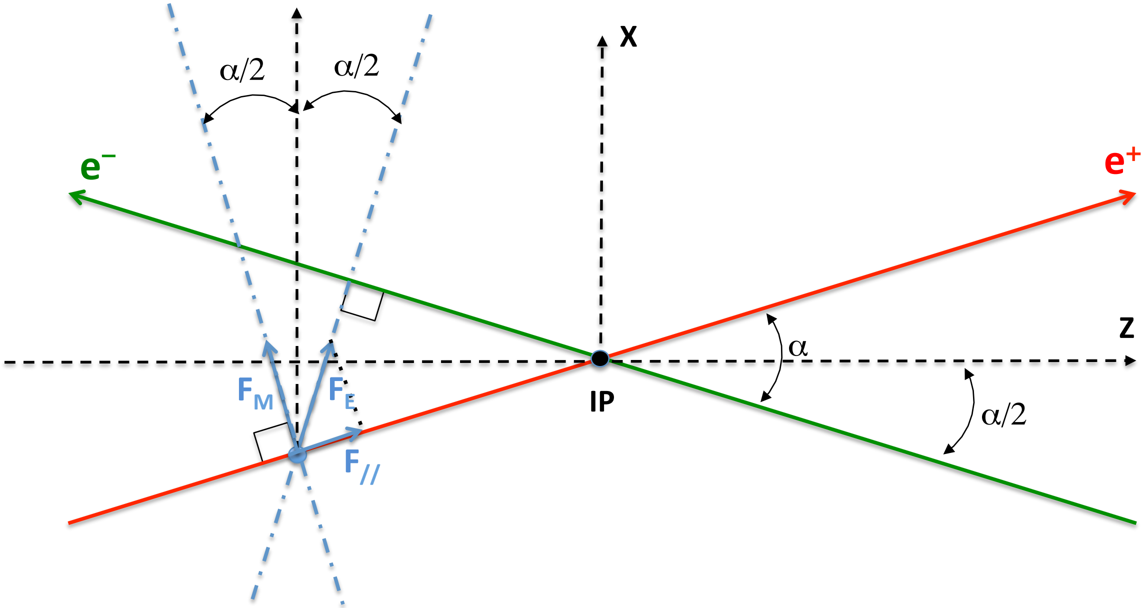
Measurements of beam-beam effects at IP

- Reminder (see presentation from Dmitry Shatilov)



$$
\begin{aligned}
F_{\parallel} &= F_E \sin\alpha = F\sin\alpha, \\
F_{\perp} &= F_M + F_E\cos\alpha = F\left(1+\cos\alpha\right), \\
F_X &= F_E \sin{}^{\alpha}/_2 + F_M \sin{}^{\alpha}/_2 = 2F\sin{}^{\alpha}/_2, \\
F_Z &= F_E \cos{}^{\alpha}/_2 - F_M \cos{}^{\alpha}/_2 = 0.
\end{aligned}
$$

- Beam-beam effects increase beam energies ($E_\pm = E^0 \pm \delta E_\pm$) and crossing angle ($\alpha = \alpha_0 \pm \delta\alpha$)
  - But does not modify centre-of-mass energy at IP ($\sqrt{s} = 2\sqrt{p_{z+}p_{z-}}$)

$$\sqrt{s} = 2\sqrt{E_+^0 E_-^0}\cos{}^{\alpha_0}/_2 = 2\sqrt{E_+E_-}\cos{}^{\alpha}/_2$$

Measurements of beam-beam effects at IP

- Numerically, for nominal FCC-ee parameters at the Z energies
  - With α = 30 mrad and $E_\pm$=45.6 GeV
    - Beam-energy increase: $\delta E_\pm$ = 60.5 keV
      - ➨ Predicted from Lifetrac (D. Shatilov), GuineaPig / numerical integation (E. Perez)

        Similar to precision of resonant depolarization measurement
    - Crossing angle increase
      - ➨ 100% correlated with beam-energy increase

$$\delta\alpha = \frac{1}{\tan \alpha/2}\left(\frac{\delta E_+}{E_+} + \frac{\delta E_-}{E_-}\right)$$

$\delta\alpha$ = 0.177 mrad

  - But why would we care, as $\sqrt{s}$ is not modified?

$$\sqrt{s} = 2\sqrt{E_+^0 E_-^0}\cos \alpha_0/2 = 2\sqrt{E_+ E_-}\cos \alpha/2$$

Centre-of-mass energy determination

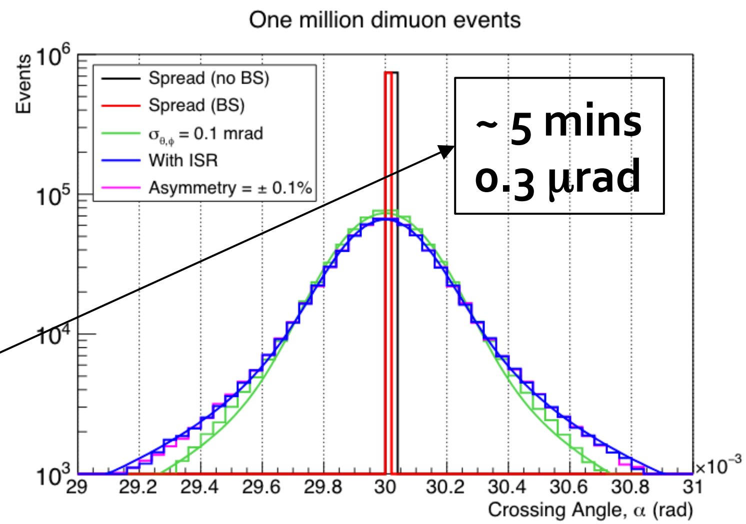
- Resonant depolarization with single, non-interacting bunches
  - Measure $E^0_\pm$ without beam-beam effects
- Difficult to measure α_0 with a precision better than 0.1 mrad
  - With beam position monitors (BPMs) placed on the last quadrupoles
- However, α can be measured at IP with $e^+e^- \to \mu^+\mu^-(\gamma)$ events in the detectors at IP
  - From total-energy momentum conservation in the transverse plane (p_x, p_y, E)
    - Directly with the muon directions $\phi^\pm$ and $\theta^\pm$

$$\alpha = 2 \arcsin \left[\frac{\sin(\varphi^- - \varphi^+) \sin\theta^+ \sin\theta^-}{\sin\varphi^- \sin\theta^- - \sin\varphi^+ \sin\theta^+} \right]$$

See my presentation in Amsterdam and the Energy Calibration paper

➨ Assumed angular resolution: 0.1 mrad

➨ Precision on α: 0.3 mrad / $\sqrt{N_{\mu\mu}}$



Again, why does it matter ?

- $\sqrt{s}$ is not affected by beam-beam effects, but …

$$\sqrt{s} = 2\sqrt{E_+^0 E_-^0}\cos{}^{\alpha_0}/_2 = 2\sqrt{E_+ E_-}\cos{}^{\alpha}/_2$$

  - We measure this … and this …
    - But not that …. or that.

- It is therefore necessary to find a way to measure $\delta\alpha$ (and therefore $\alpha_0 = \alpha - \delta\alpha$)
  - With a precision $\Delta\delta\alpha$, which translates into a precision $\Delta\sqrt{s}$

$$\frac{\Delta\sqrt{s}}{\sqrt{s}} \simeq \frac{1}{4}\alpha\delta\alpha\,\frac{\Delta\delta\alpha}{\delta\alpha} \approx 1.3\times 10^{-6}\,\frac{\Delta\delta\alpha}{\delta\alpha}.$$

    - $\Delta\delta\alpha/\delta\alpha = \pm 100\% \Rightarrow \Delta\sqrt{s} = \mp 120$ keV (with BPMs); $\Delta\delta\alpha/\delta\alpha = \pm 10\% \Rightarrow \Delta\sqrt{s} = \mp 12$ keV ;
      X ✓

Beam crossing angle increase determination

- Measure α with increasing bunch population ?
  - Collider "filling" period (bootstrapping) is ideal in principle
    - Half nominal intensity injected
    - Topped-up by steps of 10%
      - ➨ Every 52 seconds in e^+ or e^-
    - Collisions with nominal optics
      - ➨ Stabilize after a few seconds
    - Collect $\mu^+\mu^-(\gamma)$ events for 40 seconds
      - ➨ Measure crossing angle α
    - Repeat until nominal luminosity
    - Extrapolate to $N^\pm = N_{\mu\mu} = 0$
      - ➨ Infer $\delta\alpha$ (and α_0)

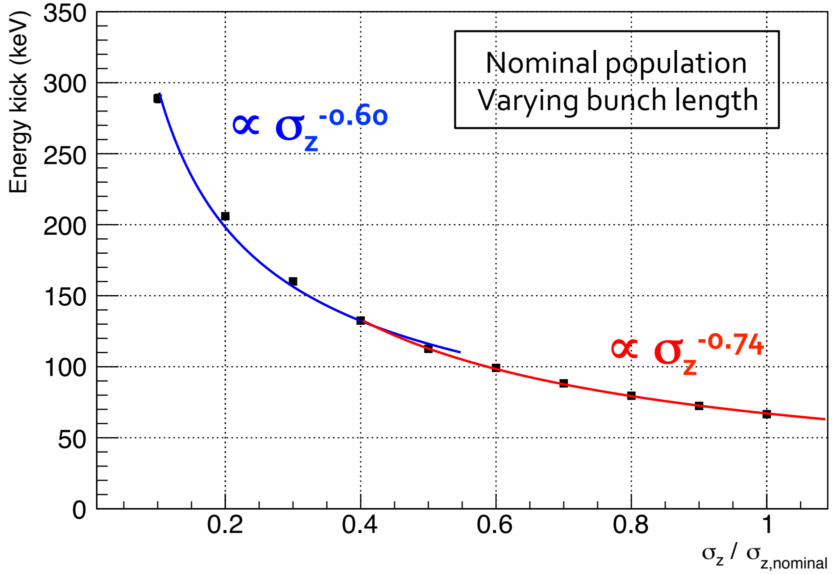
$N^+_{\rm part}$	$N^-_{\rm part}$	$\mathcal{L}$	σ^+_δ	σ^-_δ	$\sigma_{\sqrt{s}}$	δE_+	δE_-	α	$N_{\mu^+\mu^-}$
0.50	0.50	0.37	0.68	0.68	0.680	39.2	39.2	30.1147	49210
0.50	0.55	0.38	0.79	0.61	0.705	47.9	33.7	30.1193	50540
0.60	0.55	0.44	0.64	0.84	0.747	35.5	51.5	30.1273	58250
0.60	0.65	0.50	0.87	0.68	0.781	52.9	39.2	30.1347	66500
0.70	0.65	0.56	0.69	0.93	0.819	40.1	56.5	30.1413	74480
0.70	0.75	0.62	0.94	0.74	0.846	57.5	43.8	30.1480	82460
0.80	0.75	0.68	0.76	0.99	0.883	44.7	61.6	30.1553	90440
0.80	0.85	0.74	1.02	0.80	0.917	63.4	45.6	30.1593	98420
0.90	0.85	0.81	0.82	1.04	0.936	49.2	65.2	30.1673	107730
0.90	0.95	0.87	1.09	0.84	0.973	67.5	49.2	30.1707	115710
1.00	0.95	0.91	0.86	1.12	0.998	49.2	67.5	30.1707	121030
1.00	1.00	1.00	1.00	1.00	1.000	60.2	60.2	30.1760	133000

Recorded in 40 s
Bunch length / energy spread
Normalized to nominal
Crossing angle (mrad)
Beam-energy increase (keV)

- Table: Numbers predicted from Lifetrac (D. Shatilov)

Extrapolation to $N^{\pm} = 0$

- **Energy kicks $\delta E^{\pm}$ directly proportional to opposite bunch population $N^{\mp}$**
  - **Also increases when opposite bunch length decreases (charge density increases)**



- **From independent numerical integration**
  - ➨ **(Code from E. Perez)**
- **Fit to a power law in σ_z (or in σ_δ, equivalently)**

$$\delta E^{\pm} \propto \frac{N^{\mp}_{\rm part}}{\sigma_\delta^{\mp\,2/3}}.$$

➨ **Uncertainty of ±0.05 on the exponent**

Treated as systematic uncertainty in the following

Check with Lifetrac simulation

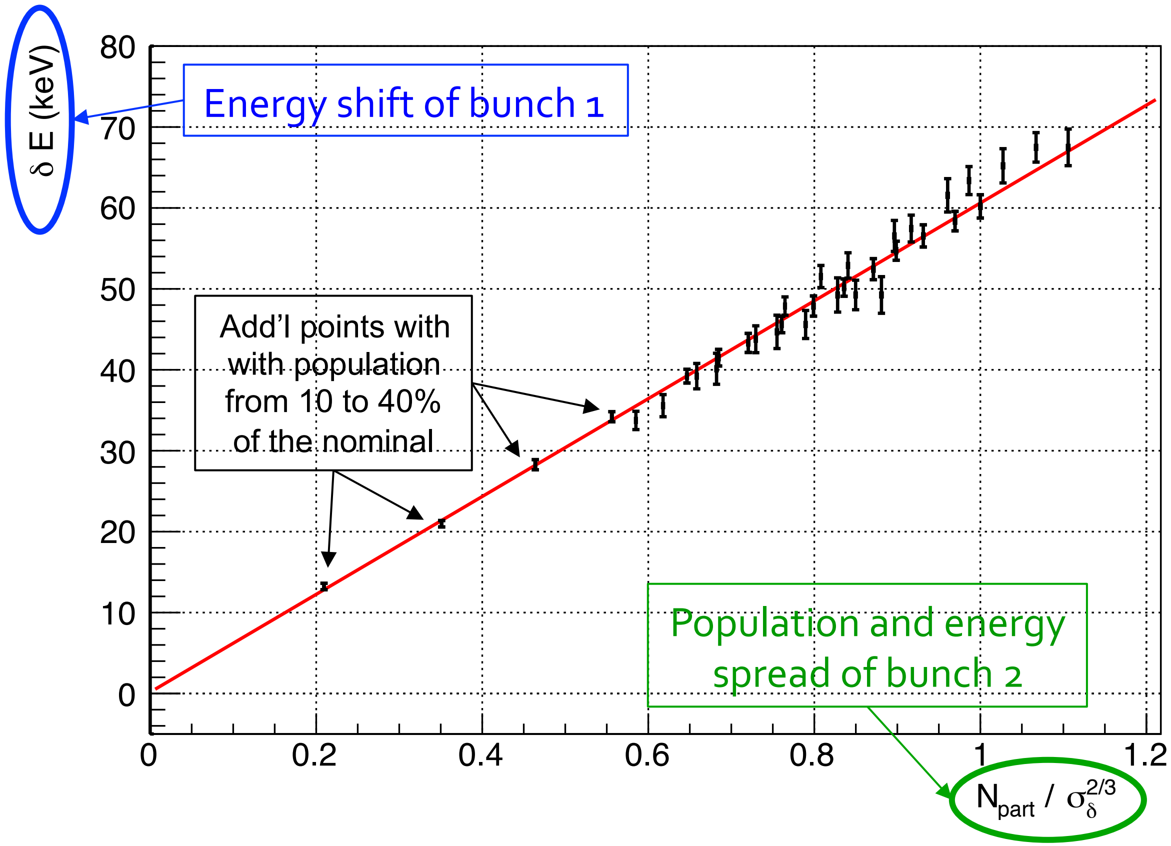
- **From the numbers in the table of Slide 5 during the filling period**

  - **Linear fit and extrapolation**

δE_0 (keV)= 0.2 ± 0.2 (stat.) ± 1.0 (syst.)

δE_{nom} (keV)= 60.5 ± 0.5 (stat.) ± 0.6 (syst.)

  - **Statistical uncertainty**
    - **From Lifetrac MC statistics**
  - **Systematic uncertainty**
    - **From σ_δ exponent uncertainty**



Measurement of $\delta\alpha$

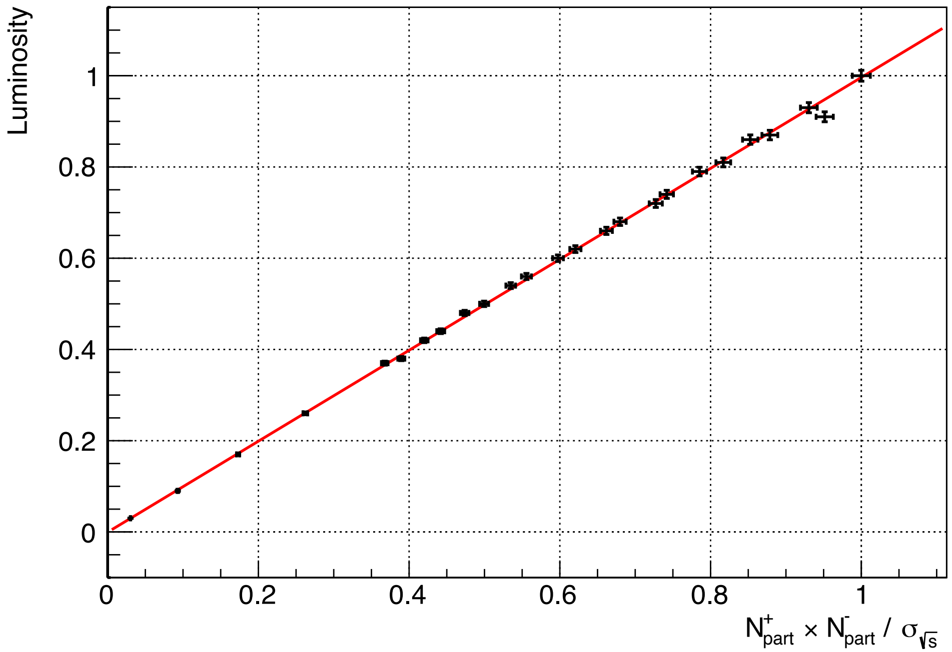
- For equal e^+ and e^- bunch populations, $\delta\alpha$ is proportional to the common δE:

$$\delta\alpha = \frac{1}{\tan {}^{\alpha}/_{2}} \left(\frac{\delta E_+}{E_+} + \frac{\delta E_-}{E_-} \right)$$

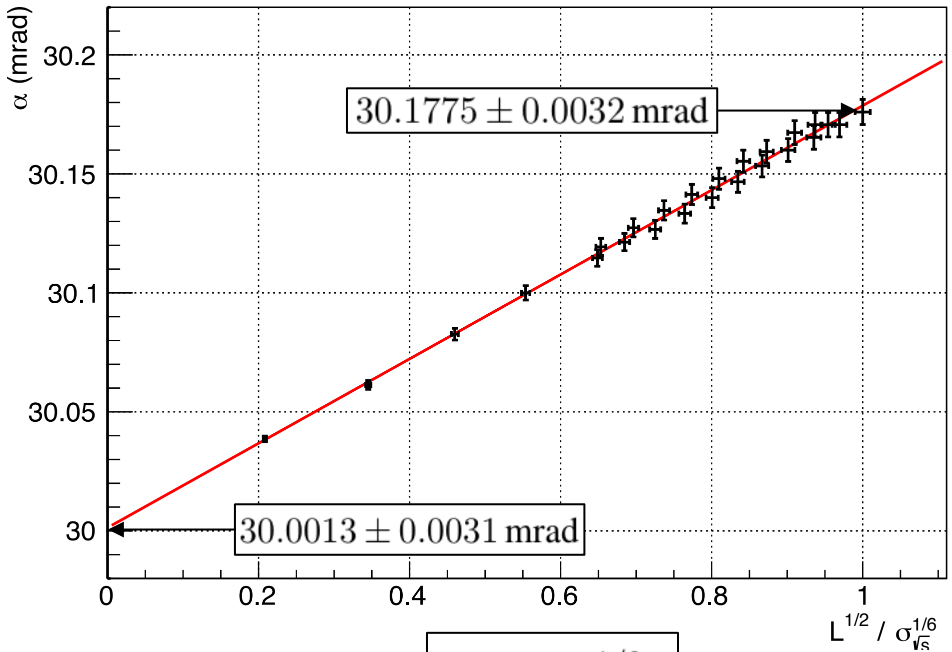
  - Therefore, $\delta\alpha$ follows the same power law as δE: $\delta\alpha \propto \frac{N_{\text{part}}}{\sigma_{\sqrt{s}}^{2/3}}$. with $\sigma_{\sqrt{s}} = \sigma_\delta^+ \oplus \sigma_\delta^-$
  - The bunch population N_{part} is in turn related to the luminosity: $\mathcal{L} \propto \frac{N^2_{\text{part}}}{\sigma_z} \Leftrightarrow \mathcal{L} \propto \frac{N^2_{\text{part}}}{\sigma_{\sqrt{s}}}$.
  - Leading to the remarkable power law: $\delta\alpha \propto \frac{\mathcal{L}^{1/2}}{\sigma_{\sqrt{s}}^{1/6}}$.
    - It turns out that the beam crossing angle, the luminosity, and the centre-of mass energy spread can be measured altogether with $\mu^+\mu^-(\gamma)$ events [see slide 10]
      - ➨ Linear fit of a vs $L^{1/2}/\sigma_{\sqrt{s}}^{1/6}$ will give in turn the values of $\delta\alpha$ and α_0

Check with Lifetrac simulation

- From the numbers in the table of Slide 5



$$\mathcal{L} \propto \frac{N_{\rm part}^2}{\sigma_z} \Leftrightarrow \mathcal{L} \propto \frac{N_{\rm part}^2}{\sigma_{\sqrt{s}}}.$$



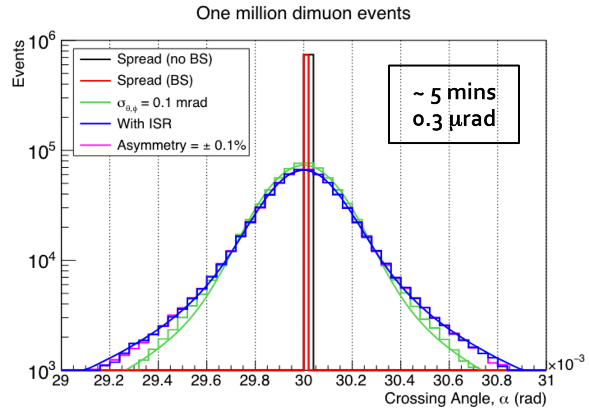
$$\delta\alpha \propto \frac{\mathcal{L}^{1/2}}{\sigma_{\sqrt{s}}^{1/6}}.$$

Measurement with μ⁺μ⁻(γ) events

- **From total energy-momentum conservation**
  - In the transverse plane $[p_x, p_y, E]$: see slide 3

$$\alpha = 2\arcsin\left[\frac{\sin\left(\varphi^- - \varphi^+\right)\sin\theta^+\sin\theta^-}{\sin\varphi^-\sin\theta^- - \sin\varphi^+\sin\theta^+}\right]$$

$$\Delta\alpha = \frac{0.3\,\mathrm{mrad}}{\sqrt{N_{\mu\mu}}}.$$

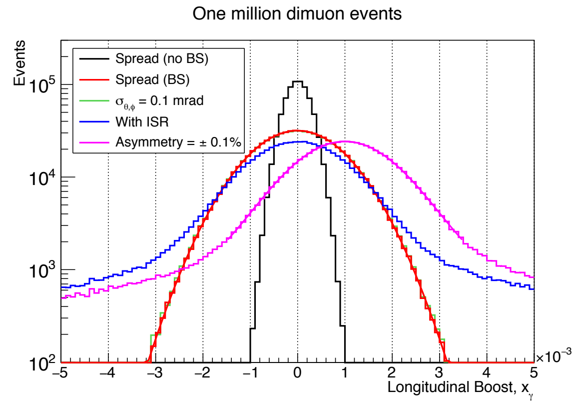


  - In the longitudinal direction $[p_z, E]$: see my presentation in Amsterdam and the Energy Calibration paper
    - Longitudinal boost distribution ~ √s spread due to σ_δ

$$x_\gamma = -\frac{x_+\cos\theta^+ + x_-\cos\theta^-}{\cos(\alpha/2) + |x_+\cos\theta^+ + x_-\cos\theta^-|},$$

$$\text{with} \quad x_\pm = \frac{\mp\sin\theta^\mp\sin\varphi^\mp}{\sin\theta^+\sin\varphi^+ - \sin\theta^-\sin\varphi^-}.$$

$$\frac{\Delta\sigma_{\sqrt{s}}}{\sigma_{\sqrt{s}}} = \frac{1}{\sqrt{N_{\mu\mu}}}$$



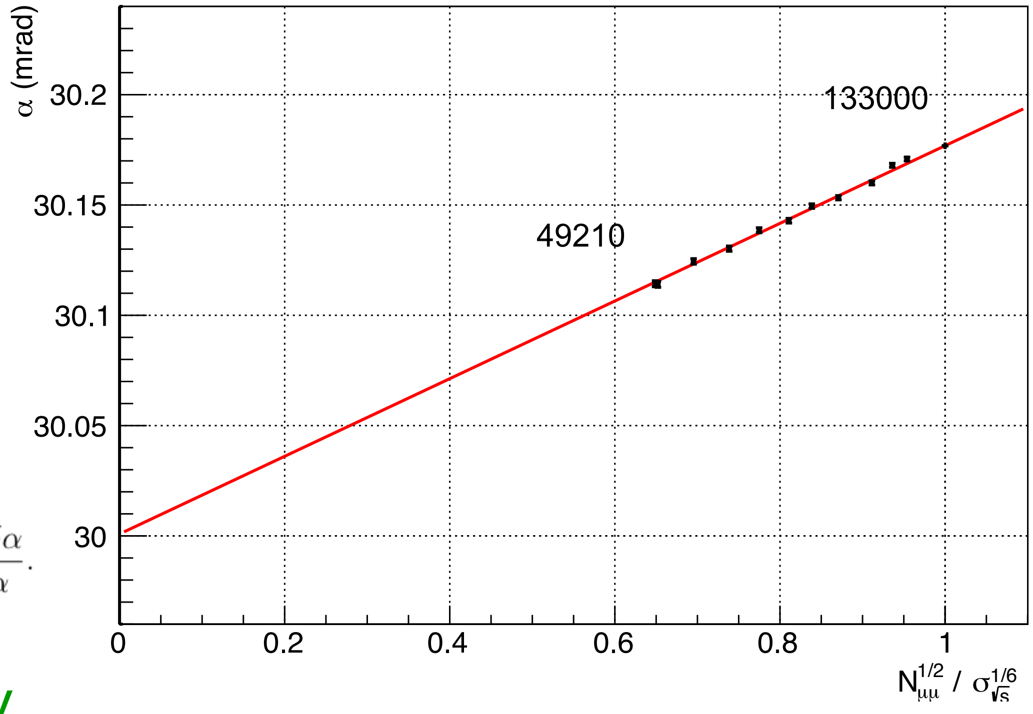
  - Luminosity directly proportional to $N_{\mu\mu}$

$$\frac{\Delta\mathcal{L}}{\mathcal{L}} = \frac{1}{\sqrt{N_{\mu\mu}}}$$

Measurements during the filling period (Z pole)

- **Measure α, $\sigma_{\sqrt{s}}$ and $N_{\mu\mu}$ for 11 steps of 40 seconds at the Z pole**
  - **Plot a versus $\sqrt{N_{\mu\mu}} / \sigma_{\sqrt{s}}^{1/6}$**
    - **And fit a straight line to the data**

$$\alpha_0 = 30.0008 \pm 0.0016(\text{stat.}) \pm 0.0031(\text{syst.})\ \text{mrad},$$
$$\delta\alpha = 0.1761 \pm 0.0016(\text{stat.}) \pm 0.0032(\text{syst.})\ \text{mrad},$$



  - **Feed α_0 back to the centre-of-mass energy**

$$\sqrt{s} = 2\sqrt{E_+^0 E_-^0}\cos\alpha_0/2 \quad \text{and} \quad \frac{\Delta\sqrt{s}}{\sqrt{s}} \simeq \frac{1}{4}\alpha\delta\alpha\,\frac{\Delta\delta\alpha}{\delta\alpha} \approx 1.3\times10^{-6}\,\frac{\Delta\delta\alpha}{\delta\alpha}.$$

    - **Uncertainty of $\sqrt{s}$ of the order of 2.5 keV**
      - ➨ **Well within the requirements, negligible w.r.t. to the beam energy uncertainty (50 keV)**

Caveats and alternative method

- The measurement requires that, during the filling period
  - The beam instabilities can be kept under control
  - The detector high voltages can be safely turned on
    - As will have to be the case during regular top-up injection in stable collisions

- What if these assumptions do not hold?
  - Use natural bunch population spread, or have half of the bunches with 99% nominal current
    - Inducing a minute loss of luminosity of 0.75%
  - Or better, use the fact that each bunch population varies between 101% and 99% of the nominal over every period of 104 seconds, with alternate $e^{\pm}$ injection every 52 seconds.
    - Measure α, $\sigma_{\sqrt{s}}$ and $N_{\mu\mu}$ every 26 seconds (just before and just after any top-up)
      - ➨ Precision on α of 0.016 mrad /√hours at the Z pole

Corresponding to a precision on √s on 10 keV/√hours at the Z pole

Off peak points and higher energies

- Dimuon rate smaller by factors 7.8 and 3.2 at 88 and 94 GeV than at 91.2 GeV
  - Precision on $\delta\alpha$ and on $\sqrt{s}$ degrades accordingly
    - 0.0053 mrad and 3.6 keV at 88 GeV
      - ➨ 32 kev/$\sqrt{}$hours with the alternative method
    - 0.0043 mrad and 3.0 keV at 94 GeV
      - ➨ 19 keV/$\sqrt{}$hours with the alternative method

Still within requirements

- Much smaller $\mu^+\mu^-$ rate and faster filling at 161 GeV and above: method cannot be used
  - At the WW threshold, the $\sqrt{s}$ uncertainty from resonant depolarization is about 300 keV
    - Significantly larger than the bias due to the crossing angle increase (< 100 keV)
      - ➨ $\delta\alpha$ can be predicted/measured after calibrating Lifetrac / BPMs at the Z pole
  - At the HZ maximum and the top-pair threshold, $\langle\sqrt{s}\rangle$ is determined in situ at the IP
    - With Zγ, WW, and ZZ events from E,p conservation and m_W & m_Z precise knowledge
      - ➨ ±1.7 MeV at 240 GeV, ±5 MeV at 350 GeV, ±2 MeV at 365 GeV

Summary

- **Beam-beam effects cause the beam energies and crossing angle to increase at the IP**
  - Beam energies are measured by resonant depolarization with non-colliding beams
    - Measurement uncertainty: $\sigma(E^0)$ ~ 50 keV – Increase at the IP: δE ~ 60 keV
  - The crossing angle is measured at the IP with $\mu\mu(\gamma)$ events
    - Measurement uncertainty: $\sigma(\alpha)$ ~ 0.3 µrad – Increase at the IP: $\delta\alpha$ ~ 177 µrad
      - ➨ Leading to a bias of −120 keV on √s

$$\sqrt{s} = 2\sqrt{E_+^0 E_-^0}\cos{\alpha_0/2} = 2\sqrt{E_+ E_-}\cos{\alpha/2}$$

  - Crossing angle increase $\delta\alpha$ can be measured at the Z pole during the filling period
    - With uncertainties $\sigma(\delta\alpha)$ = 3.5, 4.3, and 5.3 µrad at 91.2, 94, and 88 GeV
      - ➨ Leading to √s uncertainties of 2.5, 3.0, and 3.6 keV

        Trackers must be designed to take data during the filling period
    - Alternatively, can use stable collisions in between top-up injections
      - ➨ Leading to √s uncertainties of 10, 19, and 32 keV/√h
    - Such a measurement is not needed at higher energies

Backup slides

- Prepared for FCC week in Amsterdam
  - Written up in the energy calibration paper
    - See draft at https://www.overleaf.com/11630130cmkmfpvyhhgb

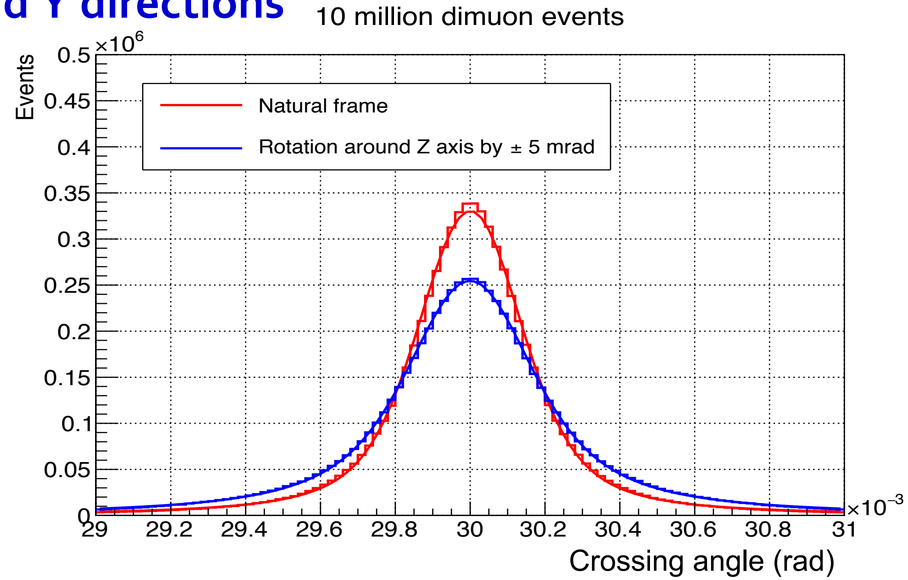
Control the angular resolution to 0.01 mrad ?

- Q: How to measure the angular resolution to 10% or better
  - For any value of θ and ϕ ?

- A: Take a muon track in dimuon events
  - Refit it with the odd hits, on the one hand, and with the even hits, on the other
    - And compare the angles
  - Need only 100 tracks in each (θ , ϕ) bin for a 10% precision
    - 10^6 dimuon events = 5 minutes at the Z pole = bins of 3×3 $(\mathrm{mrad})^2$
  - Expected to be stable in time
    - Precision (or bin size) improves with dimuon statistics

Absolute tracker alignment

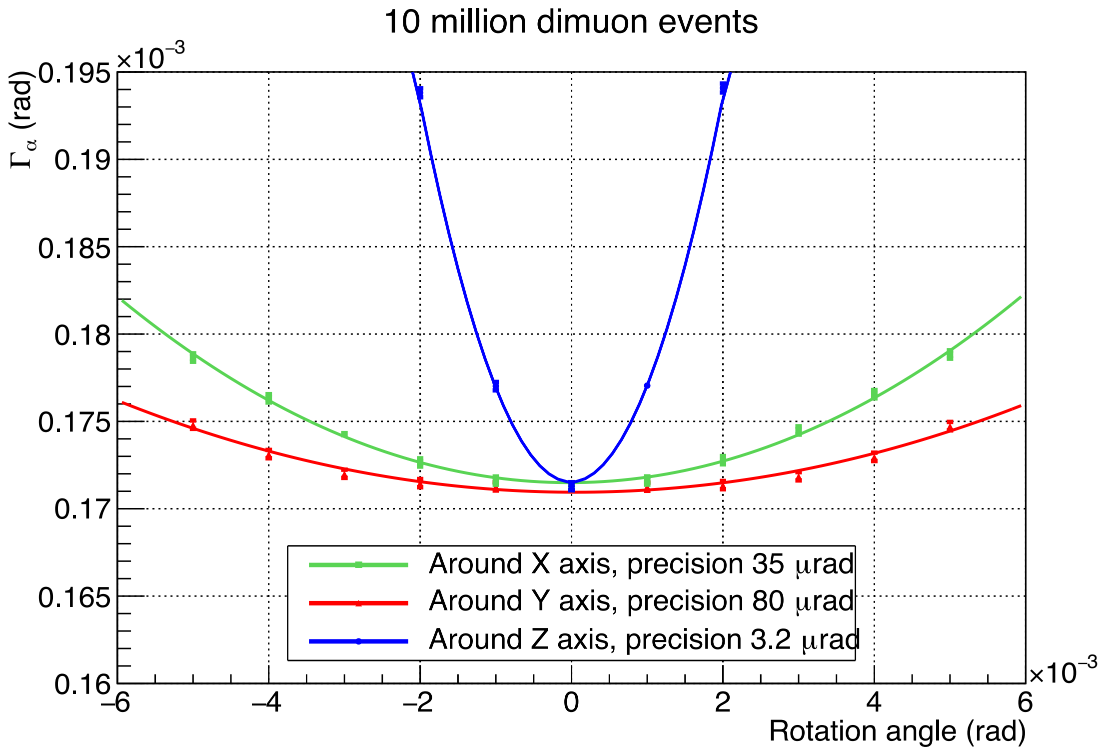
- Absolute angle determination is (usually) not an easy task
  - Requires alignment of the local (detector) frame with the natural (FCC-ee) frame
    - Z axis = solenoid axis vs bissector of the two beam axes
    - (X,Z) plane = horizontal plane vs plane containing the two beam axes
- Spread of α increases with anything happening in the transverse plane
  - E.g., rotation around the Z axis changes both X and Y directions



Similarly, rotation around the X (Y) axis changes Y (X) direction

Detector alignment

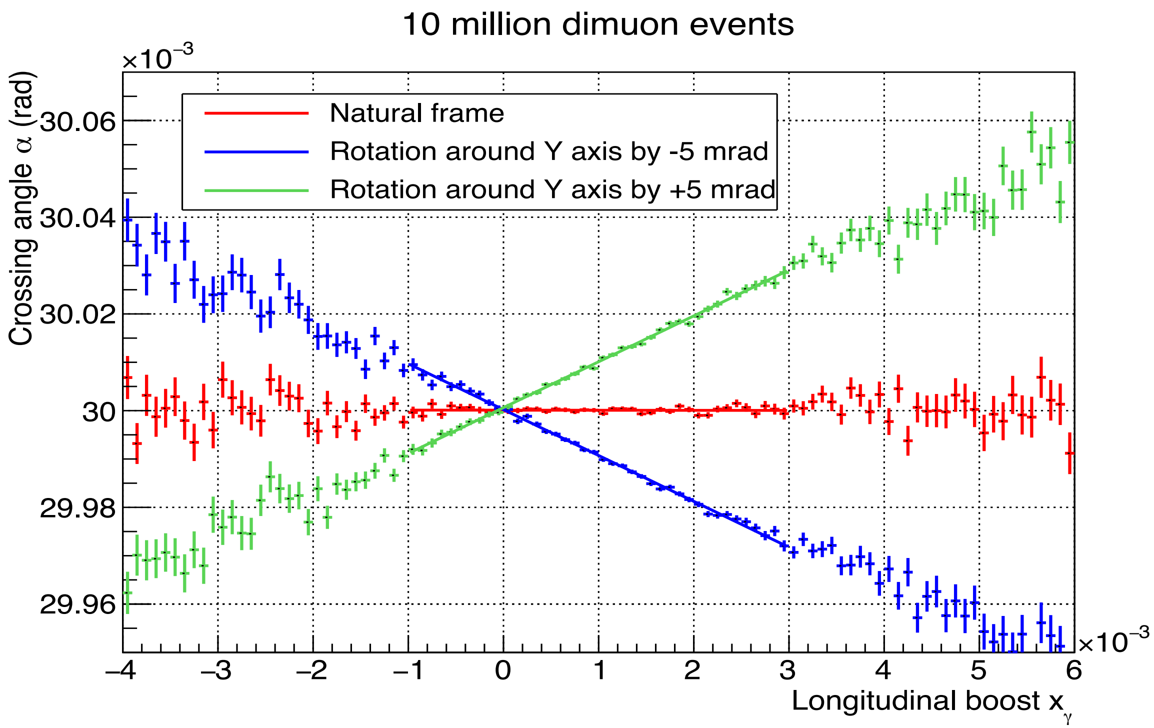
- Minimize the spread of the α distribution to find the three Euler angles



- **Note: α spread dominated by the ϕ resolution (here 0.1 mrad)**
  - **Precisions quadratically improves with the resolution in ϕ (here 0.1 mrad)**

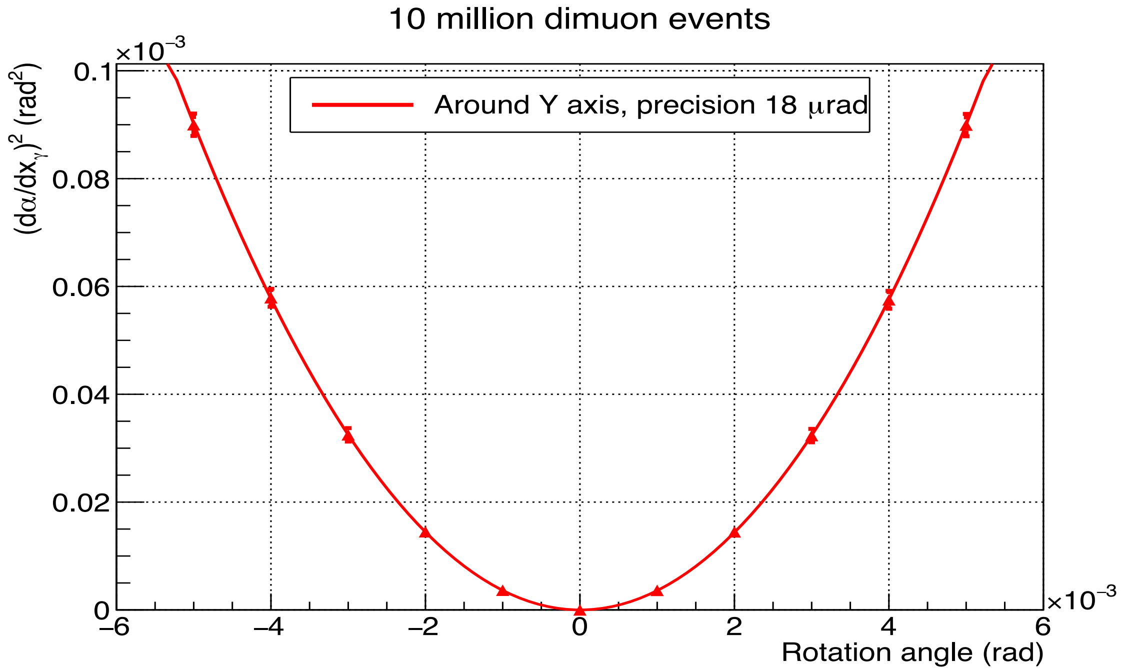
Detector alignment

- **Improve the angle corresponding to a rotation around the Y axis**
  - **X and Z information get mixed by such a rotation**
    - **Resulting in a strong (linear) correlation between x_γ and α:**



Detector alignment

- **Minimize the correlation between x_γ and α:**



  - **Improves the precision on that angle by a factor of five.**
    - **Reach a precision of 0.1 μrad on α and of 10^{-7} on x_γ**
    - **Variation of the x_γ spread already insignificant with 100 times less events**