Cross section and differential distributions for top quarks near the production threshold

Thomas Rauh

AEC, University of Bern

M. Beneke, Y. Kiyo, P. Marquard, J. Piclum, A. Penin, M. Steinhauser, arXiv:1506.06864
M. Beneke, A. Maier, TR, P. Ruiz-Femenía, arXiv:1711.10429
WHIZARD, A. Hoang, M. Stahlhofen, T. Teubner, arXiv:1712.02220
F. Simon, arXiv:1902.07246

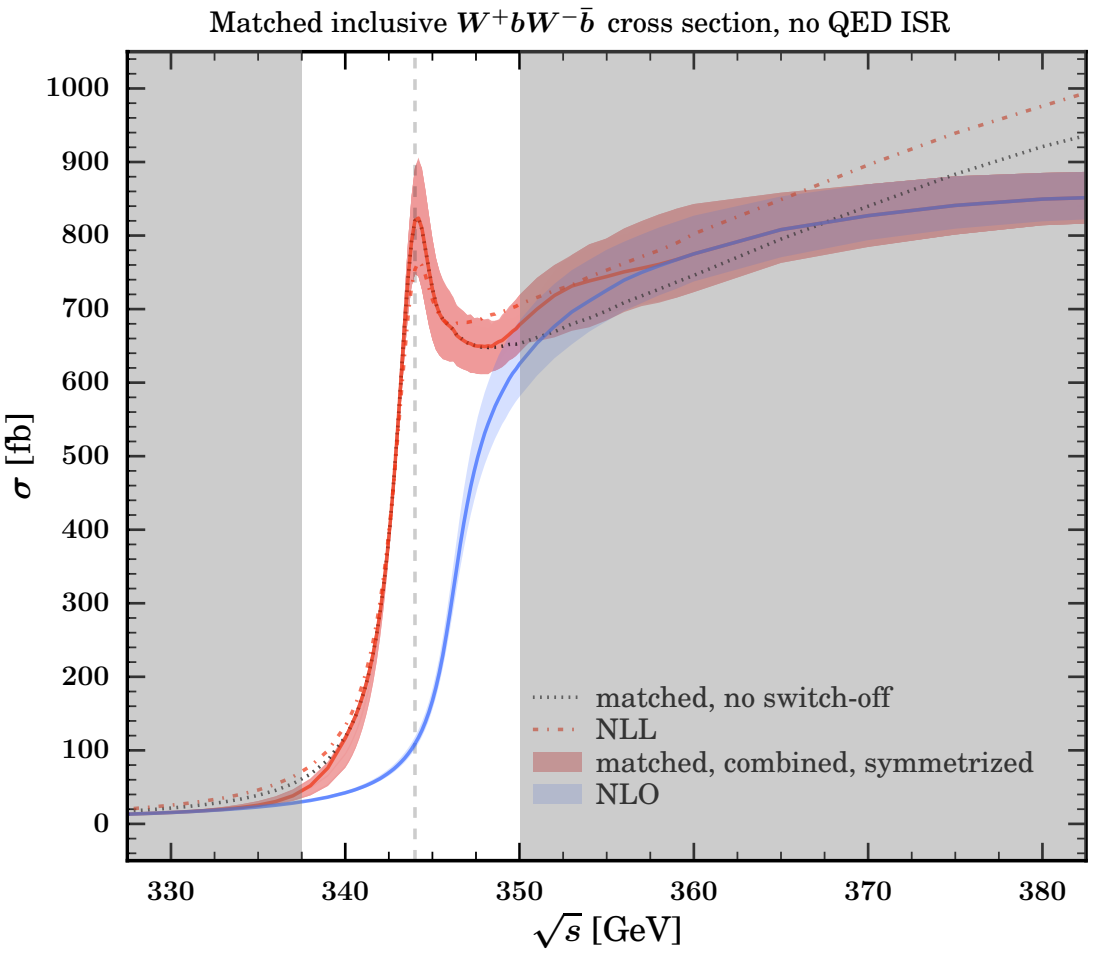
UNIVERSITÄT BERN

AEC
ALBERT EINSTEIN CENTER
FOR FUNDAMENTAL PHYSICS

Top threshold scan

Motivation

Consider the top threshold region of the $e^+e^- \to W^+W^-\bar{b}bX$ cross section:



[Bach, Chokoufé Nejad, Hoang, Kilian, Reuter, Stahlhofen, Teubner, Weiss 2017]

Top threshold scan

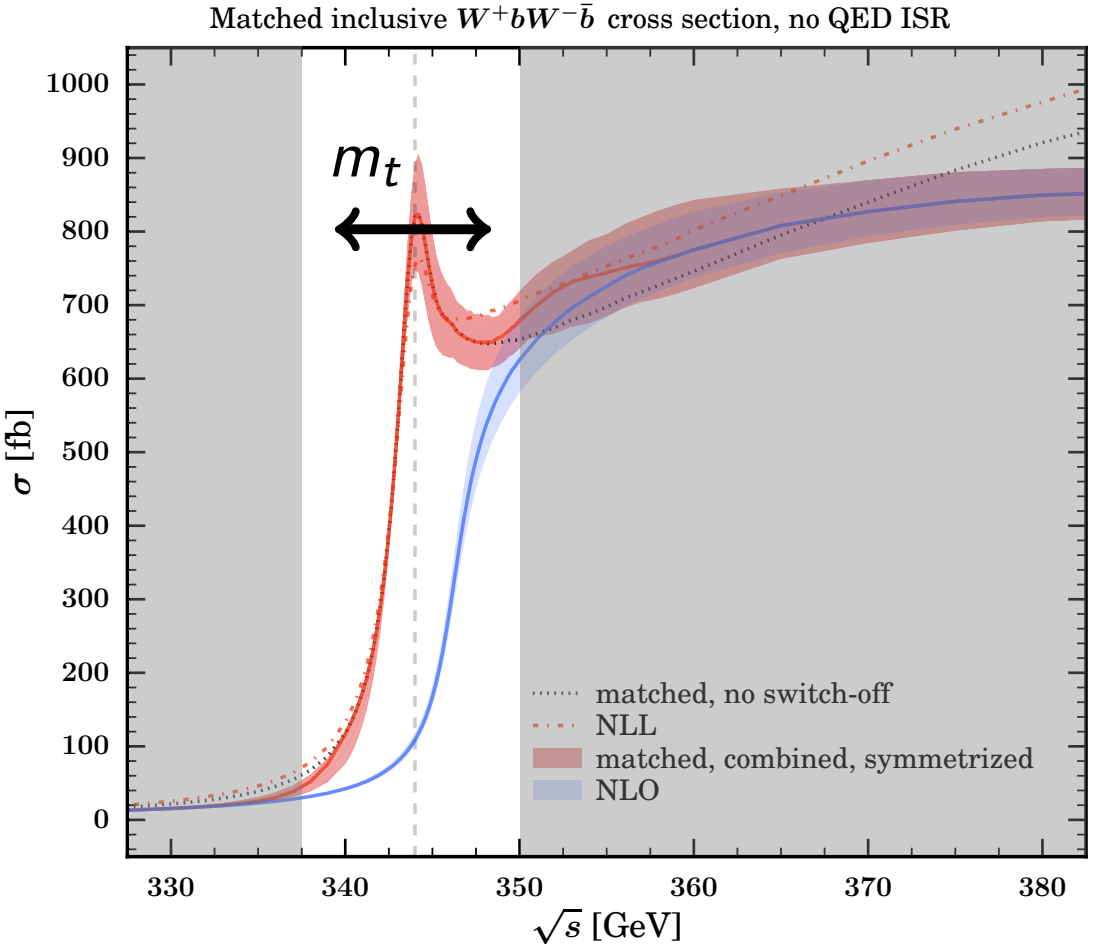
Motivation

Consider the top threshold region of the $e^+e^- \to W^+W^-\bar{b}bX$ cross section:

- Allows extremely precise determination of the top quark mass, goal:

$$\delta\overline{m}_t(\overline{m}_t) \leq 50\,\text{MeV}$$

- Sensitive to Γ_t, α_s, y_t



[Bach, Chokoufé Nejad, Hoang, Kilian, Reuter, Stahlhofen, Teubner, Weiss 2017]

Top threshold scan

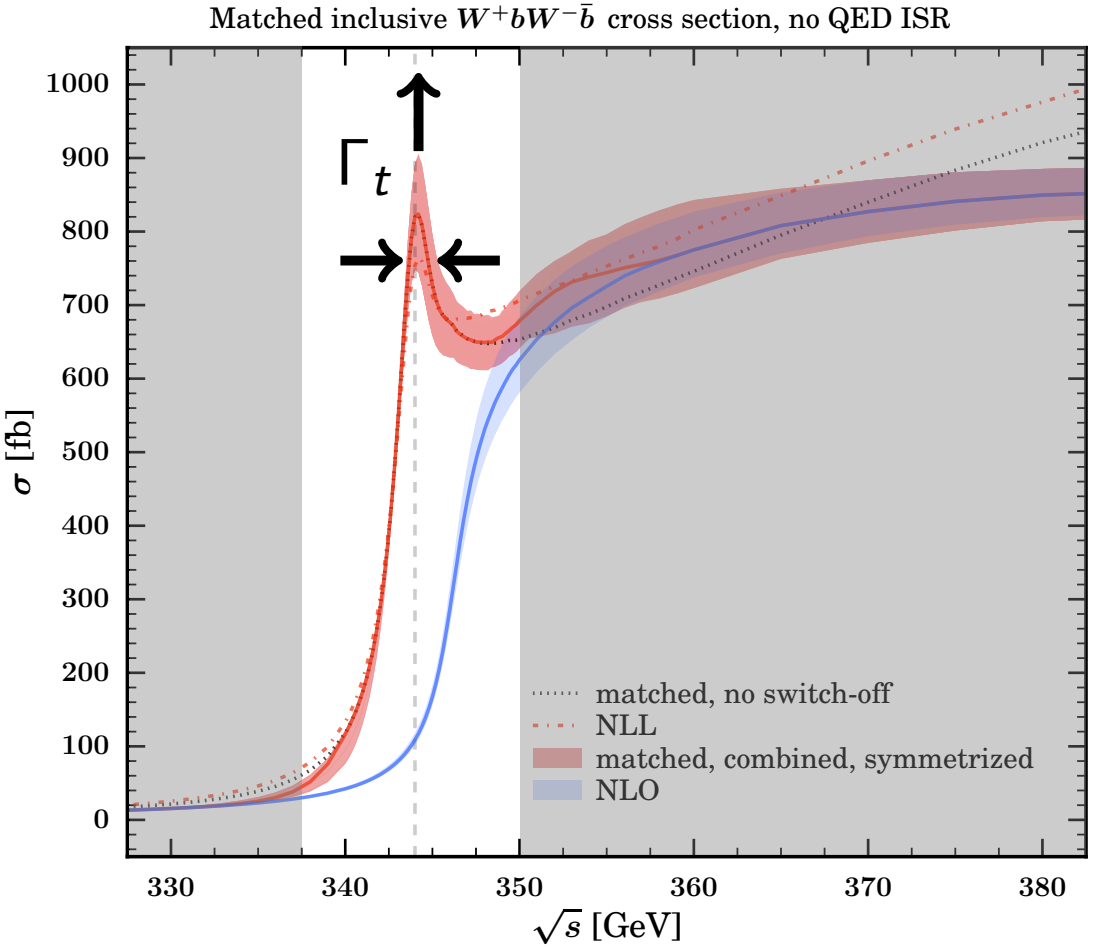
Motivation

Consider the top threshold region of the $e^+e^- \to W^+W^-\bar{b}bX$ cross section:

- Allows extremely precise determination of the top quark mass, goal:

$$\delta\overline{m}_t(\overline{m}_t) \leq 50\,\text{MeV}$$

- Sensitive to Γ_t, α_s, y_t



[Bach, Chokoufé Nejad, Hoang, Kilian, Reuter, Stahlhofen, Teubner, Weiss 2017]

Top threshold scan

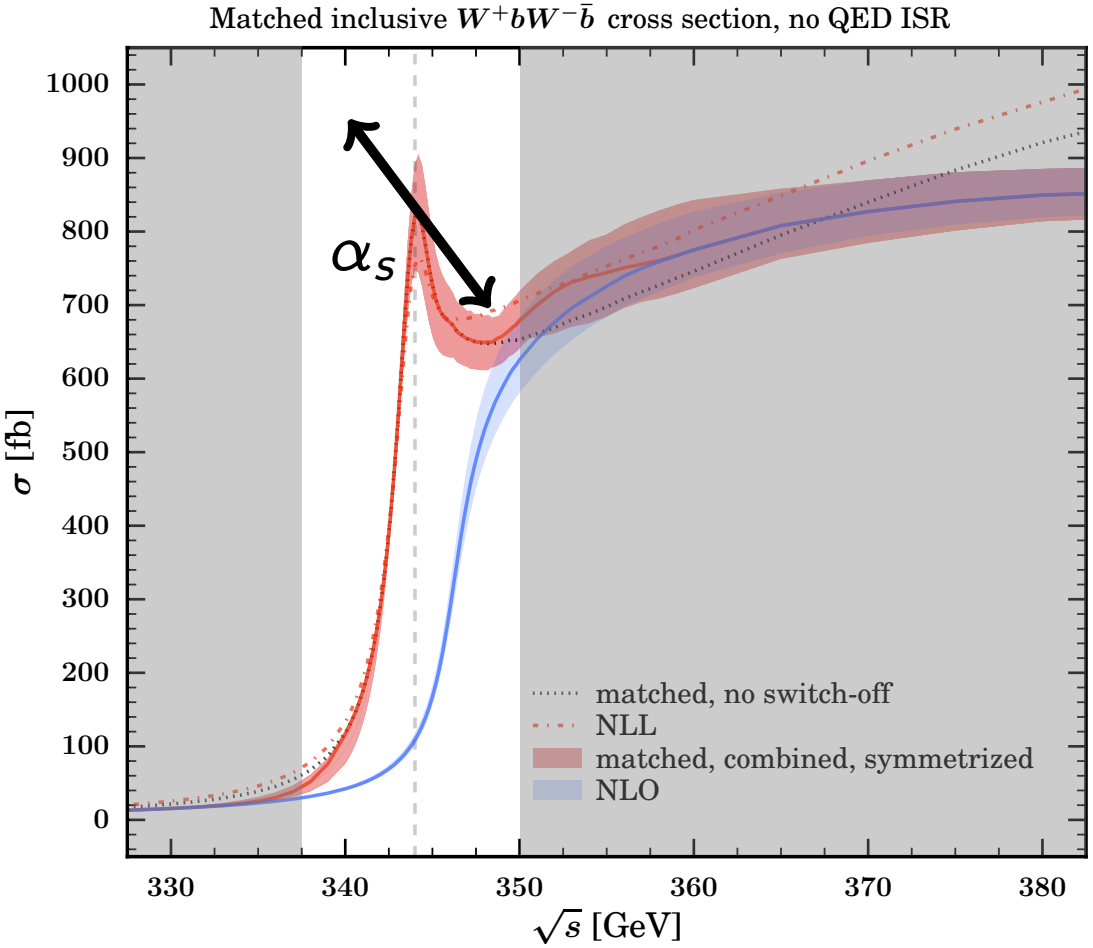
Motivation

Consider the top threshold region of the $e^+e^- \to W^+W^-\bar{b}bX$ cross section:

- Allows extremely precise determination of the top quark mass, goal:

$$\delta\overline{m}_t(\overline{m}_t) \leq 50\,\text{MeV}$$

- Sensitive to Γ_t, α_s, y_t



[Bach, Chokoufé Nejad, Hoang, Kilian, Reuter, Stahlhofen, Teubner, Weiss 2017]

Top threshold scan

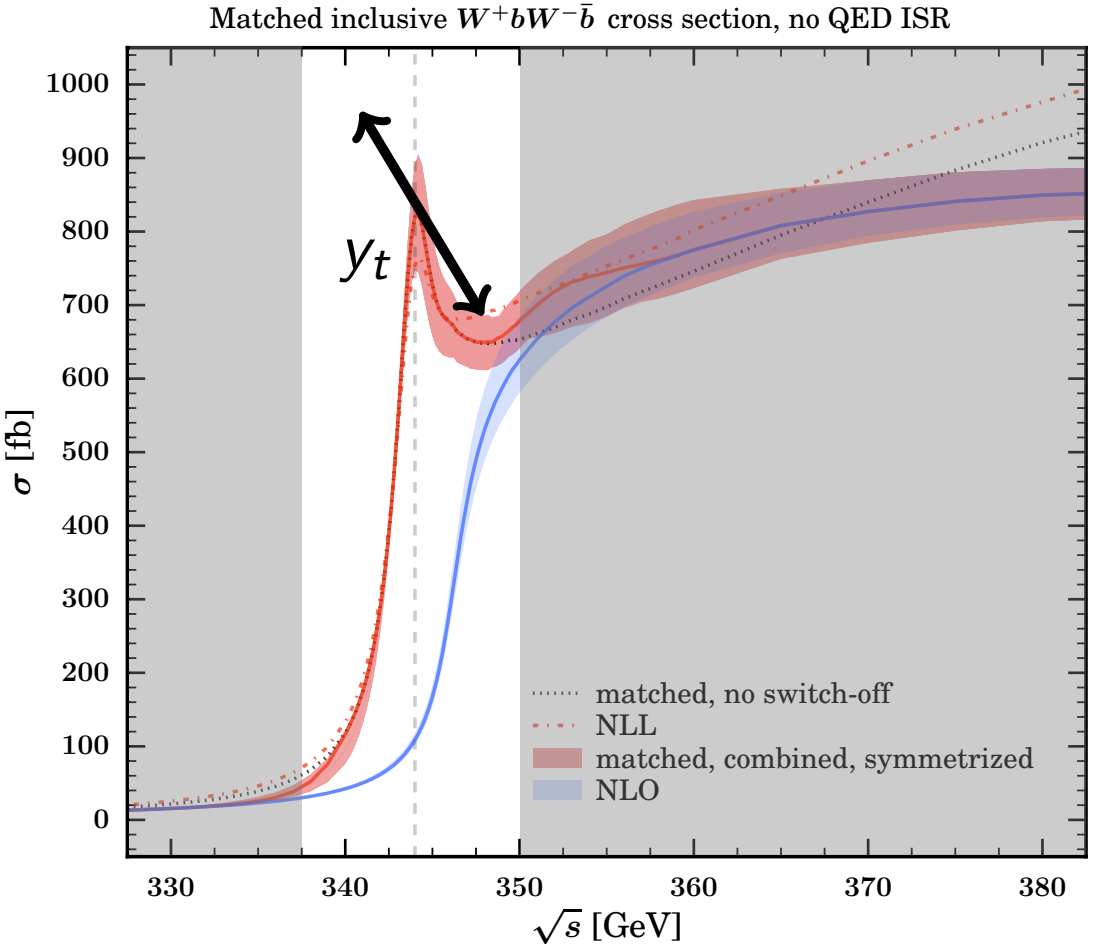
Motivation

Consider the top threshold region of the $e^+e^- \to W^+W^-\bar{b}bX$ cross section:

- Allows extremely precise determination of the top quark mass, goal:

$$\delta\overline{m}_t(\overline{m}_t) \leq 50\,\text{MeV}$$

- Sensitive to Γ_t, α_s, y_t



[Bach, Chokoufé Nejad, Hoang, Kilian, Reuter, Stahlhofen, Teubner, Weiss 2017]

Top threshold scan

Motivation

Consider the top threshold region of the $e^+e^- \to W^+W^-\bar{b}bX$ cross section:

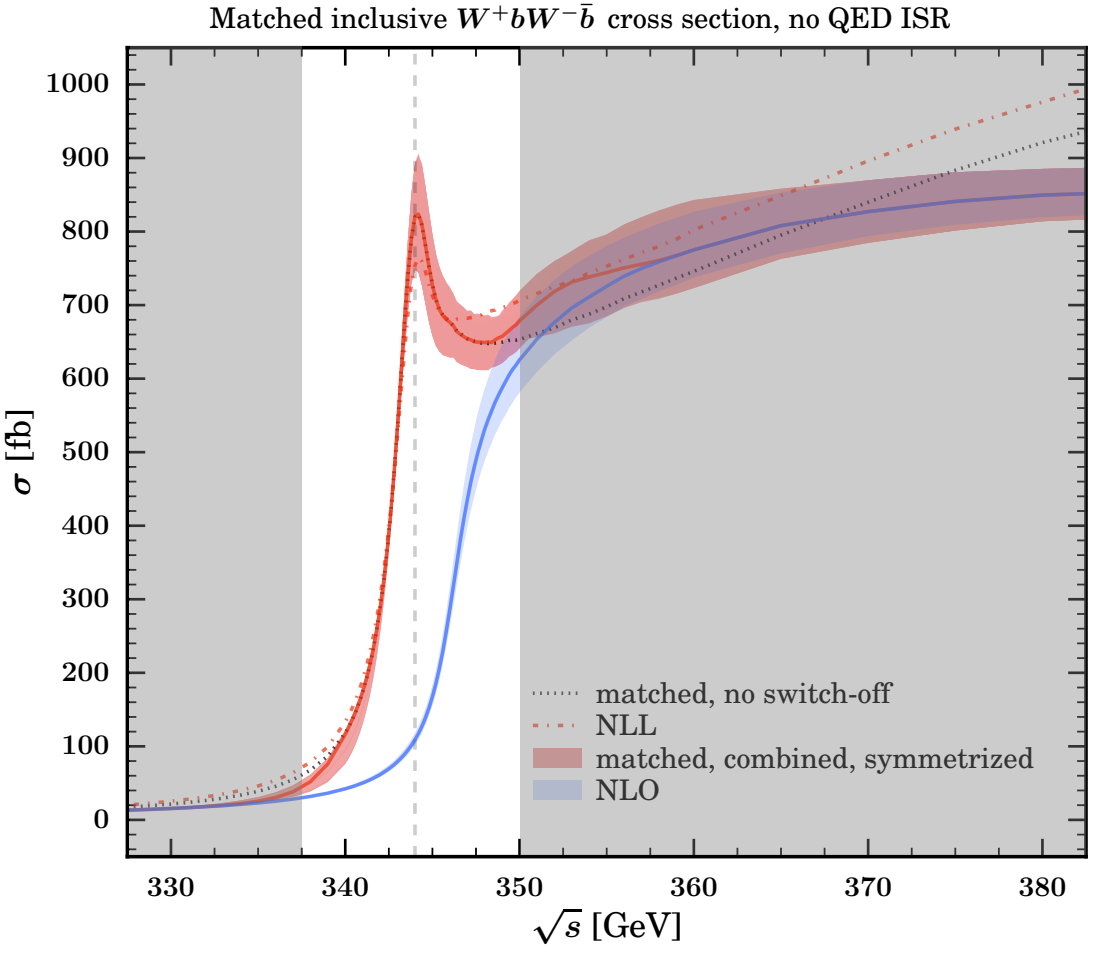
- Allows extremely precise determination of the top quark mass, goal:

$$\delta\overline{m}_t(\overline{m}_t) \leq 50\,\text{MeV}$$

- Sensitive to Γ_t, α_s, y_t

Requires very precise theory predictions:

- Inclusive result known at NNNLO QCD + NNLO SM + LL ISR + NNNLO Yukawa
- Differential result known at (N)LL + NLO QCD



[Bach, Chokoufé Nejad, Hoang, Kilian, Reuter, Stahlhofen, Teubner, Weiss 2017]

Top quarks near threshold

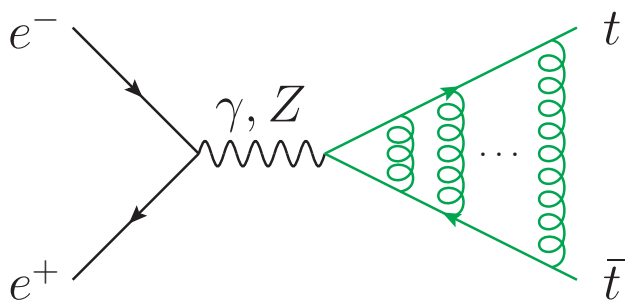
Relevant scales and Coulomb effects

Near threshold tops are non-relativistic with velocity $v \sim \alpha_s$

- Multiple scales are relevant:

hard	m_t	top mass
soft	$m_t v$	momentum
ultrasoft	$m_t v^2$	energy

- Coulomb singularities $(\alpha_s/v)^n$ from n exchanges of potential gluons



$k^0 \sim m_t v^2$, $\mathbf{k} \sim m_t v$

- Conventional perturbation theory in α_s fails
- Coulomb singularities must be resummed to all orders
- Done with potential non-relativistic QCD (PNRQCD)

[Pineda, Soto 1998; Beneke, Signer, Smirnov 1999; Brambilla, Pineda, Soto, Vairo 2000; Beneke, Kiyo, Schuller 2013]

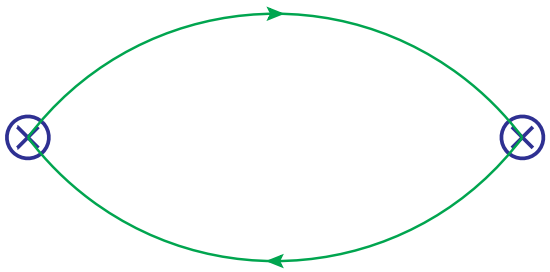
QCD cross section

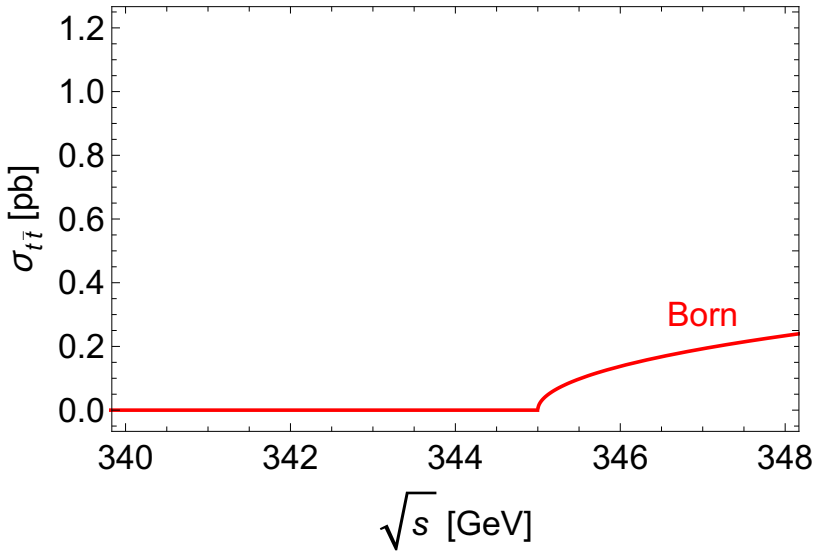
Born approximation

Total inclusive cross section from the optical theorem:

$$\sigma_{t\bar{t}}(s) = 12\pi e_t^2 f(s)\,\mathrm{Im}\left[\Pi^{(v)}(s)\right] \sim \alpha_{\mathrm{EW}}^2 v\left[1+\mathcal{O}(v^2)\right]$$

vector current correlator



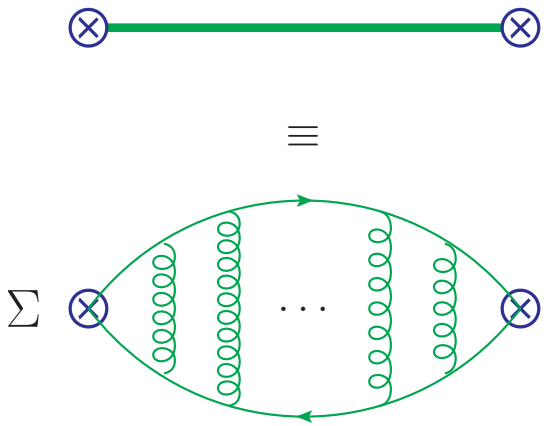


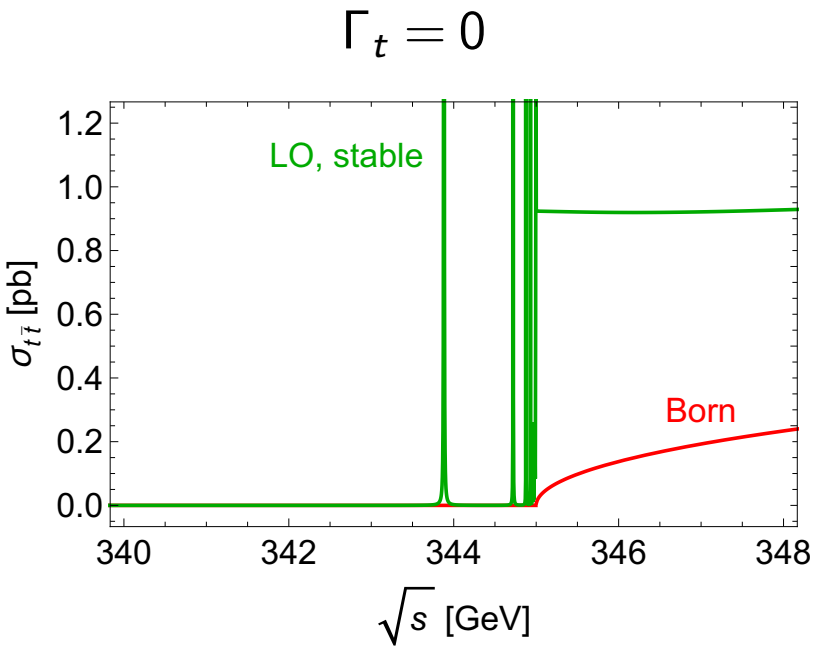
QCD cross section

Resummed cross section at LO

Coulomb resummation yields narrow toponium resonances

$$\sigma_{t\bar{t}}(s) \sim \alpha_{\mathrm{EW}}^2 v \sum_{k=0}^{\infty} \left(\frac{\alpha_s}{v} \right)^k$$



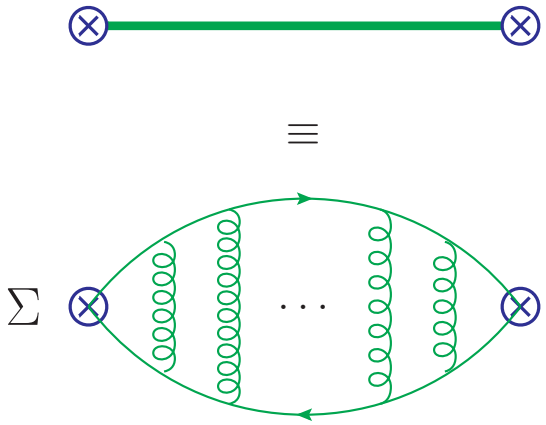


QCD cross section

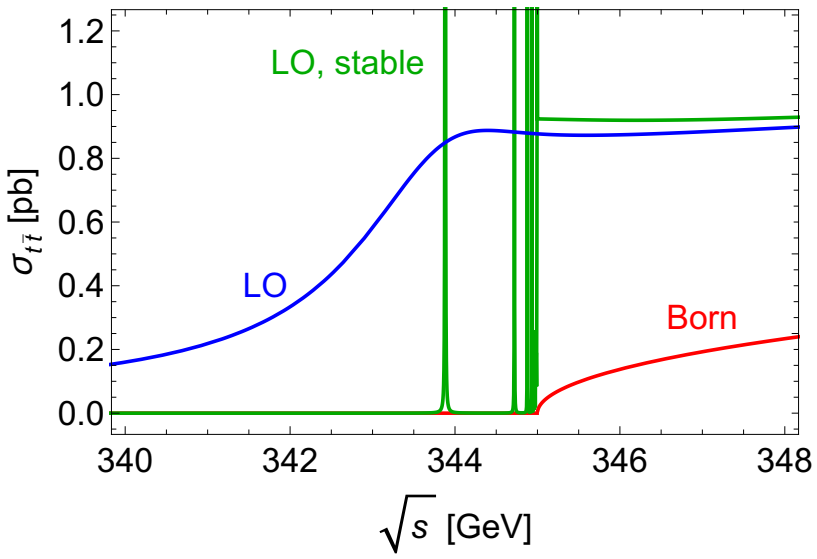
Resummed cross section at LO

Coulomb resummation yields narrow toponium resonances which are smeared out by top decays

$$\sigma_{t\bar{t}}(s) \sim \alpha_{\mathrm{EW}}^2 v \sum_{k=0}^{\infty} \left(\frac{\alpha_s}{v} \right)^k$$



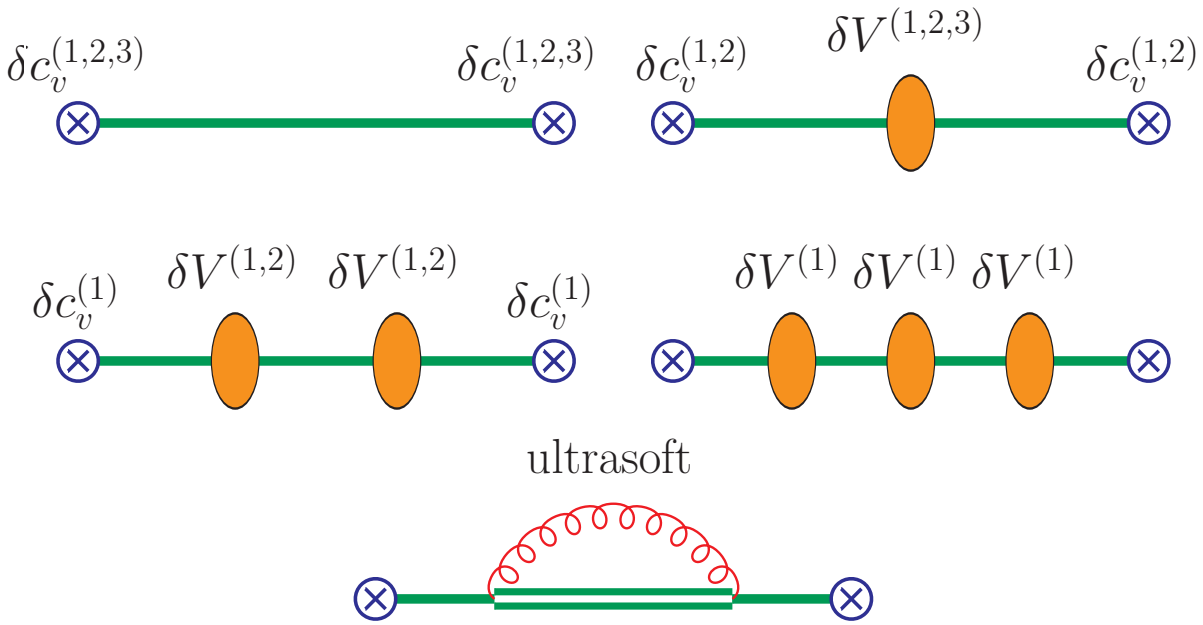
$$\Gamma_t \sim m_t \alpha_{\mathrm{EW}} \sim m_t \alpha_s^2 \sim m_t v^2$$



QCD cross section

Resummed cross section at NNNLO

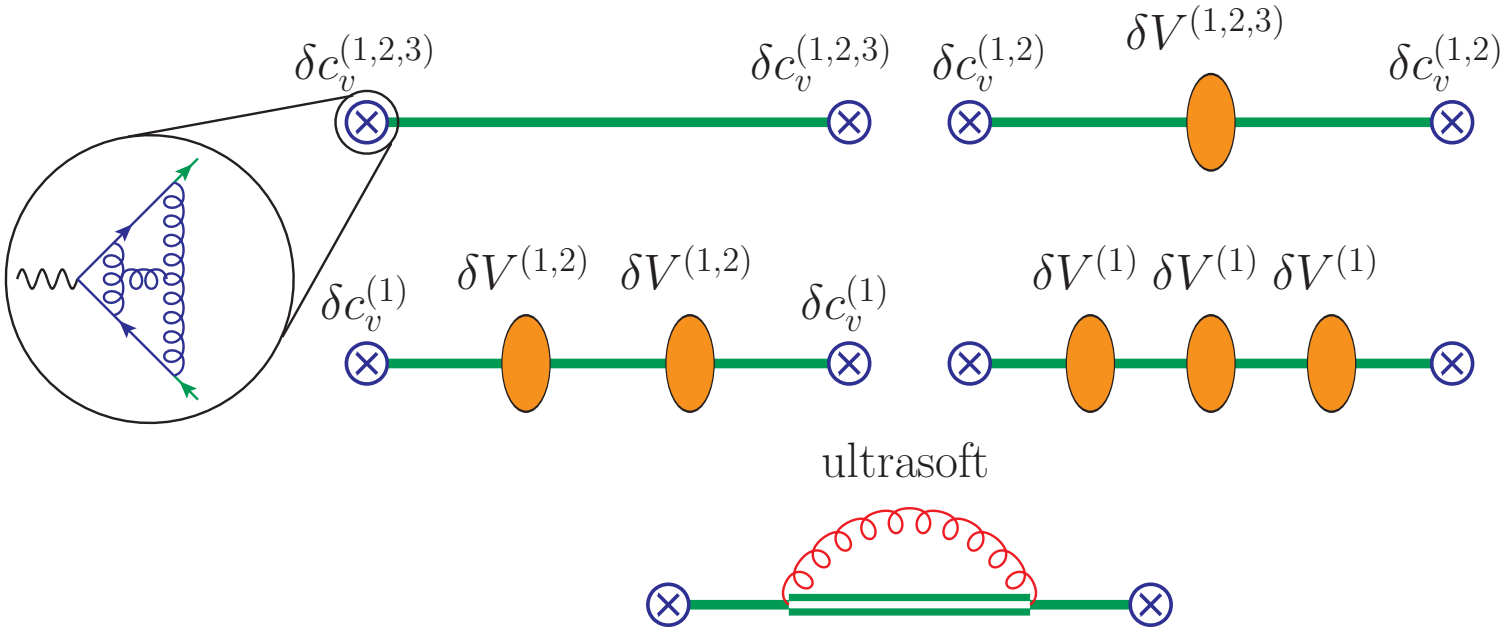
$$\sigma_{t\bar{t}}(s) \sim \alpha_{\mathrm{EW}}^2 v \sum_{k=0}^{\infty} \left(\frac{\alpha_s}{v}\right)^k \times \begin{cases} 1 & \text{LO} \\ \alpha_s, v & \text{NLO} \\ \alpha_s^2, \alpha_s v, v^2 & \text{NNLO} \\ \alpha_s^3, \alpha_s^2 v, \alpha_s v^2, v^3 & \text{NNNLO} \end{cases}$$



QCD cross section

Resummed cross section at NNNLO

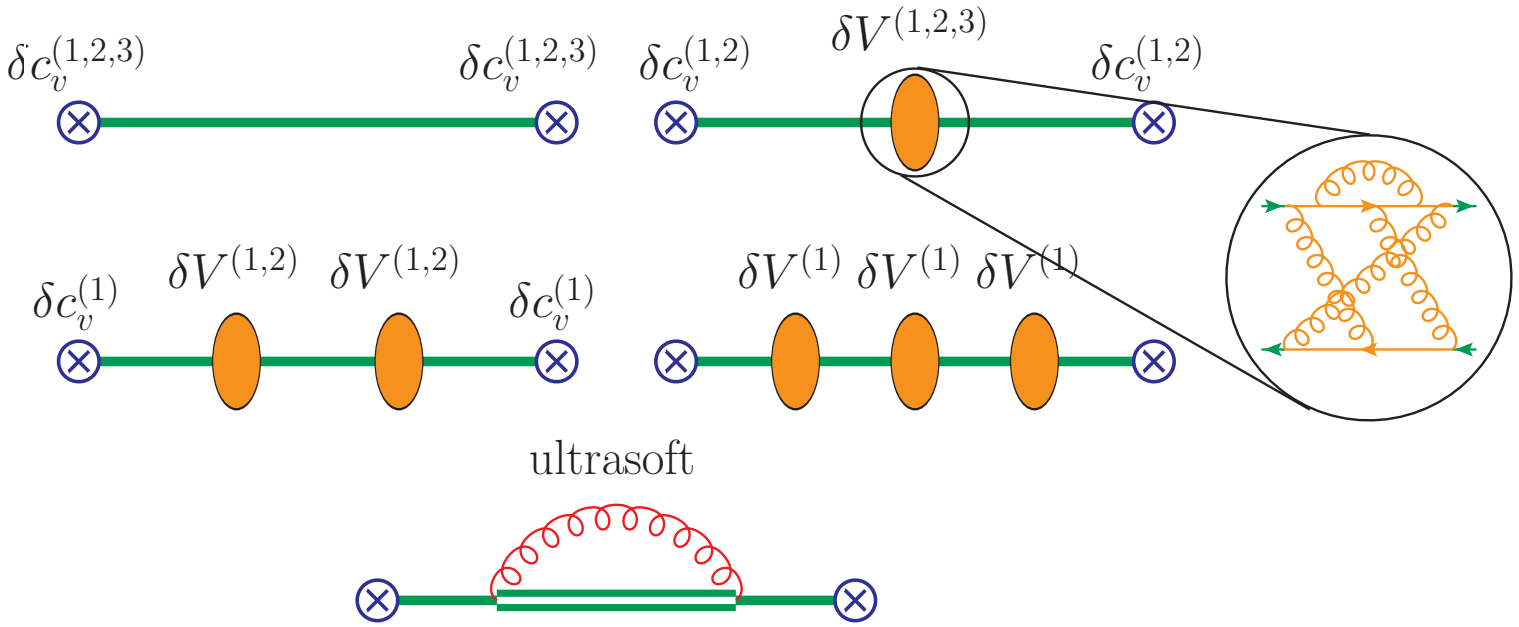
$$\sigma_{t\bar{t}}(s) \sim \alpha_{\mathrm{EW}}^2 v \sum_{k=0}^{\infty} \left(\frac{\alpha_s}{v}\right)^k \times \begin{cases} 1 & \text{LO} \\ \alpha_s, v & \text{NLO} \\ \alpha_s^2, \alpha_s v, v^2 & \text{NNLO} \\ \alpha_s^3, \alpha_s^2 v, \alpha_s v^2, v^3 & \text{NNNLO} \end{cases}$$



QCD cross section

Resummed cross section at NNNLO

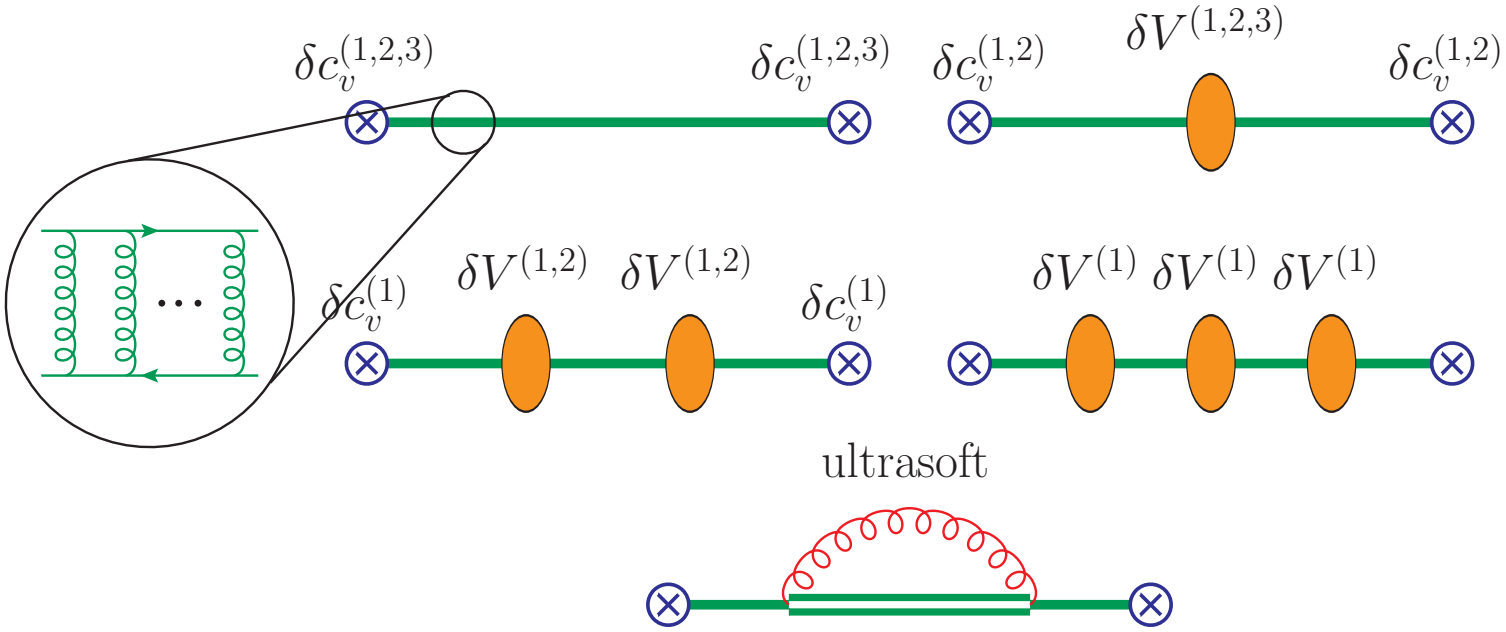
$$\sigma_{t\bar{t}}(s) \sim \alpha_{\mathrm{EW}}^2 v \sum_{k=0}^{\infty} \left(\frac{\alpha_s}{v}\right)^k \times \begin{cases} 1 & \text{LO} \\ \alpha_s, v & \text{NLO} \\ \alpha_s^2, \alpha_s v, v^2 & \text{NNLO} \\ \alpha_s^3, \alpha_s^2 v, \alpha_s v^2, v^3 & \text{NNNLO} \end{cases}$$



QCD cross section

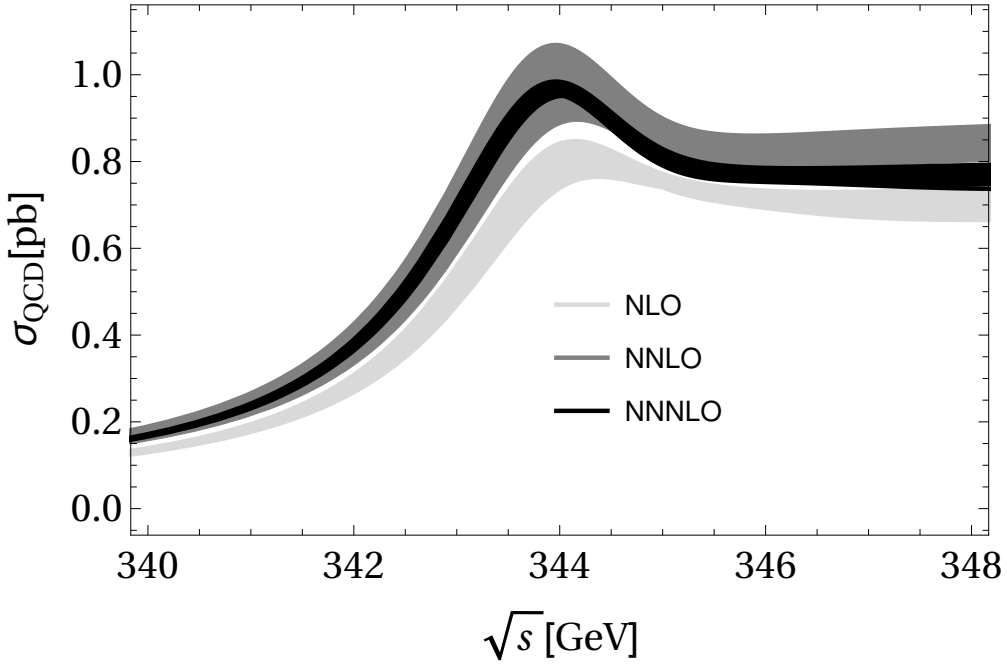
Resummed cross section at NNNLO

$$\sigma_{t\bar{t}}(s) \sim \alpha_{\mathrm{EW}}^2 v \sum_{k=0}^{\infty} \left(\frac{\alpha_s}{v}\right)^k \times \begin{cases} 1 & \text{LO} \\ \alpha_s, v & \text{NLO} \\ \alpha_s^2, \alpha_s v, v^2 & \text{NNLO} \\ \alpha_s^3, \alpha_s^2 v, \alpha_s v^2, v^3 & \text{NNNLO} \end{cases}$$



QCD cross section

Resummed cross section at NNNLO



- NNNLO S-wave [Beneke, Kiyo, Marquard, Penin, Piclum, Steinhauser 2015]
- NLO P-wave [Beneke, Piclum, TR 2013]
- QQbar_Threshold code [Beneke, Kiyo, Maier, Piclum 2016; Beneke, Maier, TR, Ruiz-Femenía 2017]

- Stabilization of perturbative expansion at NNNLO
- 3% uncertainty due to scale variation from 50 to 350 GeV
- Similar conclusions at NNLL (5% uncertainty) [Hoang, Stahlhofen 2013]

Work beyond NNLO QCD

	Effective field theory	QCD cross section	Non-QCD effects
	Pineda, Soto 1998		Fadin, Khoze 1987 Grzadkowski, Kühn, Krawczyk, Stuart 1987 Guth, Kühn 1992
	Beneke, Signer, Smirnov 1999 Brambilla, Pineda, Soto, Vairo 1999		
2000	Luke, Manohar, Rothstein 2000 Manohar, Stewart 2000	Hoang et al., 2000	
		Kniehl, Penin, Steinhauser, Smirnov 2001 Hoang, Manohar, Stewart, Teubner 2001	
	Hoang, Stewart 2002		
		Wüster 2003 Hoang 2003	
	Beneke, Chapovsky, Signer, Zanderighi 2003 Beneke, Chapovsky, Signer, Zanderighi 2004		
			Hoang, Reißer 2004
2005		Beneke, Kiyo, Schuller 2005	
		Pineda, Signer 2006 Hoang, Stahlhofen 2006	Eiras, Steinhauser 2006 Hoang, Reißer 2006
	Beneke et al., 2007	Beneke, Kiyo, Penin 2007	
	Actis, Beneke, Falgari, Schwinn, Signer 2008	Beneke, Kiyo, 2008	Kiyo, Seidel, Steinhauser 2008
		Anzai, Kiyo, Sumino 2009 Smirnov, Smirnov, Steinhauser 2009	
2010			Beneke, Jantzen, Ruiz-Femenía 2010 Hoang, Reißer, Ruiz-Femenía 2010
		Hoang, Stahlhofen 2011	Penin, Piclum 2011
	Beneke, Kiyo, Schuller 2013	Hoang, Stahlhofen 2013 Beneke, Piclum, Rauh 2013	Jantzen, Ruiz-Femenía 2013
		Marquard, Piclum, Seidel, Steinhauser 2014	Ruiz-Femenía 2014
2015		Beneke, Kiyo, Marquard, Penin, Piclum, Steinhauser 2015	Beneke, Piclum, Maier, Rauh 2015
		Lee, Smirnov, Smirnov, Steinhauser 2016	Beneke, Kiyo, Maier, Piclum 2016
		WHIZARD, Hoang, Stahlhofen, Teubner 2017	Beneke, Maier, Rauh, Ruiz-Femenía 2017
		Beneke, Kiyo, Schuller, in preparation	

Work beyond NNLO QCD

Effective field theory	QCD cross section	Non-QCD effects
Pineda, Soto 1998		Fadin, Khoze 1987
PNRQCD	**NNLO QCD**	Grzadkowski, Kühn, Krawczyk, Stuart 1987
Beneke, Signer, Smirnov 1999		Guth, Kühn 1992
Brambilla, Pineda, Soto, Vairo 1999		
Luke, Manohar, Rothstein 2000	Hoang et al., 2000	
Manohar, Stewart 2000		
vNRQCD	Kniehl, Penin, Steinhauser, Smirnov 2001	
	Hoang, Manohar, Stewart, Teubner 2001	
Hoang, Stewart 2002		
	Wüster 2003	
Beneke, Chapovsky, Signer, Zanderighi 2003	Hoang 2003	
Beneke, Chapovsky, Signer, Zanderighi 2004		
Unstable Particle ET		Hoang, Reißer 2004
	Beneke, Kiyo, Schuller 2005	
	Pineda, Signer 2006	Eiras, Steinhauser 2006
	Hoang, Stahlhofen 2006	Hoang, Reißer 2006
	Beneke, Kiyo, Penin 2007	
Beneke et al., 2007		
Actis, Beneke, Falgari, Schwinn, Signer 2008	Beneke, Kiyo, 2008	Kiyo, Seidel, Steinhauser 2008
	Anzai, Kiyo, Sumino 2009	
	Smirnov, Smirnov, Steinhauser 2009	
		Beneke, Jantzen, Ruiz-Femenía 2010
		Hoang, Reißer, Ruiz-Femenía 2010
	Hoang, Stahlhofen 2011	Penin, Piclum 2011
	NNLL QCD	
Beneke, Kiyo, Schuller 2013	Hoang, Stahlhofen 2013	Beneke, Piclum, Rauh 2013
		Jantzen, Ruiz-Femenía 2013
PNRQCD	Marquard, Piclum, Seidel, Steinhauser 2014	**NNNLO QCD**
		Ruiz-Femenía 2014
	Beneke, Kiyo, Marquard, Penin, Piclum, Steinhauser 2015	Beneke, Piclum, Maier, Rauh 2015
	Lee, Smirnov, Smirnov, Steinhauser 2016	Beneke, Kiyo, Maier, Piclum 2016
	WHIZARD, Hoang, Stahlhofen, Teubner 2017	Beneke, Maier, Rauh, Ruiz-Femenía 2017
	(N)LL+NLO diff.	**NNLO SM+LL ISR +NNNLO Yukawa**
	Beneke, Kiyo, Schuller, in preparation	

2000 — 2005 — 2010 — 2015

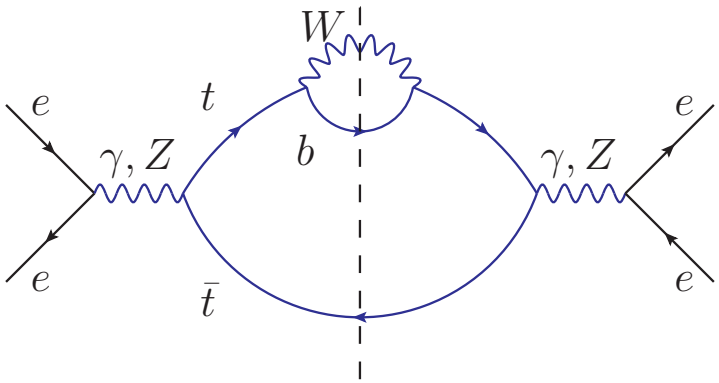
Non-QCD effects

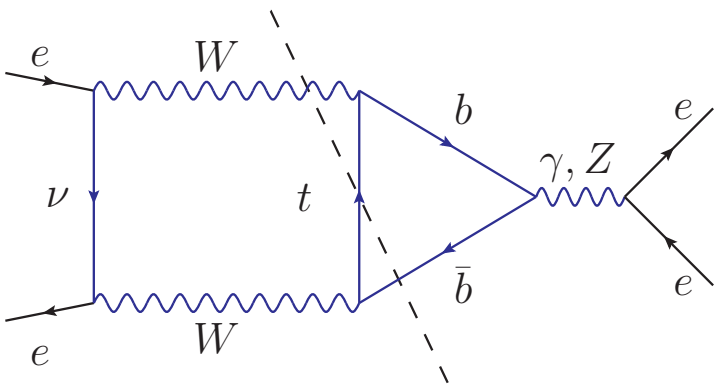
Non-resonant contributions

The physical final state is $W^+W^-\bar{b}bX$

- $\Gamma_t \sim m_t\alpha \sim m_t\alpha_s^2$ is not suppressed with respect to the ultrasoft scale
- Narrow width approximation is unphysical!
- Top decay modifies cross section in non-perturbative way (smearing of toponium resonances)

Top instability implies existence of contributions to the cross section from hard subgraphs that connect to the initial and final state





Non-QCD effects

Effective theory setup

Contributions can be organized systematically within Unstable Particle Effective Theory [Beneke, Chapovsky, Signer, Zanderighi 2003-4]

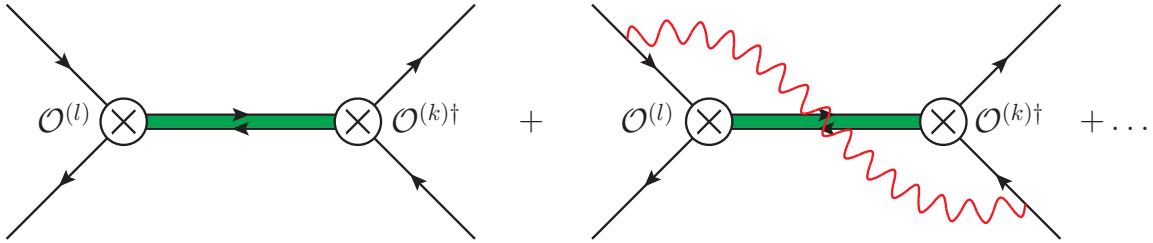
$$\sigma(s) \sim \mathrm{Im}\left\{ \sum_{k,l} C^{(k)} C^{(l)} \int d^4x \, \langle e^- e^+ | \mathrm{T}[i\mathcal{O}^{(k)\dagger}(0)\, i\mathcal{O}^{(l)}(x)] | e^- e^+ \rangle_{\mathrm{EFT}} \right.$$
$$\left. + \sum_k C_{4e}^{(k)} \langle e^- e^+ | i\mathcal{O}_{4e}^{(k)}(0) | e^- e^+ \rangle_{\mathrm{EFT}} \right\}$$

Non-QCD effects

Effective theory setup

Contributions can be organized systematically within Unstable Particle Effective Theory [Beneke, Chapovsky, Signer, Zanderighi 2003-4]

Resonant contribution involving non-rel. tops. Width resummed into propagators $E \to E + i\Gamma_t$



$$\sigma(s) \sim \mathrm{Im}\left\{ \sum_{k,l} C^{(k)} C^{(l)} \int d^4x \, \langle e^- e^+ | \mathrm{T}[i\mathcal{O}^{(k)\dagger}(0)\, i\mathcal{O}^{(l)}(x)] | e^- e^+ \rangle_{\mathrm{EFT}} \right.$$

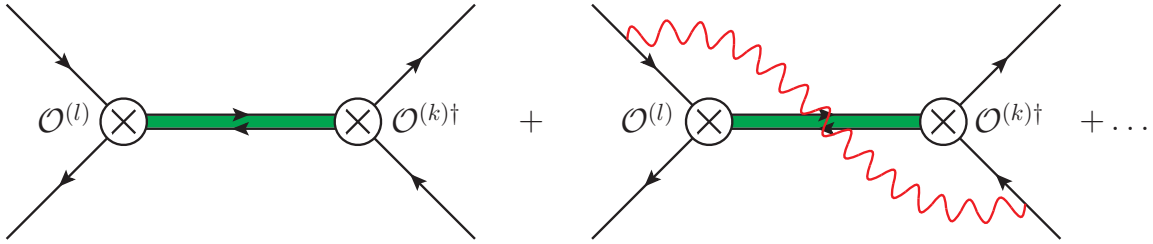
$$\left. + \sum_k C_{4e}^{(k)} \langle e^- e^+ | i\mathcal{O}_{4e}^{(k)}(0) | e^- e^+ \rangle_{\mathrm{EFT}} \right\}$$

Non-QCD effects

Effective theory setup

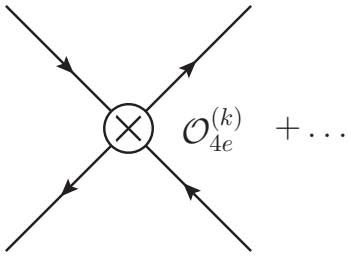
Contributions can be organized systematically within Unstable Particle Effective Theory [Beneke, Chapovsky, Signer, Zanderighi 2003-4]

Resonant contribution involving non-rel. tops. Width resummed into propagators $E \to E + i\Gamma_t$



$$\sigma(s) \sim \mathrm{Im}\left\{ \sum_{k,l} C^{(k)} C^{(l)} \int d^4x \, \langle e^- e^+ | \mathrm{T}[i\mathcal{O}^{(k)\dagger}(0)\, i\mathcal{O}^{(l)}(x)] | e^- e^+ \rangle_{\mathrm{EFT}} \right.$$
$$\left. + \sum_k C_{4e}^{(k)} \langle e^- e^+ | i\mathcal{O}_{4e}^{(k)}(0) | e^- e^+ \rangle_{\mathrm{EFT}} \right\}$$

Non-resonant contribution from $W^+W^- b\bar{b}$ production in hard process

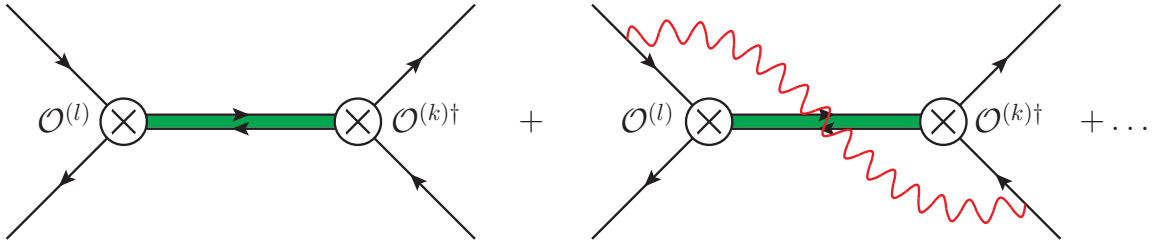


Non-QCD effects

Effective theory setup

Contributions can be organized systematically within Unstable Particle Effective Theory [Beneke, Chapovsky, Signer, Zanderighi 2003-4]

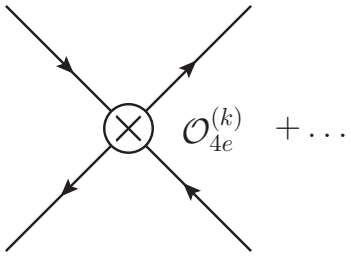
Resonant contribution involving non-rel. tops. Width resummed into propagators $E \to E + i\Gamma_t$



$$\sigma(s) \sim \mathrm{Im}\left\{ \sum_{k,l} C^{(k)} C^{(l)} \int d^4x \, \langle e^- e^+ | \mathrm{T}[i\mathcal{O}^{(k)\dagger}(0)\, i\mathcal{O}^{(l)}(x)] | e^- e^+ \rangle_{\mathrm{EFT}} \right.$$
$$\left. + \sum_k C_{4e}^{(k)} \langle e^- e^+ | i\mathcal{O}_{4e}^{(k)}(0) | e^- e^+ \rangle_{\mathrm{EFT}} \right\}$$

Both parts contain spurious divergences! Only the sum is finite. Calculations must be done in the same regularization scheme.

Non-resonant contribution from $W^+W^- b\bar{b}$ production in hard process



Public code

Implementation of NNNLO QCD + NNLO SM + LL ISR + NNNLO Yukawa results is available on HEPForge

- Home
- Download
- Documentation
  - Version 2
  - Version 1
- Changelog

QQbar_threshold

QQbar_threshold computes the top-quark pair production cross section near threshold in electron-positron annihilation at NNNLO in resummed non-relativistic perturbation theory [1, 2]. It includes Higgs, QED, electroweak and non-resonant corrections at various accuracies and a consistent implementation of initial-state radiation. Details can be found in

- M. Beneke, Y. Kiyo, A. Maier, and J. Piclum
  Near-threshold production of heavy quarks with QQbar_threshold
  Comput. Phys. Commun. 209 (2016) 96-115, arXiv:1605.03010 [hep-ph]
- M. Beneke, A. Maier, T. Rauh, and P. Ruiz-Femenía
  Non-resonant and electroweak NNLO correction to the $e^+ e^-$ top anti-top threshold
  arXiv:1711.10429 [hep-ph]

Please cite these (and possibly other articles, where the theoretical input was first computed) when QQbar_threshold is used for published work.

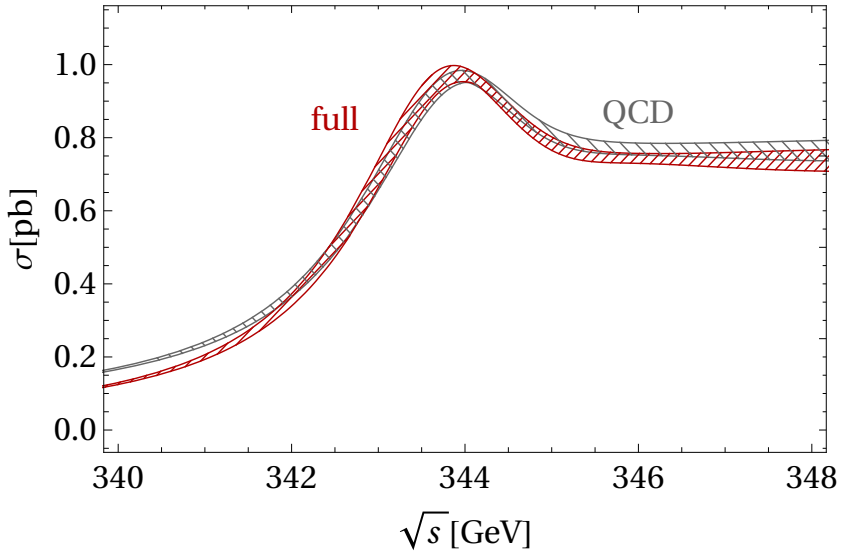
The functionality of the package can also be used to compute the bound state energies and residues of bottomonium S-wave states and high moments of the bottom production cross section at NNNLO, including the continuum (see M. Beneke, A. Maier, J. Piclum, T. Rauh, Nucl.Phys. B891 (2015) 42-72, arXiv:1411.3132 [hep-ph]).

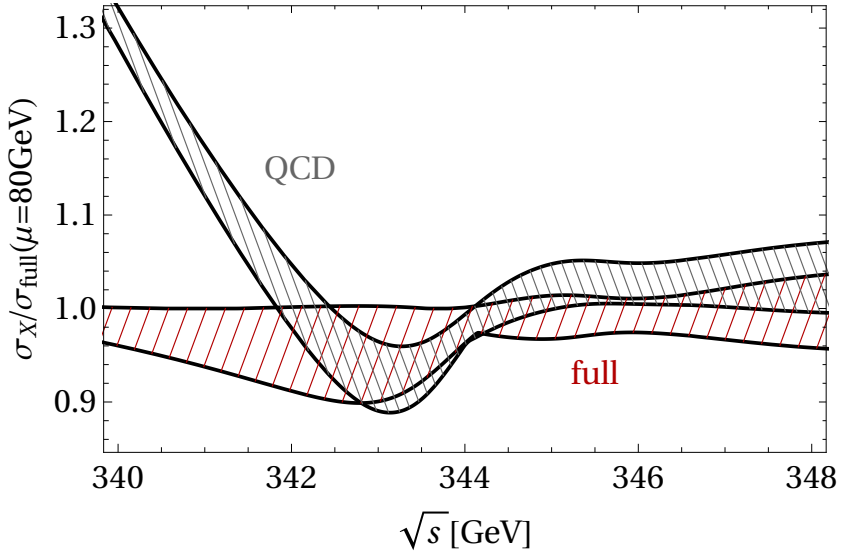
QQbar_threshold is written in C++ and Wolfram Language. It can be used as a C++ library or through a Mathematica interface.

For questions, comments, and bug reports write to qqbarthreshold@projects.hepforge.org.

Non-QCD effects

NNLO SM and NNNLO Yukawa contributions

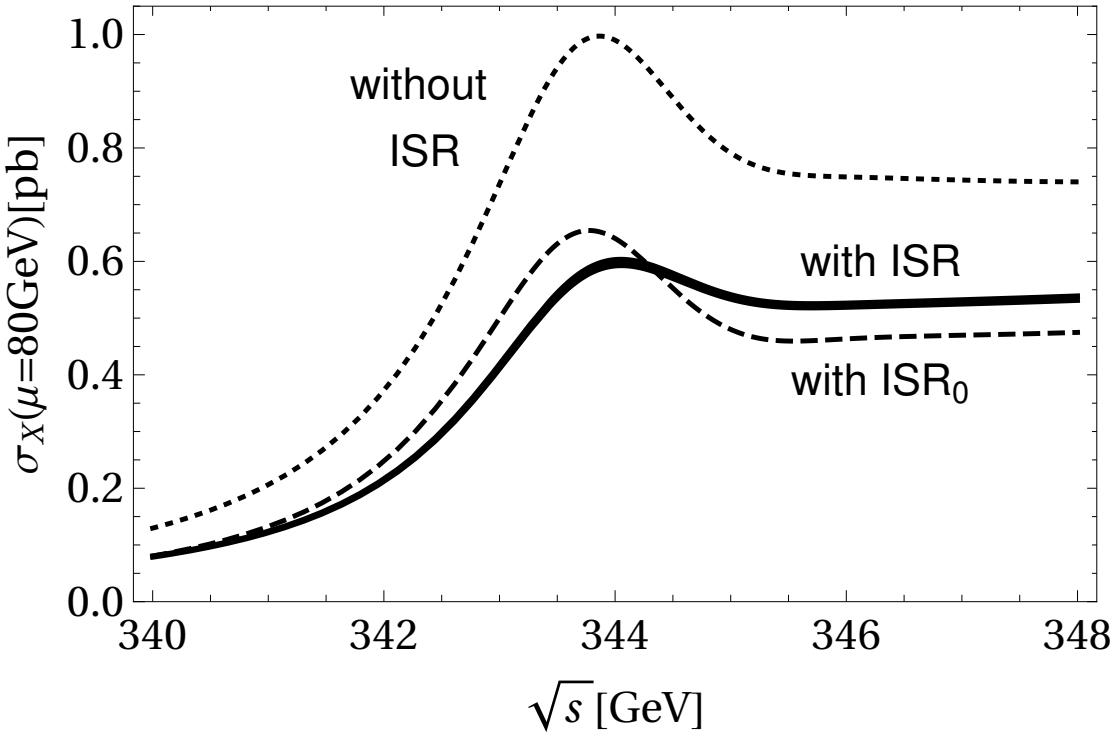




- Uncertainty due to renormalization scale variation between 50 GeV and 350 GeV
- Effects significantly larger than QCD uncertainty
- Shape changes particularly in the important region at and below threshold

Non-QCD effects

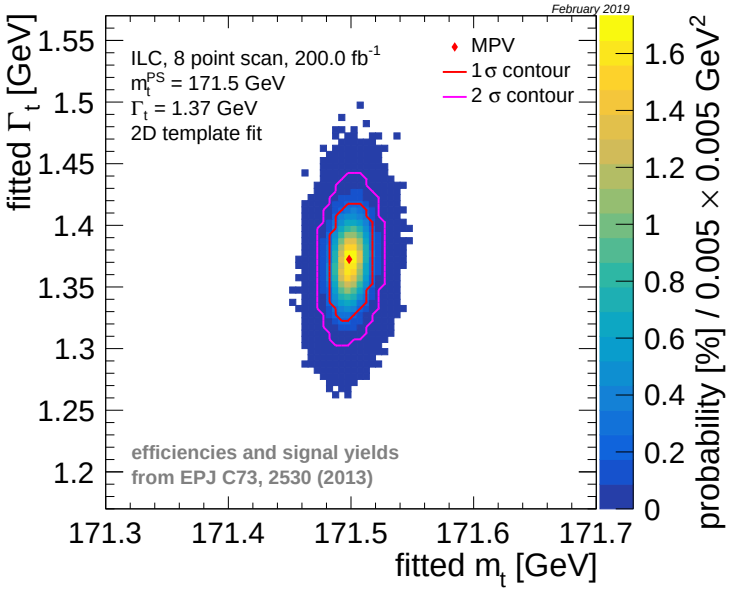
Initial state radiation

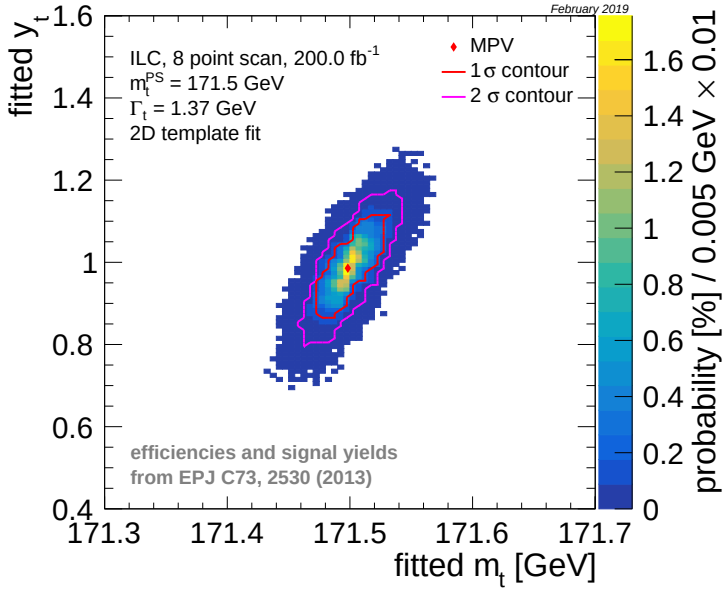


- ISR reduces cross section by 30-45 %
- Band is envelope of different LL accurate implementations
- NLL precision is a must for a lepton collider (not just for ttbar)

Determination of SM parameters

Results of a full simulation assuming ILC luminosity spectrum [Simon 2019]





parameter	8 point scan	10 point scan
1D fit		
m_t	$(\pm 10.3_{\text{(stat)}} \pm 44_{\text{(theo)}})$ MeV	$(12.2_{\text{(stat)}} \pm 40_{\text{(theo)}})$ MeV
2D fit m_t and Γ_t		
m_t	$({}^{+20.7}_{-24.3}{}_{\text{(stat)}} \pm 45_{\text{(theo)}})$ MeV	$({}^{+29.7}_{-25.3}{}_{\text{(stat)}} \pm 43_{\text{(theo)}})$ MeV
Γ_t	$({}^{+50}_{-55}{}_{\text{(stat)}} \pm 32_{\text{(theo)}})$ MeV	$({}^{+80}_{-55}{}_{\text{(stat)}} \pm 39_{\text{(theo)}})$ MeV
2D fit m_t and y_t		
m_t	$(\pm 35_{\text{(stat)}} \pm 45_{\text{(theo)}})$ MeV	$({}^{+34}_{-31}{}_{\text{(stat)}} \pm 42_{\text{(theo)}})$ MeV
y_t	${}^{+0.12}_{-0.14}{}_{\text{(stat)}} \pm 0.09_{\text{(theo)}}$	${}^{+0.128}_{-0.112}{}_{\text{(stat)}} \pm 0.132_{\text{(theo)}}$

Differential distributions

Implementation in WHIZARD [Bach, Chokoufé Nejad, Hoang, Kilian, Reuter, Stahlhofen, Teubner, Weiss 2017]

Matched cross section:

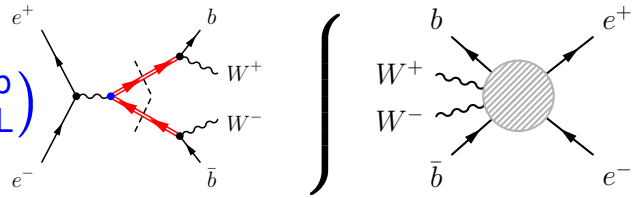
$$\sigma_{\text{matched}} = \sigma_{\text{NLO}}(\alpha_{\text{H}}) + \sigma_{\text{resum}}(f_s\alpha_{\text{H}}, f_s\alpha_{\text{S}}, f_s\alpha_{\text{US}}) - \sigma_{\text{resum}}^{\text{expand}}(f_s\alpha_{\text{H}})$$

Fixed order $W^+W^-b\bar{b}$ cross section at NLO in QCD from WHIZARD

Resummed cross section at (N)LL with form factors $\tilde{F}_{\text{NLL}} = F_{\text{NLL}} - 1$ in the resonant contribution

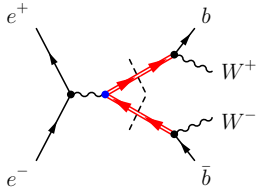
Subtraction to remove double counting

$$\sigma_{\text{NLO+NLL}} = \sigma_{\text{NLO}} + \Bigg(\left(F_{\text{NLL}} - F_{\text{NLL}}^{\text{exp}}\right)$$

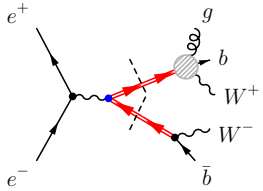


$$\Bigg)$$

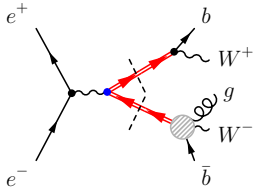
$$+ \left| \tilde{F}_{\text{NLL}} \right.$$



$$\left.\right|^2 + \left| \tilde{F}_{\text{NLL}} \right.$$

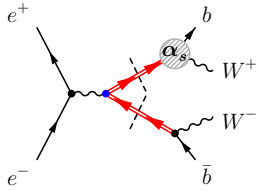


$$\left.\right|^2 + \left| \tilde{F}_{\text{NLL}} \right.$$

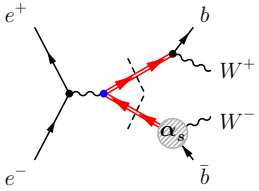


$$\left.\right|^2$$

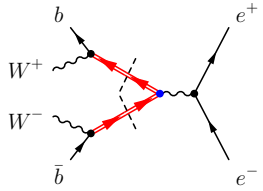
$$+ \Bigg(\tilde{F}_{\text{NLL}} \Bigg(\Bigg.$$



$$+$$



$$\Bigg. \Bigg) \Bigg(\Bigg.$$



$$\tilde{F}_{\text{NLL}} \Bigg. \Bigg)$$

Differential distributions

Implementation in WHIZARD

f_s = switch-off function to turn off resummation in relativistic regime

Matched cross section:

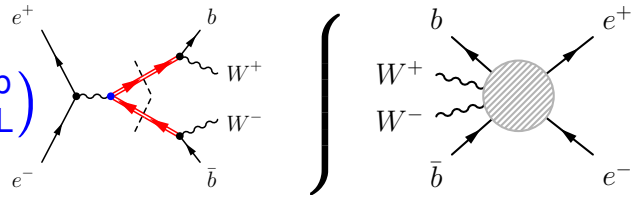
$$\sigma_{\text{matched}} = \sigma_{\text{NLO}}(\alpha_{\text{H}}) + \sigma_{\text{resum}}(f_s\alpha_{\text{H}}, f_s\alpha_{\text{S}}, f_s\alpha_{\text{US}}) - \sigma_{\text{resum}}^{\text{expand}}(f_s\alpha_{\text{H}})$$

Fixed order $W^+W^-b\bar{b}$ cross section at NLO in QCD from WHIZARD

Resummed cross section at (N)LL with form factors $\tilde{F}_{\text{NLL}} = F_{\text{NLL}} - 1$ in the resonant contribution

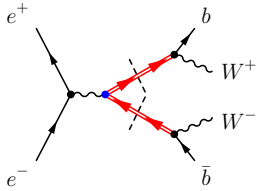
Subtraction to remove double counting

$$\sigma_{\text{NLO+NLL}} = \sigma_{\text{NLO}} + \left(\left(F_{\text{NLL}} - F_{\text{NLL}}^{\text{exp}}\right) \right.$$

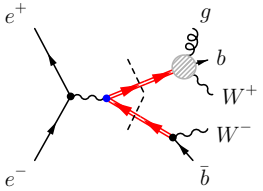


$$\left. \right)$$

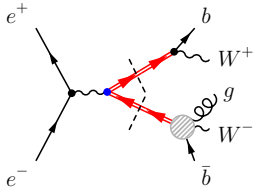
$$+ \left| \tilde{F}_{\text{NLL}} \right.$$



$$\left.\right|^2 + \left| \tilde{F}_{\text{NLL}} \right.$$

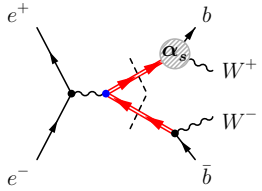


$$\left.\right|^2 + \left| \tilde{F}_{\text{NLL}} \right.$$

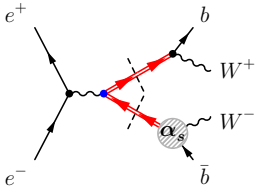


$$\left.\right|^2$$

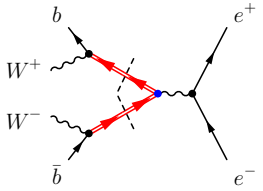
$$+ \left(\tilde{F}_{\text{NLL}} \left(\right.\right.$$



$$+$$



$$\left.\left.\right)\right.$$



$$\tilde{F}_{\text{NLL}} \left.\right)$$

Differential distributions

Implementation in WHIZARD

Matched cross section:

$$\sigma_{\text{matched}} = \sigma_{\text{NLO}}(\alpha_{\text{H}}) + \sigma_{\text{resum}}(f_s\alpha_{\text{H}}, f_s\alpha_{\text{S}}, f_s\alpha_{\text{US}}) - \sigma_{\text{resum}}^{\text{expand}}(f_s\alpha_{\text{H}})$$

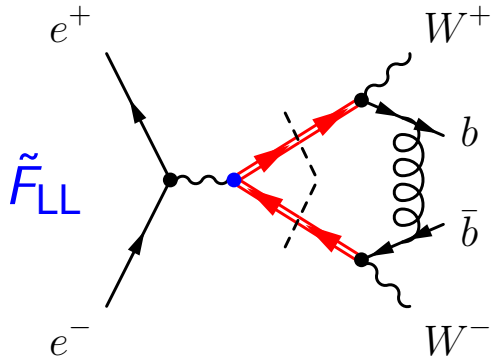
Fixed order $W^+W^-b\bar{b}$ cross section at NLO in QCD from WHIZARD

Resummed cross section at (N)LL with form factors $\tilde{F}_{\text{NLL}} = F_{\text{NLL}} - 1$ in the resonant contribution

Subtraction to remove double counting

(N)LL+NLO accuracy depending on observable:

$$\sigma \sim \alpha_{\text{EW}}^2 v \sum_{k,i} \left(\frac{\alpha_s}{v}\right)^k (\alpha_s \ln v)^i \times \begin{cases} 1 & \text{LL} \\ \alpha_s, v & \text{NLL} \end{cases}$$

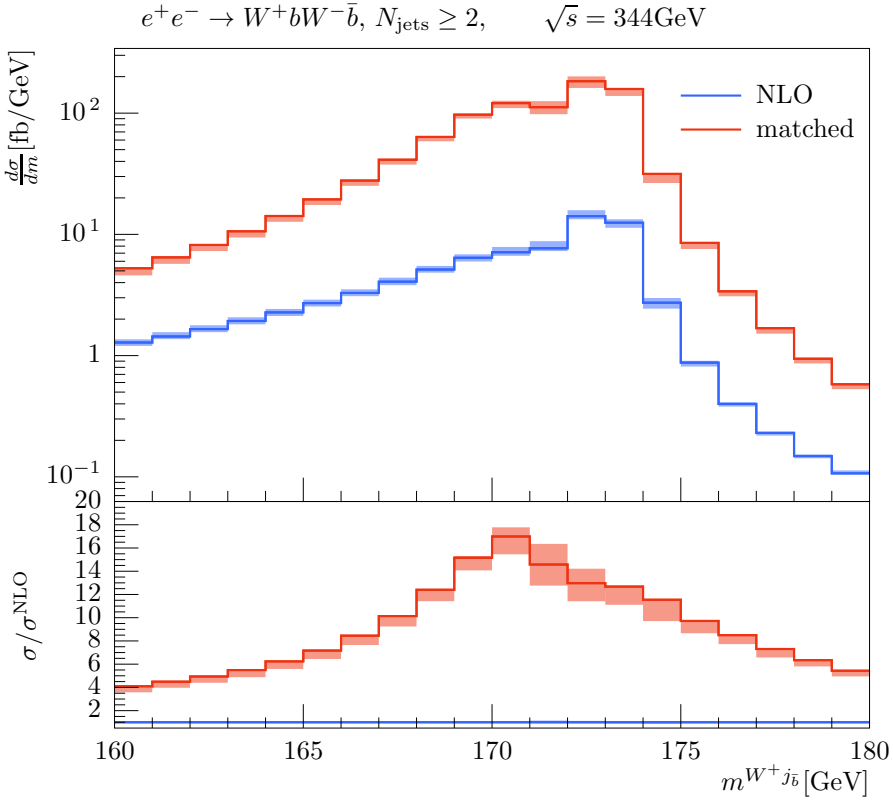


Ultrasoft gluon exchanges involving the decay products are missing, but cancel in sufficiently inclusive quantities.

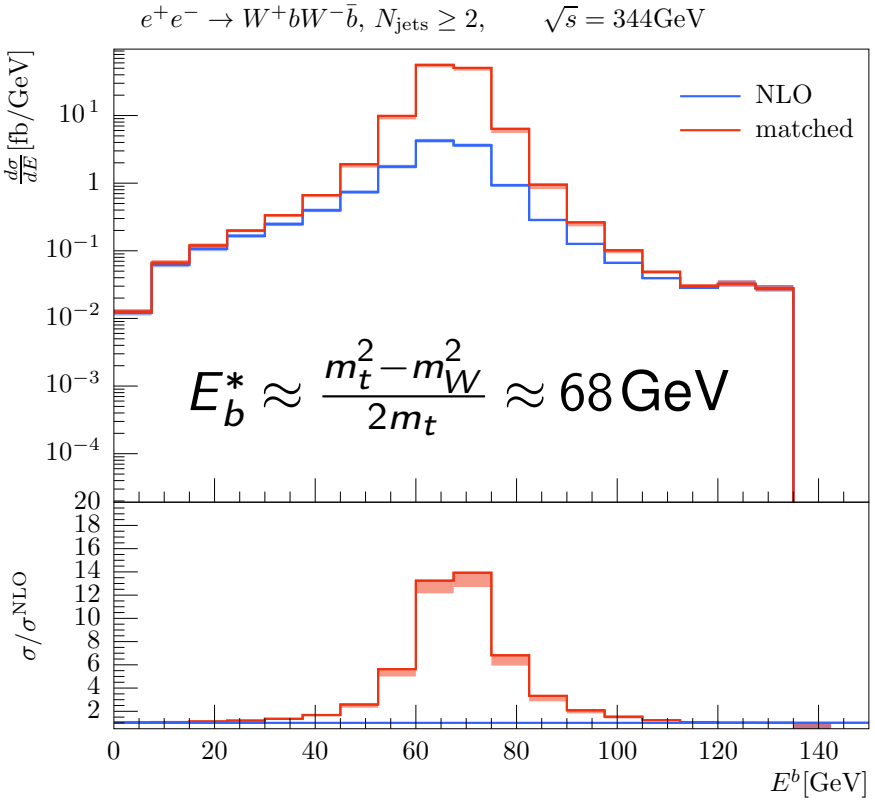
Differential distributions

Examples at the peak $\sqrt{s} = 2m_t^{1S}$

top inv. mass distribution



b-jet energy distribution

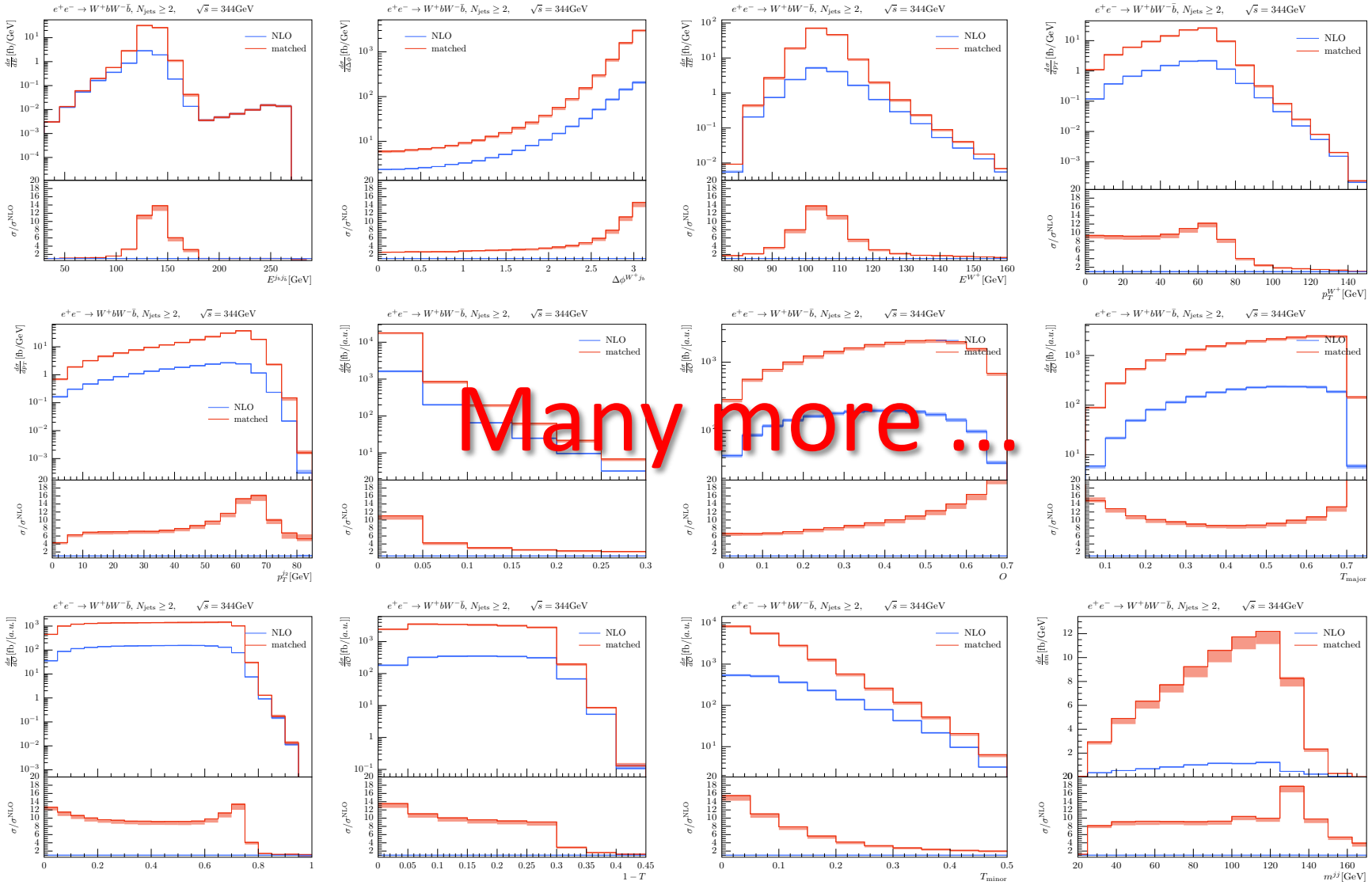


(RIVET event analysis; FASTJET generalized k_T algorithm, $R = 0.4$, $p = -1$; $E_{\text{jet}} > 1\text{GeV}$)

[Bach, Chokoufé Nejad, Hoang, Kilian, Reuter, Stahlhofen, Teubner, Weiss 2017]

Differential distributions

Top-antitop threshold in WHIZARD



Many more …

Summary & Outlook

- Determination of several SM parameters possible from scan of the total $e^+e^- \to W^+W^-\bar{b}bX$ cross section near the top threshold
- NNNLO QCD + NNLO SM + LL ISR + NNNLO Yukawa prediction known and available in QQbar_Threshold
- Theoretical uncertainty of 2-5% (energy-dependent), translates to

parameter	8 point scan	10 point scan
m_t	$(\pm 10.3_{\text{(stat)}} \pm 44_{\text{(theo)}})$ MeV	$(12.2_{\text{(stat)}} \pm 40_{\text{(theo)}})$ MeV

[Simon 2019]

- Fully differential results at (N)LL+NLO implemented in WHIZARD
- Increase precision: NNNLO+NNLL QCD, NLL ISR, N^{4}LO Yukawa, NLL differential, ...
- Phenomenology of differential distributions, parameter sensitivity

Power counting

$$\alpha_{\rm EW} \sim \alpha_t \equiv \frac{\lambda_t^2}{4\pi} \sim \alpha_s^2 \sim v^2,$$

$$\sigma_{\rm QCD\,only} \sim \alpha_{\rm EW}^2 v \sum_{k=0}^{\infty} \left(\frac{\alpha_s}{v}\right)^k \times \begin{cases} 1 & \text{LO} \\ \alpha_s, v & \text{NLO} \\ \alpha_s^2, \alpha_s v, v^2 & \text{NNLO} \\ \alpha_s^3, \alpha_s^2 v, \alpha_s v^2, v^3 & \text{NNNLO} \end{cases},$$

$$\sigma \sim \alpha_{\rm EW}^2 v \sum_{k=0}^{\infty} \left(\frac{\alpha_s}{v}\right)^k \times \begin{cases} \dfrac{\alpha_{\rm em}}{v} & \text{NLO} \\ \left(\dfrac{\alpha_{\rm em}}{v}\right)^2, \dfrac{\alpha_{\rm em}}{v} \times \{\alpha_s, v\}, \alpha_{\rm EW}, \sqrt{\alpha_{\rm EW}\alpha_t}, \alpha_t & \text{NNLO} \\ \left(\dfrac{\alpha_{\rm em}}{v}\right)^3, \left(\dfrac{\alpha_{\rm em}}{v}\right)^2 \times \{\alpha_s, v\}, \dfrac{\alpha_{\rm em}}{v} \times \{\alpha_s^2, \alpha_s v, v^2, \sqrt{\alpha_{\rm EW}\alpha_t}\}, & \\ \alpha_t \times \{\dfrac{\alpha_{\rm em}}{v}, \alpha_s, v\}, \ldots & \text{NNNLO} \end{cases}$$

$$+ \alpha_{\rm EW}^2 \times \begin{cases} \alpha_{\rm EW} & \text{NLO} \\ \alpha_{\rm EW}\alpha_s & \text{NNLO} \\ \ldots & \text{NNNLO} \end{cases},$$

Organization of the calculation

Split cross section into three separately finite parts (I), (II) and (III):

$$\sigma^{\text{NNLO}} = \underbrace{\left[\sigma_{\text{sq}} + \sigma_{\text{res, rest}}\right]}_{\text{(I)}} + \underbrace{\left[\sigma_{\text{int}}^{(\text{EP div})} + \sigma_{C^{(k)}_{\text{Abs,bare}}}\right]}_{\text{(II)}} + \underbrace{\left[\sigma_{\text{int}}^{(\text{EP fin})} + \sigma_{\text{aut}}\right]}_{\text{(III)}}.$$

- (I): computational scheme for 'squared contribution' fixed by existing QCD results (Dim reg with NDR for γ^5)
- (II): Use freedom of scheme choice to simplify calculation (some parts done in four dimensions)
- (III): Endpoint finite part of 'interference contribution' must be computed consistent with MadGraph

Divergence structure

			UV finite	IR finite	EP finite
(I)			✓	✓	✓
	σ_{sq}		✓	✓	–
		$\sigma_{\text{sq}}^{(h_{1a},\ldots,h_{1g})}$	✓	–	–
		$\sigma_{\text{sq}}^{(g_1,\ldots,g_6)}$	✓	–	$\star$
	$\sigma_{\text{res, rest}}$		✓	✓	–
		σ_{QCD}	✓	✓	–
		$\sigma_{\text{P-wave}}$	✓	✓	–
		σ_{H}	✓	✓	✓
		$\sigma_{\delta V_{\text{QED}}}$	✓	✓	✓
		σ_{Γ}	✓	✓	–
		$\sigma_{C_{\text{EW}}^{(k)}}$	✓	✓	✓
		$\sigma_{C_{\text{Abs},Z_t}^{(k)}}$	✓	✓	–
		$\sigma_{\text{IS}}^{\text{conv}}$	✓	✓	✓
(II)			✓	✓	✓
	$\sigma_{\text{int}}^{(\text{EP div})}$		✓	✓	–
	$\sigma_{C_{\text{Abs,bare}}^{(k)}}$		✓	✓	–
(III)			✓	✓	✓
	$\sigma_{\text{int}}^{(\text{EP fin})}$		–	✓	✓
	σ_{aut}		–	✓	✓

Non-QCD effects

Top-quark decay width

$$\mathcal{L}_{\text{bilinear}} = \psi^\dagger \left[i\partial^0 + \frac{\vec{\partial}^2}{2m_t} + \frac{i\Gamma_t}{2} + \frac{(\vec{\partial}^2 + i m_t \Gamma_t)^2}{8m_t^3} + \dots \right] \psi + \text{anti-quark}$$

- $\psi^\dagger \frac{i\Gamma_t}{2} \psi$: same order as kinetic term, shifts $E \to E + i\Gamma_t \qquad (E = \sqrt{s} - 2m_t)$
  Causes divergences at NNLO: $\sigma \supset \sigma_0 \text{Im}\left[\frac{E}{\epsilon}\right] \to \sigma_0 \text{Im}\left[\frac{E+i\Gamma_t}{\epsilon}\right]$

Non-QCD effects

Top-quark decay width

$$\mathcal{L}_{\text{bilinear}} = \psi^\dagger \left[i\partial^0 + \frac{\vec{\partial}^2}{2m_t} + \frac{i\Gamma_t}{2} + \frac{(\vec{\partial}^2 + i m_t \Gamma_t)^2}{8m_t^3} + \dots \right] \psi + \text{anti-quark}$$

- $\psi^\dagger \frac{i\Gamma_t}{2} \psi$: same order as kinetic term, shifts $E \to E + i\Gamma_t \qquad (E = \sqrt{s} - 2m_t)$
  Causes divergences at NNLO: $\sigma \supset \sigma_0 \text{Im}\left[\frac{E}{\epsilon}\right] \to \sigma_0 \text{Im}\left[\frac{E+i\Gamma_t}{\epsilon}\right]$
- $-\psi^\dagger \frac{\Gamma_t^2}{8m_t} \psi$: additional shift $\to E + i\Gamma_t - {\Gamma_t}^2/(8m_t)$, but treated perturbatively

Non-QCD effects

Top-quark decay width

$$\mathcal{L}_{\text{bilinear}} = \psi^\dagger \left[i\partial^0 + \frac{\vec{\partial}^2}{2m_t} + \frac{i\Gamma_t}{2} + \frac{(\vec{\partial}^2 + i m_t \Gamma_t)^2}{8m_t^3} + \dots \right] \psi + \text{anti-quark}$$

- $\psi^\dagger \frac{i\Gamma_t}{2} \psi$: same order as kinetic term, shifts $E \to E + i\Gamma_t \qquad (E = \sqrt{s} - 2m_t)$
  Causes divergences at NNLO: $\sigma \supset \sigma_0 \text{Im}\left[\frac{E}{\epsilon}\right] \to \sigma_0 \text{Im}\left[\frac{E+i\Gamma_t}{\epsilon}\right]$
- $-\psi^\dagger \frac{\Gamma_t^2}{8m_t} \psi$: additional shift $\to E + i\Gamma_t - {\Gamma_t}^2/(8m_t)$, but treated perturbatively
- $\psi^\dagger \frac{i\Gamma_t \vec{\partial}^2}{4m_t^2} \psi$: time dilatation, reduces toponium width $\Gamma_n = 2\Gamma_t - \frac{\Gamma_t \alpha_s^2 C_F^2}{4n^2} + \dots$

Non-Hermitian Hamiltonian $H \Rightarrow$ eigenstates do not form a basis

$$H\,|n\rangle = \mathcal{E}_n\,|n\rangle\,, \qquad H^\dagger\,|\tilde{m}\rangle = \tilde{\mathcal{E}}_m\,|\tilde{m}\rangle\,, \qquad \tilde{\mathcal{E}}_n = \mathcal{E}_n^* = (E_n - i\Gamma_n/2)^*$$

$\uparrow$ exponentially decaying states

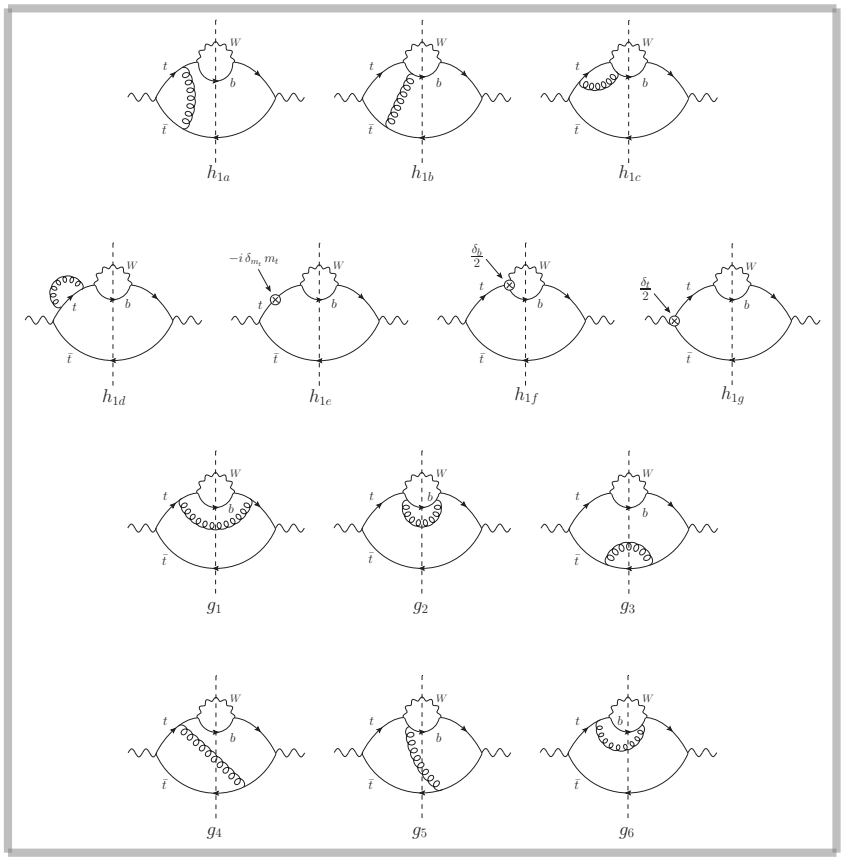
$\uparrow$ exponentially growing states

$$\langle n\,|\,\tilde{m}\rangle = \delta_{nm}$$

Non-relativistic Green function: $G(E) = \left\langle \vec{0} \middle| \hat{G}(E) \middle| \vec{0} \right\rangle = \sum\!\!\!\!\!\!\int_n \frac{\psi_n(\vec{0})\psi_{\tilde{n}}^*(\vec{0})}{\mathcal{E}_n - E}$

NNLO non-resonant contribution

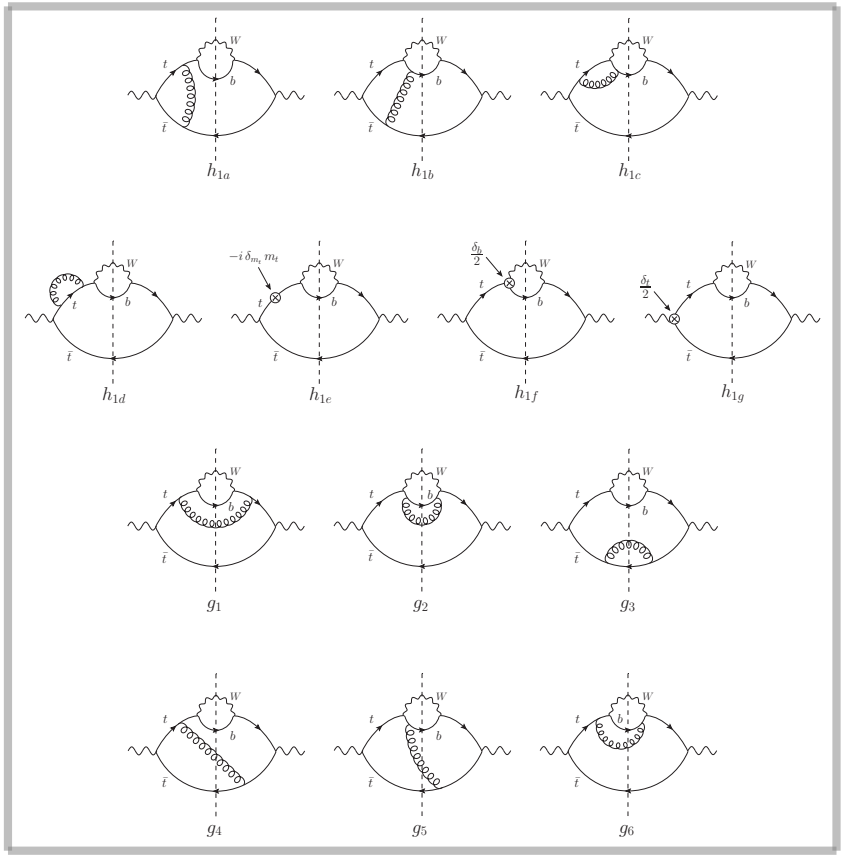
Contains endpoint divergences when the hard tops go on-shell [Jantzen, Ruiz-Femenía '13]



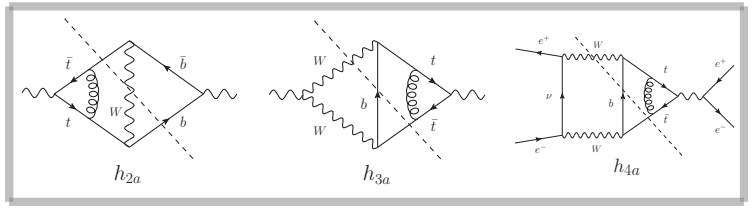
'Squared contribution': Gluon corrections to h1, endpoint divergent but UV & IR finite

NNLO non-resonant contribution

Contains endpoint divergences when the hard tops go on-shell [Jantzen, Ruiz-Femenía '13]



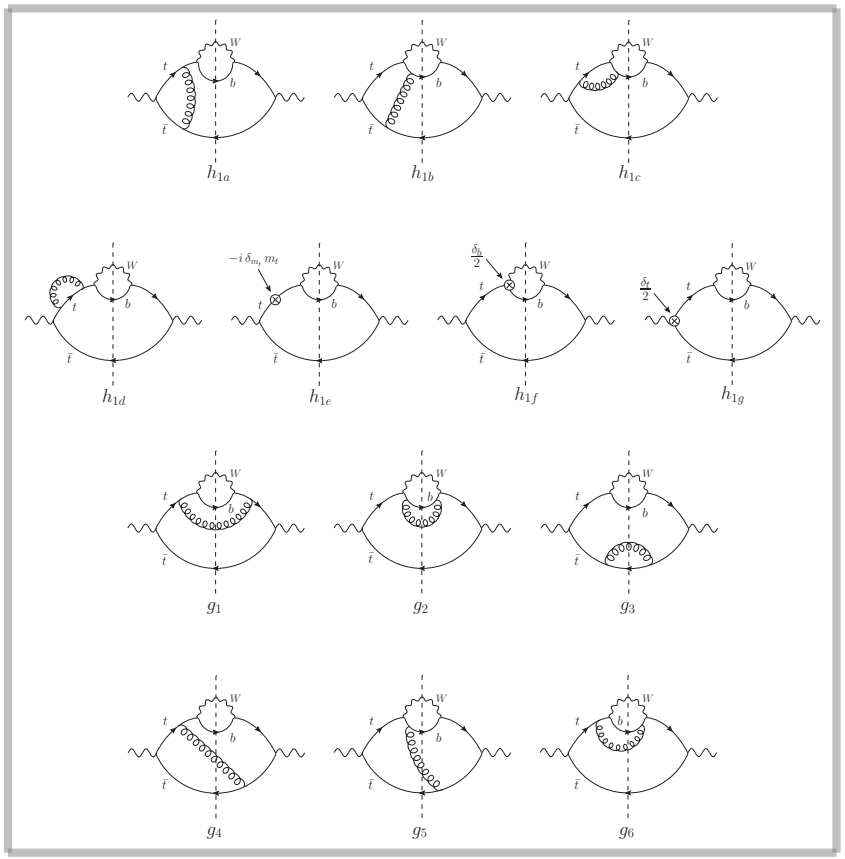
'Squared contribution': Gluon corrections to h1, endpoint divergent but UV & IR finite



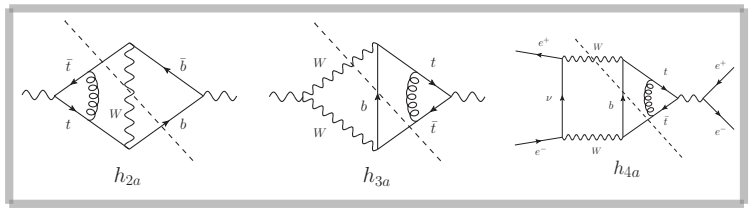
'Interference contribution': endpoint & UV divergent

NNLO non-resonant contribution

Contains endpoint divergences when the hard tops go on-shell [Jantzen, Ruiz-Femenía '13]



'Squared contribution': Gluon corrections to h1, endpoint divergent but UV & IR finite



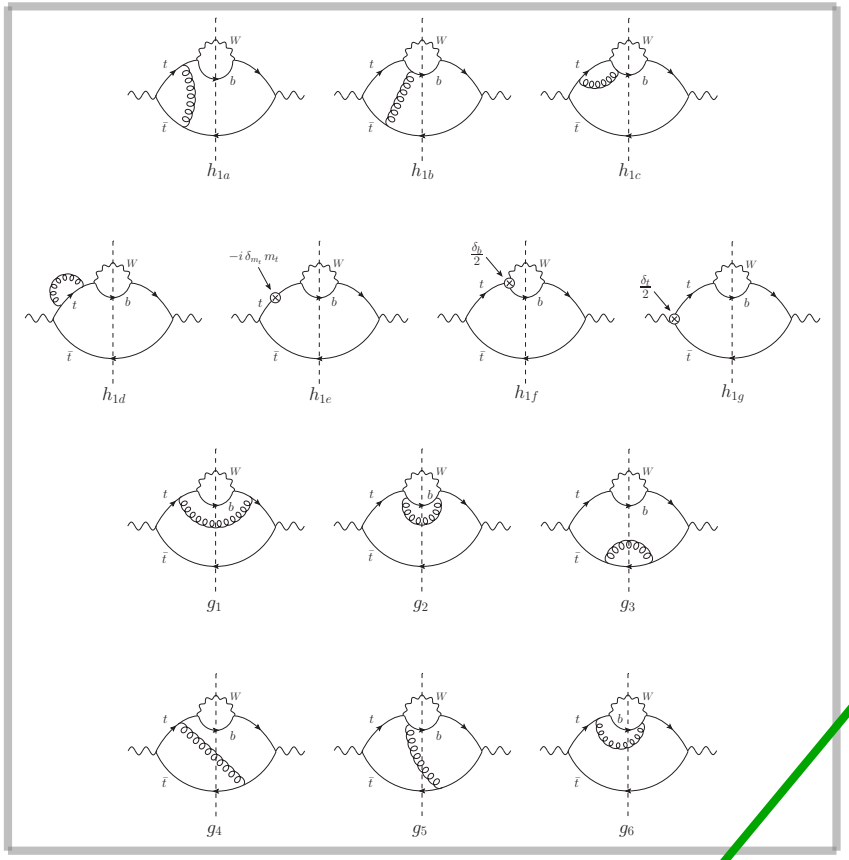
'Interference contribution': endpoint & UV divergent

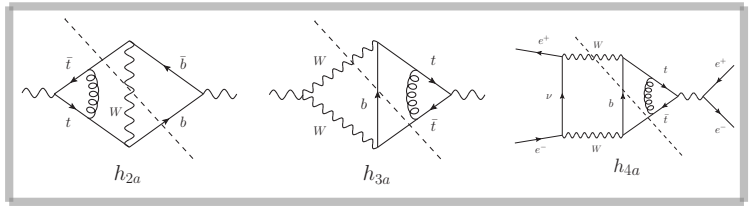
+ O(100) endpoint finite diagrams (not drawn)

'Automated contribution': endpoint finite but UV divergent, computed with automated tools (MadGraph)

NNLO non-resonant contribution

Contains endpoint divergences when the hard tops go on-shell [Jantzen, Ruiz-Femenía '13]





'Interference contribution': endpoint & UV divergent

cancel with resonant part

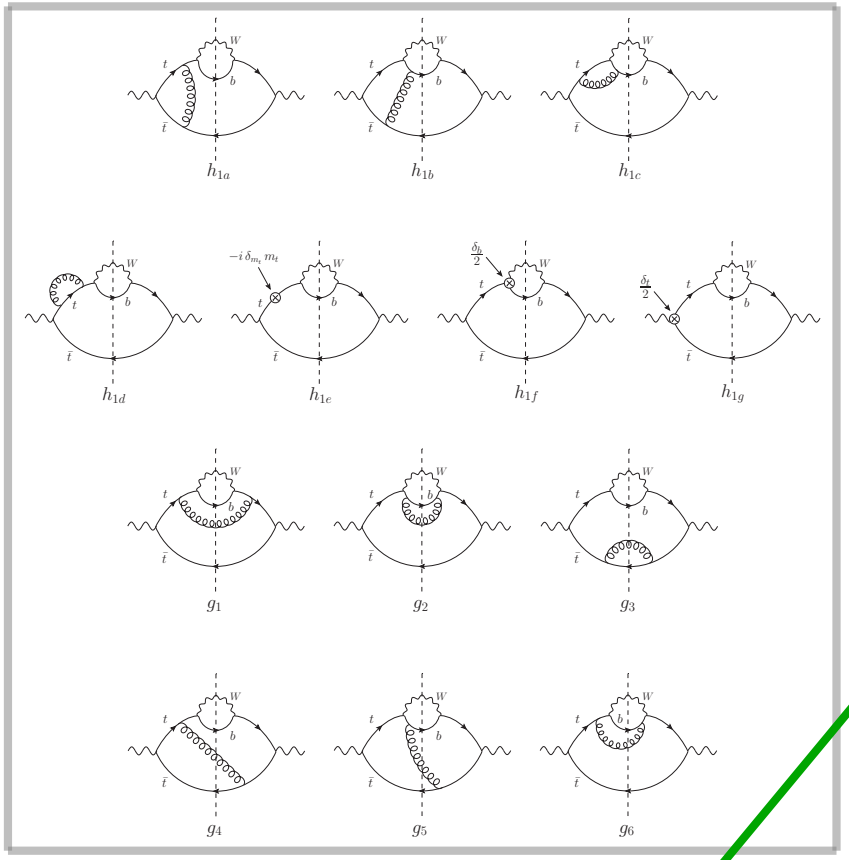
+ O(100) endpoint finite diagrams (not drawn)

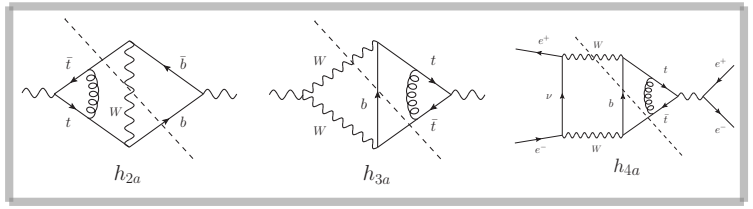
'Squared contribution': Gluon corrections to h1, endpoint divergent but UV & IR finite

'Automated contribution': endpoint finite but UV divergent, computed with automated tools (MadGraph)

NNLO non-resonant contribution

Contains endpoint divergences when the hard tops go on-shell [Jantzen, Ruiz-Femenía '13]





'Interference contribution': endpoint & UV divergent

cancel with resonant part

cancel

+ O(100) endpoint finite diagrams (not drawn)

'Squared contribution': Gluon corrections to h1, endpoint divergent but UV & IR finite

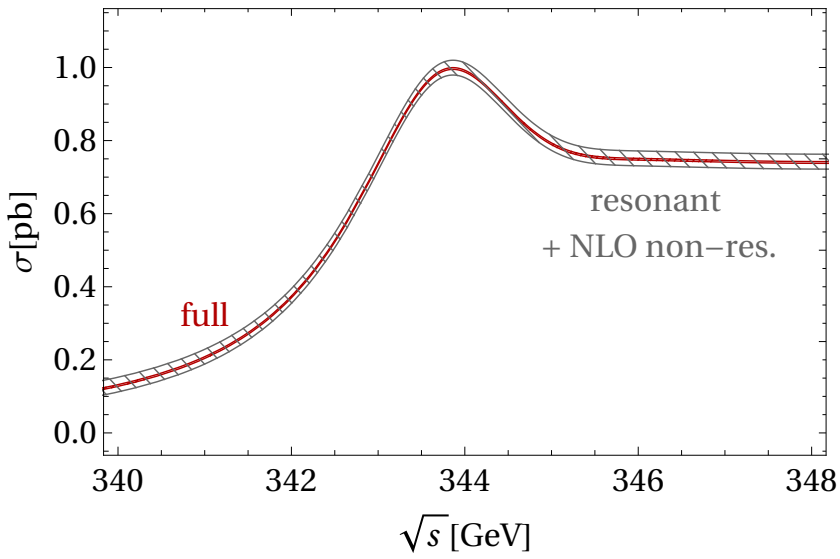
'Automated contribution': endpoint finite but UV divergent, computed with automated tools (MadGraph)

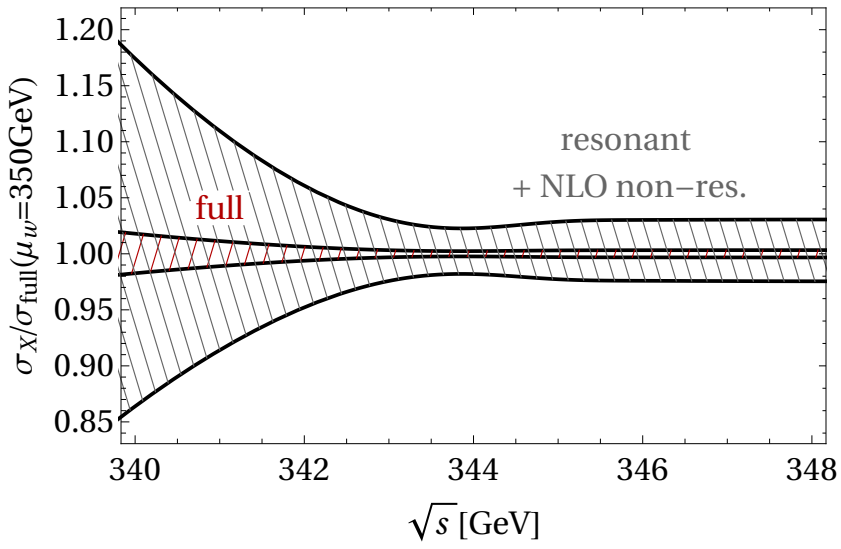
Dependence on μ_w scale

Regularizing width-related/endpoint divergences dimensionally splits some of the large logarithms by introducing the scale μ_w

$$\sigma_{\text{full}} \supset \ln v = \underbrace{\ln \frac{\mu_w}{m_t}}_{\subset \sigma_{\text{non-res}}} + \underbrace{\ln \frac{m_t v}{\mu_w}}_{\subset \sigma_{\text{res}}} .$$

The dependence on μ_w cancels exactly at a given order.





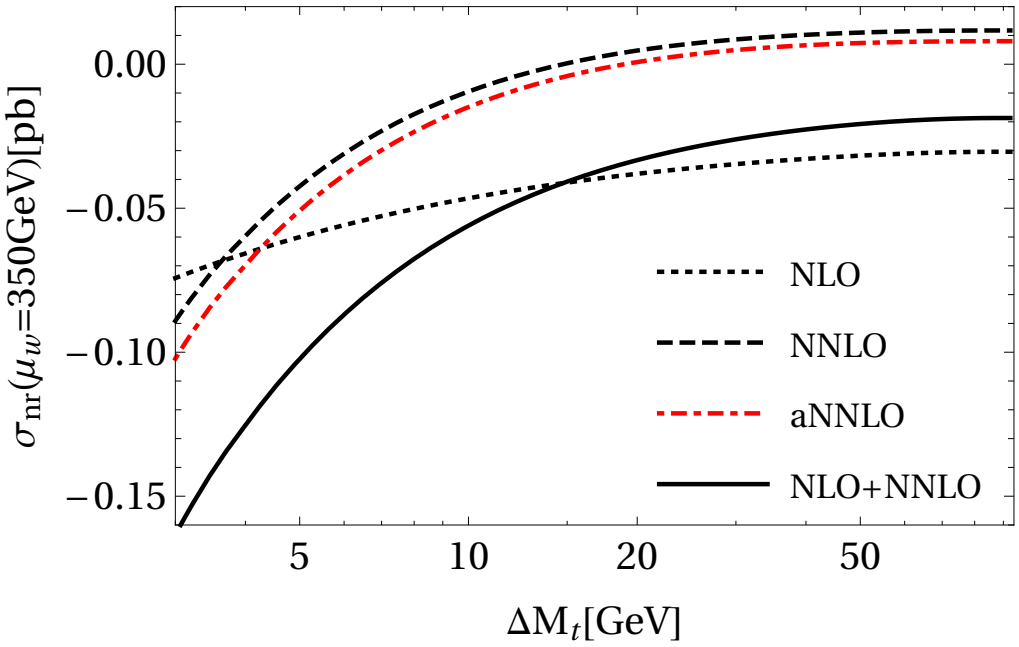
We choose a central scale of $\mu_w = 350\,\text{GeV}$ to minimize the unknown logarithms from the NNNLO non-resonant part.

Invariant mass cut

Consider "loose" invariant mass cuts

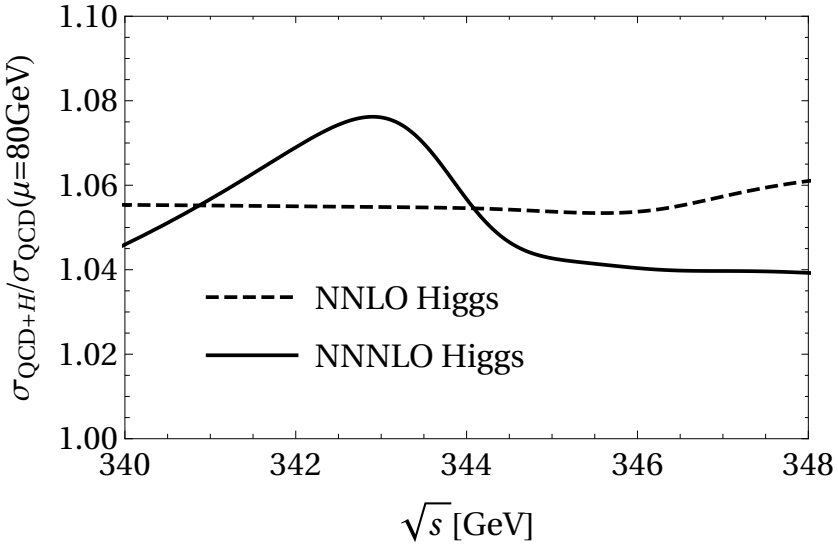
$$(m_t - \Delta M_t)^2 \leq p_{t,\bar{t}}^2 \leq (m_t + \Delta M_t)^2,$$

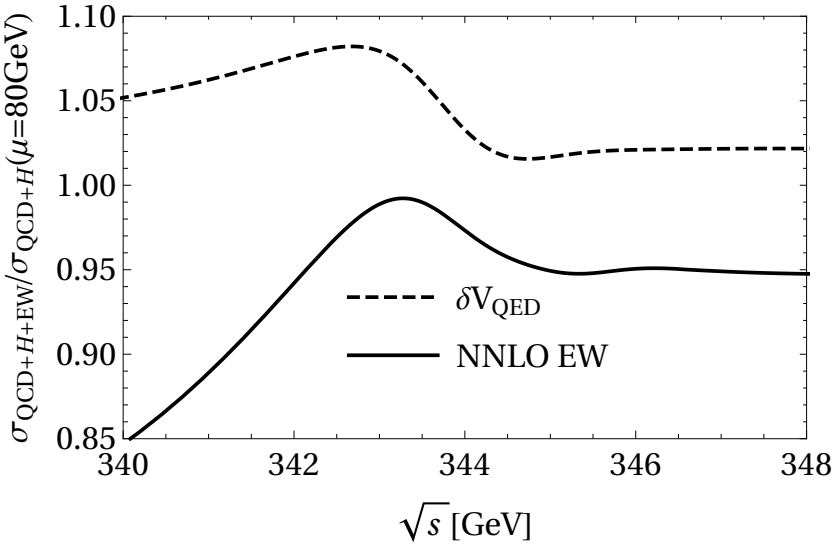
with $\Delta M_t \gg \Gamma_t$. Since the off-shellness in the resonant part is parametrically of the order Γ_t they only affect the non-resonant part:

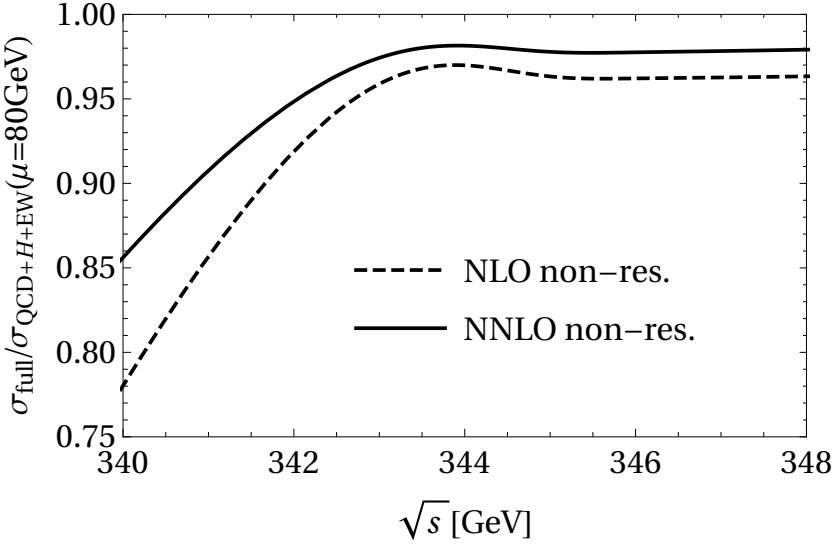


Non-QCD effects

Individual contributions

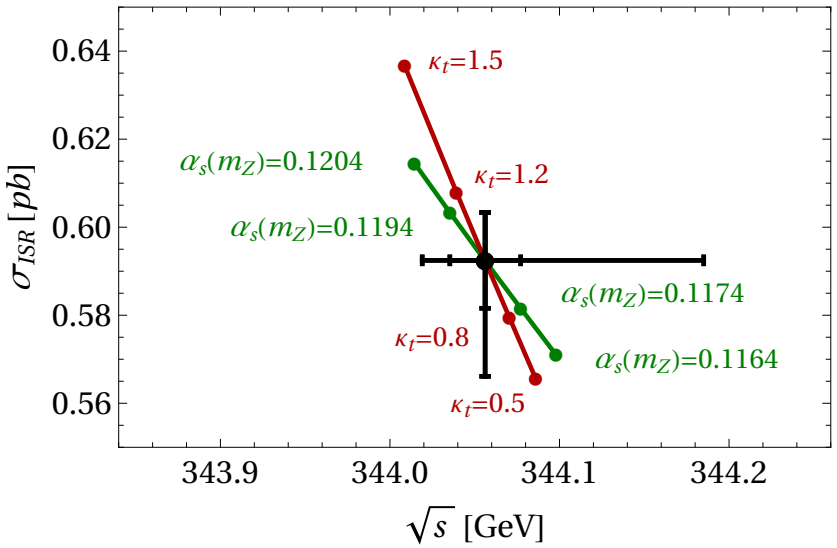






Determination of SM parameters

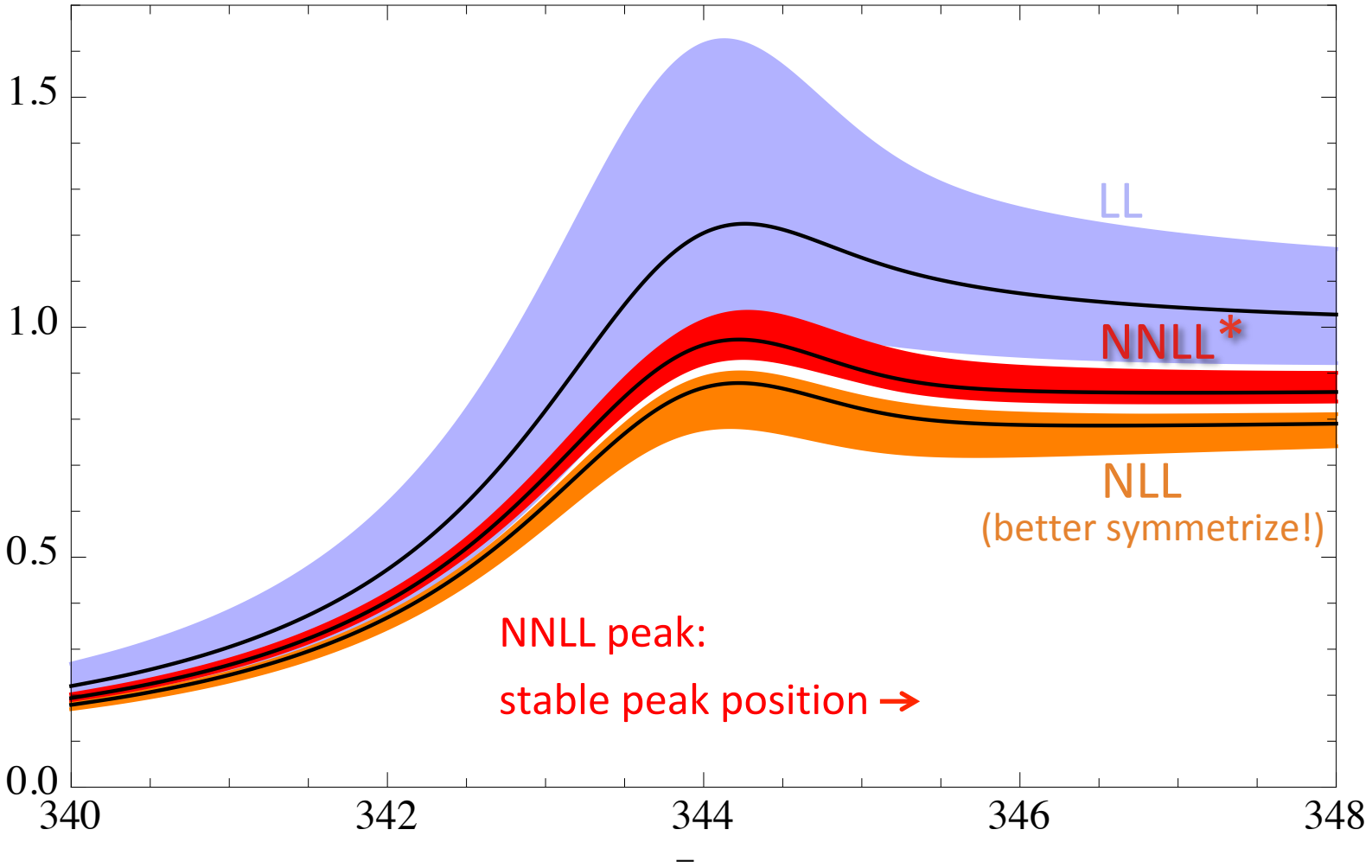
Correlation between top Yukawa and strong coupling



Peak height and width

- Estimate theory uncertainty by determining what parameter shift is needed to obtain curves outside the scale variation band
- Naive expectation: $\delta\kappa_t \approx^{+20}_{-25}$ % and $\delta\alpha_s \approx 0.0015$
- Effects from variation of Yukawa coupling and strong coupling very similar
- Need full simulation to see how well they can be disentangled

Total cross section



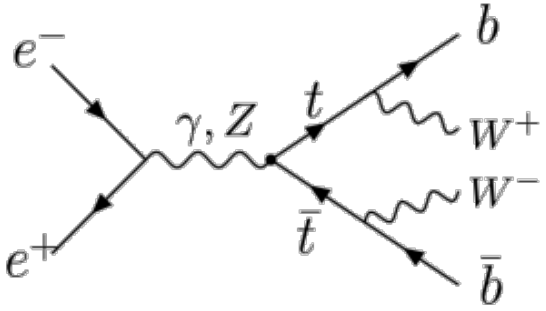
[Hoang, MS, '13]

Implementation of threshold resummation

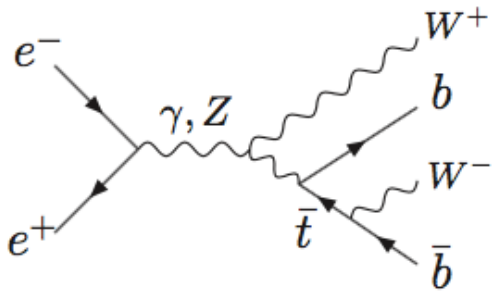
Top-antitop threshold in WHIZARD

Resolve realistic final state:

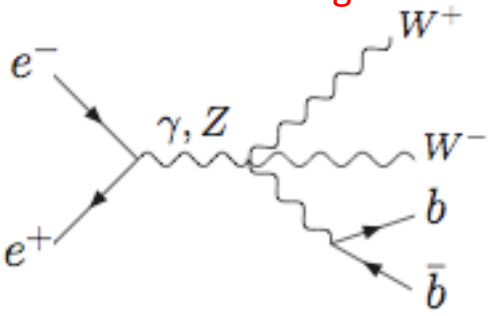
double-resonant "signal"



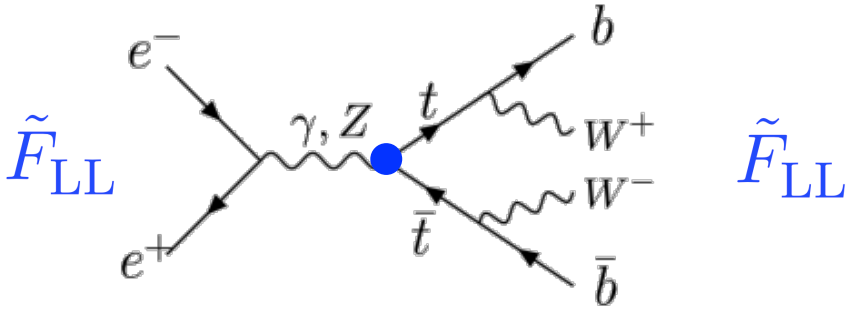
single-resonant background



non-resonant background



<u>Idea:</u> add threshold resummation via form factor:

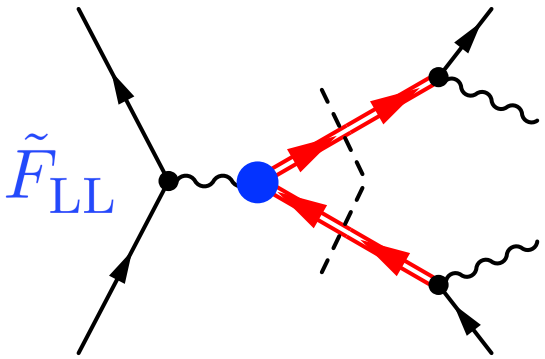


(N)LL S/P-wave G-function from TOPPIK code
[Jezabek, Teubner]

avoids double counting

Top-antitop threshold in WHIZARD

Ensure gauge invariance: Double Pole Approximation



gauge invariant onshell MEs

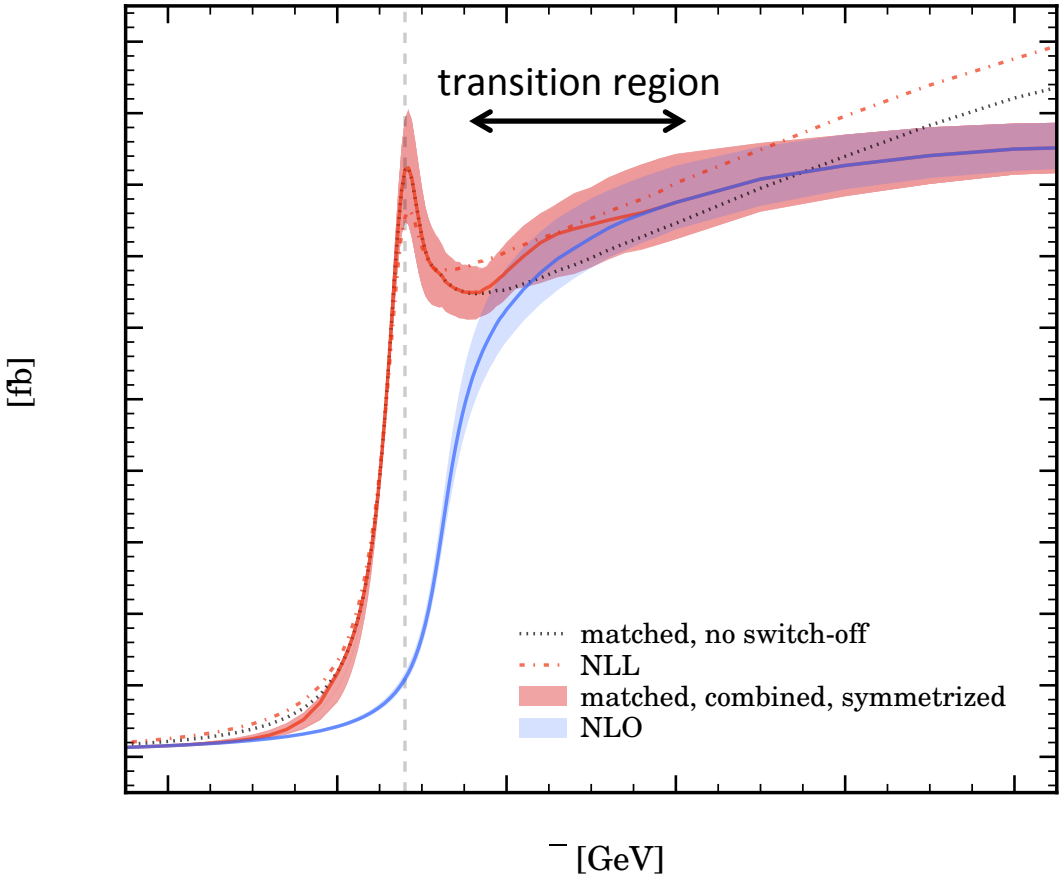
$\tilde{F}_{\mathrm{LL}}$

- For use onshell projection:
- For use momenta as if
- Directions of original 3-momenta retained!

(N)LL + NLO matched cross section

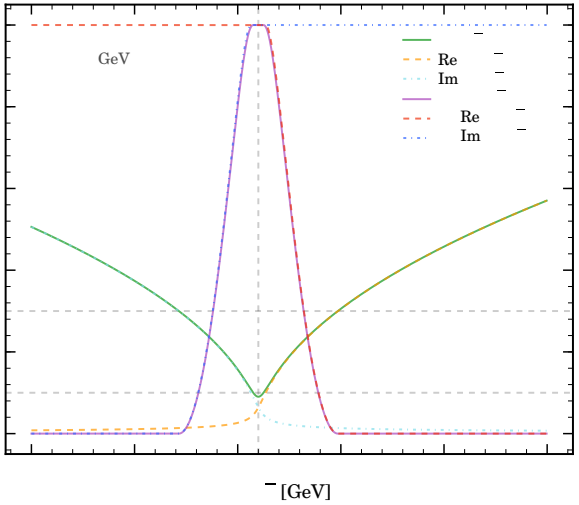
Top-antitop threshold in WHIZARD

Matching NLO+NLL with relativistic NLO continuum:



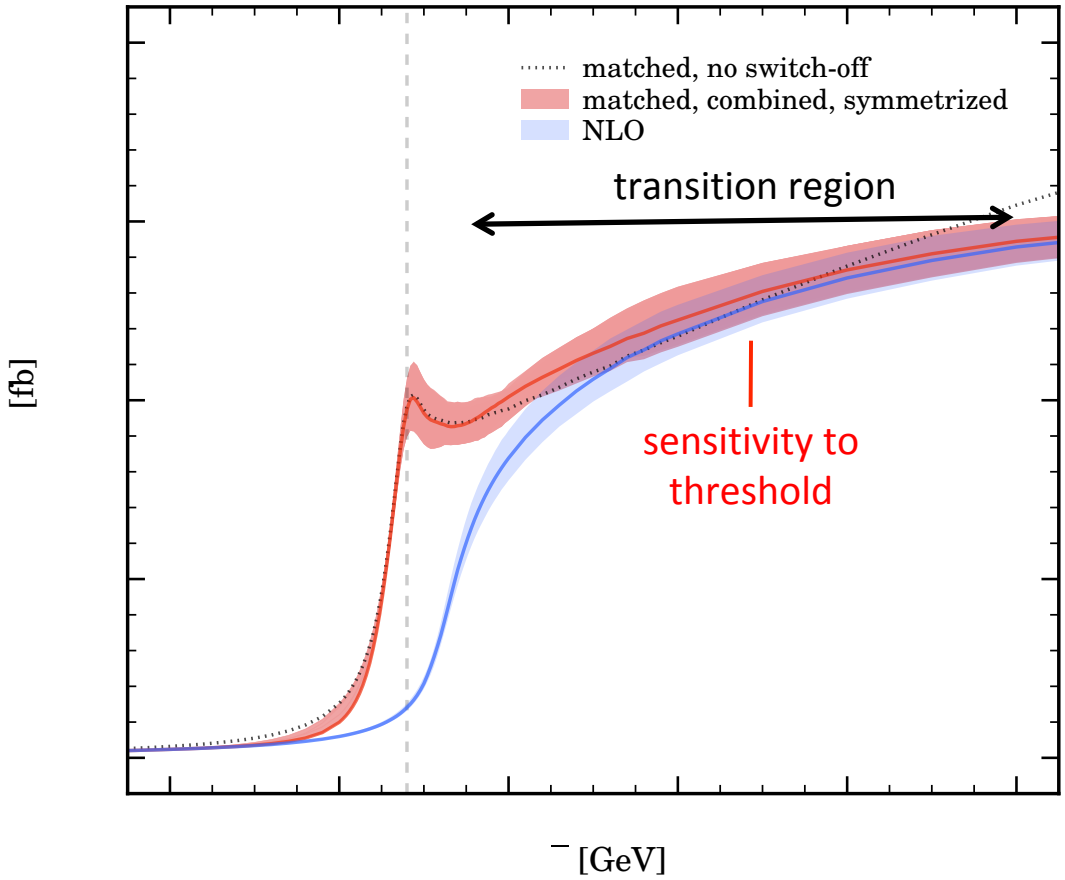
- Usual scale variations
- Symmetrize
- Variation of switch-off parameters
- Take envelope

switch-off function f_s:

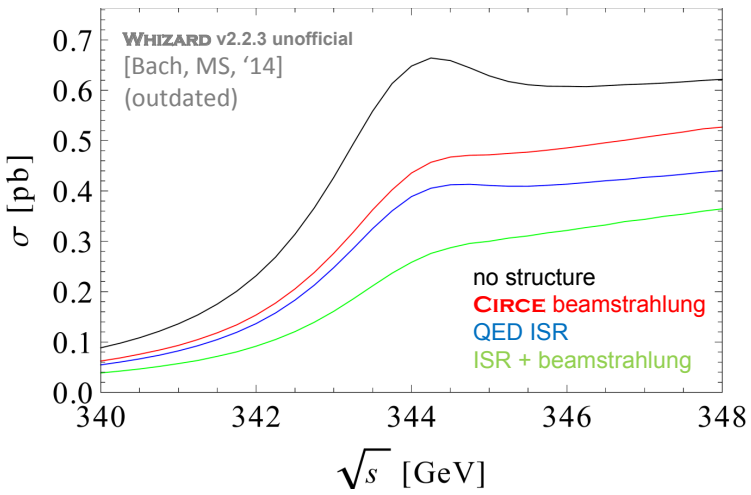


Top-antitop threshold in WHIZARD

Including QED ISR via convolution with structure function:



Further beam effects straightforward:



... polarization of colliding leptons can also be taken into account.