First Steps to the Optimization of Undulator Parameters for 125 GeV Drive Beam

by Manuel Formela

Overview

- Introducing formulas for:
  - Power absorbed by the undulator vessel in form of photons P_{vessel}
  - Number of produced e^+ by e^-e^+- pair production in a Ti-6% Al-4%V target
- Undulator scheme used in the RDR
- Reproducing values for already calculated P_{vessel} for the RDR set-up
- Calculations of N_{e^+} for various parameter values for $K, \lambda, l_u, N_{hcell}$
- Dropping some parameter combinations due to restraints in N_{e^+} and P_{vessel}
- Outlook into possible future

Radiated Synchrotron Energy Spectral Density per Solid Angle per Electron

Formulas taken from: Kincaid, Brian M. "A short-period helical wiggler as an improved source of synchrotron radiation." *Journal of Applied Physics* 48.7 (1977): 2684-2691.

First approximations:
- relativistic ($\gamma \gg 1$)
- far field ($R \gg \lambda_\gamma$)
- pointlike charge ($V_{e^-} \to 0$)

Photon frequency

$$\frac{dI(\omega)}{d\Omega} = \frac{d^2W(\omega)}{d\Omega d\omega} = \frac{e^2\omega^2}{14\pi^3\epsilon_0 c}\left|\int_{-\infty}^{+\infty} \hat{n}\times(\hat{n}\times\vec{\beta})e^{i\omega\left(t-\frac{\hat{n}\vec{r}(t)}{c}\right)}dt\right|^2$$

For helical trajectory:

Opening angle

$$\frac{dI(\omega)}{d\Omega} = \frac{\mathrm{e}^2\omega^2K^2}{4\pi^3\epsilon_0 c\omega_u^2\gamma^2}\sum_{n=1}^{\infty}\left[J_n'^2(x_1) + \left(\frac{\gamma\theta}{K} - \frac{n}{x_1}\right)^2 J_n^2(x_1)\right]\frac{\sin^2\left[N_u\pi\left(\frac{\omega}{\omega_1} - n\right)\right]}{\left(\frac{\omega}{\omega_1} - n\right)^2}$$

2nd approximations:
- small (radiation) angle ($|\theta| \ll 1 \Rightarrow \cos\theta \approx 1, \sin\theta \approx \theta$); this is reasonable, because the radiation cone has according to theory an Opening angle of $\theta \approx 1/\gamma$
- Many undulator periods ($N_u \gtrsim 100$)
- reasonably small undulator parameter ($K \lesssim 1 \to K/\gamma \ll 1$)

$$\frac{dI(\omega)}{d\Omega} = \frac{e^2\omega^2K^2}{4\pi^3\epsilon_0 c\omega_u^2\gamma^2}\sum_{n=1}^{\infty}\left[J_n'^2(x_1) + \left(\frac{\gamma\theta}{K} - \frac{n}{x_1}\right)^2 J_n^2(x_1)\right]\frac{\sin^2\left[N_u\pi\left(\frac{\omega}{\omega_1} - n\right)\right]}{\left(\frac{\omega}{\omega_1} - n\right)^2}$$

Approximation $\sin^2(N\pi y)/y^2 \to N\pi\delta(y)$:

1. $$\frac{dW}{d\Omega} = \int_0^{\infty}\frac{dI(\omega)}{d\Omega}d\omega \approx \frac{N_u e^2\omega_u K^2 8\gamma^4}{4\pi\epsilon_0 c(1+K^2+\gamma^2\theta^2)^3}\sum_{n=1}^{\infty} n^2\left[J_n'^2(x_n) + \left(\frac{\gamma\theta}{K} - \frac{n}{x_n}\right)^2 J_n^2(x_n)\right]$$ Radiated energy per solid angle

2. $$\frac{dW}{d\omega} = \int\frac{dI(\omega)}{d\Omega}d\Omega \approx \frac{N_u e^2K^2 r}{\epsilon_0 c}\sum_{n=1}^{\infty} n^2\left[J_n'^2(y_n) + \left(\frac{\alpha_n}{K} - \frac{n}{y_n}\right)^2 J_n^2(y_n)\right]H(\alpha_n^2)$$ Radiated energy spectral density

Numerical integration leads to:

1. $$P_{vessel} = \dot{N}_{e^-}\int\frac{dW}{d\Omega}d\Omega = 2\pi\dot{N}_{e^-}\int_{\theta_1}^{\pi}\sin\theta\frac{dW}{d\theta}d\theta$$ Power deposited in the undulator vessel

2. $$N_{e^+} = \frac{1}{\hbar}\int_0^{\infty}\frac{1}{\omega}\frac{dW}{d\omega}\left(1 - e^{-d\rho\sigma(\omega)}\right)d\omega$$ Positron number produced by all photons

Target thickness (d)

Target density (ρ)

Cross section for e^-e^+-pair production by photon of target material ($\sigma(\omega)$)

Undulator set up (RDR, BCD)

Taken from: Scott, Duncan J. "An Investigation into the Design of the Helical Undulator for the International Linear Collider Positron Source“

Undulator aperture = 5.85 mm

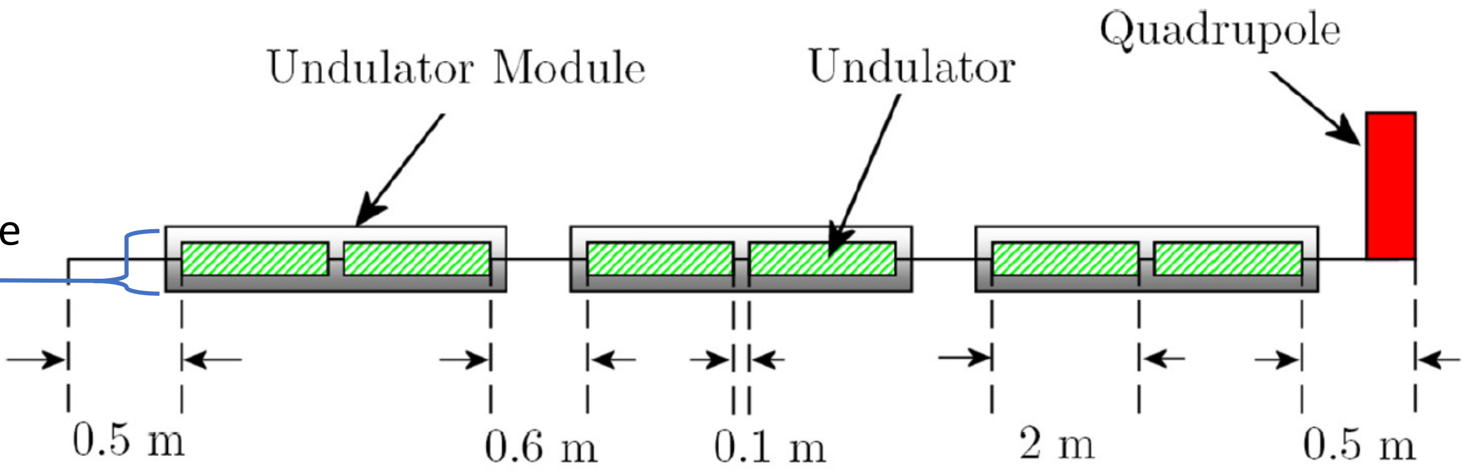
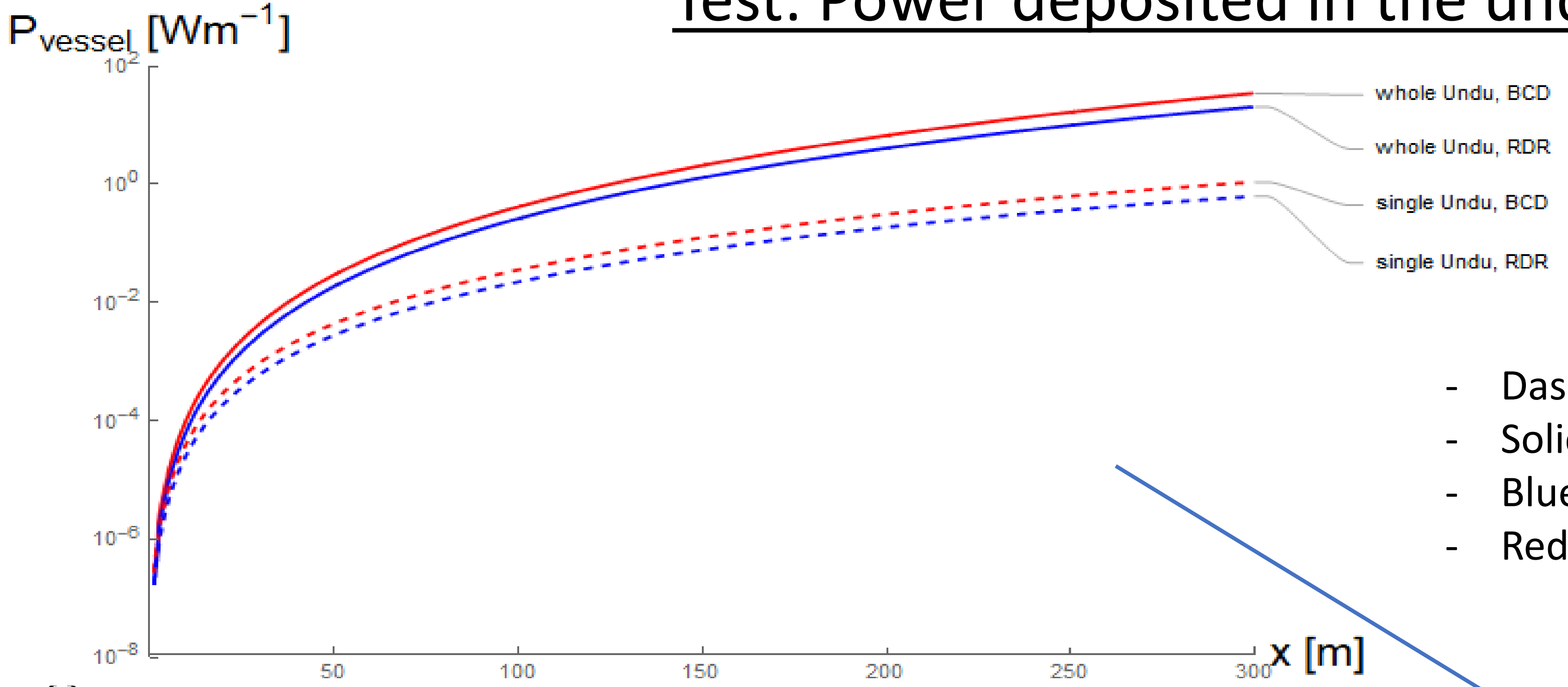


Figure 5.2: Schematic layout of a half-cell of the undulator line for incident SR power calculation.

of such half-cell will be arranged in a row to form the full undulator (with 240 m of total magnetic length)

Parameter	Unit	ILC-BCD	ILC-RDR
Undulator Period	mm	10	11.5
Undulator K		1	0.92
Undulator Aperture	mm	5.85[a]	5.85
Undulator Length	m	2	2
N_e		2.82×10^{-14}	2.82×10^{-14}

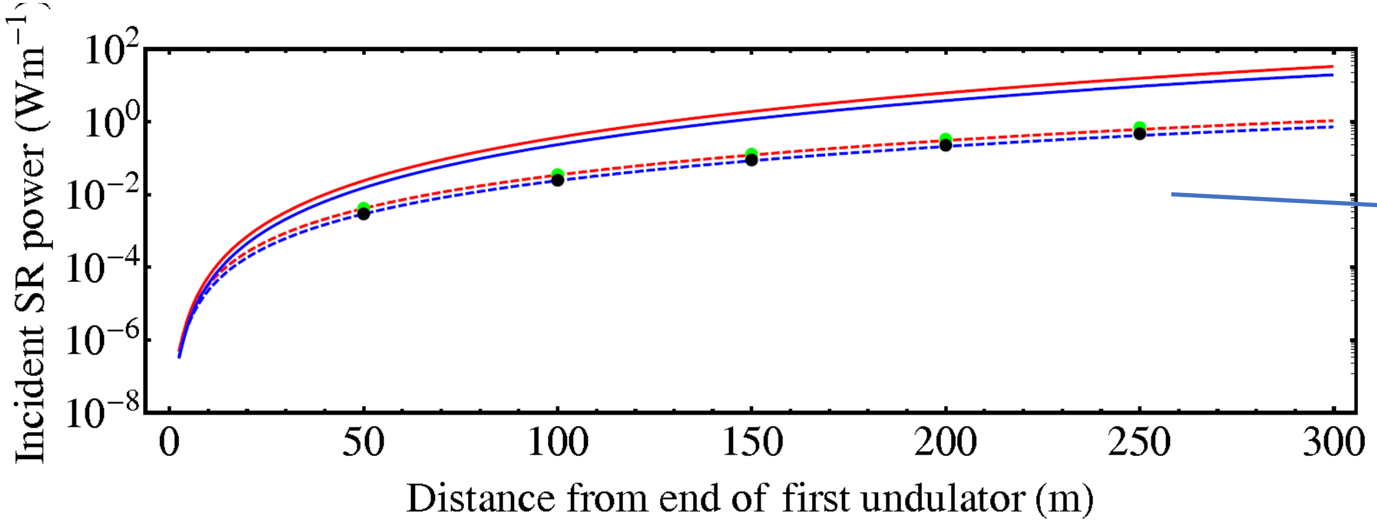
Test: Power deposited in the undulator vessel



- Dashed lines: single undulator piece
- Solid lines: whole undulator scheme
- Blue: RDR parameters
- Red: BCD parameters

Current calculations

In good agreement



Duncan J Scott's calculations

Undulator mask (consisting of collimators with aperture of 4.4 mm) for preventing heating of the vessel due to photon absorption.

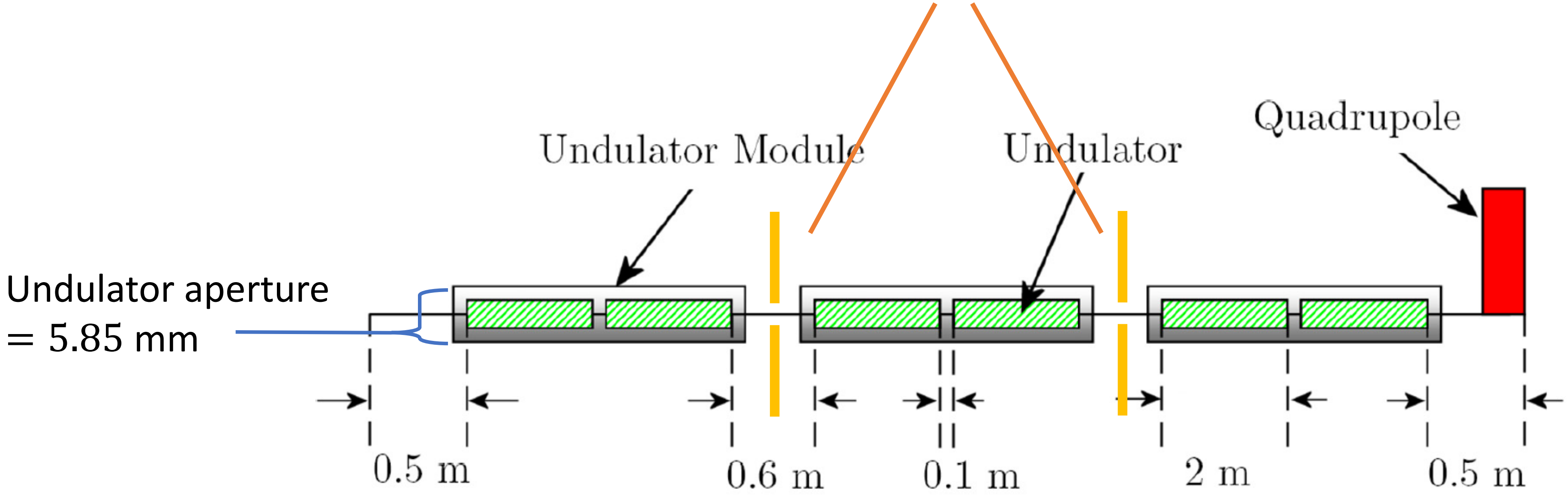
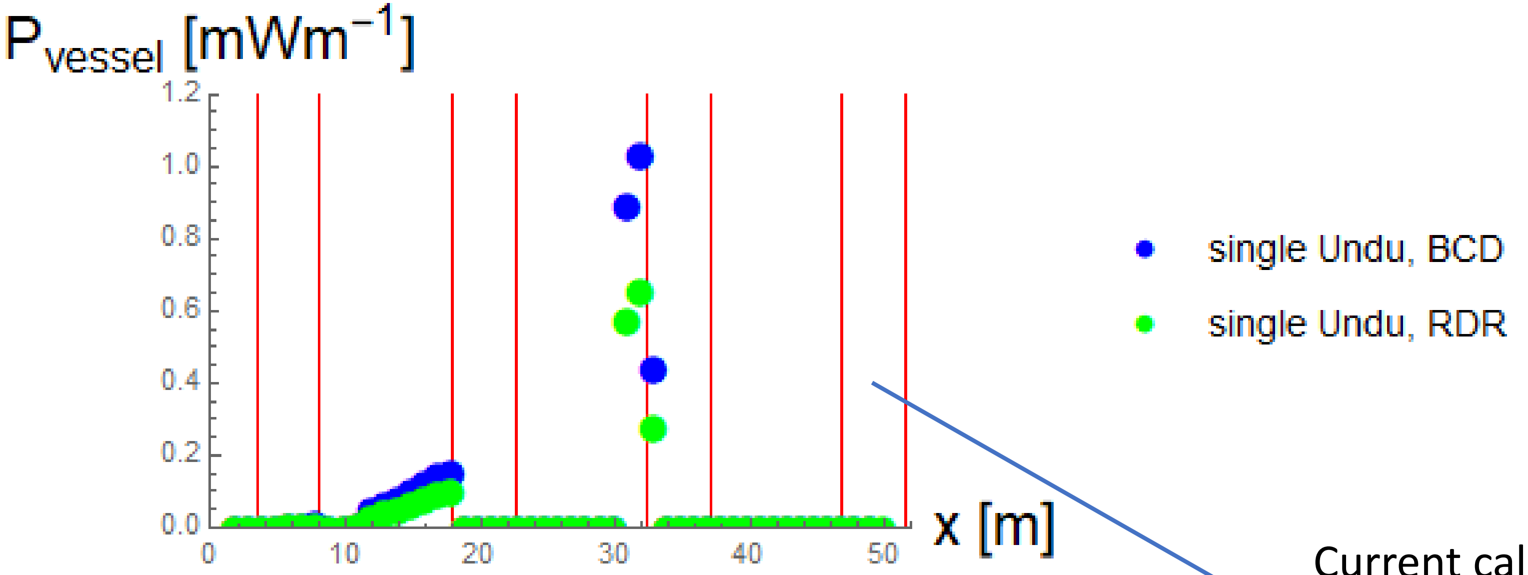


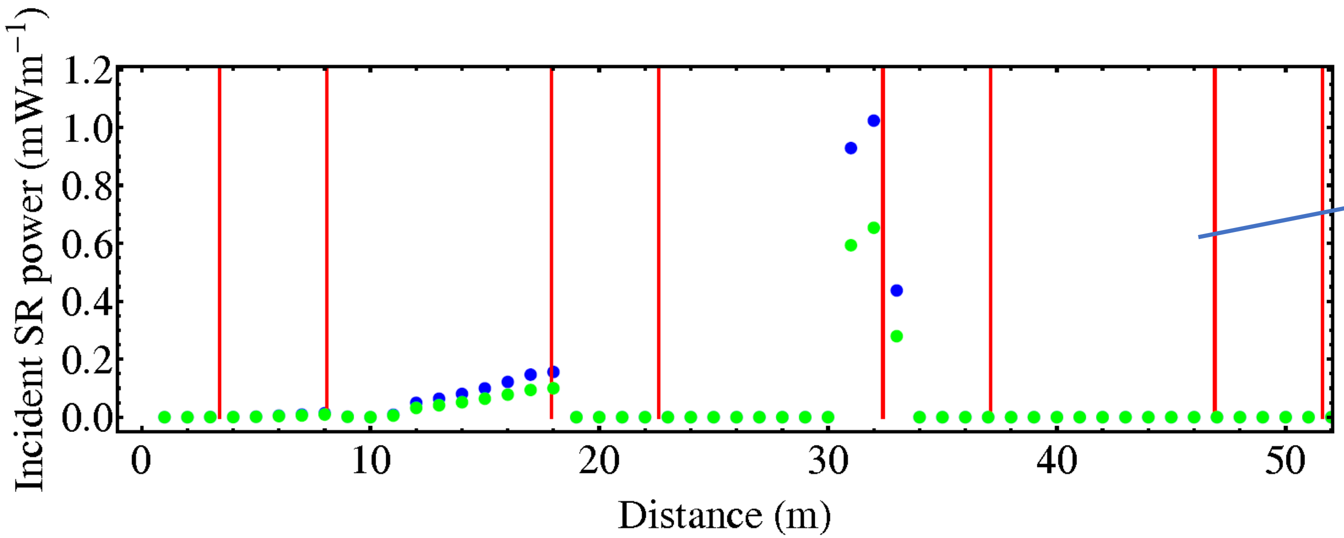
Figure 5.2: Schematic layout of a half-cell of the undulator line for incident SR power calculation.

The limit of maximal absorped power is 1 Wm^{-1} (according to Duncan J Scott, who in turn names the source to be private communication with T Bradshaw)



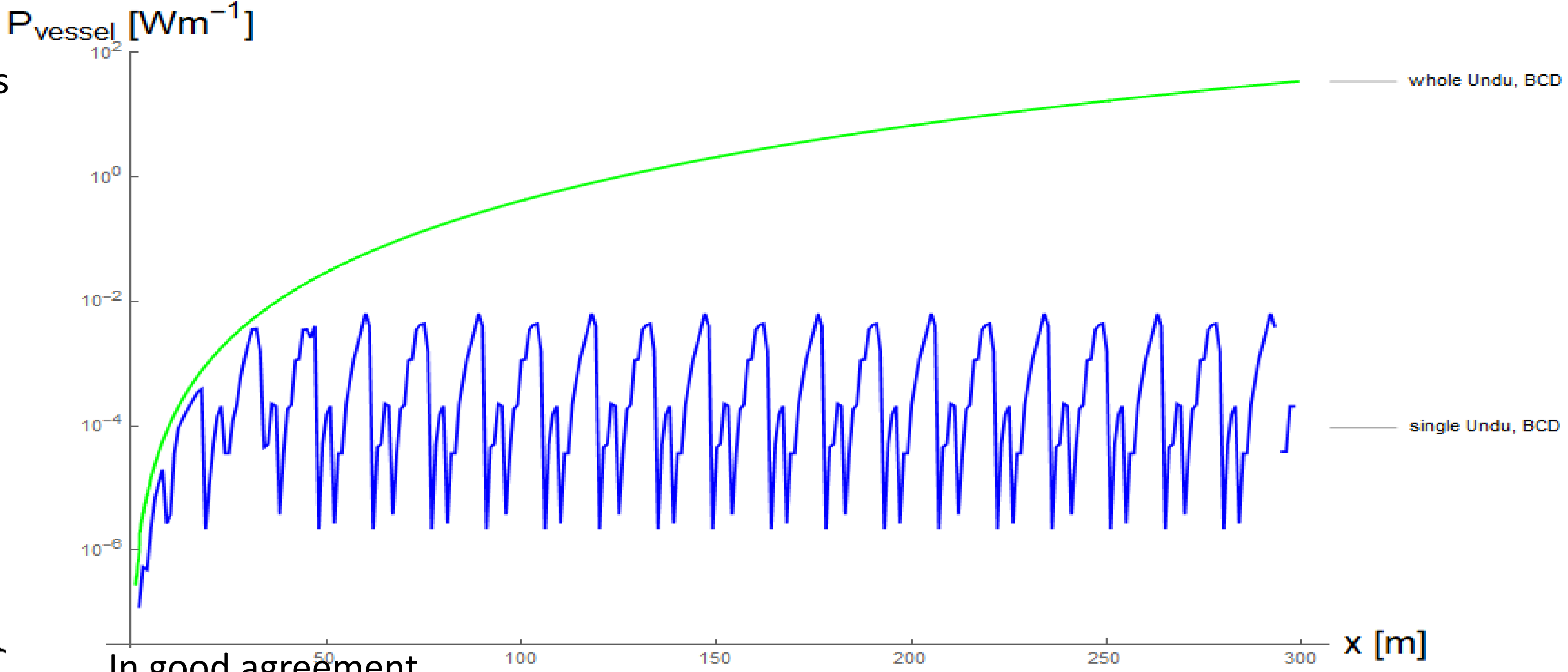
Current calculations

In good agreement



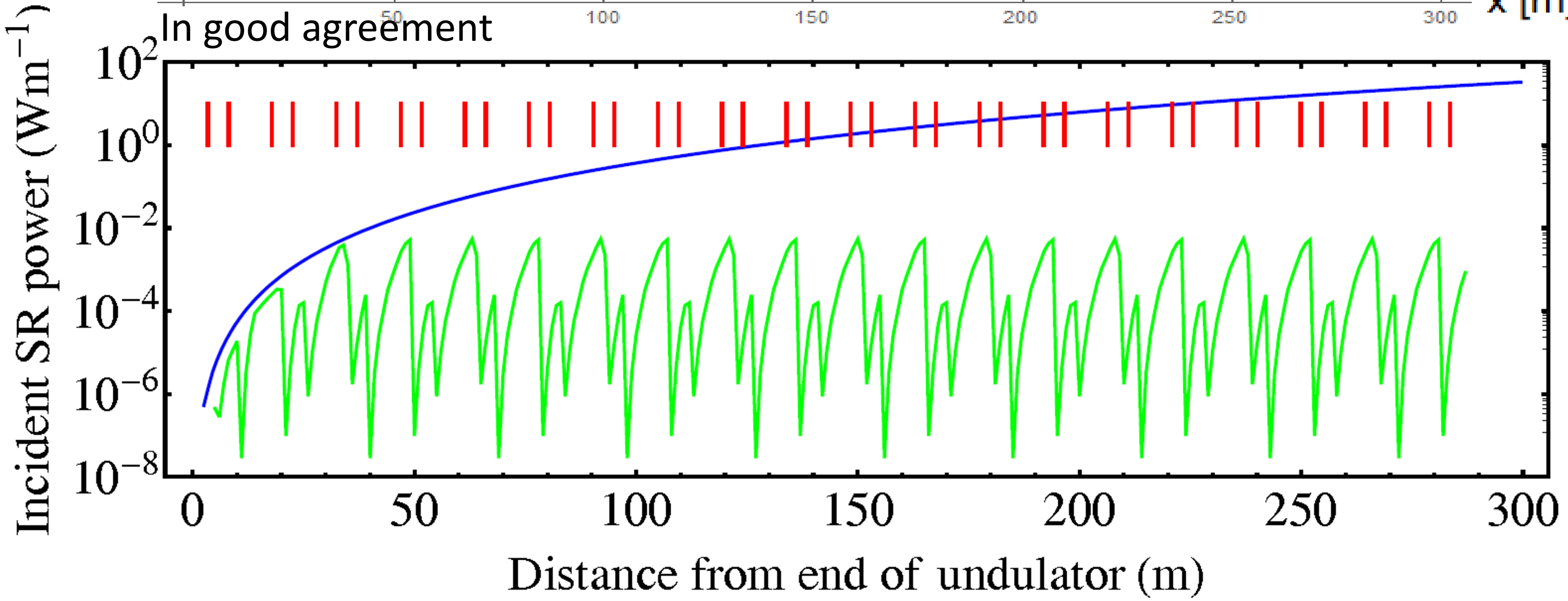
Duncan J Scott‘s calculations

Current calculations

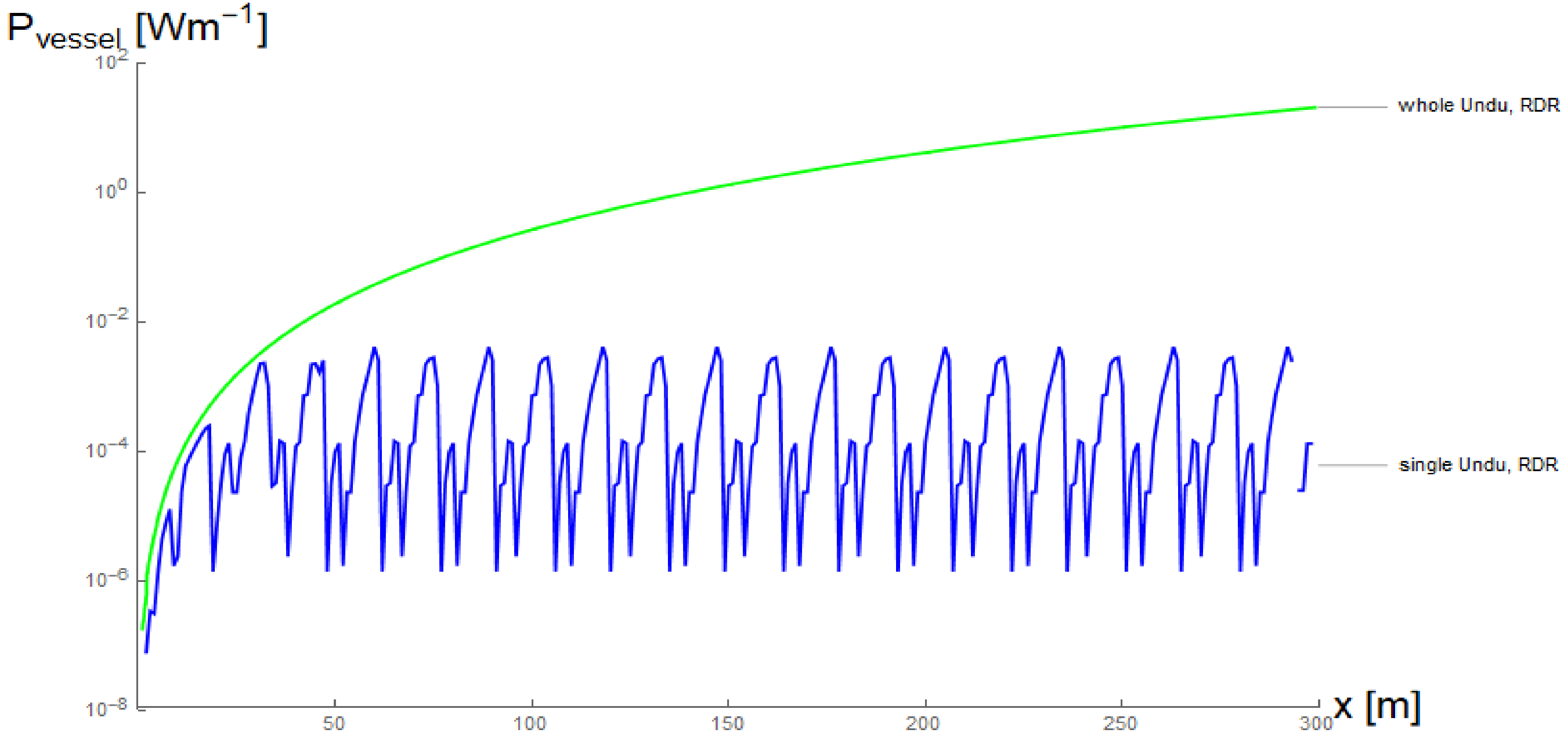


Peridocity and peak values are in good agreement; Disagreement in dip values and local shape of the graph

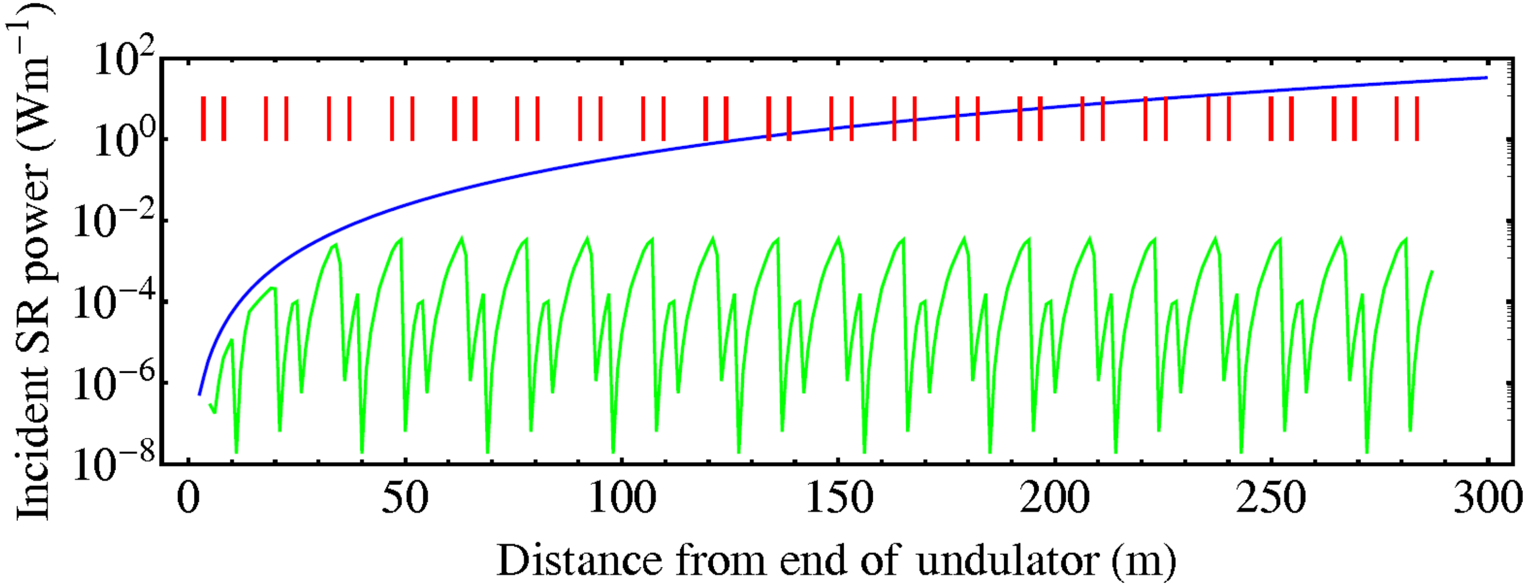
In good agreement



Duncan J Scott‘s calculations



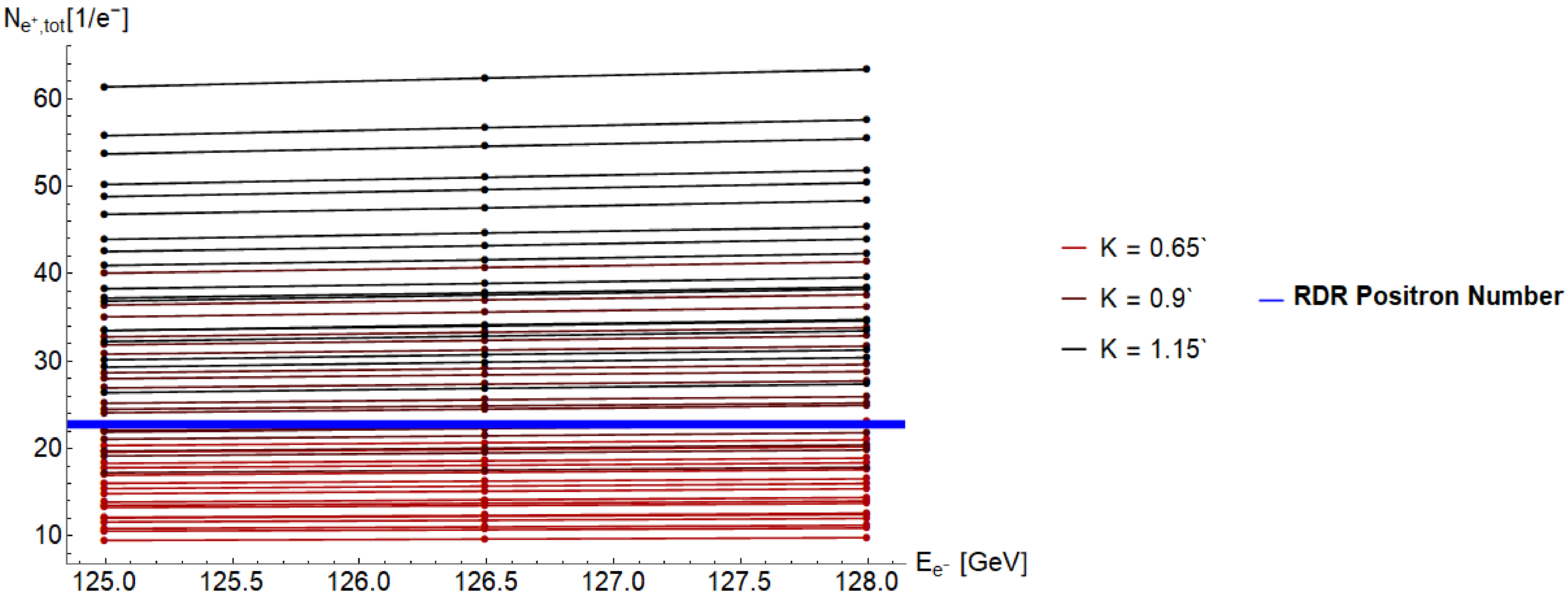
P_vessel [Wm^-1]
whole Undu, RDR
single Undu, RDR
x [m]



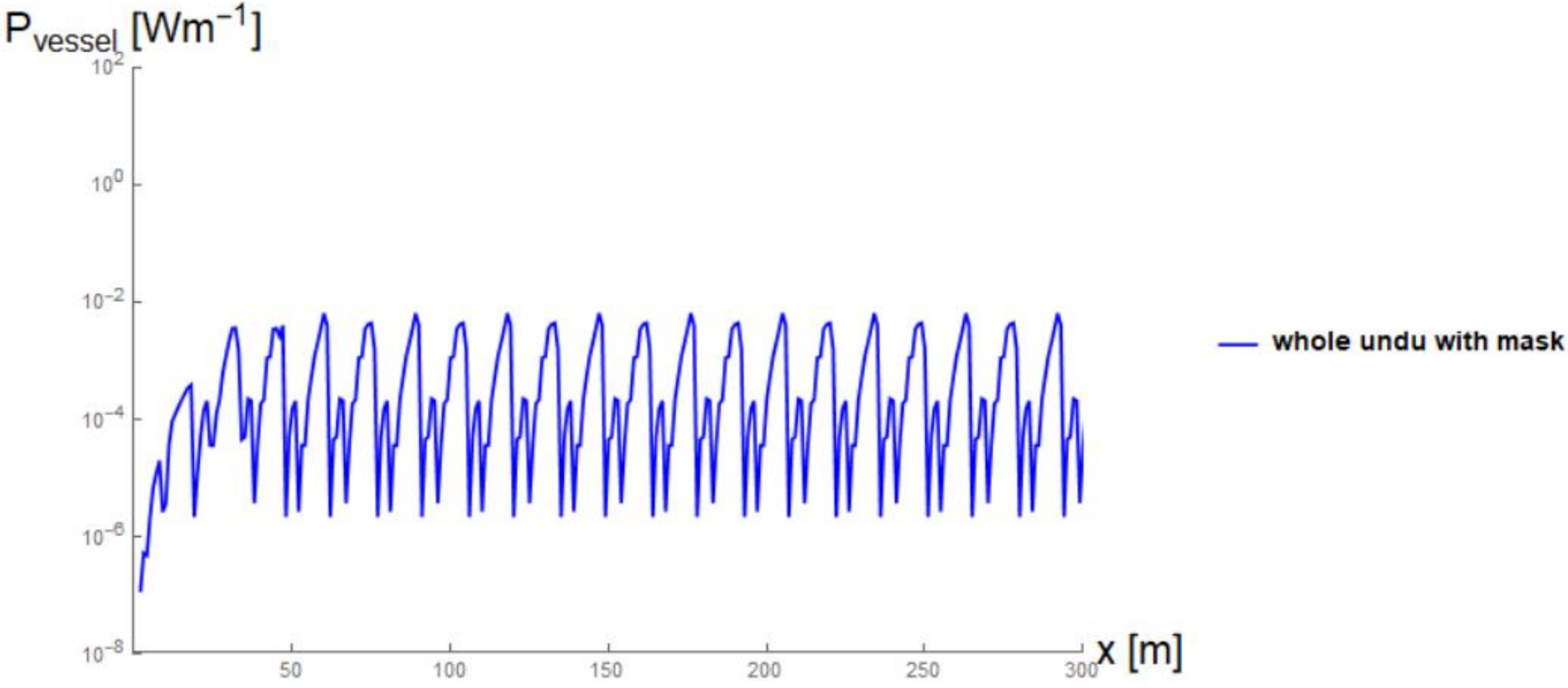
Incident SR power (Wm^-1)
Distance from end of undulator (m)

Examined parameter combination for the positron number

$K = 0.65, 0.9, 1.15, \quad \lambda_u = 8.5, 10, 11.5\ \text{mm}, \quad l_u = 1.75, 2\ \text{m}, \quad N_{hcell} = 18, 20, 22$



K [1]	lambda [mm]	l_u [m]	N_hcell [1]
1.15	8.5	2	22
1.15	8.5	2	20
1.15	8.5	1.75	22
1.15	8.5	2	18
1.15	8.5	1.75	20
1.15	10	2	22
1.15	8.5	1.75	18
1.15	10	2	20
1.15	10	1.75	22
0.9	8.5	2	22
1.15	10	2	18
1.15	10	1.75	20
1.15	11.5	2	22
0.9	8.5	2	20
0.9	8.5	1.75	22
1.15	11.5	2	20
0.9	8.5	2	18
1.15	11.5	1.75	22
0.9	8.5	1.75	20
0.9	10	2	22
1.15	11.5	2	18
1.15	11.5	1.75	20
0.9	8.5	1.75	18
0.9	10	2	20
0.9	10	1.75	22
1.15	11.5	1.75	18
0.9	10	2	18
0.9	10	1.75	20
0.9	11.5	2	22



$K = 1.15, \quad \lambda = 8.5\ \mathrm{mm}, \quad l_u = 2\mathrm{m}, \quad N_{hcell} = 22 \sim 264\ \mathrm{m}$ magnetic length

Possible future improvements

- Drop a single or multiple approximations ($\gamma \gg 1$, $|\theta| \ll 1$, $N_u \gtrsim 100$, $K \lesssim 1$, $R \gg \lambda_\gamma$, $V_{e^-} \to 0$, $\sin^2(N\pi y)/y^2 \to \mathrm{N}\pi\delta(y)$, etc.)
- Correcting possible flaws in the undulator mask considerations
- For N_{e^+}: Numerical integration over a solid angle, that only covers the target instead of the full $\theta = 0 - \pi$

$$\frac{dW}{d\omega} = \int \frac{dI(\omega)}{d\Omega} d\Omega \approx \frac{\mathrm{N_u e^2} K^2 r}{\epsilon_0 c} \sum_{n=1}^{\infty} n^2 \left[J_n'^2(y_n) + \left(\frac{\alpha_n}{K} - \frac{n}{y_n} \right)^2 J_n^2(y_n) \right] H(\alpha_n^2)$$

- Examining more intermediate parameter values between the upper and lower limits
- Adding more criteria for the optimazation besides lower limit for N_{e^+} and upper limit for P_{vessel}

Thank you for your attention