From matrix / tensor models to black holes

Fidel I. Schaposnik Massolo

based on arXiv:1707.03431 with T. Azeyanagi and F. Ferrari

Physique Théorique et Mathématique
Université Libre de Bruxelles

Desirable features of a black hole model

A) Macroscopic space-time description [Schwarzschild - 1916; Kerr - 1963; ...]

- Definition of the horizon [Finkelstein - 1958; ...]
- Description of the interior [Kruskal - 1960; Penrose, Hawking - 1965, 1970; ...]
- Entropy $S = A/4G_N$ [Bekenstein - 1972; Bardeen, Carter, Hawking - 1973; ...]

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- Loss of time-reversal invariance
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Reverse engineering black holes

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Can we study black holes starting from (B) and getting to (A) through holography?

Guiding principles

Existence of parameter N

Loss of time-reversal invariance / Unitarity problems

$$\Downarrow$$

Thermodynamical irreversibility (limit $N \to \infty$)

Chaotic dynamics

$$F_\beta(t) = \left\langle \hat{O}(0)\hat{O}(t)\hat{O}(0)\hat{O}(t) \right\rangle_{\beta,\text{con.}} \propto e^{\lambda_L t}$$

Lyapunov exponent saturates bound for black holes

$$\lambda_L \leq \frac{2\pi}{\beta}$$

[Maldacena, Shenker, Stanford - 2015]

SYK model [Sachdev, Ye - 1993; Kitaev - 2015]

N Majorana fermions $\psi_1, \dots, \psi_N$ in $0+1$ dim. with Hamiltonian

$$H = \sum_{i<j<k<l} J_{ijkl}\, \psi_i\, \psi_j\, \psi_k\, \psi_l$$

Quenched disorder

$$\langle \cdot \rangle \equiv \sum_{J_{ijkl}} p(J_{ijkl})\, \langle \cdot \rangle_{J_{ijkl}} \qquad \text{with} \qquad \sigma^2(J_{ijkl}) \propto J^2$$

Some nice features

- Approximate conformal symmetry in IR $\implies$ $\mathrm{NAdS}_2/\mathrm{NCFT}_1$
- Analytical treatment for $N \to \infty$
- Explicit numerics for small N ($|\mathcal{H}| = 2^{N/2}$)
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Not a proper Quantum Field Theory :-(

Vector and matrix models: an overview

	Large D vector models	**Large N matrix models**
Field content	ϕ_μ with $\mu = 1, \ldots, D$	${X^a}_b$ with $a, b = 1, \ldots, N$
Symmetry	$O(D)$	$U(N)^2$ or $U(N)$
Interactions	$(\vec{\phi}^2)^k$ for $k = 1, \ldots$	$\mathrm{Tr}(XX^\dagger XX^\dagger \cdots), \ldots$
Diag. scaling	$D^{V-P+\varphi} = D^{1-\ell}$	$N^{V-P+f} = N^{2-2g}$
Leading	cacti diagrams (auxiliary tree-level)	planar diagrams
Applications	cond. mat. ph., CFT, higher spin gravity	nucl. ph., QCD, string theory

New large N, D limit [Ferrari - 2017; Ferrari, Rivasseau, Valette - 2017]

$O(D) \times U(N)^2$ **model for a vector of complex matrices**

Interaction vertices are $V_B = \mathrm{Tr}\left(X_{\mu_1} X^{\dagger}_{\mu_2} \cdots X_{\mu_{2s-1}} X^{\dagger}_{\mu_{2s}} \right)$

Usual scaling $S = ND \left(\frac{1}{2} \mathrm{Tr}(X_\mu X^{\dagger}_{\mu}) + \sum_B t_B V_B(X_\mu) \right)$

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Enhance 't Hooft coupling t_B for V_B as

$$t_B = \lambda_B D^{l(B)} \qquad \text{with} \qquad l(B) \geq 0$$

In the $N \to \infty$ limit

$$F = \sum_{g \geq 0} N^{2-2g} F_g$$

In the $D \to \infty$ limit (g fixed)

$$F_g = \sum_{k \geq 0} D^{1+g-k/2} F_{g,k}$$

New large N, D limit: example

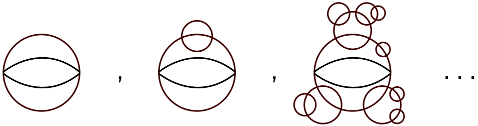
Enhanced couplings $\Longrightarrow$ more diagrams contribute order-by-order

$$\mathrm{Tr}(X_\mu X_\mu^\dagger X_\nu X_\nu^\dagger) \quad\Longrightarrow\quad I(B) = 0$$

$$\mathrm{Tr}(X_\mu X_\nu^\dagger X_\mu X_\nu^\dagger) \quad\Longrightarrow\quad I(B) = 1/2$$

$$\mathrm{Tr}(X_\rho X_\mu^\dagger X_\rho X_\sigma^\dagger X_\mu X_\sigma^\dagger) \quad\Longrightarrow\quad I(B) = 1$$

For $\sqrt{D}\,\mathrm{Tr}(X_\mu X_\nu^\dagger X_\mu X_\nu^\dagger)$ dominating diagrams are melons ($\equiv$ SYK)



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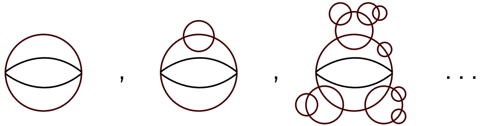


Intuition: Genus g restriction kills many diagrams, so we can enhance some couplings and still have well-defined $D \to \infty$ limit

Quartic models for fermionic matrices

Fermionic matrices in $0+1$ dimensions

$$(\psi_\mu^\dagger)^a{}_b = (\psi_{\mu a}^b)^\dagger \quad \text{with} \quad \left\{\psi_{\mu b}^a, (\psi_\nu^\dagger)^c{}_d\right\} = \frac{1}{ND}\delta_{\mu\nu}\delta_d^a\delta_b^c$$

- **Model** I**:** $O(D) \times U(N)^2$

$$H_\mathrm{I} = ND\,\mathrm{Tr}\Big(m\,\psi_\mu^\dagger\psi_\mu + \tfrac{1}{2}\lambda\sqrt{D}\psi_\mu\psi_\nu^\dagger\psi_\mu\psi_\nu^\dagger\Big)$$

- **Model** II**:** $O(D) \times U(N)$

$$H_\mathrm{II} = ND\,\mathrm{Tr}\Big\{m\,\psi_\mu^\dagger\psi_\mu + \tfrac{1}{2}\sqrt{D}\left(\lambda\psi_\mu^\dagger\psi_\nu\psi_\mu\psi_\nu + \lambda^*\psi_\mu^\dagger\psi_\nu^\dagger\psi_\mu^\dagger\psi_\nu\right)\Big\}$$

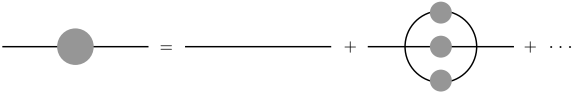
Melonic-dominated models $\Longrightarrow$ **Physics similar to SYK**

Procedure

Fermionic perturbation theory

- $m \gg \lambda \quad \Longrightarrow \quad$ Exp. around decoupled fermionic oscillators
- $T \gg \lambda \quad \Longrightarrow \quad$ Non-standard (SYK-like) perturbation theory

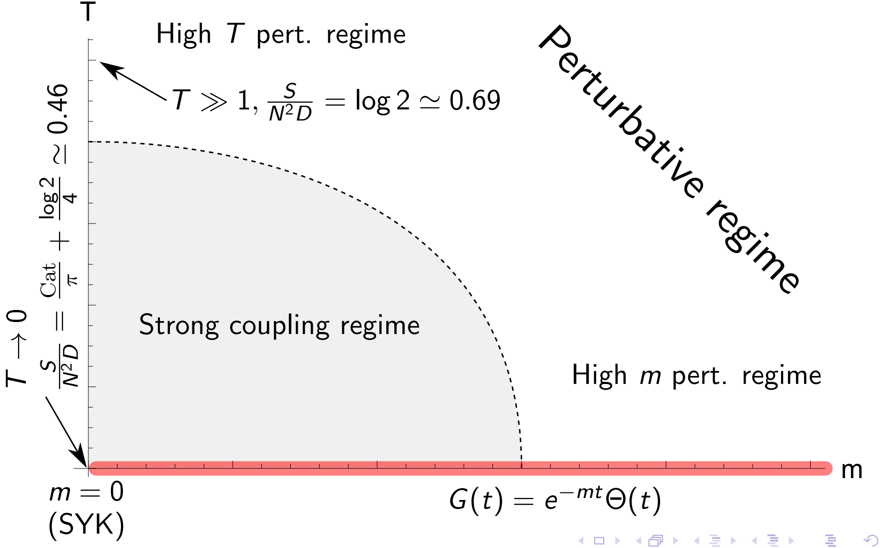
Feynman diagram structure $\Longleftrightarrow$ Schwinger-Dyson equations



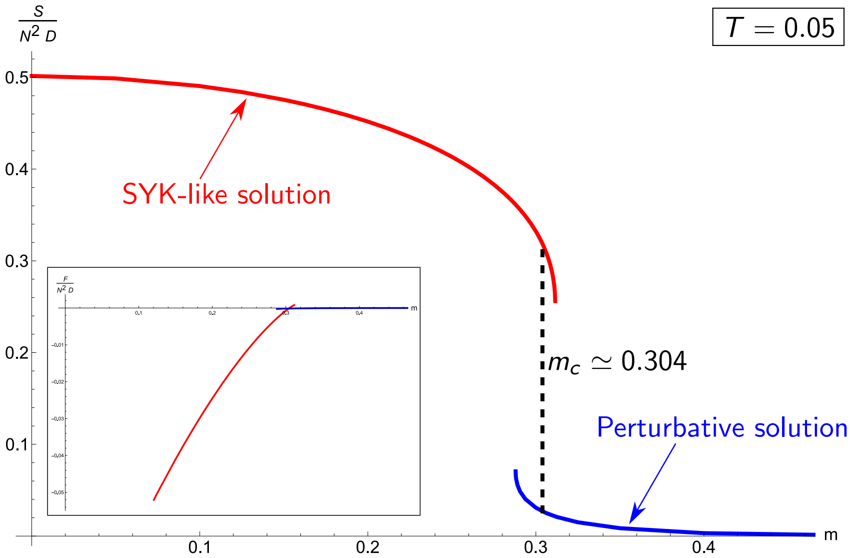
$$G_k^{-1} = m - i\omega_k + \Sigma_k \qquad \Sigma(t) = \lambda^2 G(t)^2 G(-t)$$

$$G(t) = \frac{1}{\beta} \sum_k G_k e^{-i\omega_k t} \qquad \omega_k = \frac{2\pi}{\beta} k \quad \text{with} \quad k \in \mathbb{Z} + \tfrac{1}{2}$$

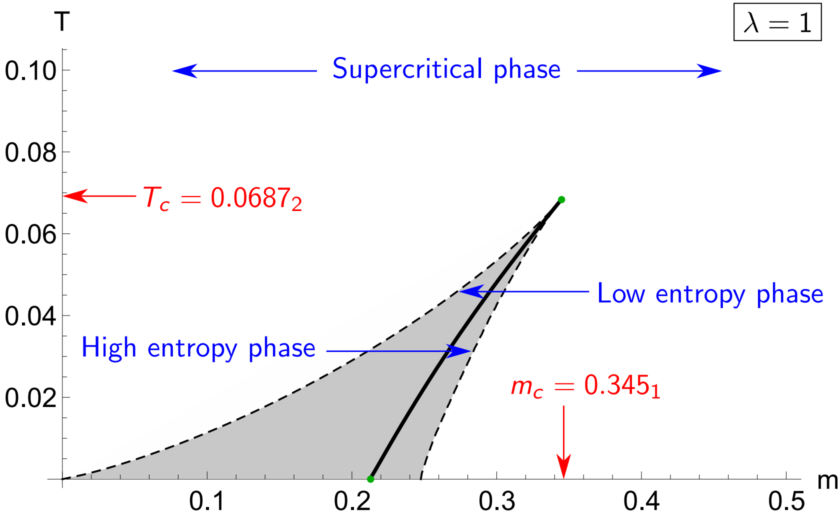
Phase diagram structure



Building the phase diagram



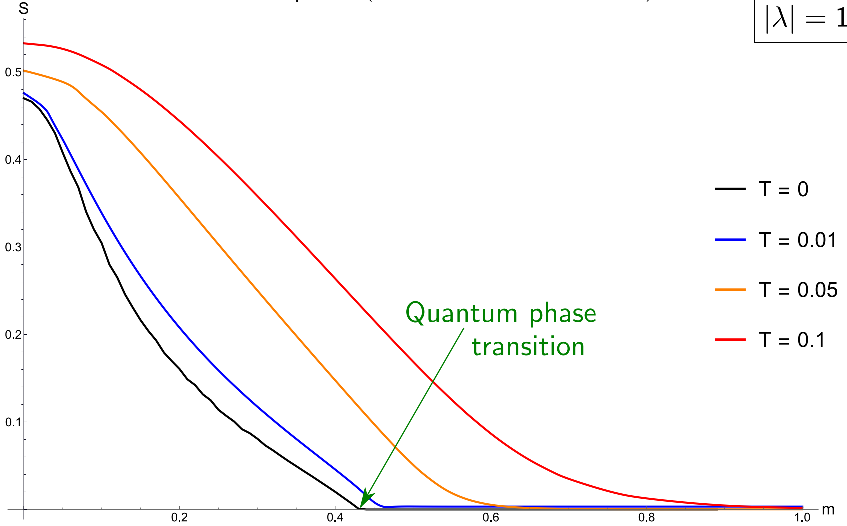
Phase diagram: (m, T) plane



Fermionic model II

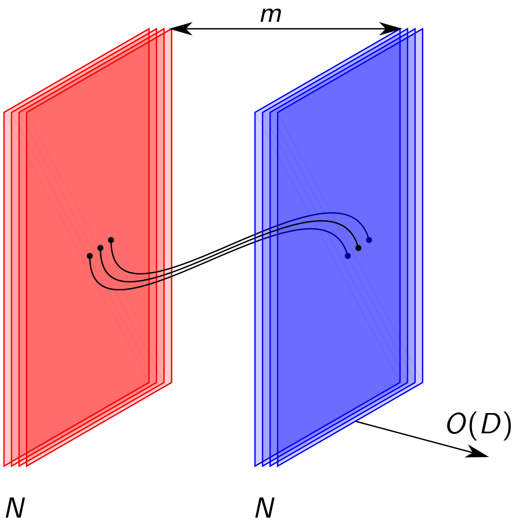
$$\Sigma(t) = -\frac{1}{4}|\lambda|^2 \left(G(t)^3 + 3G(t)G(-t)^2\right)$$

$|\lambda| = 1$



Possible interpretation

Stringy description of gravitational collapse



Finite N analysis

$\psi^{a}_{\mu b}$ has $D \times N^2$ fermionic degrees of freedom

$$\Downarrow$$

Hilbert space is 2^{DN^2} dimensional :-(

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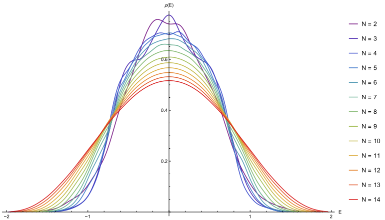
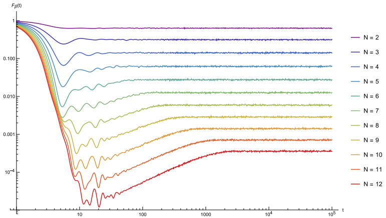
Disordered models with N Dirac fermions ($|\mathcal{H}| = 2^N$)

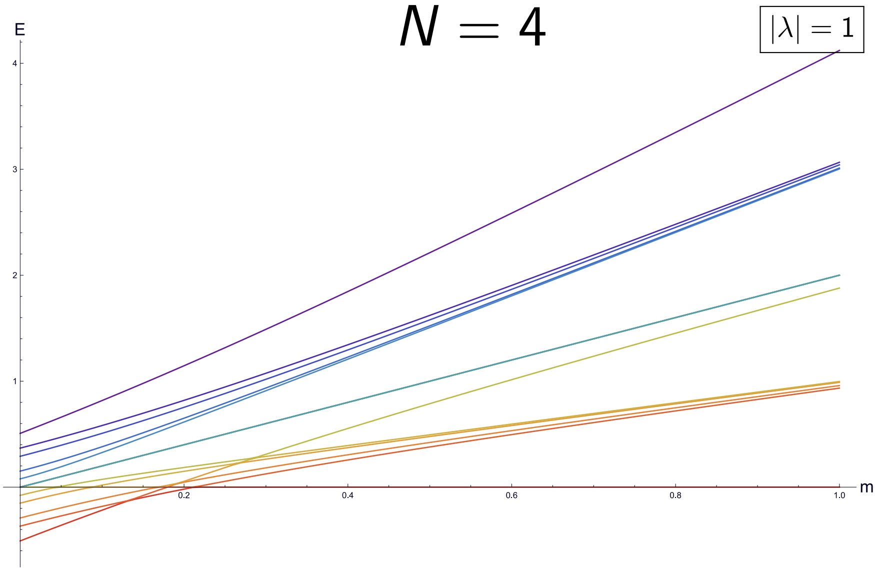
$$\tilde{H}_{\rm I} = m\chi^\dagger_i \chi^i + J^{ij}_{kl}\chi^\dagger_i \chi^\dagger_j \chi^k \chi^l$$

$$\tilde{H}_{\rm II} = m\chi^\dagger_i \chi^i + J^i_{jkl}\chi^\dagger_i \chi^j \chi^k \chi^l + J^l_{ijk}\chi^\dagger_i \chi^\dagger_j \chi^\dagger_k \chi^l$$





Finite N spectrum



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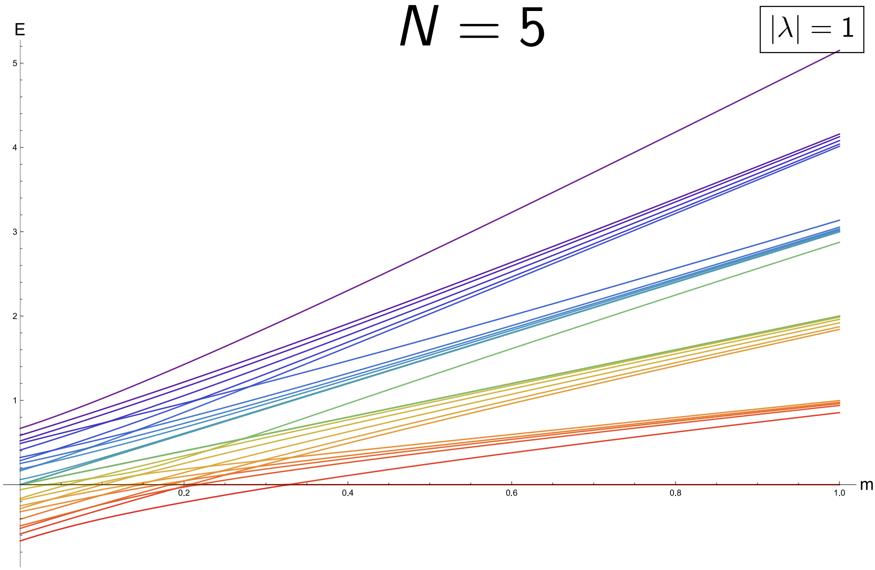

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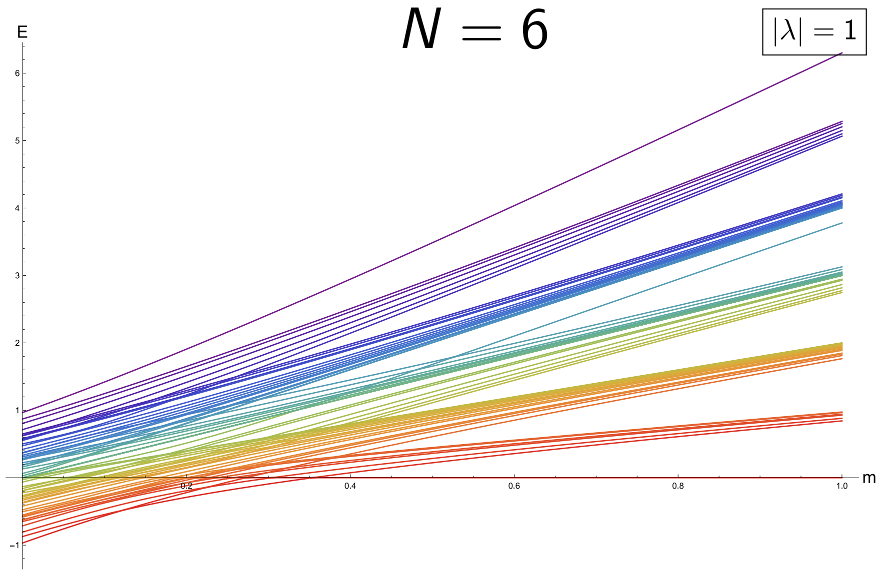

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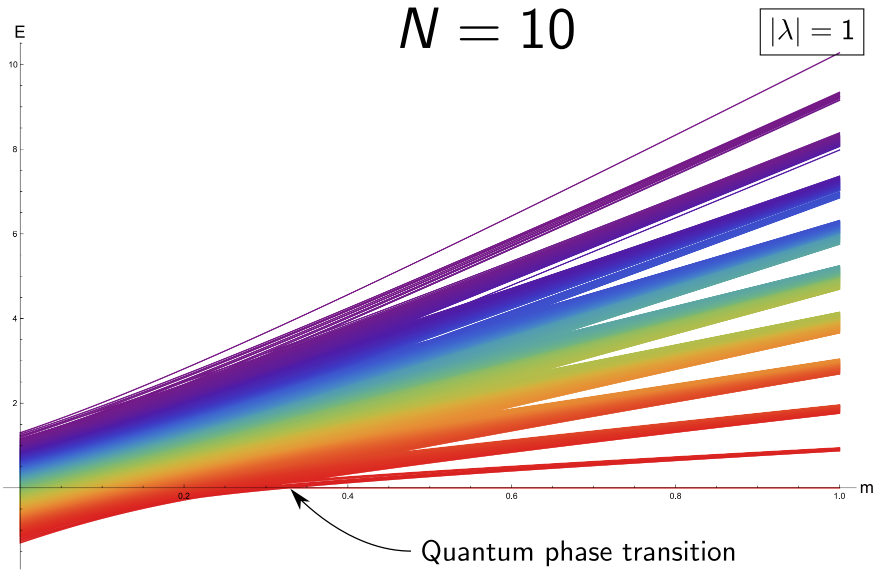
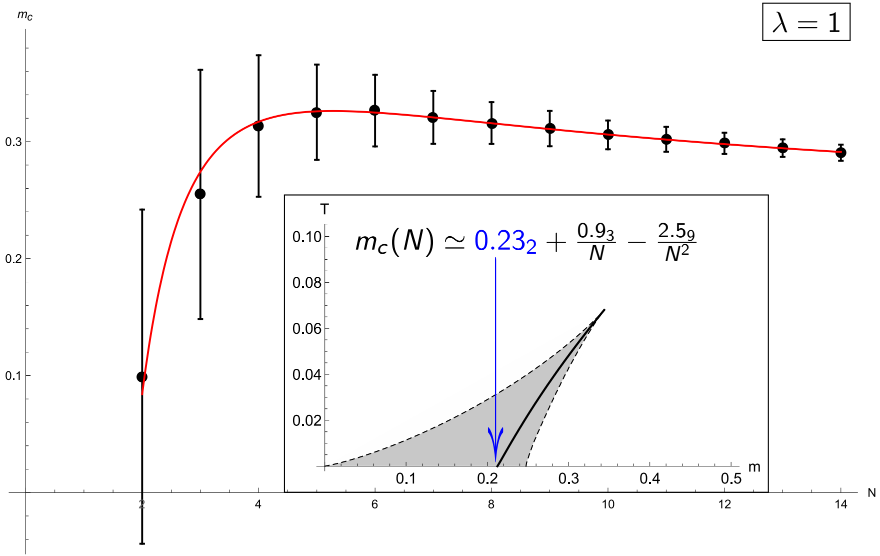
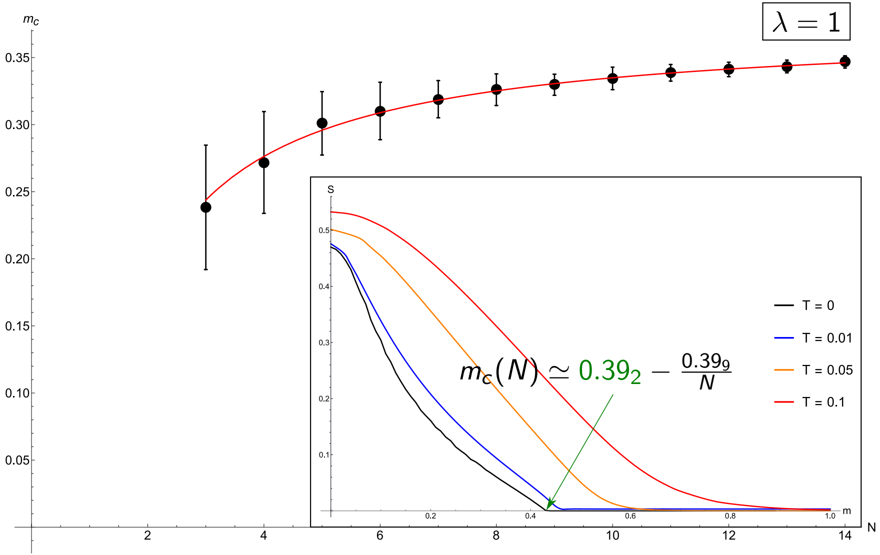


Quantum critical mass: model I

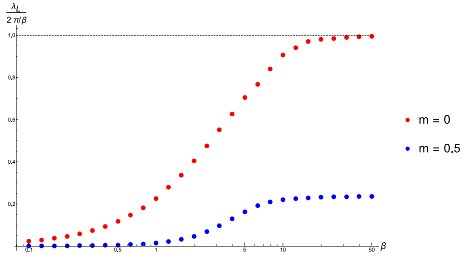


Quantum critical mass: model II



Outlook

1. Real-time physics



2. Effective description à la Landau
3. Supersymmetric models
4. Holographic picture

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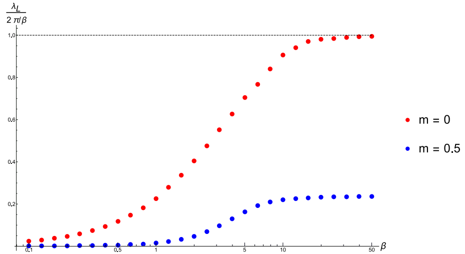

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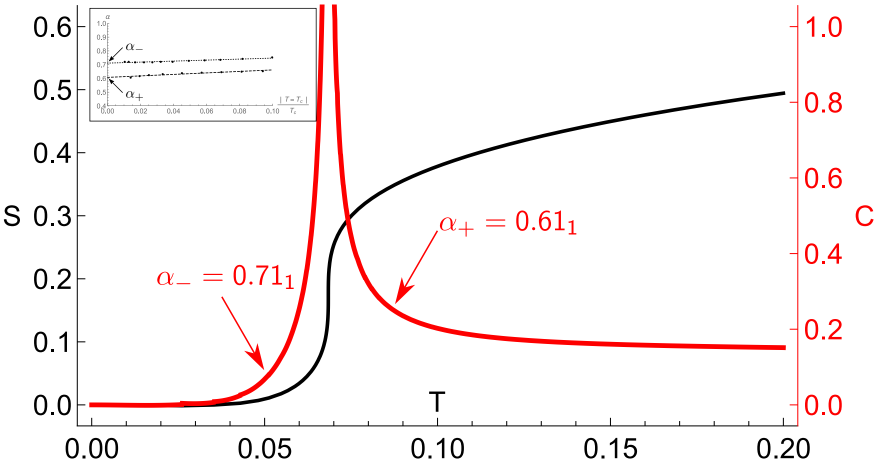
Takeaway: quantum black hole playground (in a computer :-)

Thanks!

$S(T)$ and $C_m = T\partial S/\partial T$ at $m = m_c$

New strongly coupled critical point

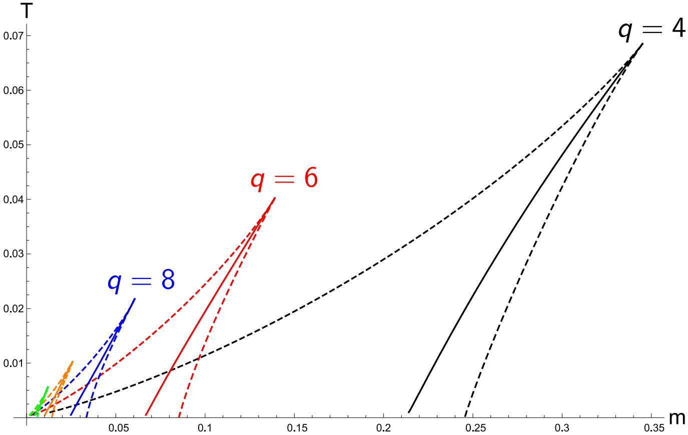
$\lambda = 1$



Generalizations and extensions

q-body interacting generalization of the fermionic model I

$$\Sigma(t) = \lambda^2 G(t)^2 G(-t) \quad \Longrightarrow \quad \Sigma(t) = (-1)^{\frac{q}{2}} \lambda^2 G(t)^{\frac{q}{2}} G(-t)^{\frac{q}{2}-1}$$



Bosonic models

$O(D) \times U(N)^2$ **purely bosonic models of Hermitian matrices**

$$H_{\rm III} = ND\,\mathrm{Tr}\left(\frac{m^2}{2}X_\mu X_\mu + \frac{\lambda^3}{4}\sqrt{D}X_\mu X_\nu X_\mu X_\nu\right)$$

$$H_{\rm IV} = ND\,\mathrm{Tr}\left(\frac{m^2}{2}X_\mu X_\mu + \lambda^4 D\, X_\rho X_\mu X_\rho X_\sigma X_\mu X_\sigma\right)$$

Crucial differences:

- **Classical limit $T \gg \lambda$:**

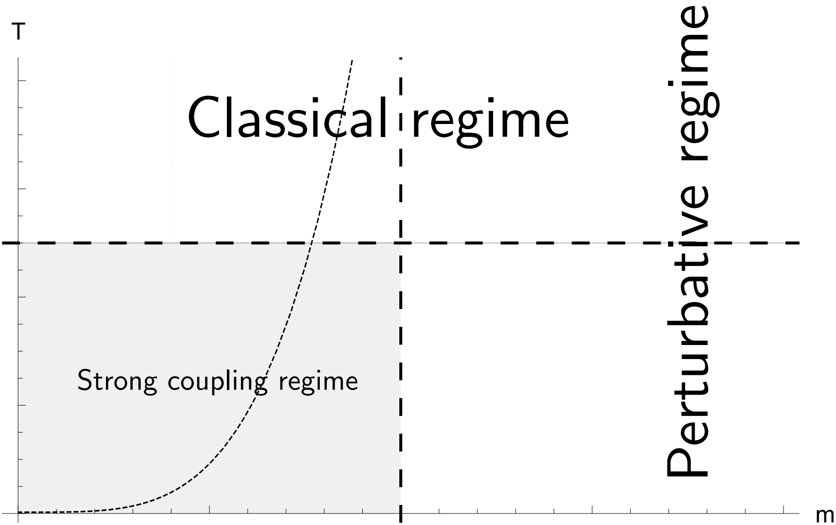
  Strongly coupled zero mode $\Longrightarrow$ No high T pert. theory

  (Classical eq. for zero mode of (III): $x^{-1} = m^2 - \frac{\lambda^6}{\beta^2}x^3$)

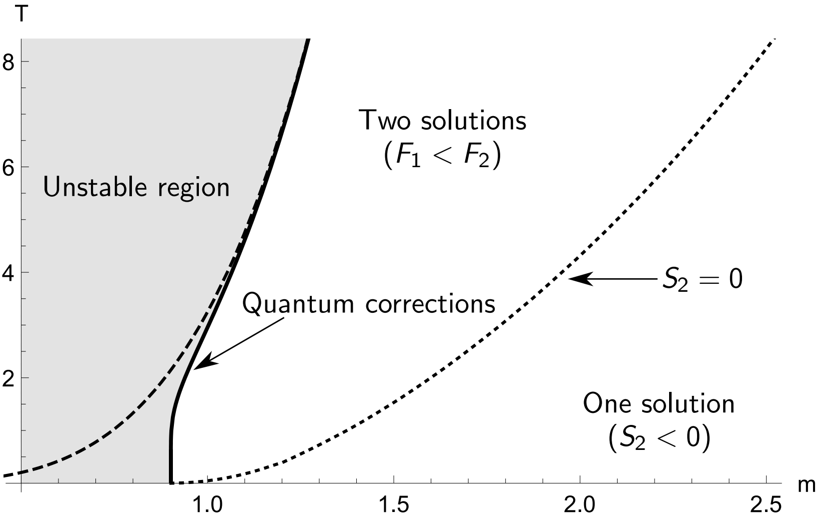
- **Instabilities:** Potential for (III) is unstable

  No gap $\Longrightarrow$ IR divergences

Phase diagram structure



Phase diagram for (III): (m, T) plane



$(q_1, q_2) = (4, 8)$ domain wall

$m = 0; T = 10^{-4}$