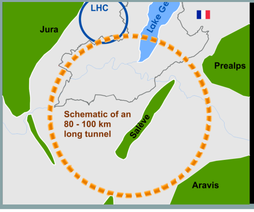

QED at the Z pole: Challenges

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Polish Academy of Sciences

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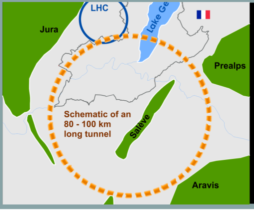
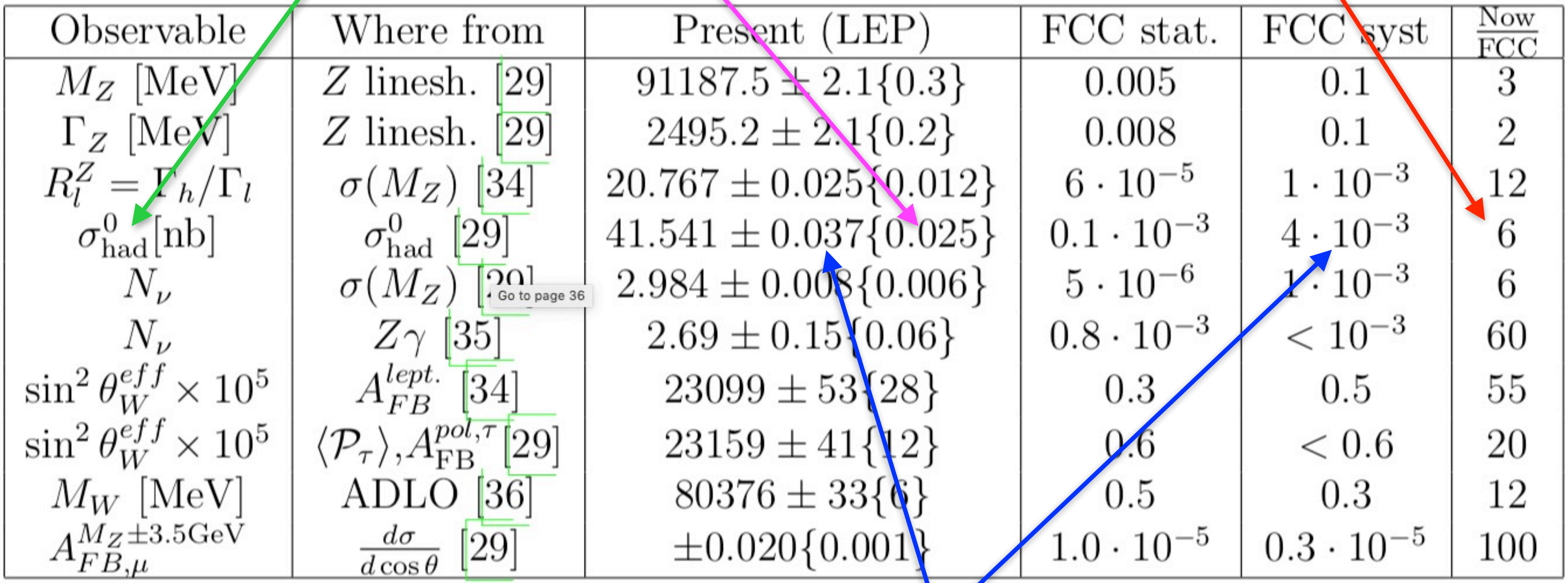


Explaining Master Table of https://arxiv.org/abs/1903.09895

What are PSEUDO-OBSERVABLEs (POs)?

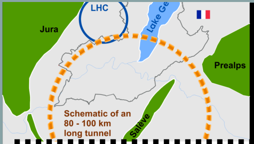
Desired improvement factor for QED!

What is QED-induced uncertainty in PO?



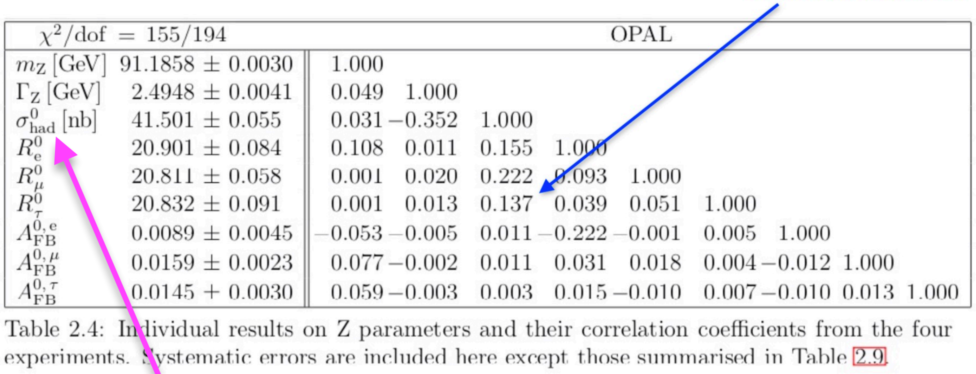
Observable	Where from	Present (LEP)	FCC stat.	FCC syst	Now/FCC
M_Z [MeV]	Z linesh. [29]	$91187.5 \pm 2.1\{0.3\}$	0.005	0.1	3
Γ_Z [MeV]	Z linesh. [29]	$2495.2 \pm 2.1\{0.2\}$	0.008	0.1	2
$R_l^Z = \Gamma_h/\Gamma_l$	$\sigma(M_Z)$ [34]	$20.767 \pm 0.025\{0.012\}$	$6 \cdot 10^{-5}$	$1 \cdot 10^{-3}$	12
$\sigma^0_{\rm had}$[nb]	$\sigma^0_{\rm had}$ [29]	$41.541 \pm 0.037\{0.025\}$	$0.1 \cdot 10^{-3}$	$4 \cdot 10^{-3}$	6
N_ν	$\sigma(M_Z)$ [29]	$2.984 \pm 0.008\{0.006\}$	$5 \cdot 10^{-6}$	$1 \cdot 10^{-3}$	6
N_ν	$Z\gamma$ [35]	$2.69 \pm 0.15\{0.06\}$	$0.8 \cdot 10^{-3}$	$< 10^{-3}$	60
$\sin^2\theta_W^{eff} \times 10^5$	$A_{FB}^{lept.}$ [34]	$23099 \pm 53\{28\}$	0.3	0.5	55
$\sin^2\theta_W^{eff} \times 10^5$	$\langle\mathcal{P}_\tau\rangle, A_{\rm FB}^{pol,\tau}$ [29]	$23159 \pm 41\{12\}$	0.6	< 0.6	20
M_W [MeV]	ADLO [36]	$80376 \pm 33\{6\}$	0.5	0.3	12
$A_{FB,\mu}^{M_Z \pm 3.5\rm GeV}$	$\frac{d\sigma}{d\cos\theta}$ [29]	$\pm 0.020\{0.001\}$	$1.0 \cdot 10^{-5}$	$0.3 \cdot 10^{-5}$	100

How LEP and FCC-ee exp. precisions do compare?



What are EW pseudo-observables (EWPOs)?

Correlation matrix

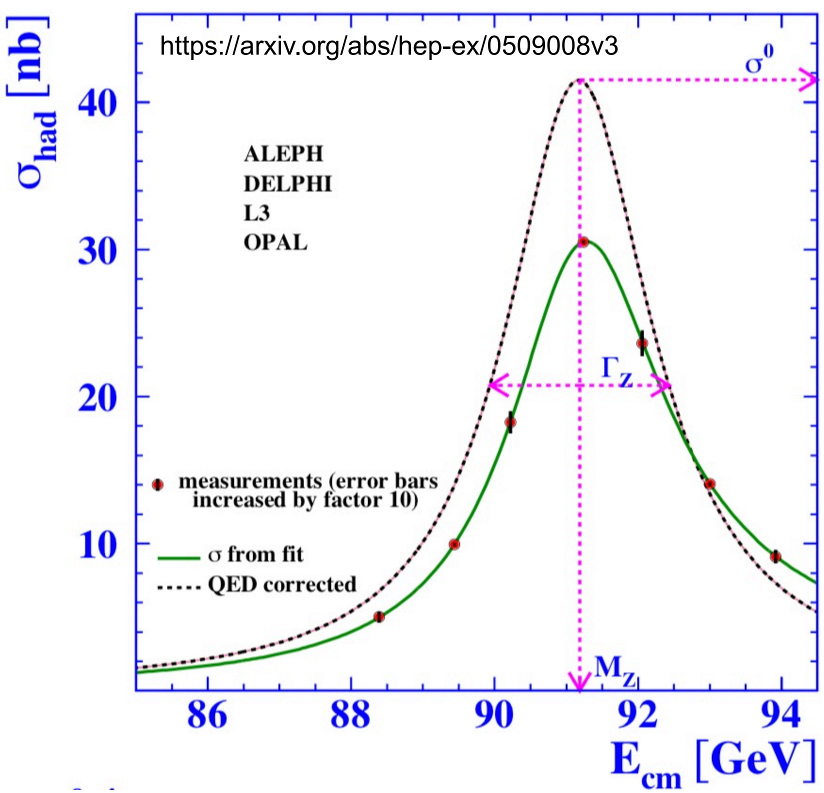


$\chi^2/\mathrm{dof} = 155/194$		OPAL								
m_Z [GeV]	91.1858 ± 0.0030	1.000								
Γ_Z [GeV]	2.4948 ± 0.0041	0.049	1.000							
σ^0_{had} [nb]	41.501 ± 0.055	0.031	−0.352	1.000						
R^0_e	20.901 ± 0.084	0.108	0.011	0.155	1.000					
R^0_μ	20.811 ± 0.058	0.001	0.020	0.222	0.093	1.000				
R^0_τ	20.832 ± 0.091	0.001	0.013	0.137	0.039	0.051	1.000			
$A^{0,e}_{FB}$	0.0089 ± 0.0045	−0.053	−0.005	0.011	−0.222	−0.001	0.005	1.000		
$A^{0,\mu}_{FB}$	0.0159 ± 0.0023	0.077	−0.002	0.011	0.031	0.018	0.004	−0.012	1.000	
$A^{0,\tau}_{FB}$	$0.0145 + 0.0030$	0.059	−0.003	0.003	0.015	−0.010	0.007	−0.010	0.013	1.000

Table 2.4: Individual results on Z parameters and their correlation coefficients from the four experiments. Systematic errors are included here except those summarised in Table 2.9.

Example of basic 9 EWPO's at LEP1, without lepton universality

Example of EWPO: σ^0_{had}

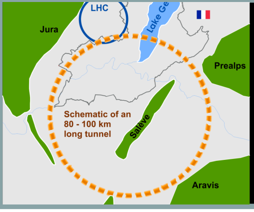


Experimental $\sigma_{\mathrm{had}}(s_i)$ measured at 7 energies $E^{(i)}_{\mathrm{cm}} = s_i^{1/2}$

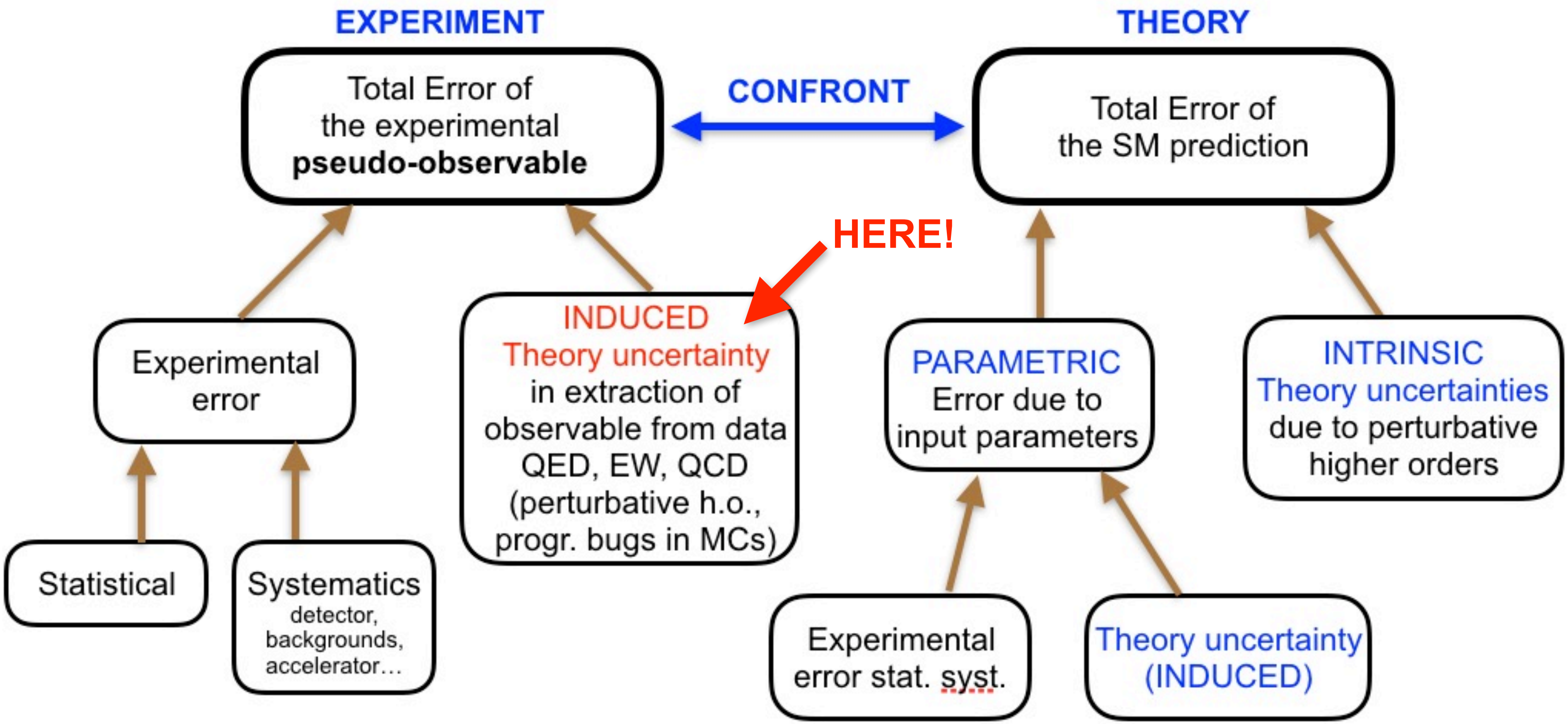
are fit using 1-D convolution formula $\sigma(s) = \int_0^1 dz\ \sigma^{Born}(zs)\ \rho_{QED}(z)$

and $\sigma^0_{\mathrm{had}} = \sigma^{Born}_{\mathrm{had}}(M_Z)$ is calculated <u>afterwards</u>! Z Mass and width from the same fit.

Induced QED uncertainty (next slide) enters through ρ_{QED}



Where is QED-induced uncertainty of PO in the landscape of theory and exp. errors?



EXPERIMENT

Total Error of
the experimental
pseudo-observable

CONFRONT

THEORY

Total Error of
the SM prediction

HERE!

Experimental
error

INDUCED
Theory uncertainty
in extraction of
observable from data
QED, EW, QCD
(perturbative h.o.,
progr. bugs in MCs)

PARAMETRIC
Error due to
input parameters

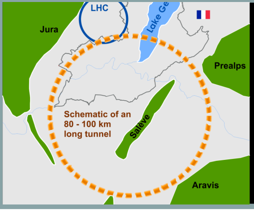
INTRINSIC
Theory uncertainties
due to perturbative
higher orders

Statistical

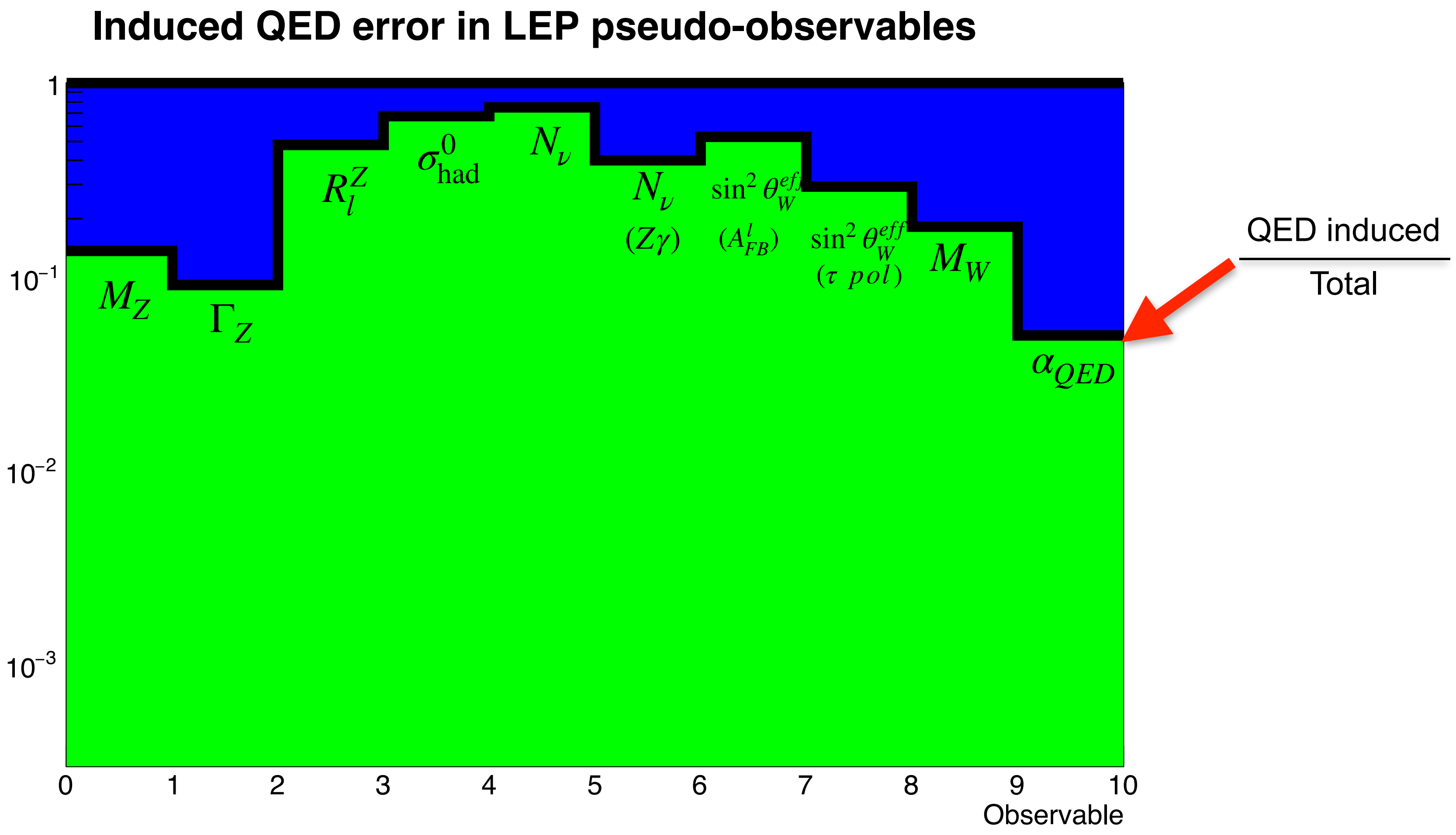
Systematics
detector,
backgrounds,
accelerator…

Experimental
error stat. syst.

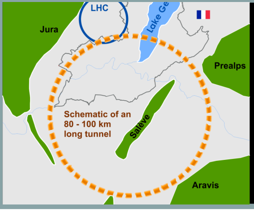
Theory uncertainty
(INDUCED)



Induced QED error in LEP pseudo-observables?



In LEP experiments QED uncertainty was safely below pure experimental errors



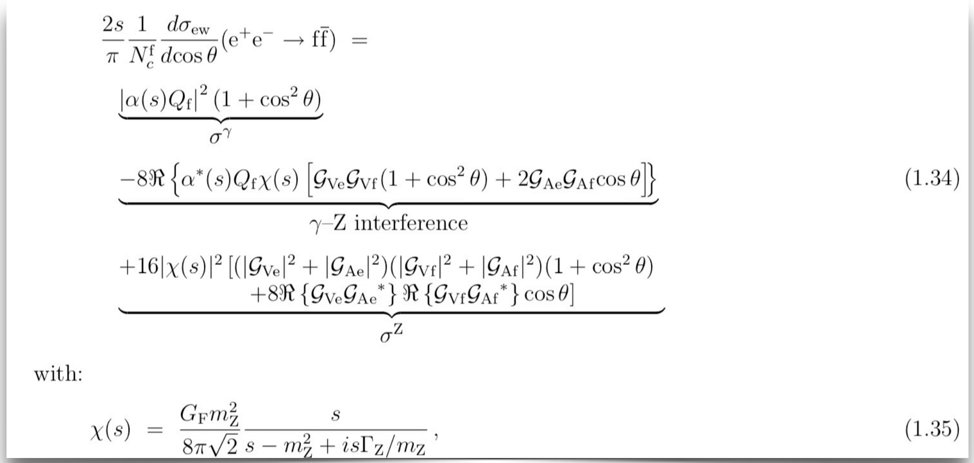
What are EW pseudo-observables (EWPOs)?

Example of charge asymmetry is more complicated:

$$A_{FB}^{\mu,0} = \left.\frac{\int_F d\sigma^{Born} - \int_B d\sigma^{Born}}{\int_F d\sigma^{Born} + \int_B d\sigma^{Born}}\right|_{s=M_Z^2}$$

calculated using $\frac{d\sigma^{Born}(s)}{d\cos\theta}[g_V^\mu, g_A^\mu]$

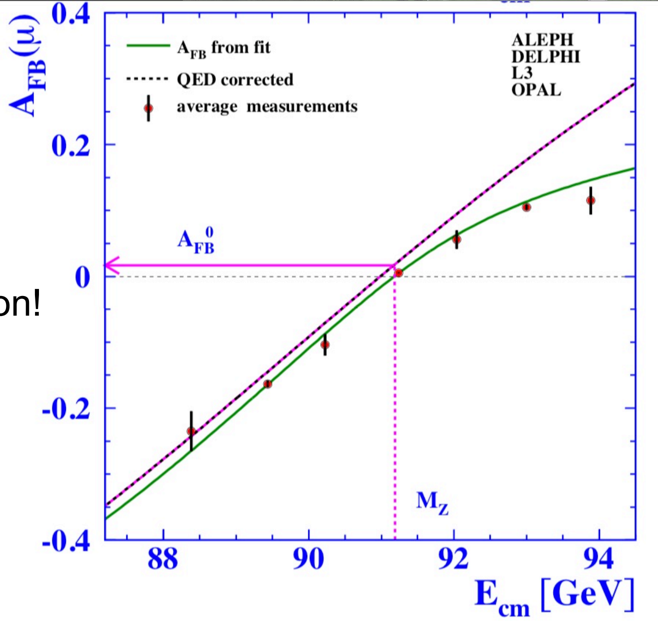
Eff. Born is central in EWPO construction!



$$\begin{aligned}\frac{2s}{\pi}\frac{1}{N_c^{\mathrm{f}}}\frac{d\sigma_{\mathrm{ew}}}{d\cos\theta}(\mathrm{e^+e^-}\to\mathrm{f\bar{f}}) \;=\; & \\ & \underbrace{|\alpha(s)Q_{\mathrm{f}}|^2(1+\cos^2\theta)}_{\sigma^\gamma} \\ & \underbrace{-8\Re\left\{\alpha^*(s)Q_{\mathrm{f}}\chi(s)\left[\mathcal{G}_{\mathrm{Ve}}\mathcal{G}_{\mathrm{Vf}}(1+\cos^2\theta)+2\mathcal{G}_{\mathrm{Ae}}\mathcal{G}_{\mathrm{Af}}\cos\theta\right]\right\}}_{\gamma\text{–Z interference}} \qquad (1.34) \\ & \underbrace{\begin{matrix}+16|\chi(s)|^2\left[(|\mathcal{G}_{\mathrm{Ve}}|^2+|\mathcal{G}_{\mathrm{Ae}}|^2)(|\mathcal{G}_{\mathrm{Vf}}|^2+|\mathcal{G}_{\mathrm{Af}}|^2)(1+\cos^2\theta)\right. \\ \left.+8\Re\{\mathcal{G}_{\mathrm{Ve}}\mathcal{G}_{\mathrm{Ae}}{}^*\}\Re\{\mathcal{G}_{\mathrm{Vf}}\mathcal{G}_{\mathrm{Af}}{}^*\}\cos\theta\right]\end{matrix}}_{\sigma^{\mathrm{Z}}}\end{aligned}$$

with:

$$\chi(s) \;=\; \frac{G_{\mathrm{F}}m_{\mathrm{Z}}^2}{8\pi\sqrt{2}}\frac{s}{s-m_{\mathrm{Z}}^2+is\Gamma_{\mathrm{Z}}/m_{\mathrm{Z}}}, \qquad (1.35)$$

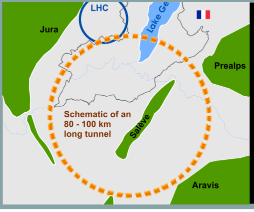


https://arxiv.org/abs/hep-ex/0509008v3

Z coupling constants in the effective Born $g_{V,A}^f = \Re(\mathcal{G}_{Vf,Af})$

are fit to $A_{FB}^\mu(s_i), \sigma(s_i)$ at several s_i using convolution formula

$$\frac{d\sigma^\mu}{d\cos\theta^*}(s,\theta^*) = \mathbf{CONV}\left\{\frac{d\sigma_\mu^{Born}(s)}{d\cos\theta}, \rho_{QED}\right\}, \qquad \theta^* \neq \theta$$



What are EW pseudo-observables (EWPOs)?

From experimental **DATA to EWPO —** effective Born is central object!

$$\boxed{A_{FB}^{e,\mu,\tau}(s_i),\ \sigma^{h,e,\mu,\tau}(s_i), P_\tau(s_i)\dots}$$

Fit (MINUIT) using eff. Born

$$\boxed{g^f_{V,A} = \Re(\mathcal{G}_{Vf,Af})}$$

pocket calculator

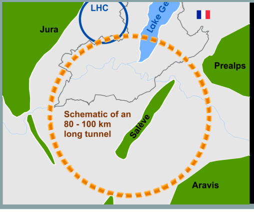
$$
\begin{aligned}
A_{\rm FB}^{0,\,\rm f} &= \frac{3}{4}\mathcal{A}_{\rm e}\mathcal{A}_{\rm f} & \mathcal{A}_{\rm f} &= \frac{g_{\rm Lf}^2 - g_{\rm Rf}^2}{g_{\rm Lf}^2 + g_{\rm Rf}^2} = \frac{2g_{\rm Vf}g_{\rm Af}}{g_{\rm Vf}^2 + g_{\rm Af}^2}\\
A_{\rm LR}^0 &= \mathcal{A}_{\rm e} & & \\
A_{\rm LRFB}^0 &= \frac{3}{4}\mathcal{A}_{\rm f} & \frac{g_{\rm Vf}}{g_{\rm Af}} &= 1 - \frac{2Q_{\rm f}}{T_3^{\rm f}}\sin^2\theta_{\rm eff}^{\rm f}\\
\langle \mathcal{P}_\tau^0 \rangle &= -\ \mathcal{A}_\tau & & \\
A_{\rm FB}^{\rm pol,0} &= -\frac{3}{4}\mathcal{A}_{\rm e}\,. & \sigma_{\rm f\bar f}^0 &= \frac{12\pi}{m_{\rm Z}^2}\,\frac{\Gamma_{\rm ee}\Gamma_{\rm f\bar f}}{\Gamma_{\rm Z}^2}.
\end{aligned}
$$

$$\Gamma_{\rm f\bar f} = N_c^{\rm f}\frac{G_{\rm F}m_{\rm Z}^3}{6\sqrt{2}\pi}\left(|\mathcal{G}_{\rm Af}|^2 R_{\rm Af} + |\mathcal{G}_{\rm Vf}|^2 R_{\rm Vf}\right) + \Delta_{\rm ew/QCD}\,.$$

Two key points:

1. The convolution formula approximates QED, including (at LEP) $\mathcal{O}(\alpha^1), \mathcal{O}(L_e^2\alpha^2), \mathcal{O}(L_e^3\alpha^3), \mathcal{O}(L_e^2\alpha^1)$, etc. (It may include 1-st order IFI.)
   Most likely <u>will be replaced by the Monte Carlo</u> to attain FCC-ee precision.

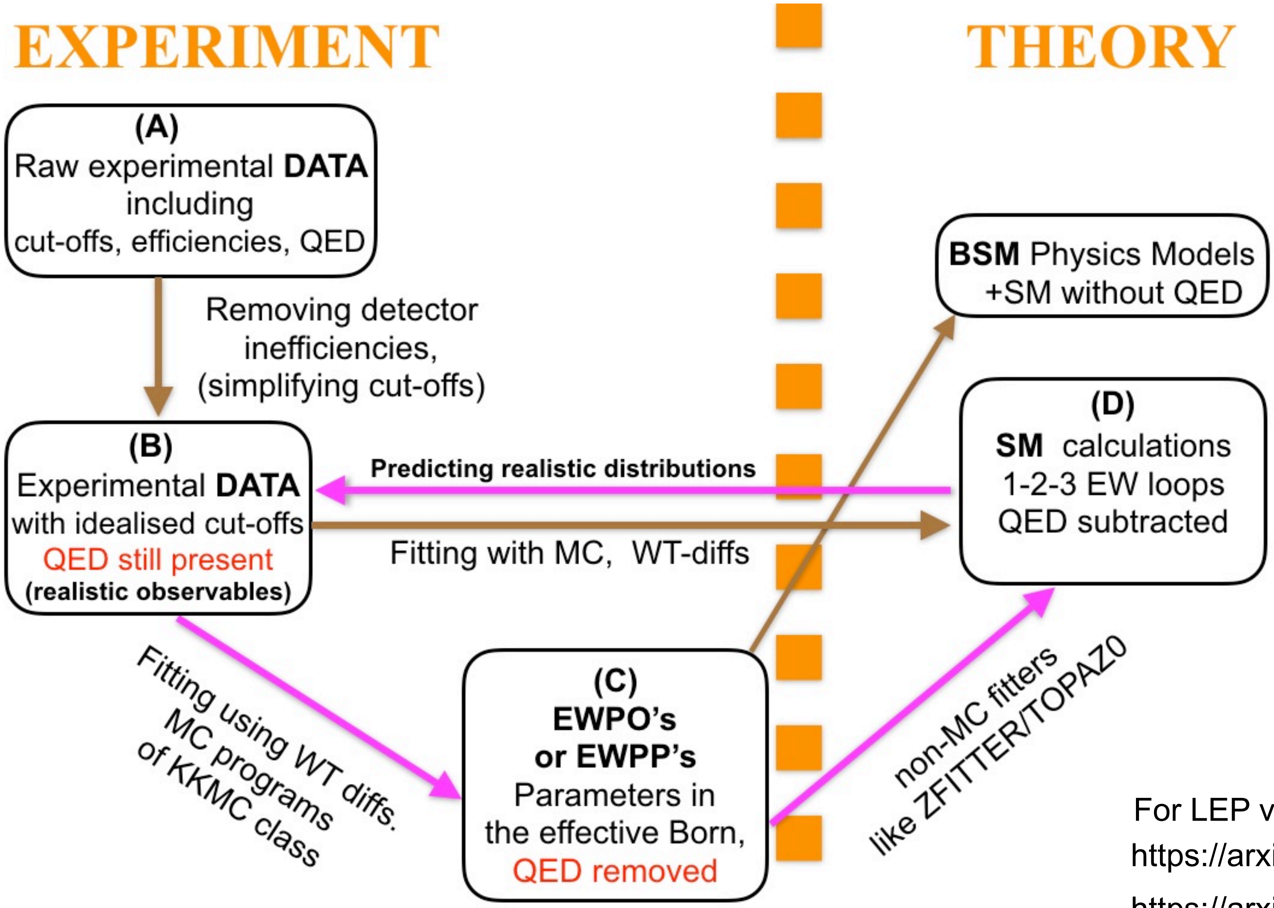
2. The role of the effective Born is to encapsulate/represent data within exp. precision in the (SM) Model independent way. At FCC-ee precision it may necessarily <u>include more of h.o. SM</u> (EW boxes?), then just only imaginary parts of g_V, g_A !!!



Validating/testing Pseudo-Observables at FCC-ee

https://arxiv.org/abs/1903.09895

Basic circular test (B)->(C)->(D)->(B) will be at FCC-ee the same as in LEP



EXPERIMENT

THEORY

(A)
Raw experimental **DATA**
including
cut-offs, efficiencies, QED

Removing detector
inefficiencies,
(simplifying cut-offs)

(B)
Experimental **DATA**
with idealised cut-offs
QED still present
(realistic observables)

Predicting realistic distributions

Fitting with MC, WT-diffs

BSM Physics Models
+SM without QED

(D)
SM calculations
1-2-3 EW loops
QED subtracted

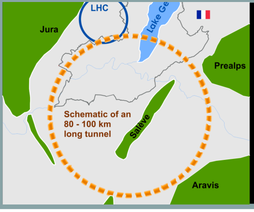
Fitting using WT diffs.
MC programs
of KKMC class

(C)
EWPO's
or EWPP's
Parameters in
the effective Born,
QED removed

non-MC fitters
like ZFITTER/TOPAZ0

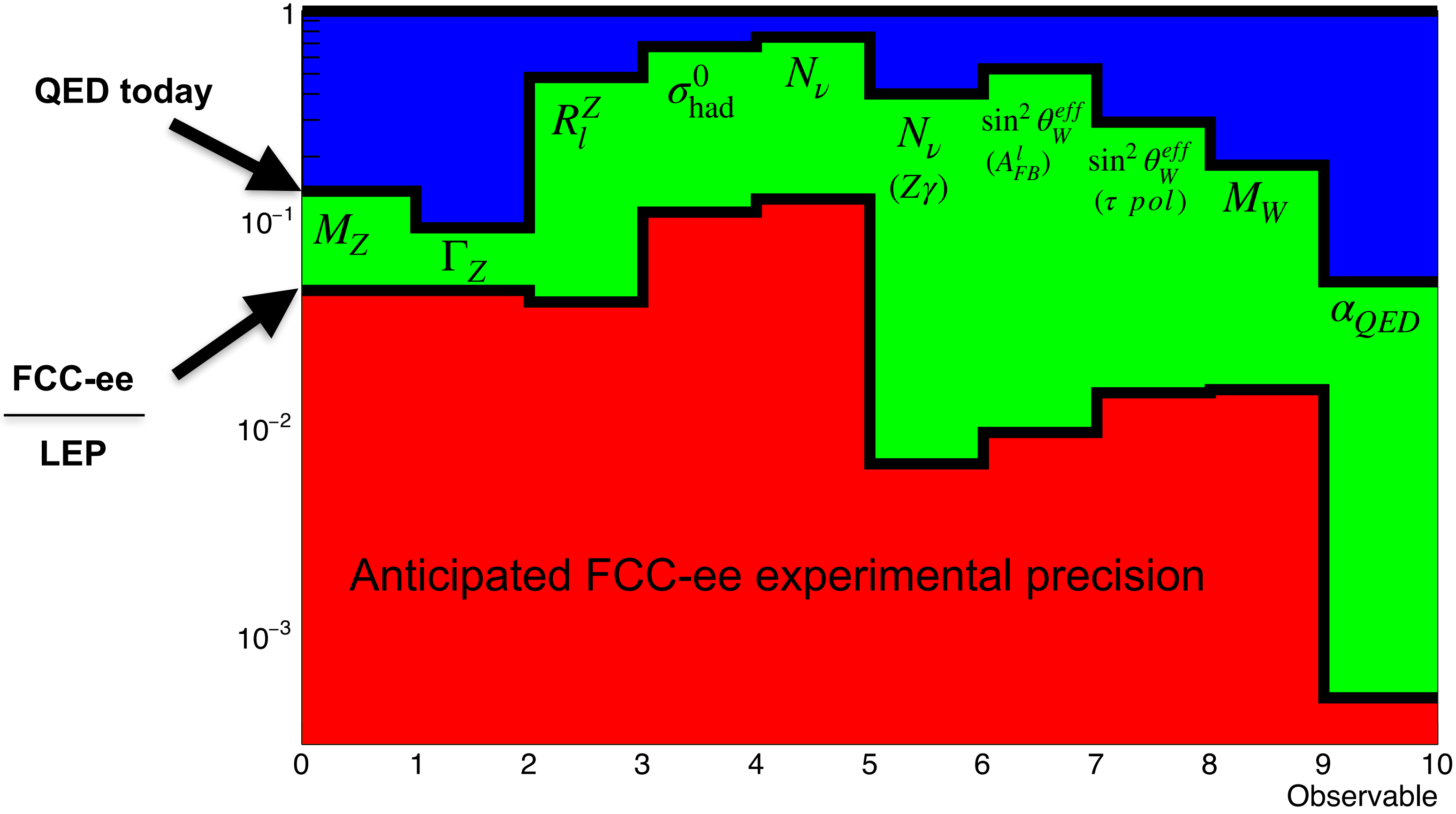
For LEP version see:
https://arxiv.org/abs/hep-ph/9902452
https://arxiv.org/abs/hep-ex/0509008v3

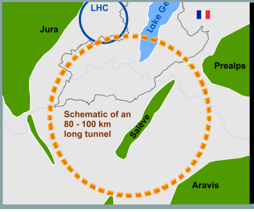
Main difference with LEP is Monte Carlo use in steps (B)->(C) and (B)->(D) instead of progs like ZFITTER/TOPAZ0



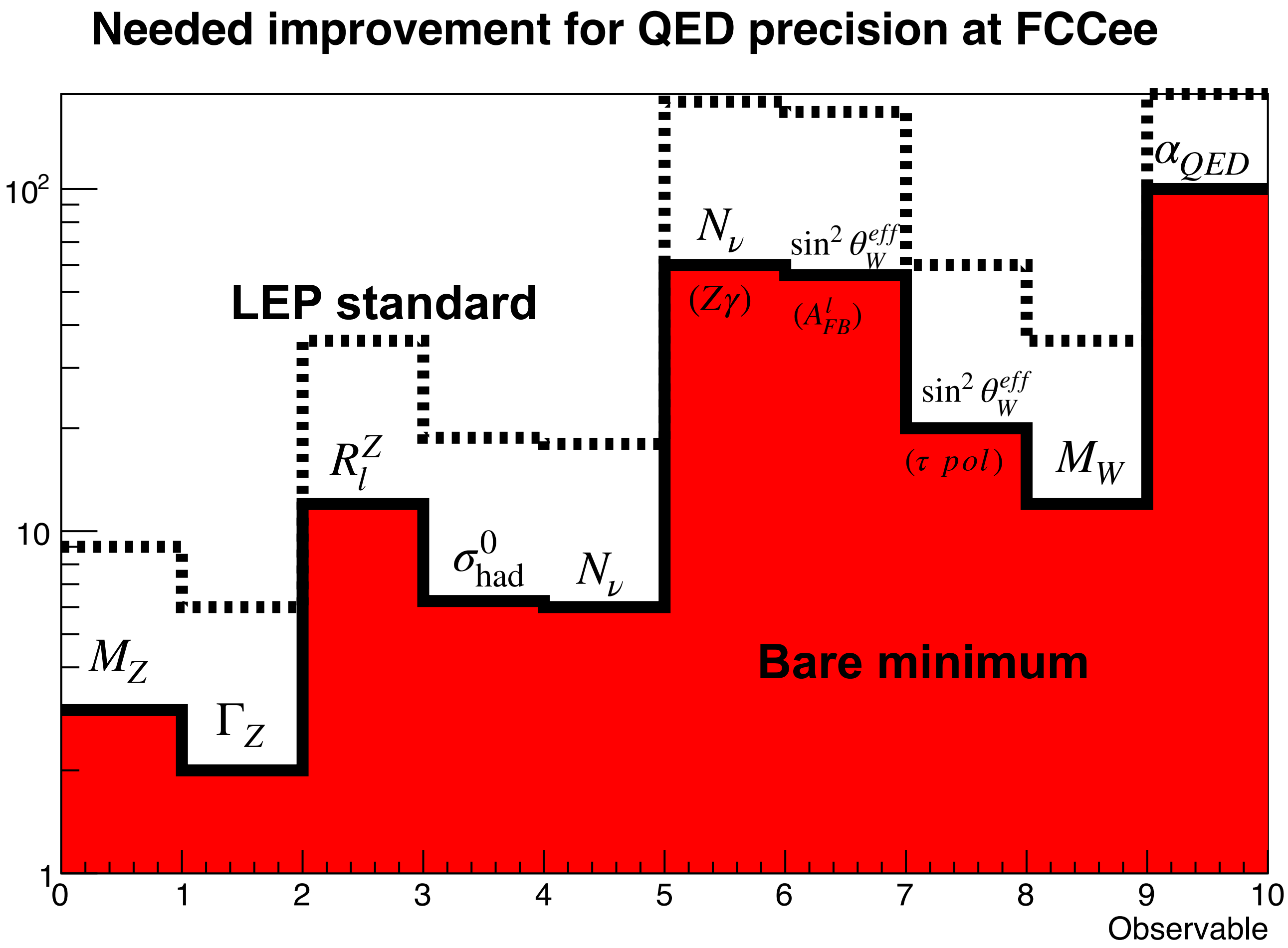
At the FCC-ee exp. precisions present QED uncertainty is unacceptable!

Current QED precision vs. FCCee exp. error

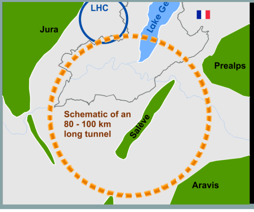




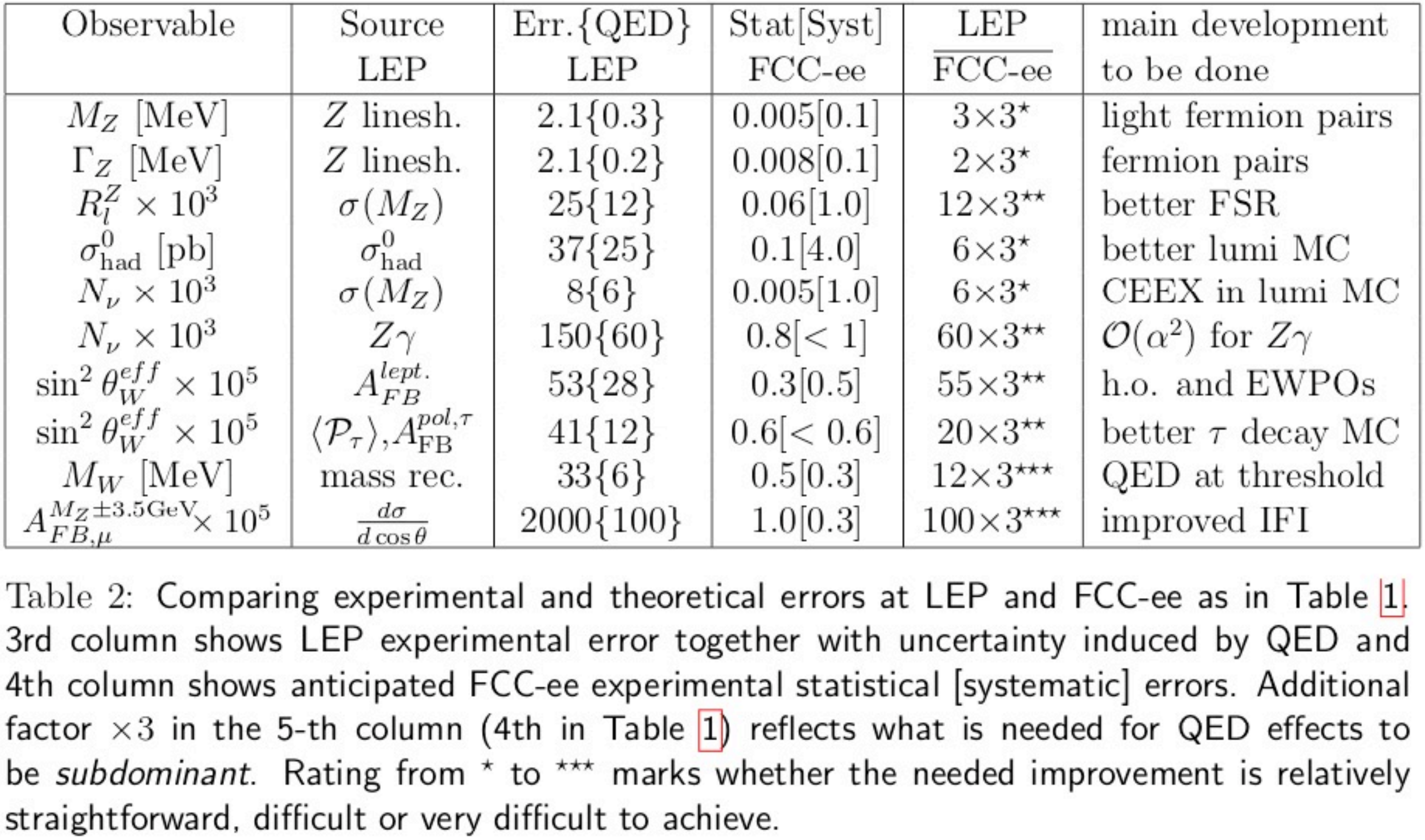
Desired improvement factor for QED uncertainty at FCC-ee



Depending on the observable factor 6-200 improvements needed!



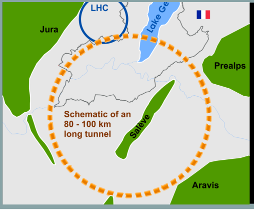
The same but with difficulty rating and planing what to be done?



Observable	Source LEP	Err.{QED} LEP	Stat[Syst] FCC-ee	$\frac{\text{LEP}}{\text{FCC-ee}}$	main development to be done
M_Z [MeV]	Z linesh.	2.1{0.3}	0.005[0.1]	$3\times3^{\star}$	light fermion pairs
Γ_Z [MeV]	Z linesh.	2.1{0.2}	0.008[0.1]	$2\times3^{\star}$	fermion pairs
$R_l^Z \times 10^3$	$\sigma(M_Z)$	25{12}	0.06[1.0]	$12\times3^{\star\star}$	better FSR
$\sigma^0_{\rm had}$ [pb]	$\sigma^0_{\rm had}$	37{25}	0.1[4.0]	$6\times3^{\star}$	better lumi MC
$N_\nu \times 10^3$	$\sigma(M_Z)$	8{6}	0.005[1.0]	$6\times3^{\star}$	CEEX in lumi MC
$N_\nu \times 10^3$	$Z\gamma$	150{60}	0.8[< 1]	$60\times3^{\star\star}$	$\mathcal{O}(\alpha^2)$ for $Z\gamma$
$\sin^2\theta_W^{eff} \times 10^5$	$A_{FB}^{lept.}$	53{28}	0.3[0.5]	$55\times3^{\star\star}$	h.o. and EWPOs
$\sin^2\theta_W^{eff} \times 10^5$	$\langle\mathcal{P}_\tau\rangle, A_{\rm FB}^{pol,\tau}$	41{12}	0.6[< 0.6]	$20\times3^{\star\star}$	better τ decay MC
M_W [MeV]	mass rec.	33{6}	0.5[0.3]	$12\times3^{\star\star\star}$	QED at threshold
$A_{FB,\mu}^{M_Z\pm3.5{\rm GeV}} \times 10^5$	$\frac{d\sigma}{d\cos\theta}$	2000{100}	1.0[0.3]	$100\times3^{\star\star\star}$	improved IFI

Table 2: Comparing experimental and theoretical errors at LEP and FCC-ee as in Table 1. 3rd column shows LEP experimental error together with uncertainty induced by QED and 4th column shows anticipated FCC-ee experimental statistical [systematic] errors. Additional factor $\times 3$ in the 5-th column (4th in Table 1) reflects what is needed for QED effects to be *subdominant*. Rating from * to *** marks whether the needed improvement is relatively straightforward, difficult or very difficult to achieve.

S.J. and M. Skrzypek arXiv:1903.09895 [hep-ph]



More details for selected observables

QED in Z line-shape: $\sigma_{tot}(s), M_Z, \Gamma_Z, R_l$

Present (LEP)

No cut-offs (except on $\sum E_\gamma$)

QED err. according to ADLO 2005: $\delta M_Z, \delta\Gamma_Z \simeq 0.2 - 0.3$ MeV

σ_{had} ISR: $\mathcal{O}(\alpha^1 L_e^1, \alpha^1, \alpha^2 L_e^2, \alpha^2 L_e^1, \alpha^3 L_e^3)_\gamma$ $\mathcal{O}(\alpha^2 L^2, \alpha^2 L^1, \alpha^3 L^3)_{pairs}$
Phys.Lett. B456 (1999) 77

σ_{lept} ISR+FSR

Non-MC implementation, 1-d or 2-d convolution
Initial-final interference (IFI) neglected

Simplified idealised cut-offs

ZFITTER and TOPAZ0 non-MC programs

AND

MC event generators: KORALZ, KKMC, BHWIDE

Arbitrary realistic cut-offs

MC event generators: KORALZ, KKMC, BHWIDE

FCC-ee

No cut-offs

exp . $\delta M_Z, \delta\Gamma_Z \leq 0.1$ MeV, QED ≤ 0.03 MeV

Factor ~10 improvement in QED is needed!

LEP simplistic convolution may survive only for σ_{had} provided pairs improved, $\mathcal{O}(\alpha^2 L_e^0, \alpha^3 L_e^2, \alpha^4 L_e^4)_\gamma$ are added and mixed QCD-QED corrections are improved.

For leptons MC will *take over* due to IFI and pairs

Simplified idealised cut-offs

Only MC event generators of the KKMC class or better will be able to match FCC-ee precision

Arbitrary realistic cut-offs

Only MC event generators of the KKMC class or better:

Upgrades of the matrix element:
$\mathcal{O}(\alpha^2 L_e^1)$ penta-boxes, $\mathcal{O}(\alpha^3 L_e^3)$ in CEEX m.e.

Inventing new MC approach for light fermion pairs.

Provisions for SM parameter fitting
and extracting new EWPOs from data

For luminosity uncertainty see next…

Charge and spin asymmetries at mZ

Present (LEP)

FCC-ee

Charge asymmetry

QED err. at LEP: $\delta A^{\mu}_{FB}(M_Z) \simeq 50 \cdot 10^{-5}$

translates into $\delta \sin^2\theta^{eff}_W \simeq 28 \cdot 10^{-5}$

[Conservative estimate based on comparisons of KKMC, ZFITTER, KORALZ, Phys. Ref. D63 (2001) 113009]

However, the effects due to ISR, IFI, EW boxes, imaginary parts of Z couplings, gamma exch. background are genuinely of order $\delta A^{\mu}_{FB}(M_Z) \simeq 10 \cdot 10^{-5}$

FCC-ee exp. error $\delta A^{\mu}_{FB}(M_Z) \simeq 1 \cdot 10^{-5}$, $\delta \sin^2\theta^{eff}_W \simeq 0.5 \cdot 10^{-5}$

Factor ~ 50-150 improvement in QED is needed!

Once they are mastered with 10% precision, the way to $\delta A^{\mu}_{FB}(M_Z) \simeq 1 \cdot 10^{-5}$ is open!

KKMC with complete $\mathcal{O}(\alpha^2)$ matrix element, soft photon resummation including IFI, EW corrections is already there. One needs the same for Bhabha!

The biggest challenge is, may be, the consistent definition of $\sin^2\theta^{eff}_W$ at the FCC-ee precision!

Spin asymmetries

$\langle \mathscr{P}_\tau \rangle$ and $A^{pol,\tau}_{FB}$ at LEP were worth $\delta \sin^2\theta^{eff}_W \simeq 41 \cdot 10^{-5}$

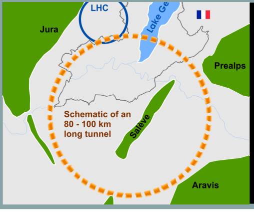
including QED induced uncertainty due to photon emissions in tau decays $\delta \sin^2\theta^{eff}_W \simeq 12 \cdot 10^{-5}$

QED err. is small due to weak dependence on CMS energy.

Expected FCC-ee exp. error $\delta \sin^2\theta^{eff}_W \simeq 0.6 \cdot 10^{-5}$

Factor ~ 20-60 improvement in QED is needed!

To be studied:
- polarimeter biases due to decay chanel cross-talk and photon emission in tau decays
- QED effects in tau-pair production
- exploiting super-Belle tau decay data in order to calibrate tau decay MC simulation

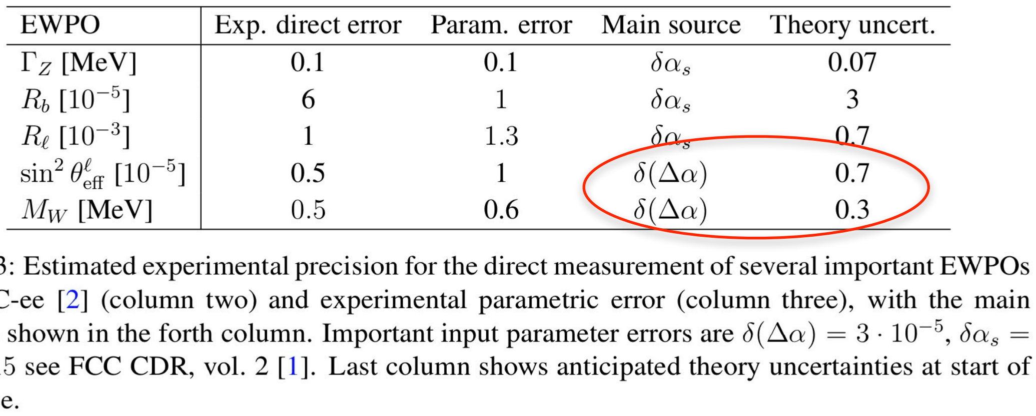


$\alpha_{QED}(M_Z)$ from $A_{FB}(M_Z \pm 3.5GeV)$

- Determination of $\alpha_{QED}(M_Z) = \alpha(0)/(1-\Delta\alpha)$ with precision ~3x10^{-5} critical for SM fits.
- Table of **parametric uncertainty** with

$$\delta M_Z \simeq 0.1MeV, \quad \delta m_t \simeq 50MeV$$
$$\delta\alpha_s \simeq 2\cdot 10^{-4}, \quad \delta(\Delta\alpha) \simeq 5\cdot 10^{-5}$$

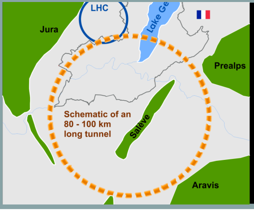
http://arxiv.org/abs/arXiv:1901.02648



EWPO	Exp. direct error	Param. error	Main source	Theory uncert.
Γ_Z [MeV]	0.1	0.1	$\delta\alpha_s$	0.07
R_b [10^{-5}]	6	1	$\delta\alpha_s$	3
R_ℓ [10^{-3}]	1	1.3	$\delta\alpha_s$	0.7
$\sin^2\theta^\ell_{\rm eff}$ [10^{-5}]	0.5	1	$\delta(\Delta\alpha)$	0.7
M_W [MeV]	0.5	0.6	$\delta(\Delta\alpha)$	0.3

Table 3: Estimated experimental precision for the direct measurement of several important EWPOs at FCC-ee [2] (column two) and experimental parametric error (column three), with the main source shown in the forth column. Important input parameter errors are $\delta(\Delta\alpha) = 3\cdot 10^{-5}$, $\delta\alpha_s = 0.00015$ see FCC CDR, vol. 2 [1]. Last column shows anticipated theory uncertainties at start of FCC-ee.

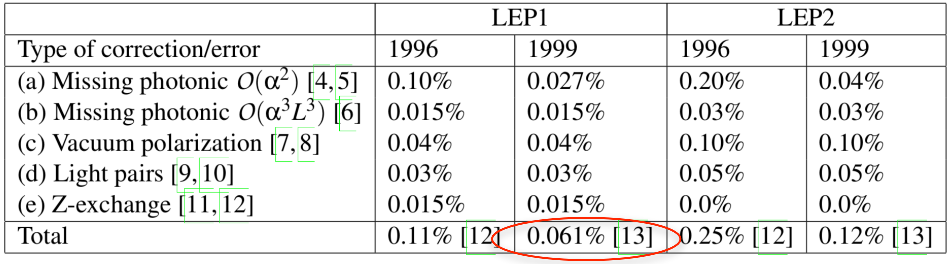
- Measuring $A_{FB}(M_Z \pm 3.5GeV)$ with precision 3x10^{-5} , **factor 200 more precisely** than at LEP was proposed in order to get $\alpha_{QED}(M_Z)$ with the needed precision ~10^{-5}.
  P. Janot, JHEP11,164 (2017) arXiv:1512.05544
- QED Initial-Final state interference **IFI** is the main obstacle!
- IFI cancels partly in the difference $A_{FB}(M_Z \pm 3.5GeV)$, but ~1% effect remains.
  Can one control IFI in A_{FB} with the precision 3x10^{-5} ???
- In arXiv:1801.08611 Phys. Rev. D (S.J. and S.Yost)
  it was shown that using **KKMC** and new **KKfoam** programs one may get precision $\leq 10^{-4}$



Low angle Bhabha (luminosity) at FCCee

arXiv:1902.05912

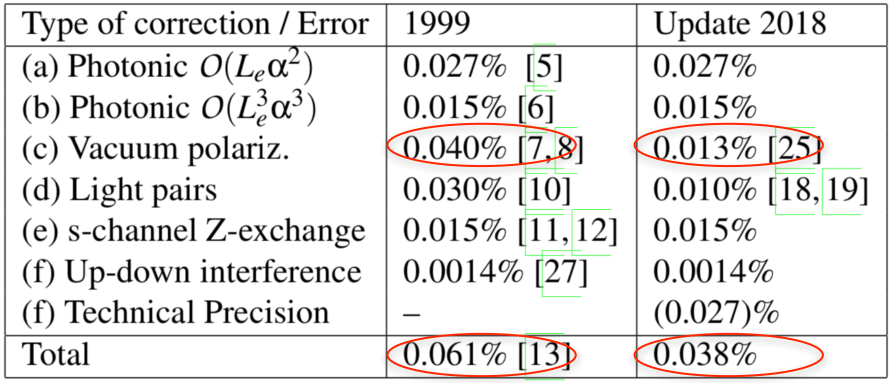
- LEP legacy, lumi TH error budget



	LEP1		LEP2	
Type of correction/error	1996	1999	1996	1999
(a) Missing photonic $O(\alpha^2)$ [4,5]	0.10%	0.027%	0.20%	0.04%
(b) Missing photonic $O(\alpha^3 L^3)$ [6]	0.015%	0.015%	0.03%	0.03%
(c) Vacuum polarization [7,8]	0.04%	0.04%	0.10%	0.10%
(d) Light pairs [9,10]	0.03%	0.03%	0.05%	0.05%
(e) Z-exchange [11,12]	0.015%	0.015%	0.0%	0.0%
Total	0.11% [12]	0.061% [13]	0.25% [12]	0.12% [13]

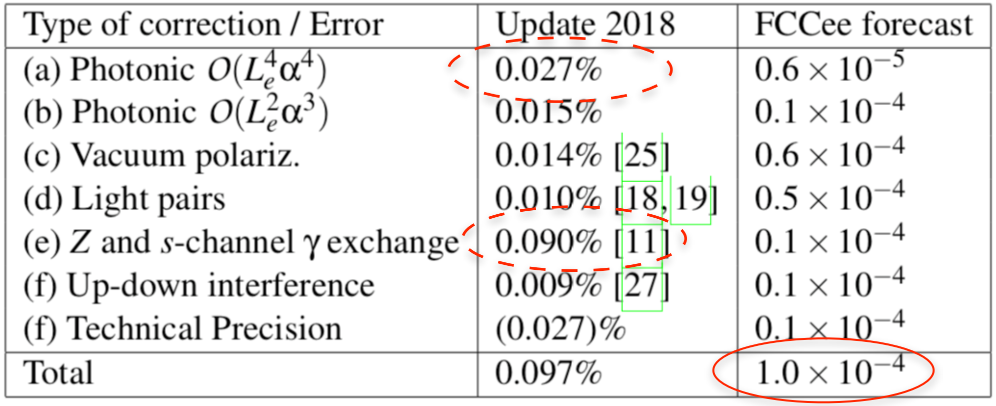
Table 1: Summary of the total (physical+technical) theoretical uncertainty for a typical calorimetric detector. For LEP1, the above estimate is valid for a generic angular range within 1°-3° (18-52 mrads), and for LEP2 energies up to 176 GeV and an angular range within 3°-6°. Total uncertainty is taken in quadrature. Technical precision included in (a).

LEP lumi update 2018



Type of correction / Error	1999	Update 2018
(a) Photonic $O(L_e \alpha^2)$	0.027% [5]	0.027%
(b) Photonic $O(L_e^3 \alpha^3)$	0.015% [6]	0.015%
(c) Vacuum polariz.	0.040% [7,8]	0.013% [25]
(d) Light pairs	0.030% [10]	0.010% [18,19]
(e) s-channel Z-exchange	0.015% [11,12]	0.015%
(f) Up-down interference	0.0014% [27]	0.0014%
(f) Technical Precision	–	(0.027)%
Total	0.061% [13]	0.038%

- By the time of FCC-ee VP contribution will be merely 0.006%
- QED corrections and Z contrib. come back to front!
- Z contr. easy to master, even if rises at FCC-ee, because (28-58)->(64-86) mrad.
- Our FCC-ee forecast is 0.01% provided QED m.e. and VP are improved.



Type of correction / Error	Update 2018	FCCee forecast
(a) Photonic $O(L_e^4 \alpha^4)$	0.027%	0.6×10^{-5}
(b) Photonic $O(L_e^2 \alpha^3)$	0.015%	0.1×10^{-4}
(c) Vacuum polariz.	0.014% [25]	0.6×10^{-4}
(d) Light pairs	0.010% [18,19]	0.5×10^{-4}
(e) Z and s-channel γ exchange	0.090% [11]	0.1×10^{-4}
(f) Up-down interference	0.009% [27]	0.1×10^{-4}
(f) Technical Precision	(0.027)%	0.1×10^{-4}
Total	0.097%	1.0×10^{-4}

Z invisible width from peak cross section and radiative return

Present (LEP)

FCC-ee

Peak cross section

QED err. of luminosity $\frac{\delta\mathscr{L}}{\mathscr{L}} = \frac{\delta\sigma^0_{had}}{\sigma^0_{had}} \simeq 0.06\,\%$

dominates LEP exp. error $N_\nu \simeq 2.984 \pm 0.008\ \{\pm 0.006\}_{QED}$

$$R^0_{\rm inv} = \left(\frac{12\pi R^0_\ell}{\sigma^0_{\rm had} m^2_{\rm Z}}\right)^{\frac{1}{2}} - R^0_\ell - (3+\delta_\tau)\,,$$

$$R^0_{\rm inv} = N_\nu \left(\frac{\Gamma_{\nu\overline{\nu}}}{\Gamma_{\ell\ell}}\right)_{\rm SM}.$$

FCC-ee exp. error (syst.) $\delta N_\nu \simeq 0.001$

Factor ~10 improvement in luminosity is needed!

$\frac{\delta\mathscr{L}}{\mathscr{L}} \simeq 10^{-4} \rightarrow \delta N_\nu \simeq 8 \cdot 10^{-4}$ seems achievable.

Radiative return I

$$e^+e^- \rightarrow \nu\bar{\nu}\gamma$$

$N_\nu \simeq 2.69 \pm 0.15\ \{\pm 0.06\}_{QED}$

Limited by poor LEP statistics at 161GeV

Expected FCC-ee exp. error of $\sigma_{\nu\bar{\nu}\gamma}$ not yet established,
most likely: $\delta\sigma/\sigma \simeq 0.03\,\% \rightarrow \delta N_\nu \simeq 0.001$

Future luminosity error 0.01% looks ok.

Estimate of h.o. QED effects using KKMC is merely 0.02% (unpublished).

Altogether $\delta N_\nu \simeq 0.001$ seems achievable:)
(Factor ~60 improvement in QED rather easy.)

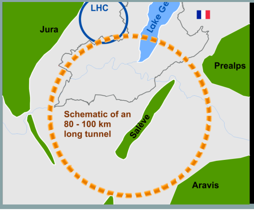
Radiative return II

Measuring ratio $R = \frac{\sigma_{\nu\bar{\nu}\gamma}}{\sigma_{\mu^+\mu^-\gamma}}$

Luminosity error drops out!

QED uncertainty due to FSR in $\sigma_{\mu^+\mu^-\gamma}$ rated at 0.03% (unpublished study using KKMC).

Again $\delta N_\nu \simeq 0.001$



Summary

- Major effort is needed to improve SM/QED predictions for FCC-ee observables by factor 10-200
- In particular QED corrections for asymmetries near Z has to be improved by factor up to 200
- New algorithms of extracting EW pseudo-observables from experimental data has to be worked out and cross-checked
- Increased role of MC event generators is anticipated

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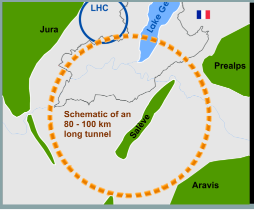
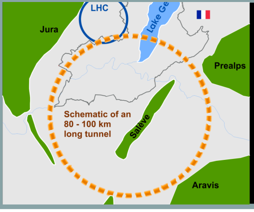
*This work is partly supported by the Polish National Science Center grant 2016/23/B/ST2/03927 and the CERN FCC Design Study Programme.


LHC
Lake Ge
Jura
Prealps
Schematic of an
80 - 100 km
long tunnel
Salève
Aravis

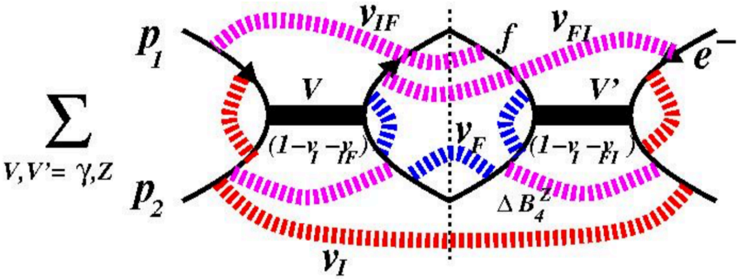
Reserve



5-dim convolution formula including IFI

NEW analytical exponentiation formula for ISR+FSR+IFI

Eq.(90) in [JWW2001] and in older Frascati works, implemented recently in **KKfoam**



$$\frac{d\sigma}{d\Omega}(s,\theta,v_{\max}) = \sum_{V,V'=\gamma,Z} \int d\theta\; dv_I\; dv_F\; dv_{IF}\; dv_{FI}\; \theta(v_I - v_F - v_{IF} - v_{FI} < v_{\max})$$

$$\times\; F(\gamma_I)\gamma_I v_I^{\gamma_I-1}\; F(\gamma_F)\gamma_I v_F^{\gamma_F-1}\;\; F(\gamma_{IF})\gamma_{IF} v_{IF}^{\gamma_{IF}-1}\; F(\gamma_{FI})\gamma_{FI} v_{IF}^{\gamma_{FI}-1}$$

$$\times\; e^{2\alpha\Delta B_4^V}\mathcal{M}_V^{(0)}(s(1-v_I-v_{IF}),\theta)\; [e^{2\alpha\Delta B_4^{V'}}\mathcal{M}_{V'}^{(0)}(s(1-v_I-v_{FI}),\theta)]^*\; \big[1+\mathrm{NIR}(\mathrm{v_I},\mathrm{v_F})\big],$$

- Convolution of **four** radiator functions (instead of two)!
- Extra virtual formfactor ΔB_4^Z due to IFI for resonant contrib.
- $\gamma_I = Q_e^2 \frac{\alpha}{\pi}[\frac{s}{m_e^2} - 1]$, $\gamma_{IF} = \gamma_{FI} = Q_e Q_f \frac{\alpha}{\pi}\ln\frac{1-\cos\theta}{1+\cos\theta}$, $F(\gamma) = \frac{e^{-C_E\gamma}}{\Gamma(1+\gamma)}$

arXiv:1801.08611 **[hep-ph]** To appear in Phys. Rev. D