^\circ$-$3^\circ$ (18-52 mrad), and for LEP2 energies up to 176 GeV and an angular range within $3^\circ$-$6^\circ$. Total uncertainty is taken in quadrature. Technical precision included in (a).*

• **By the time of FCC-ee VP contribution will be merely 0.006%**

• **QED corrections and Z contrib. come back to front!**

• **Z contr. easy to master, even if rises at FCC-ee, because (28-58)->(64-86) mrad.**

• **Our FCC-ee forecast is 0.01% provided QED m.e. and VP are improved.**

![arXiv:1902.05912](Low angle Bhabha (luminosity) at FCCee arXiv:1902.05912)

**LEP lumi update 2018**

<table>
<thead>
<tr>
<th>Type of correction / Error</th>
<th>1999</th>
<th>Update 2018</th>
<th>FCCee forecast</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Photonic $O(L\alpha^2)$</td>
<td>0.027% [5]</td>
<td>0.027%</td>
<td>$0.6 \times 10^{-5}$</td>
</tr>
<tr>
<td>(b) Photonic $O(L^3\alpha^3)$</td>
<td>0.015% [6]</td>
<td>0.015%</td>
<td>$0.1 \times 10^{-4}$</td>
</tr>
<tr>
<td>(c) Vacuum polariz.</td>
<td>0.040% [7,8]</td>
<td><strong>0.013% [25]</strong></td>
<td>$0.6 \times 10^{-4}$</td>
</tr>
<tr>
<td>(d) Light pairs</td>
<td>0.030% [10]</td>
<td>0.010% [18, 19]</td>
<td>$0.5 \times 10^{-4}$</td>
</tr>
<tr>
<td>(e) s-channel Z-exchange</td>
<td>0.015% [11,12]</td>
<td>0.015%</td>
<td>$0.1 \times 10^{-4}$</td>
</tr>
<tr>
<td>(f) Up-down interference</td>
<td>0.0014% [27]</td>
<td>0.0014%</td>
<td>$0.1 \times 10^{-4}$</td>
</tr>
<tr>
<td>(f) Technical Precision</td>
<td>(0.027)%</td>
<td>(0.027)%</td>
<td>$1.0 \times 10^{-4}$</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>0.061% [13]</strong></td>
<td><strong>0.038%</strong></td>
<td>$1.0 \times 10^{-4}$</td>
</tr>
</tbody>
</table>
Z invisible width from peak cross section and radiative return

Present (LEP)

QED err. of luminosity  \[ \frac{\delta \mathcal{L}}{\mathcal{L}} = \frac{\delta \sigma_{\text{had}}^0}{\sigma_{\text{had}}^0} \approx 0.06\% \]
dominate LEP exp. error  \[ N_\nu \approx 2.984 \pm 0.008 \{\pm 0.006\}_{QED} \]

Radiative return I

\[ e^+ e^- \rightarrow \nu \bar{\nu} \gamma \]
\[ N_\nu \approx 2.69 \pm 0.15 \{\pm 0.06\}_{QED} \]

Limited by poor LEP statistics at 161GeV

FCC-ee

Peak cross section

FCC-ee exp. error (syst.)  \[ \delta N_\nu \approx 0.001 \]
Factor ~10 improvement in luminosity is needed!
\[ \frac{\delta \mathcal{L}}{\mathcal{L}} \approx 10^{-4} \rightarrow \delta N_\nu \approx 8 \cdot 10^{-4} \text{ seems achievable.} \]

Radiative return II

Measuring ratio  \[ R = \frac{\sigma_{\nu \bar{\nu} \gamma}}{\sigma_{\mu^+\mu^-\gamma}} \]
Luminosity error drops out!

QED uncertainty due to FSR in  \[ \sigma_{\mu^+\mu^-\gamma} \text{ rated at } 0.03\% \]
(unpublished study using KKMC).
Again  \[ \delta N_\nu \approx 0.001 \]
Summary

- Major effort is needed to improve SM/QED predictions for FCC-ee observables by factor 10-200
- In particular QED corrections for asymmetries near Z has to be improved by factor up to 200
- New algorithms of extracting EW pseudo-observables from experimental data has to be worked out and cross-checked
- Increased role of MC event generators is anticipated

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Reserve
5-dim convolution formula including IFI

NEW analytical exponentiation formula for ISR+FSR+IFI

Eq.(90) in [JWW2001] and in older Frascati works, implemented recently in KKfoam

\[
\frac{d\sigma}{d\Omega}(s, \theta, v_{max}) = \sum_{V, V' = \gamma, Z} \int d\theta \ dv_F \ dv_{IF} \ dv_{FI} \ \theta(v_I - v_F - v_{IF} - v_{FI} < v_{max}) \\
\times F(\gamma_I) \gamma_{II}^{\gamma_{II}^{-1}} \ F(\gamma_F) \gamma_{IF}^{\gamma_{IF}^{-1}} \ F(\gamma_{IF}) \gamma_{FI}^{\gamma_{FI}^{-1}} \ F(\gamma_{FI}) \gamma_{FII}^{\gamma_{FII}^{-1}} \\
\times e^{2\alpha \Delta B_4^Z} \ M^{(0)}_V(s(1 - v_I - v_{IF}), \theta) \ [e^{2\alpha \Delta B_4^Z} \ M^{(0)}_{V'}(s(1 - v_I - v_{FI}), \theta)]^* \ [1 + \text{NIR}(v_I, v_F)],
\]

- Convolution of four radiator functions (instead of two)!
- Extra virtual formfactor $\Delta B_4^Z$ due to IFI for resonant contrib.
- $\gamma_I = Q_e^2 \frac{\alpha}{\pi} \left[ \frac{s}{m_e^2} - 1 \right]$, $\gamma_{IF} = \gamma_{FI} = Q_e Q_i \frac{\alpha}{\pi} \ln \frac{1 - \cos \theta}{1 + \cos \theta}$, $\gamma(\gamma) = \frac{e^{-C_E \gamma}}{f(1 + \gamma)}$

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