Beam polarization for energy calibration in FCC-ee

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Contents:

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Introduction

- *Resonant de-polarization* has been proposed for accurate beam energy calibration ($\ll$ 100 keV) at 45 and 80 GeV beam energy.
  It relies on the relationship $\boldsymbol{\nu_{spin}} = \boldsymbol{a\gamma}$ [a].

- Beam polarization is obtained "for free" through Sokolov-Ternov effect.
  The effect is in practice restricted to a limited range of values of machine size and beam energy because
  - of the build-up rate
  - it is jeopardized by machine imperfections (spin/orbital motion resonances) which affects the reachable level of polarization in particular at high energy.

- 10% beam polarization is estimated to be enough for the purpose of energy calibration.

[a] $\boldsymbol{a}$ = gyromagnetic anomaly

Sokolov-Ternov polarization

Beam get vertically polarized in the ring guiding field

$$P_{\infty}^{\mathrm{ST}} = 92.3\% \qquad \tau_p^{-1} = \frac{5\sqrt{3}}{8}\frac{r_e \gamma^5 \hbar}{m_0 C}\oint \frac{ds}{|\rho|^3}$$

For FCC-e^+e^- with $\rho \simeq$ 10424 m,it is

E (GeV)	τ_{pol} (h)	$\tau_{10\%}$ (*) h
45	256	29
80	14	1.6

(*) Time needed to reach P=10% for energy calibration

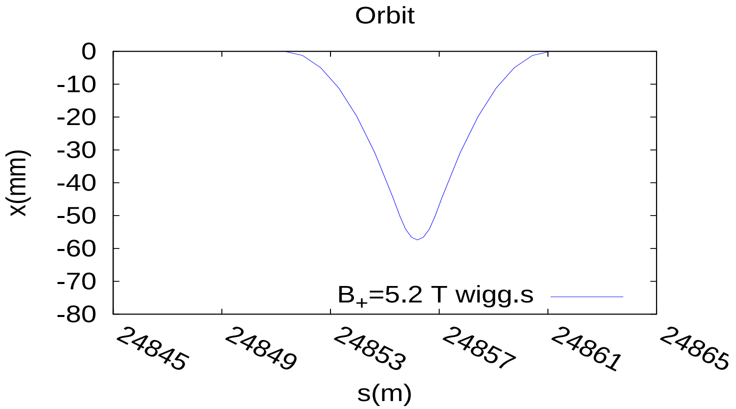
$$\tau_{10\%} = -\tau_p \times \ln(1 - 0.1/P_{\infty})$$

Polarization wigglers

τ_p is reduced by introducing *wigglers*, a chain of horizontal bending magnets with alternating field sign.

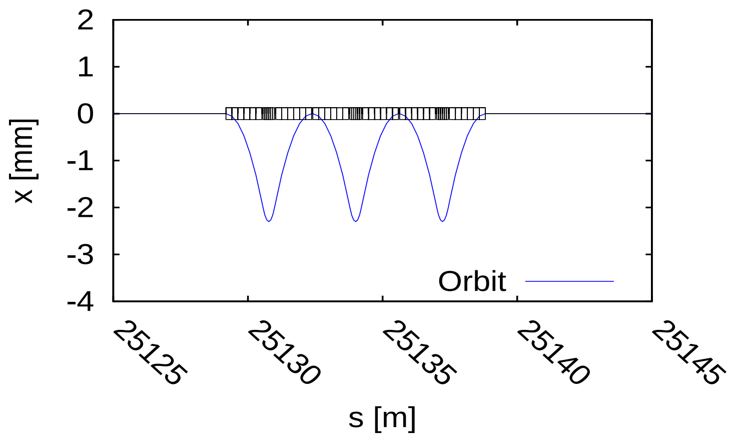
LEP-like

(orbit for B^+=5.2 T)



3 periods

(orbit for B^+=1.7 T)



- Smaller impact on ϵ_x.
- Energy spread as with previous design for the same τ_p.

Polarization in real storage rings

A perfectly planar machine (w/o solenoids) is always *spin transparent*.

Sokolov-Ternov effect in the guiding dipole field	Perturbations (v-bends, vertical orbit in quads etc.)
↓	↓
Polarisation	Depolarisation
↘	↙

Equilibrium polarisation $(< P_\infty^{\mathrm{ST}})$

Spin diffusion is larger at high energy and may be particularly large when spin and orbital motions are in resonance

$$\nu_{spin} \pm mQ_x \pm nQ_y \pm pQ_s = \text{integer}$$

- ϵ_y must be small and $\delta\hat{n}_0$ minimised
  - closed orbit, spurious D_y and betatron coupling must be well corrected!

Computational tools

Accurate simulations are necessary for evaluating the polarization level to be expected in presence of misalignments.

- MAD-X used for simulating quadrupole misalignments and orbit correction
- SITROS (by J. Kewisch) used for computing the resulting polarization.
  - Tracking code with 2th order orbit description and non-linear spin motion.
  - Used for HERA-e in the version improved by M. Böge and M. Berglund.
  - It contains SITF (fully 6D) for analytical polarization computation with *linearized* spin motion.
    - Useful tool for preliminary checks before embarking in time consuming tracking.
    - Computation of polarization related to the 3 degree of freedom separately: useful for disentangling problems!

Simulations for a toy ring

Preliminary studies with a simplified optics (FODO cells and dispersion-free regions for wigglers) have shown that large polarization could be achieved at 45 GeV (even with very large wiggler fields) and at 80 GeV, in presence of misalignments.

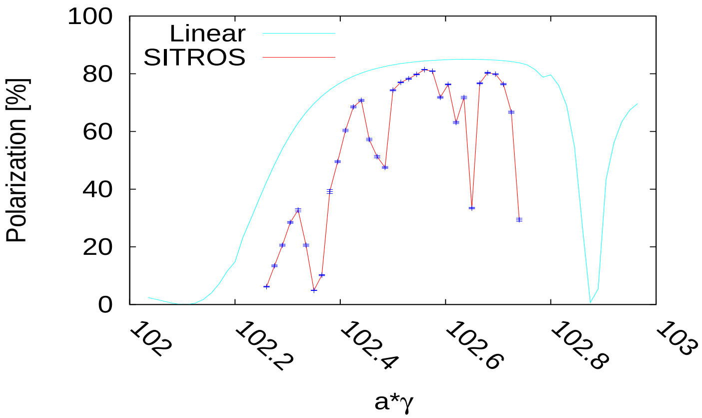
45 GeV beam energy

δy^{Q}_{rms}=200 μm

with BPMs errors

SVD+ harmonic bumps

$|\delta\hat{n}_0|_{rms}$=6.2 mrad



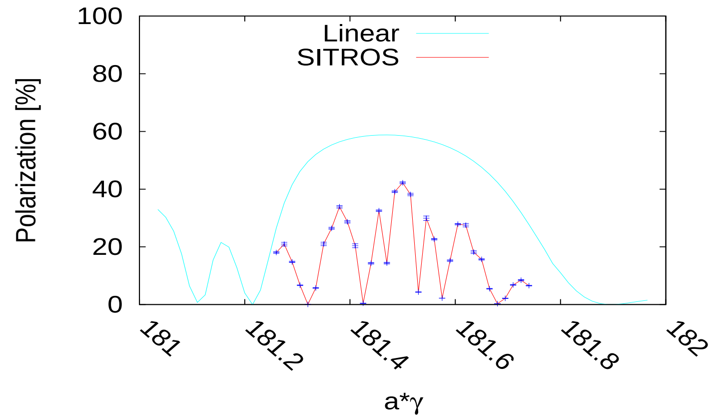
80 GeV beam energy

δy^{Q}_{rms}=200 μm

with BPMs errors

SVD+ harmonic bumps

$|\delta\hat{n}_0|_{rms}$ =14 mrad



Simulations for the "actual" FCC-ee

FCC-$e^{\pm}$ design relies on ultra-flat beams.

(January 2018)

	Z	WW
Beam energy [GeV]	45	80
FODO	$60^0/60^0$	$60^0/60^0$
ϵ_x [nm]	0.27	0.84
ϵ_y [pm]	1	1.7
β_x^* [m]	0.15	0.2
β_y^* [mm]	0.8	1
σ_x^* [μm]	6.4	13
σ_y^* [nm]	28	41

For squeezing β_y^* strong quadrupoles are needed in the IR where β_y is large.

$\rightsquigarrow$ Large impact on chromaticity and response to misalignments in the vertical plane.

Additional related problems

- Beam offsets in the strong IRs sextupoles may produce betatron coupling.
- Small offsets of the IRs quads may lead to an anti-damped machine.

Simulations of orbit distortions

2017 90/90 deg optics

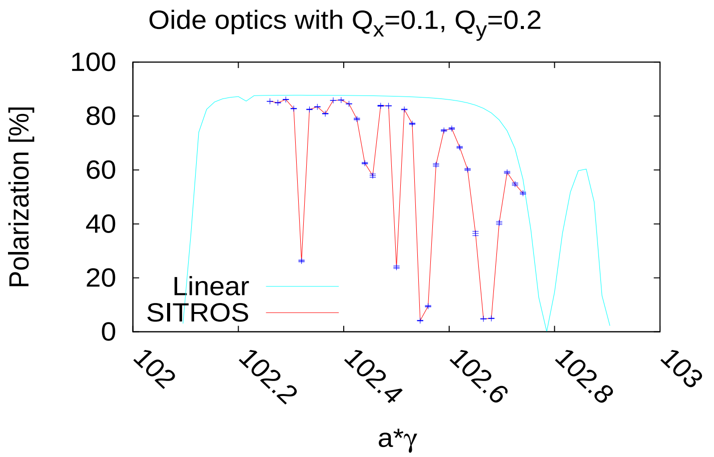
45 GeV

45 GeV case with 4 wigglers (LEP-like).

δy^Q_{rms}=200μm, no BPMs errors

y_{rms}=0.049 mm

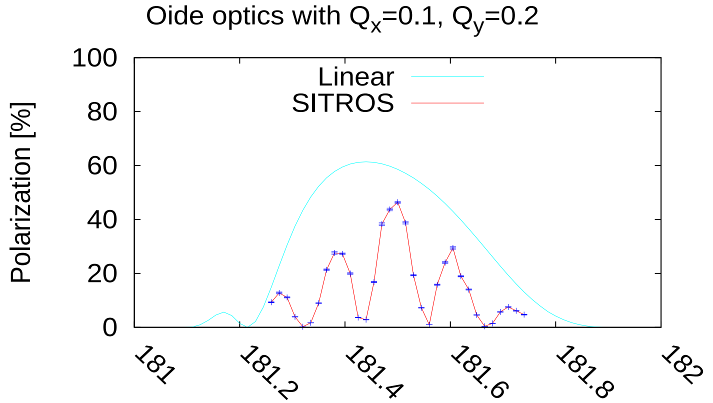
$|\delta\hat{n}|_{0,rms}$=0.4 mrad, no harmonic bumps

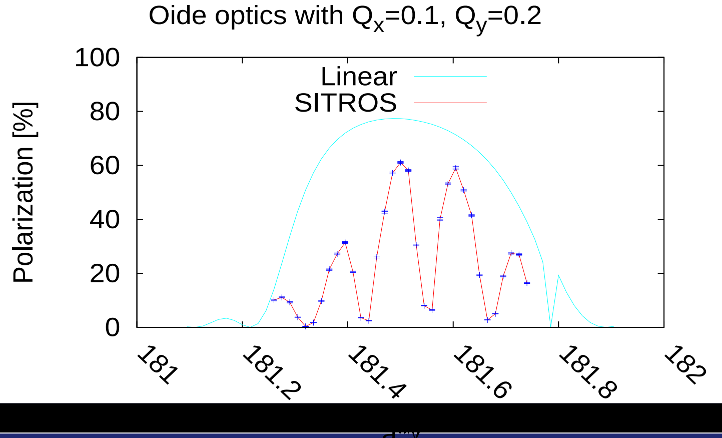


Same error realization at 80 GeV

$|\delta\hat{n}|_{0,rms}$=2 mrad

80 GeV with harmonic bumps





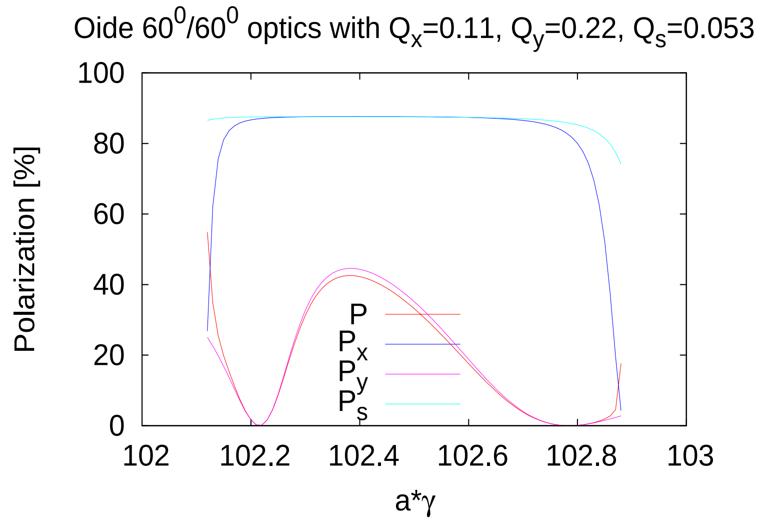
After introducing BPM errors and quadrupole radial offsets and roll angles, misalignments had to be decreased! Set of errors assumed:

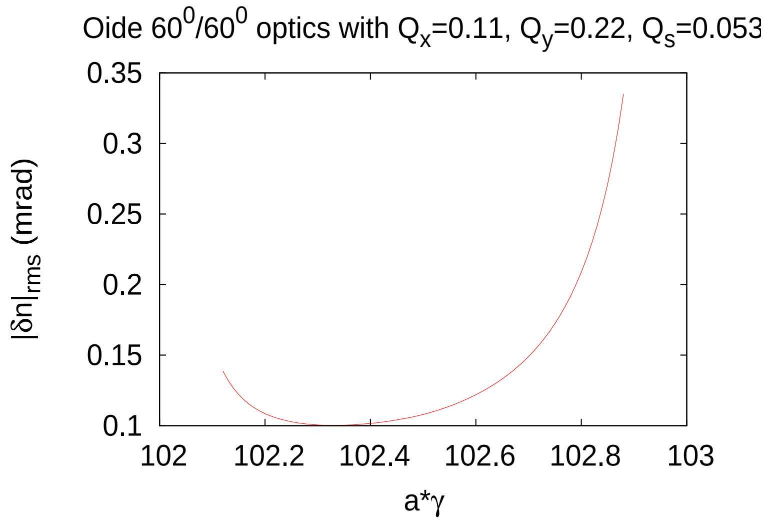
	IR Quads	IR BPMs	other Quads	other BPMs
δx (μm)	10	10	30	30
δy (μm)	10	10	30	30
$\delta\theta$ (μrad)	10	10	30	30
calibration	-	1%	-	1%

- Although the resulting orbit after correction is in the order of few microns, the vertical emittance may result above specs.
  - 289 skew quadrupoles introduced for minimizing spurious vertical dispersion and betatron coupling when needed.

Some seeds show a small $\boldsymbol{P_y}$ despite small $\boldsymbol{\epsilon_y}$ and $\boldsymbol{D_y}$.

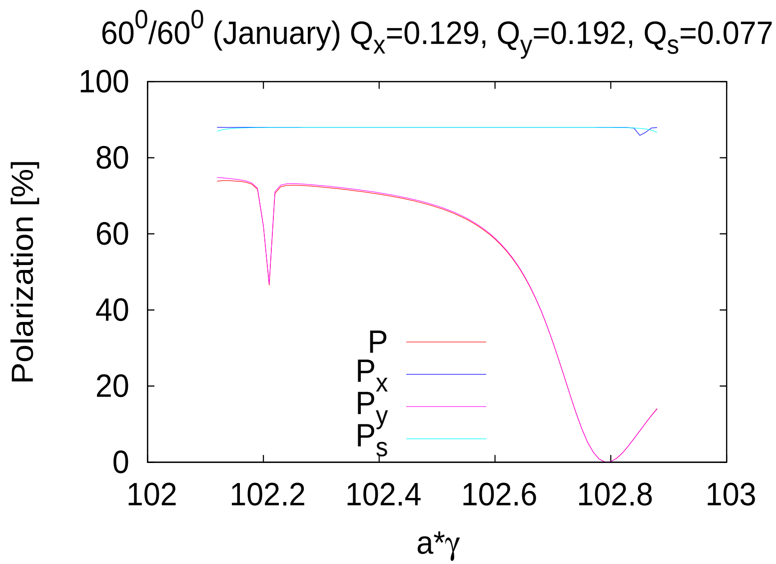
An example (October 2017 60/60 deg optics):

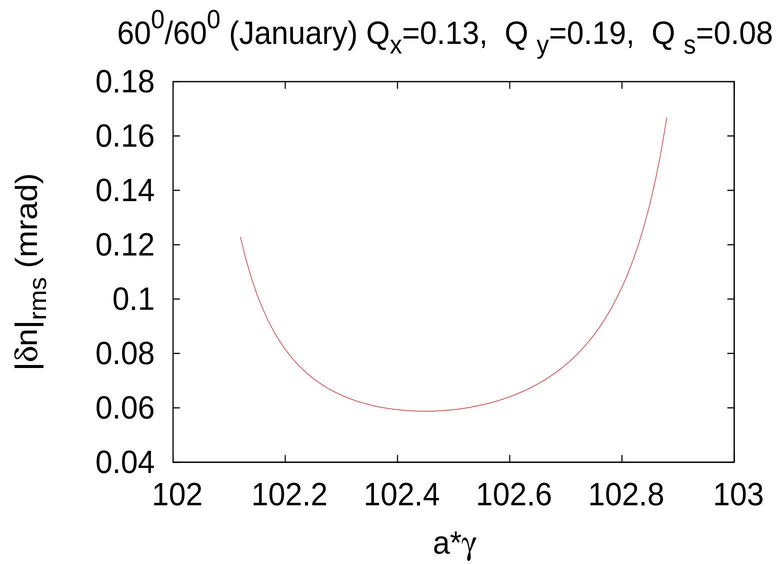




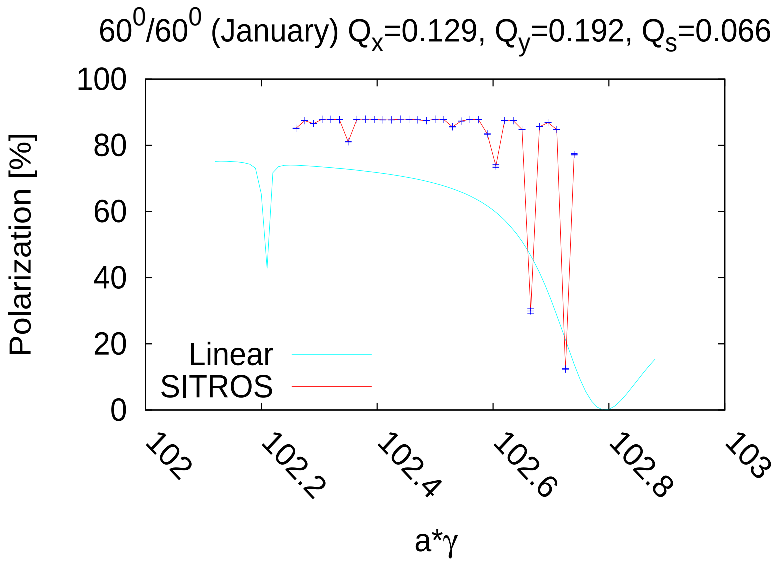
An example ($60^0/60^0$ January 2018 optics), 8 wigglers, $\tau_{10\%}$=2.7 h.

	x_{rms} (μm)	y_{rms} (μm)	D^y_{rms} (mm)	ϵ_x (nm)	ϵ_y (pm)	$\lvert C^-\rvert$
w/o skews	26	13	2	0.215	0.5	0.0014





P_y limiting polarization, but P_{lin} large enough at 45 GeV.



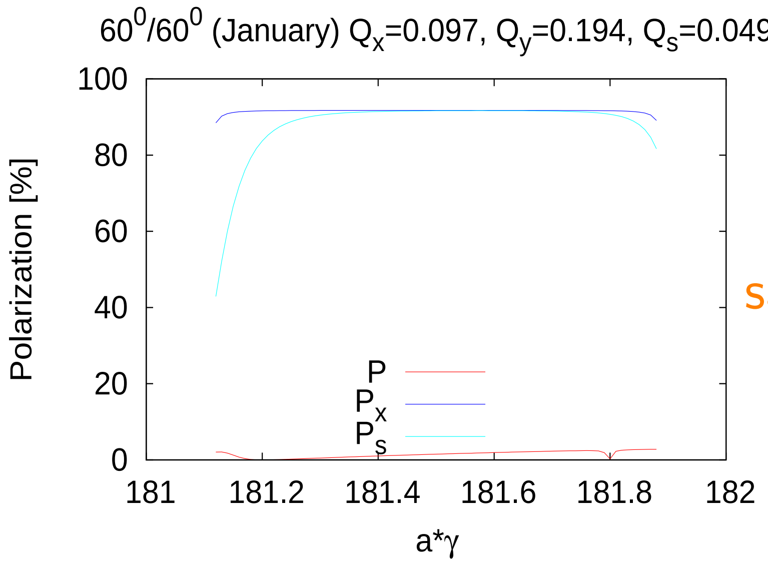
Beam size at IP

	σ_x (μm)	σ_y (nm)	σ_ℓ (mm)
analytical	5.716	23.9	3.909
SITROS Tracking	8.629	43.6	3.890

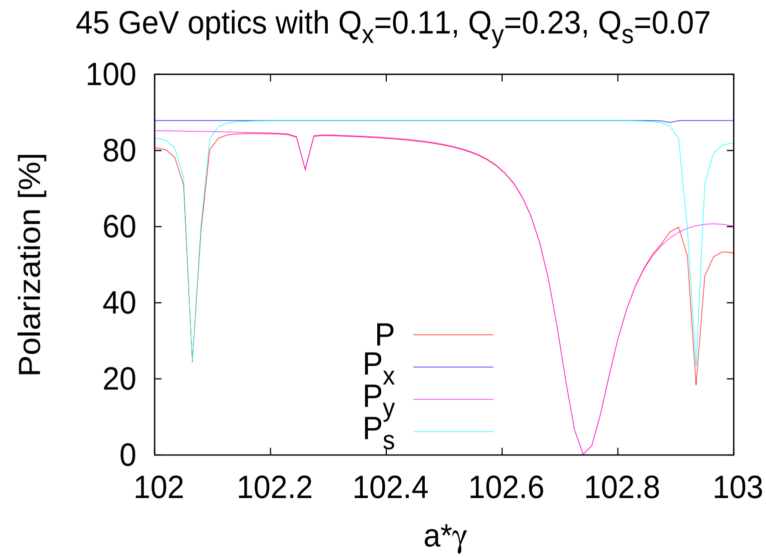
Polarization – 80 GeV

- Larger spin diffusion than at 45 GeV.
- Larger $\boldsymbol{\delta\hat{n}_0}$ for the *same* mis-alignments.
- Larger energy spread.

Some seeds show $P_y \simeq 0$ despite small ϵ_y and D_y.



⇒ same optics at 45 GeV



x_{rms}	y_{rms}	D^y_{rms}	ϵ_x	ϵ_y	$\lvert C^-\rvert$
(μm)	(μm)	(mm)	(nm)	(pm)	
144	11	2	0.792	0.1	< 0.001

x_{rms}	y_{rms}	D^y_{rms}	ϵ_x	ϵ_y	$\lvert C^-\rvert$
(μm)	(μm)	(mm)	(nm)	(pm)	
26	11	2	0.222	0.5	0.0014

Very small ϵ_y w/o resorting to skew quadrupoles, but P few percent at 80 GeV in linear approximation, limited by the vertical motion...

- Correctors added after each bending magnet for correcting sawtooth effect: linear polarization shows no improvement.

Spin diffusion in linear approximation:

$$\frac{\partial \hat{n}}{\partial \delta}(\vec{u};s) = \vec{d}(s) = \frac{1}{2}\Im\Big\{(\hat{m}_0 + i\hat{l}_0)^* \sum_{k=\pm x,\pm y,\pm s} \Delta_k\Big\}$$

with

$$\Delta_{\pm x,\pm y} = (1+a\gamma)\frac{e^{\mp i\mu_{x,y}}}{e^{2i\pi(\nu\pm Q_{x,y})}-1}\frac{[-D \pm i(\alpha D + \beta D')]_{x,y}}{\sqrt{\beta_{x,y}}} \, J_{x,y}$$

$$\equiv f_{x,y}$$

$$\Delta_{\pm s} = (1+a\gamma)\frac{e^{\pm i\mu_s}}{e^{2i\pi(\nu\pm Q_s)}-1} \, J_s$$

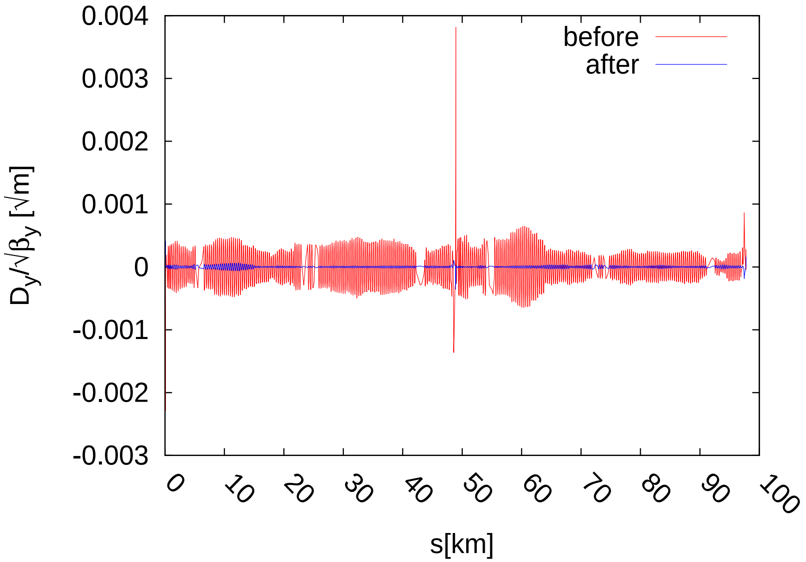
$$J_{\pm x,\pm y} = \int_s^{s+L} ds'(\hat{m}_0 + i\hat{l}_0)\cdot\left\{\begin{array}{c}\hat{y}\sqrt{\beta_x}\\ \hat{x}\sqrt{\beta_y}\end{array}\right\} K e^{\pm i\mu_{x,y}}$$

$$J_s = \int_s^{s+L} ds'(\hat{m}_0 + i\hat{l}_0)\cdot(\hat{y}D_x + \hat{x}D_y)\, K$$

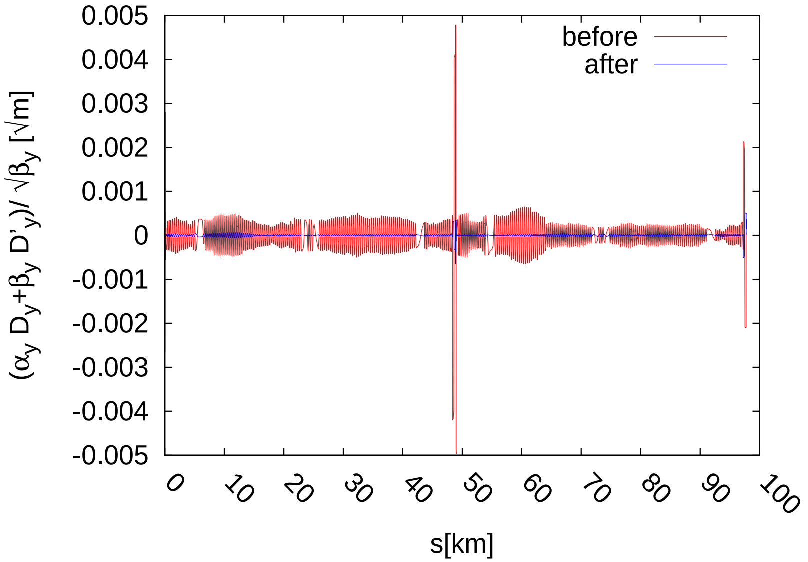
In some short regions f_y is much larger than in the rest of the ring.

- Attempts of correcting the f_y "spikes" with the skew quadrupoles were unsuccessful $\longrightarrow$ vertical correctors used instead.

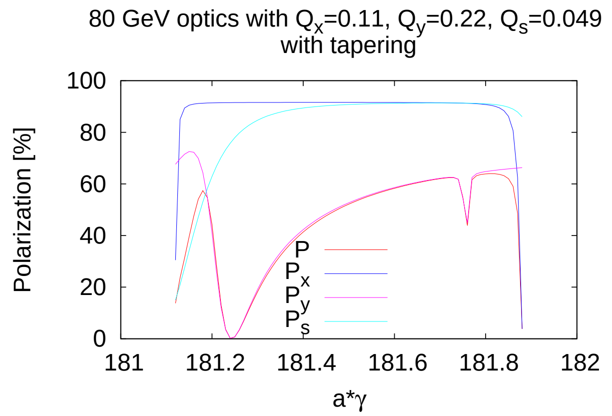
$\Re(f_y)$

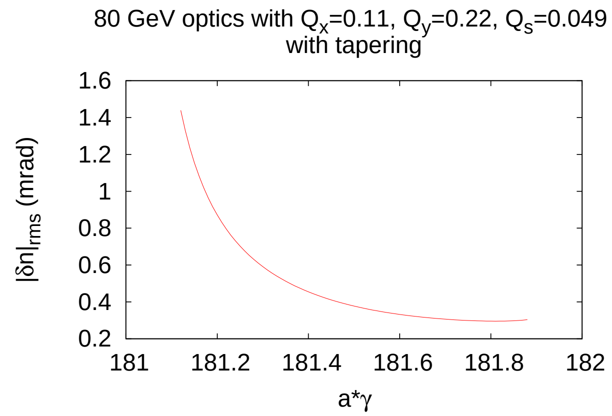


$\Im(f_y)$



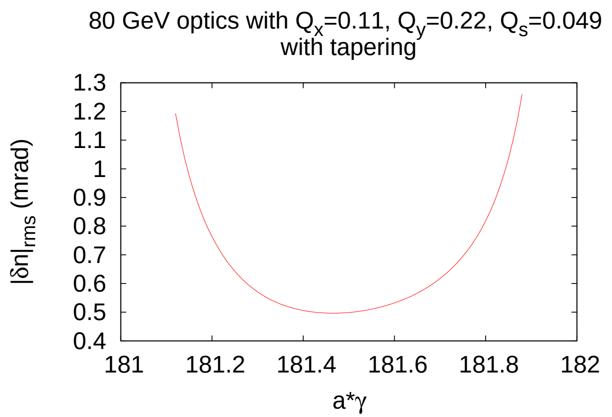
Polarization improved after f_y correction!

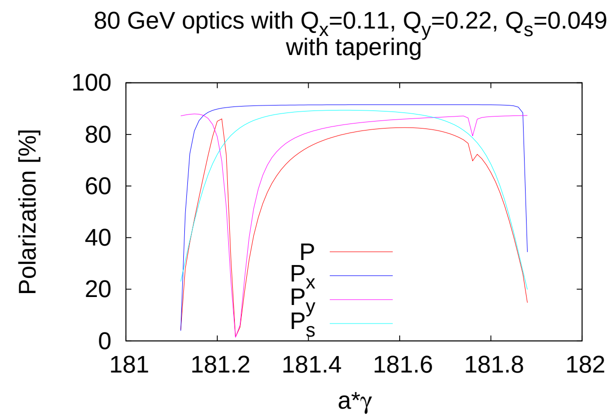




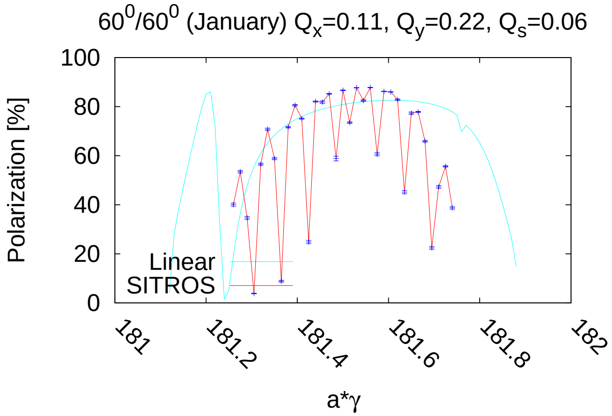
The larger $\hat{n}_0$ tilt at lower energy may be the reason of the "asymmetry".

- Harmonic bumps reduces $\delta\hat{n}_0$ and polarization improved further.





Polarization from tracking for this error realization + corrections



V= 980 MV

	$\boldsymbol{\sigma_x}$	$\boldsymbol{\sigma_y}$	$\boldsymbol{\sigma_\ell}$
	(μm)	(nm)	(mm)
analytical	12.57	35.47	2.52
SITROS Tracking	12.26	51.10	2.53

Latest simulations

Optics files by T. Charles with misalignments:

	IR Quads	other Quads	Sexts
δx (μm)	50	100	100
δy (μm)	50	100	100
$\delta \theta$ (μrad)	50	100	100

- BPMs are supposed perfectly aligned to the near-by quadrupole and perfectly calibrated.
- Tune shift and coupling are corrected by 1204 normal + 1204 skew *thin lenses* quadrupoles.

SITROS can't treat thin lenses $\longrightarrow$ replaced by 5 mm long quadrupoles, in lack of more space. Code edited for dropping

- magnets shorter than 10 mm in emittance and damped transport matrix calculation;
- quadrupole component of misaligned sextupoles in the closed orbit calculation (for compatibility with MADX).

For some seeds the thin lenses substitution went well:

Seed 13, with radiation, B_w=0

	x_{rms} (μm)	y_{rms} (μm)	ϵ_x (pm)	ϵ_y (pm)
MADX (thin)	23	22	276.4	0.04
MADX (thick)	35	22	278.4	0.04

Seed 13, with radiation , B_w for $\tau_{10\%}$=1.7 h

	x_{rms} (μm)	y_{rms} (μm)	ϵ_x (nm)	ϵ_y (pm)
MADX (thin)	23	22	239.7	0.114
MADX (thick)	35	22	241.5	0.114

Seed 1, with radiation, B_w=0

	x_{rms} (μm)	y_{rms} (μm)	ϵ_x (pm)	ϵ_y (pm)
MADX (thin)	35	21	278.2	0.366
MADX (thick)	35	21	280.2	0.375

Seed 1, with radiation , B_w for $\tau_{10\%}$=1.7 h

	x_{rms} (μm)	y_{rms} (μm)	ϵ_x (pm)	ϵ_y (pm)
MADX (thin)	35	21	242.8	0.281
MADX (thick)	35	21	244.6	0.288

Seed 1, with radiation and 8 wigglers

	Q_x	Q_y	x_{rms} (μm)	y_{rms} (μm)	ϵ_x (nm)	ϵ_y (pm)
MADX (thick)	0.1457	0.2181	34.9	21.5	0.245	0.288
SITF	0.1459	0.2175	34.4	20.7	0.231	10.3 (*)

(*) Due to CV798 ! Dropping it is ϵ_y=0.34 pm. Why MADX gives 0.288 pm?

Seed 13, with radiation and 8 wigglers

	Q_x	Q_y	x_{rms} (μm)	y_{rms} (μm)	ϵ_x (nm)	ϵ_y (pm)
MADX (thick)	0.1447	0.2097	35.2	22.1	0.241	0.112
SITF	0.1447	0.2099	35.2	21.3	0.231	0.394

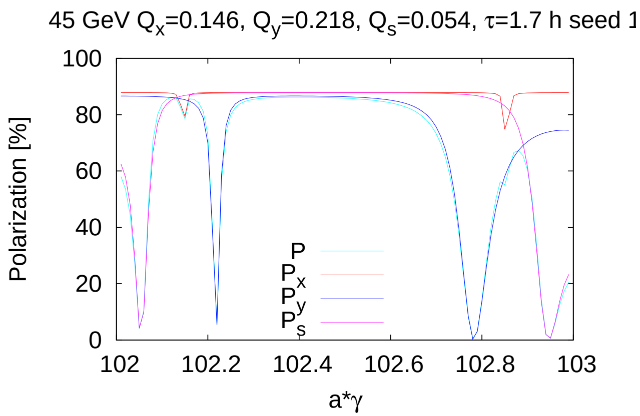
For some seeds the substitution with 5 mm lenses did not work.

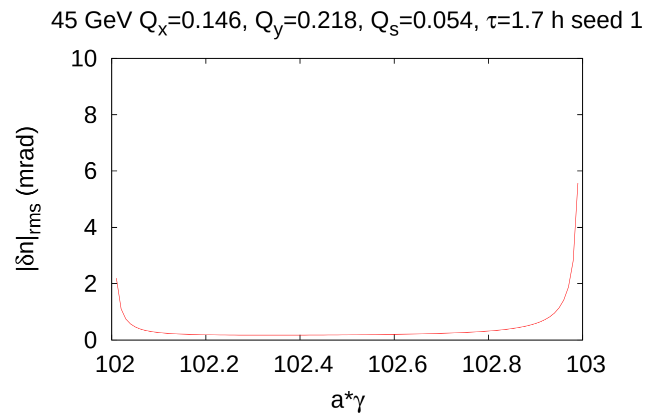
Seed 17, 45 GeV

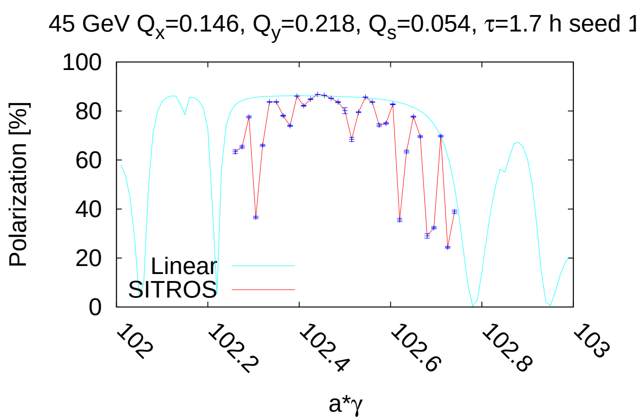
	x_{rms} (μm)	y_{rms} (μm)	J_x	J_y	J_s	ϵ_x (nm)	ϵ_y (pm)
MADX (thin)	34.3	21.7	1.001	1.000	1.998	0.240	0.14
MADX (thick)	35.4	23.3	1.200	1.395	1.402	0.234	84.5

Those seeds have been skipped.

Tessa 45 GeV optics with 8 wigglers for $\tau_{10\%}$=1.7 h, seed 1.



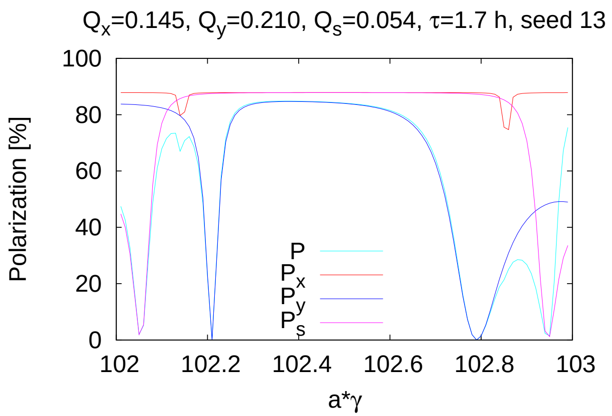


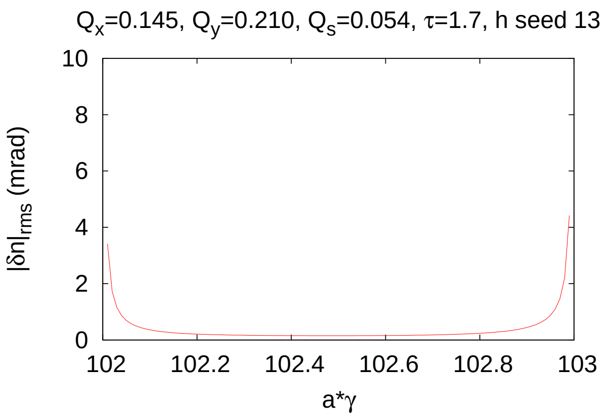


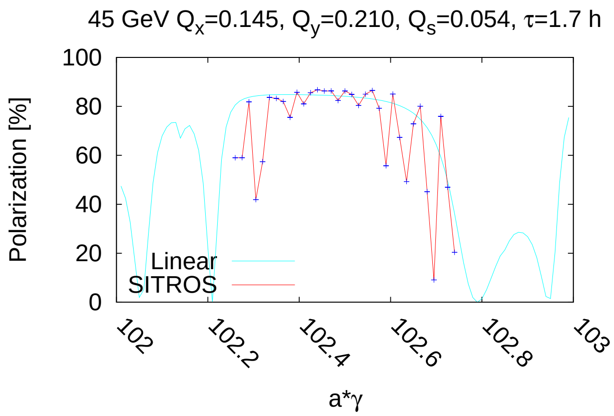
Beam size at IP1

	σ_x	σ_y	σ_ℓ
	(μm)	(nm)	(mm)
analytical	5.994	97.6	5.857
SITROS Tracking	7.274	14.9	5.917

Tessa 45 GeV optics with 8 wigglers for $\tau_{10\%}$=1.7 h, seed 13.



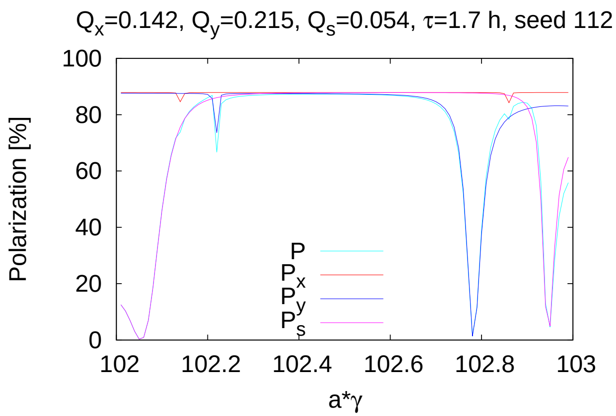


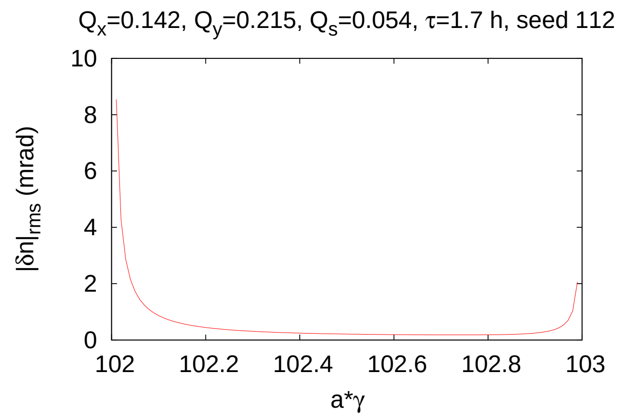


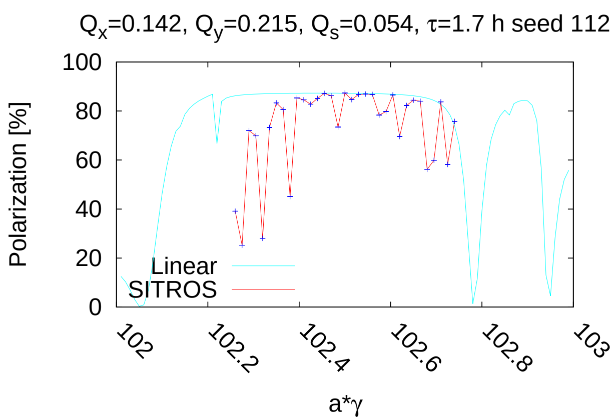
Beam size at IP1

	σ_x (μm)	σ_y (nm)	σ_ℓ (mm)
analytical	5.966	19.7	5.721
SITROS Tracking	7.114	21.2	5.681

Tessa 45 GeV optics with 8 wigglers for $\tau_{10\%}$=1.7 h, seed 112.



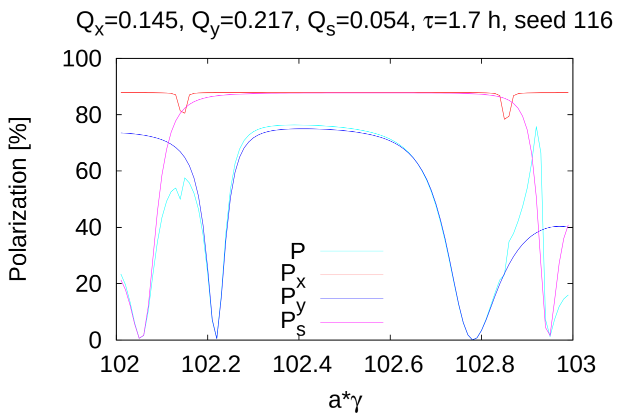


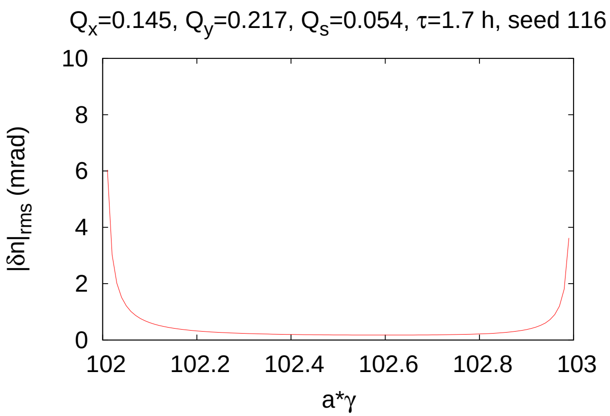


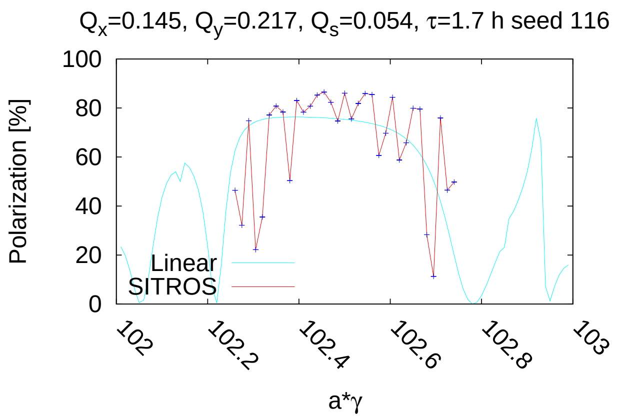
Beam size at FRF.1

	σ_x (μm)	σ_y (μm)	σ_ℓ (mm)
analytical	188.7	1.021	5.738
SITROS Tracking	264.1	1.947	5.717

Tessa 45 GeV optics with 8 wigglers for $\tau_{10\%}$=1.7 h, seed 116.







Beam size at FRF.1

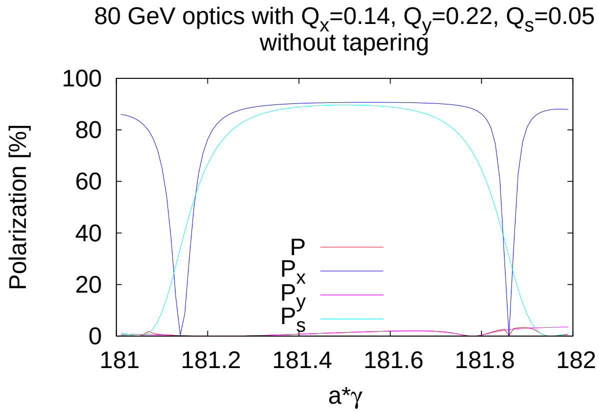
	σ_x (μm)	σ_y (μm)	σ_ℓ (mm)
analytical	183.3	9.863	5.855
SITROS Tracking	259.7	3.313	5.915

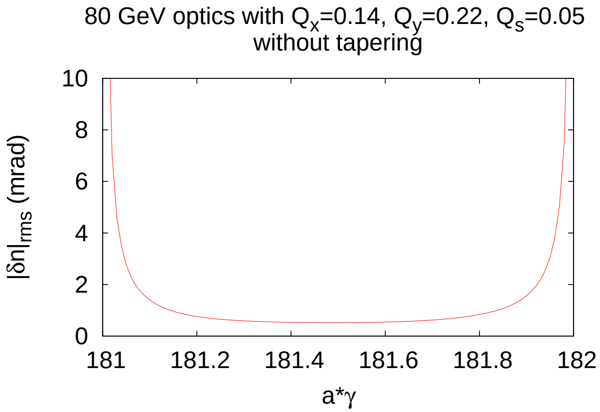
80 GeV

The same 45 GeV optics have been scaled to 80 GeV

- no wigglers
- no tapering (from previous simulations it seemed not crucial):
  - main circuits adjusted for compensating the sextupoles feed-down effect.

Seed 13



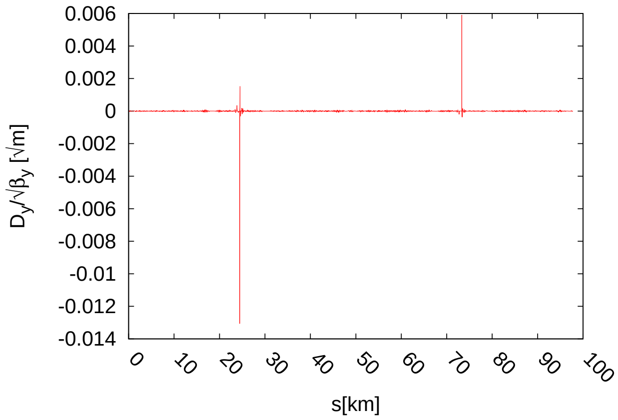


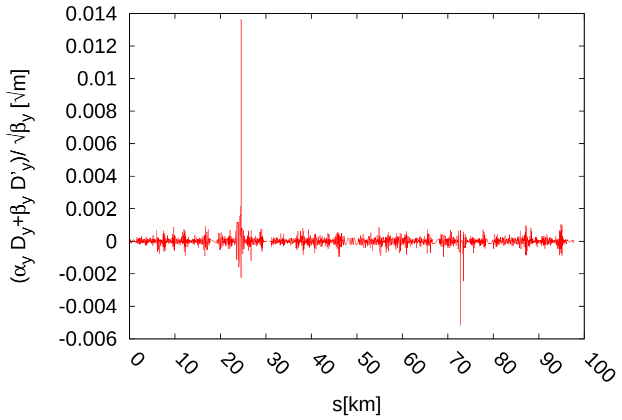
Why is $\boldsymbol{P_y} \simeq 0$?

Although the orbit is well corrected there are small regions where

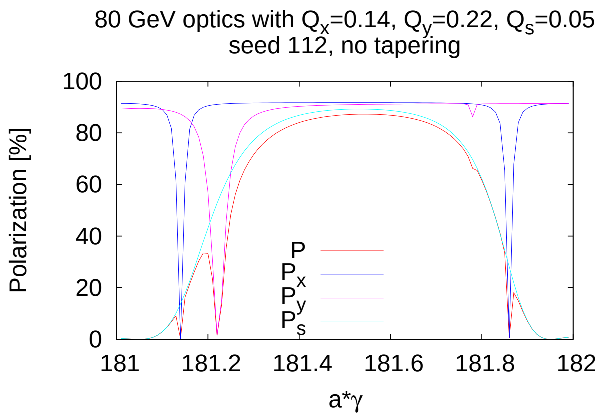
$$\frac{[-D_y \pm i(\alpha_y D_y + \beta D'_y)]}{\sqrt{\beta_y}}$$

is large.

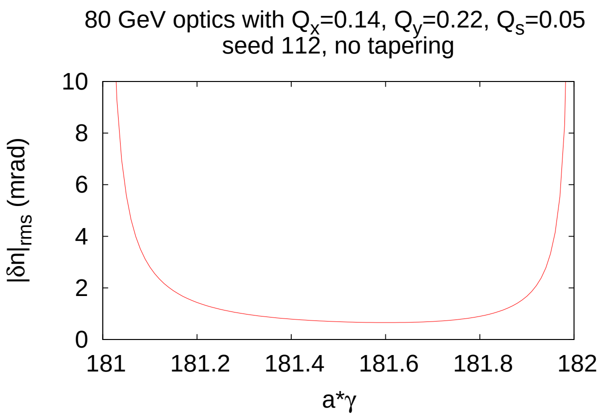




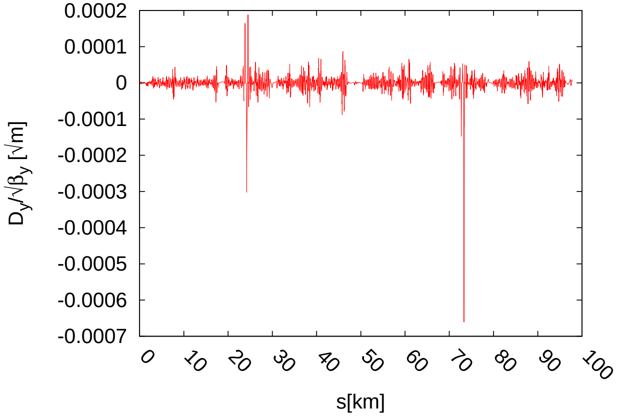
Seed 112



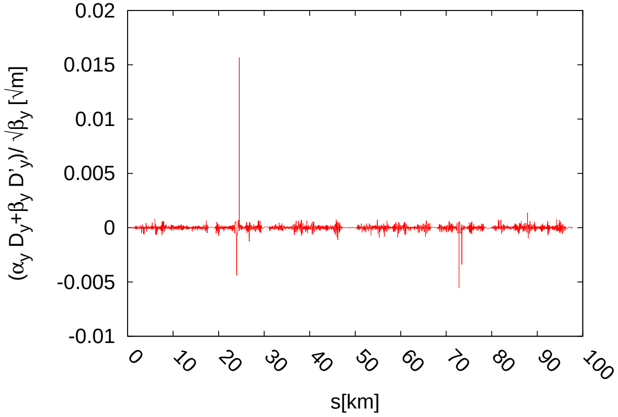
80 GeV optics with Qx=0.14, Qy=0.22, Qs=0.05
seed 112, no tapering
Polarization [%]
a*γ
P
Px
Py
Ps



80 GeV optics with Qx=0.14, Qy=0.22, Qs=0.05
seed 112, no tapering
|δn|rms (mrad)
a*γ

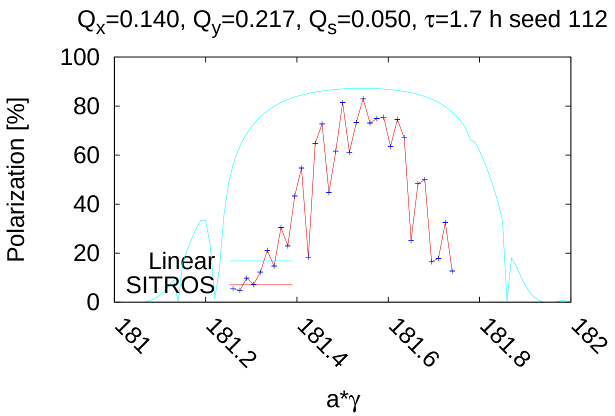


Dy/√βy [√m]
s[km]



(αy Dy+βy D'y)/ √βy [√m]
s[km]

Seed 112



Beam size at FRF.1

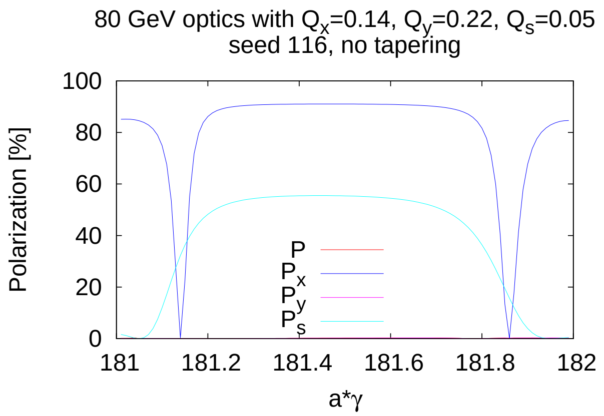
	$\boldsymbol{\sigma_x}$ (μm)	$\boldsymbol{\sigma_y}$ (μm)	$\boldsymbol{\sigma_\ell}$ (mm)
analytical	344.9	1.670	3.321
SITROS Tracking	248.7	3.164	3.309

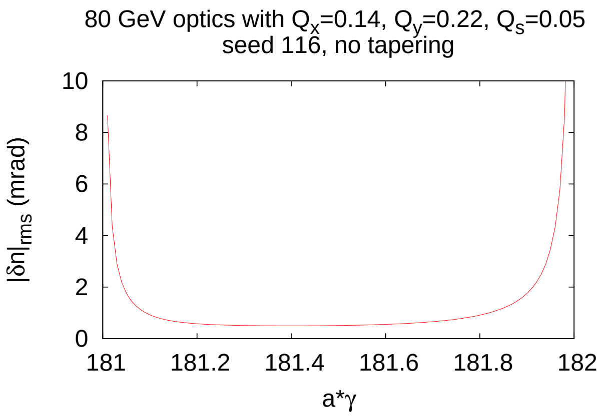
$\Downarrow$

Extrapolating tracking results to IP1

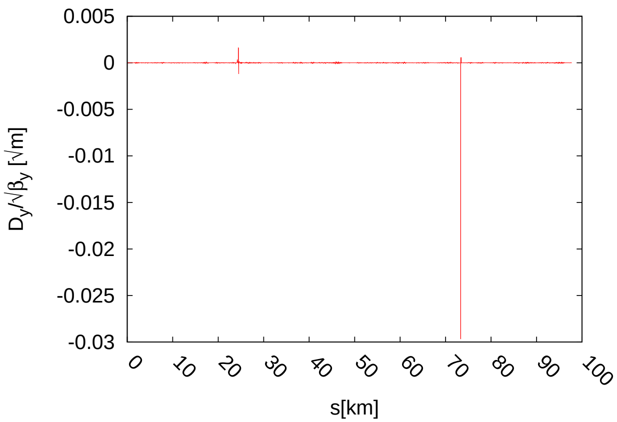
	$\boldsymbol{\sigma_x}$ (μm)	$\boldsymbol{\sigma_y}$ (μm)	$\boldsymbol{\sigma_\ell}$ (mm)
analytical	11.9	0.011	3.311
SITROS Tracking	8.58	0.208	3.299

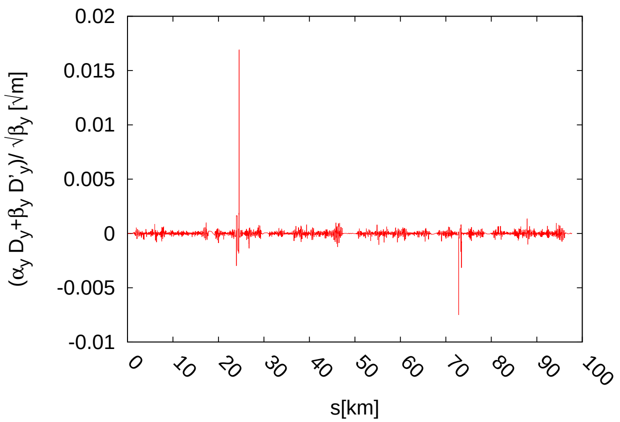
Seed 116





$\boldsymbol{P_y} \simeq 0$. Same problem as seed13.





Some considerations on energy calibration through resonant depolarization

It is based on the relationship $\boldsymbol{\nu_{spin} = a\gamma}$ and gives a measure of the *average* energy. To be considered

- Beam energy dependence on machine azimuth (for instance: *sawtooth* effect).
- The non-colliding bunches used for resonant depolarization may have a different energy than the colliding ones.

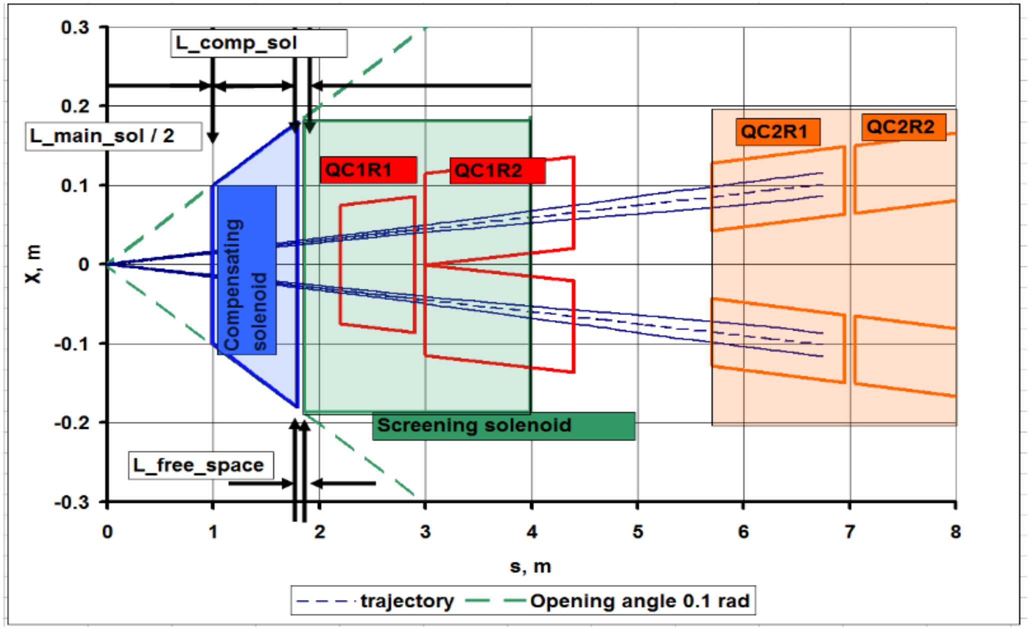
These questions are addressed in M. Koratzinos talk.

In addition: the relationship $\boldsymbol{\nu_{spin} = a\gamma}$ is valid for a *planar* ring, *w/o solenoids* and neglecting electric fields (term $\vec{\boldsymbol{\beta}} \times \vec{\boldsymbol{\mathcal{E}}}_{\boldsymbol{RF}}$ in BMT-equation).

Experiment solenoids

- tilt the polarization axis $\hat{\boldsymbol{n}}_{\boldsymbol{0}}$
- shift the spin tune breaking the $\boldsymbol{a\gamma}$ relationship.

B_s=2 T, ℓ_s^m=2 m, θ_{cross}=30 mrad



(S. Sinyatkin, FCCee IR Workshop 2017)

45 GeV:

x_{rms}=14.9 μm

y_{rms}=0.9 μm

$|\delta\hat{n}_0|_{rms}$=0.010 mrad

P_{lin}=88%

80 GeV:

x_{rms}=4.7 μm

y_{rms}=0.5 μm

$|\delta\hat{n}_0|_{rms}$=0.001 mrad

P_{lin}=88%

For the actual configuration, with 2 compensated solenoids at 45.156 GeV it is (SLIM) $\Delta\nu_{spin} \simeq 1.6\times10^{-6}$ ie $\Delta E \simeq$0.71 KeV.

The effect of the experiment solenoids is negligible and can be *measured*.

Effect of RF electric field (term $\vec{\beta} \times \vec{\mathcal{E}}_{RF}$ in BMT-equation)[a]

	ΔE (KeV)
45 GeV	2 $\times y'_{rms}$
80 GeV	16 $\times y'_{rms}$

y'_{rms} = rms slope in mrad. With

$$< y'_{rms} > \simeq \sqrt{\frac{< \gamma_y >}{< \beta_y >}} < y_{rms} > \simeq 0.1 < y_{rms} >$$

The contribution from the RF electric field is small for a well corrected orbit.

[a] From Yu. I. Eidelman et al. formulas

Spin tune shift due to closed orbit distortions. First order:

$$\Delta\nu_s^{(1)} = \frac{1}{2\pi}R(a\gamma+1)\int_0^{2\pi} d\theta(\hat{n}_0\cdot\hat{y})x''_{co}$$

that is $\Delta\nu_s^{(1)}$=0 always for a planar designed ring ($\hat{n}_0\cdot\hat{y}$=1). Second order:

$$\Delta\nu_s^{(2)} = \frac{1}{4\pi}R^2(a\gamma+1)^2\Im\Big[\frac{1}{e^{-i2\pi\nu_s^0}-1}\int_0^{2\pi} d\theta\, h^*(\theta)y''_{co}\int_\theta^{\theta+2\pi} d\theta' h(\theta')y''_{co}\Big]$$

with

$$h(\theta) = (\hat{m}_0 + i\hat{l}_0)\cdot\hat{x}$$

$$y'' = -K(y-\delta_y^Q) + \left(\frac{\Delta B}{B\rho}\right)_{cor}$$

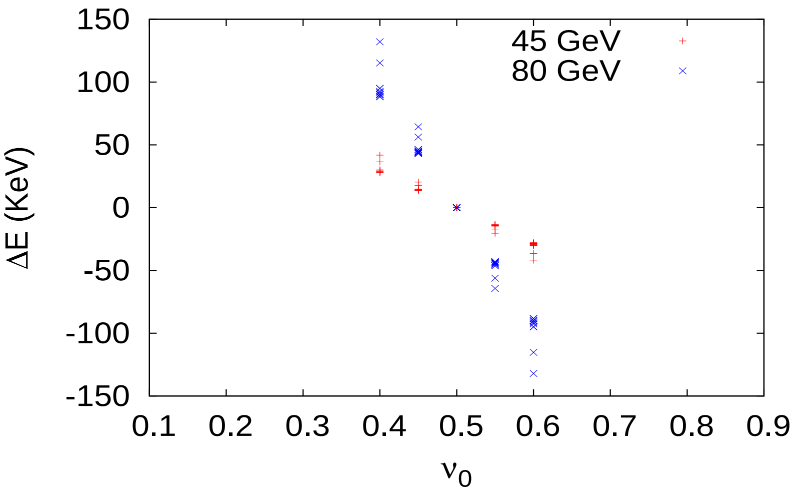
Using a simplified machine model (R. Assmann and J. P. Koutchouk)

$$<\Delta\nu_s>= \frac{\cot\pi\nu_s^0}{8\pi}(a\gamma)^2\Big[<\Sigma_q(K\ell)_q^2 y_q^2> + <\Sigma_k\theta_k^2>\Big]$$

$y_q \equiv K(y-\delta_y^Q)$.

That expression gives $\boldsymbol{\Delta\nu}$=0 at $\boldsymbol{\nu_0}$=0.5.

Evaluating it over 10 seeds ($\boldsymbol{\beta^*_y}$=1 mm optics, cases w/o BPMs errors)



The spin tune changes can be also computed directly by SITF (linear). For the machine with BPMs (*) errors at ν_0=0.5

	ΔE (KeV)	
	svd	+hb
45 GeV	36	52
80 GeV	162	135

(*) Simulations with "small" errors.

The effect is not zero at 0.5, unlike expected from the simplified ring model.

Summary

Due to the demanding IR optics design and the machine size, establishing a closed orbit and keeping a stable machine look challenging.

- Beam polarization is obtained "for free" through Sokolov-Ternov effect.
  - At 45 GeV wigglers are required to get $\tau_{10\%} \approx$ 2-3 h.
    They do not harm polarization.
- P_∞ depends on how well is the machine aligned/corrected, requirements becoming stricter at high the energy.
  - Extremely well corrected orbit/optics is required for a large chromatic machine with β_y^*=0.8 - 1 mm as FCC-ee to work and meet required performance.
    - This benefits also polarization, but a special attention may be needed for $[-D_y \pm i(\alpha_y D_y + \beta_y D'_y)]/\sqrt{\beta_y}$
      in particular at 80 GeV.

THANK YOU!

Polarization wigglers

τ_p may be reduced by introducing wigglers:

$$\tau_p^{-1} = F\gamma^5 \Big[\int_{dip} \frac{ds}{|\rho_d|^3} + \int_{wig} \frac{ds}{|\rho_w|^3} \Big] \qquad F \equiv \frac{5\sqrt{3}}{8} \frac{r_e \hbar}{m_0 C}$$

Polarization

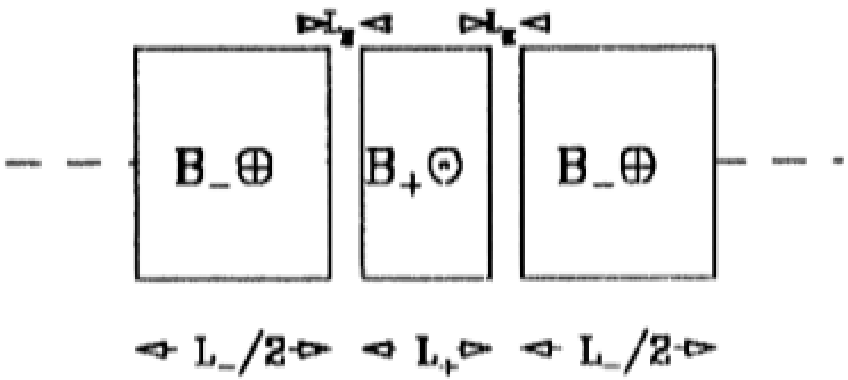
$$P_\infty = \frac{8}{5\sqrt{3}} \frac{\oint ds \frac{\hat{B}\cdot\hat{n}_0}{|\rho|^3}}{\oint ds \frac{1}{|\rho|^3}} \propto \tau_p \Big[\int_{dip} ds \, \frac{\hat{B}_d \cdot \hat{n}_0}{|\rho_d|^3} + \int_{wig} ds \, \frac{\hat{B}_w \cdot \hat{n}_0}{|\rho_w|^3} \Big]$$

$\hat{n}_0 \equiv \hat{y}$ in a perfectly planar ring.

Constraints:

- $x' = 0$ outside the wiggler $\Rightarrow \int_{wig} ds \, B_w = 0$ (vanishing field integral)
- $x = 0$ outside the wiggler $\Rightarrow \int_{wig} ds \, sB_w = 0$ (true for symmetric field)
- P large $\Rightarrow \int_{wig} ds \, B_w^3$ must be large

The LEP polarization wigglers:



For 4 LEP-like wigglers with $\boldsymbol{B_+/B_-}(=\boldsymbol{L_-/L_+})\simeq$ 6 and $\boldsymbol{B^+}$=0.7 T it is $\boldsymbol{\tau_{10\%}}\simeq$2.9 h at 45 GeV.

Horizontal emittance

$$\epsilon_x = C_q \gamma^2 \frac{\mathcal{I}_5}{J_x \mathcal{I}_2} \qquad \mathcal{I}_2 \equiv \oint ds \frac{1}{\rho^2}$$

$$\mathcal{I}_5 \equiv \oint ds \frac{\beta_x D_x'^2 + 2\alpha_x D_x D_x' + \gamma_x D_x^2}{|\rho|^3}$$

Even if located where nominally D_x=0, wigglers may increase the horizontal emittance

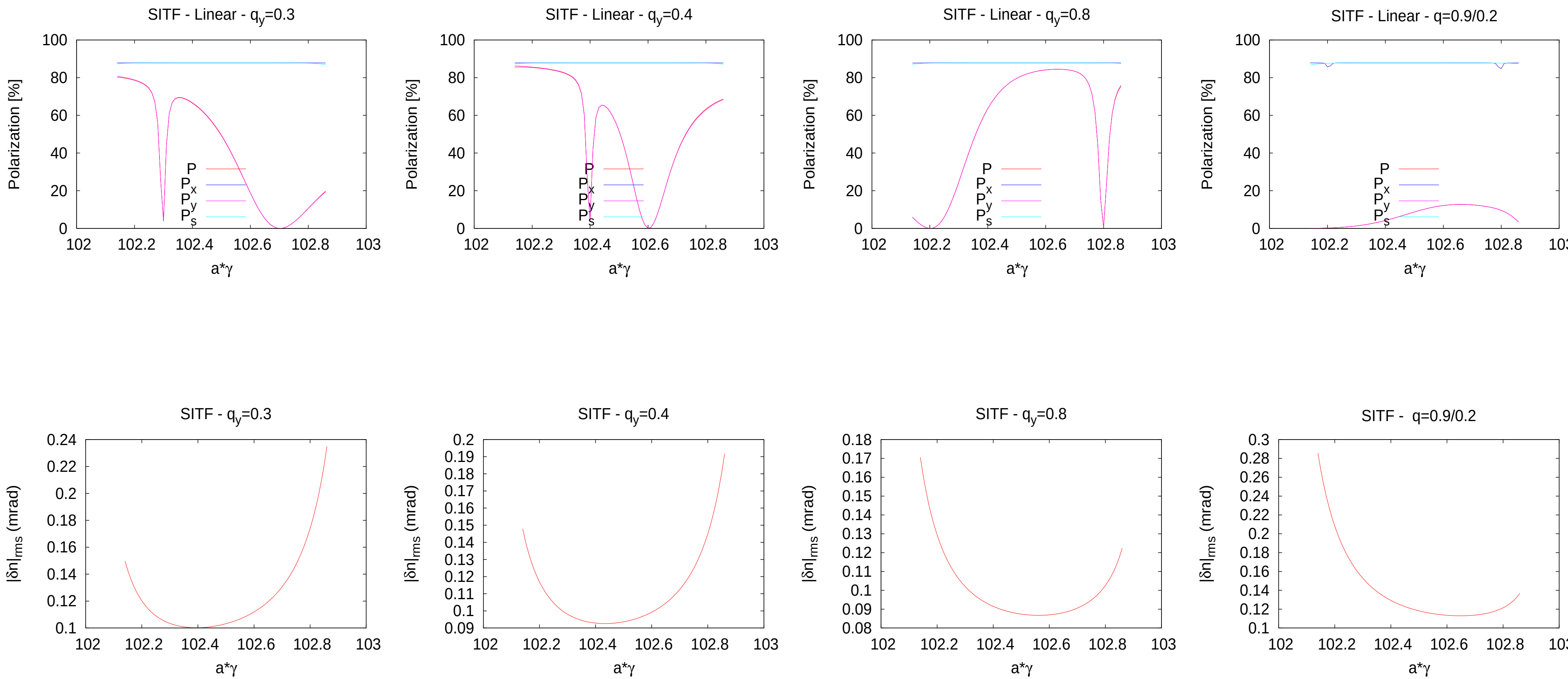
$$\Delta\mathcal{I}_5 \simeq \frac{1}{15\pi^3} \frac{< \beta_x >_w \ell_w}{\rho_w^5} \lambda_w$$

The effect is small for the $60^0/60^0$ deg FODO.

For the 1 mm β^* optics ($90^0/90^0$ deg FODO) the horizontal emittance at 45 GeV increases from 90 pm to 500 pm.

The emittance increase can be mitigated by choosing a shorter wiggler period, λ_w.

Tune scan



Why is 0.1/0.8 better then 0.1/0.2 ?

Polarization formulas

The Derbenev-Kondratenko polarization rate

$$\tau_{\mathrm{DK}}^{-1} = \frac{5\sqrt{3}}{8}\frac{r_e\gamma^5\hbar}{m_0 C}\oint < \frac{1}{|\rho|^3}\left[1 - \frac{2}{9}(\hat{n}\cdot\hat{v})^2 + \frac{11}{18}\left(\frac{\partial\hat{n}}{\partial\delta}\right)^2\right] >$$

may be written as

$$\tau_{\mathrm{DK}}^{-1} = \tau_p^{-1} \simeq \tau_{\mathrm{BKS}}^{-1} + \tau_d^{-1}$$

with

$$\tau_{\mathrm{BKS}}^{-1} = \frac{5\sqrt{3}}{8}\frac{r_e\gamma^5\hbar}{m_0 C}\oint ds \frac{1}{|\rho|^3}\left[1 - \frac{2}{9}(\hat{n}\cdot\hat{v})^2\right]$$

and

$$\tau_d^{-1} = \frac{5\sqrt{3}}{8}\frac{r_e\gamma^5\hbar}{m_0 C}\oint ds < \frac{1}{|\rho|^3}\left[\frac{11}{18}\left(\frac{\partial\hat{n}}{\partial\delta}\right)^2\right] >$$

Similarly for P_∞

$$\vec{P}_{\rm DK} = \hat{n}_0 \frac{8}{5\sqrt{3}} \frac{\oint ds < \frac{1}{|\rho|^3} \hat{b} \cdot (\hat{n} - \frac{\partial \hat{n}}{\partial \delta}) >}{\oint ds < \frac{1}{|\rho|^3} \left[1 - \frac{2}{9} (\hat{n} \cdot \hat{v})^2 + \frac{11}{18} (\frac{\partial \hat{n}}{\partial \delta})^2 \right] >} \qquad \hat{b} \equiv \vec{v} \times \dot{\vec{v}} / |\vec{v} \times \dot{\vec{v}}|$$

$$P_\infty = P_{\rm DK} \simeq P_{\rm BKS} \frac{\tau_d}{\tau_{\rm BKS} + \tau_d} = P_{\rm BKS} \frac{\tau_p}{\tau_{\rm BKS}}$$

Approximations done

- $\hat{n} \cdot \hat{v}$ is evaluated on the closed orbit
- $\hat{b} \cdot \frac{\partial \hat{n}}{\partial \delta}$ has been neglected. In general it is small.