Statistics in Polarized Positrons Sources

Twelve years after PosiPol 2006

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Outline

- Preliminaries
- Statistical issues in polarized positron production
  - Recoils affect electrons generating gammas
  - Fluctuation of positron energy losses in the conversion target
- Non–Gaussian statistics for those processes
- Theoretical study and practical yield

When it began

First PosiPols (K. Mönig, L. Rinolfi, J. Urakawa, . . .) concentrated mainly around Compton–based sources

- A key problem – the electron beam kinetics –
  - big recoils: up to a few percents of energy goes away with a gamma
  - stochastic (quantum) nature of recoils
- The diffusion coefficient in Fokker–Planck (Belyaev–Budker for storage rings) kinetic equation ?
  - $\overline{\omega^2} = 7/5\,\overline{\omega}^2$ or smaller $\overline{(\omega - \overline{\omega})^2} = 2/5\,\overline{\omega}^2$?
  - simulations had reduced optimism: $\overline{\omega^2}$ (Bulyak, Gladkikh, Omori, Rinolfi, Skomorokhov, Urakawa, Zimmermann)
- Another challenge – positron's polarization degree
  - gammas polarization correlates with its random energy/angle. Preselection: collimation of gammas
  - positrons polarization correlates with its random energy, which decreases while traversing the conversion target

Statistical specifics of positron sources

Non Gaussian statistics in gamma generation and positron production

gamma source

- Highly populated electron bunch, up to 10^{10} (perfect ensemble statistics)
- Short period of radiation, small ratio gammas/electron, < 300 (time averaging insufficient)

positron converter

- Logarithmic dependence on the gammas energy
- Ionization losses independent of a positron energy
- $\Rightarrow$ efficiency of conversion directly proportional to the gammas energy

Problem setup: quantum losses

Electrons/positrons lost their energy due to recoils:
Electrons: emission of gammas
Positrons: ionization losses in the converter
Goal: To determine the final spectrum of electrons/positrons

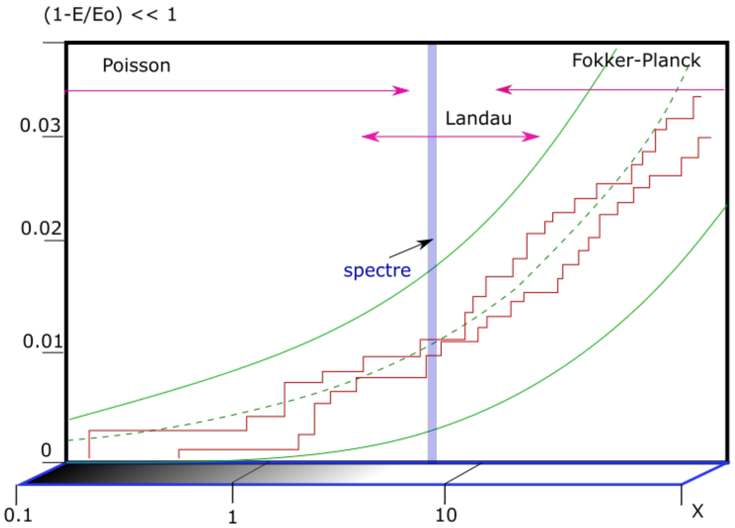
Model

- Independent identically distributed (i.i.d.) recoils
- Compact support of the recoil spectrum $w(\omega)$, $0 < \omega_{min} \leq \omega \leq \omega_{max} < \infty$
- Normalised spectrum: $\int_{-\infty}^{\infty} w(\omega)\, d\omega = 1$
- Small magnitude of a recoil, $\omega_{max} \ll \gamma_{e/p}$

Simplifying suggestion: the recoil spectrum independent of the particle energy

Scheme of the process

“Subordinate to compound Poisson”



Rigorous Solutions for Arbitrary Recoil Spectra

Characteristic functions, Bulyak, Shul'ga (2016, 2017)

evolution of spectrum

$$\hat{f} = \hat{f}_0 \exp[x(\check{w} - 1)]$$

straggling function

$$\hat{S}_x = \hat{w} e^{x(\hat{w}-1)}$$

moments of spectrum

$$\overline{\gamma}(x) = \overline{\gamma_0} - x\,\overline{\omega}$$
$$\mathrm{Var}[\gamma](x) = \overline{(\gamma - \overline{\gamma})^2} = \mathrm{Var}[\gamma_0] + x\,\overline{\omega^2}$$
$$\ldots$$

SF moments

$$\overline{\epsilon} = (1 + x)\,\overline{\omega}$$
$$\mathrm{Var}[\epsilon] = (1 + x)\,\overline{\omega^2} - \overline{\omega}^2$$
$$\ldots$$

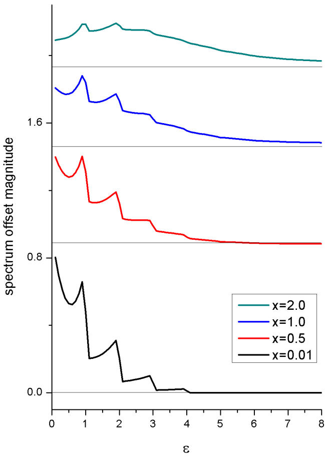
$\hat{g}$ is the Fourier transform, $\check{g}$ the inverse Fourier transform.

Density of energy losses distribution defined by

- recoil spectrum $w(\omega)$
- average number of recoils $x = \int_0^z P(z')\,dz' / \overline{\omega}$

Analysis of straggling function

Central Limit Theorem: if $x \to +\infty$ distribution $\to$ Gaussian for i.i.d.



SF in helical undulator, $K = 1$

the CLT does not hold, why?

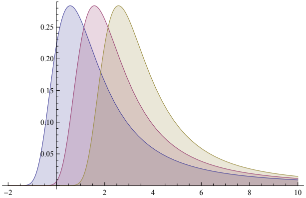
- moderate number of events: $x \leq 10^4 \ll \infty$
- sensitivity to the initial conditions
- longer relaxation time, $x_{\mathrm{relax}} > x$

Problem: intermediate straggling function evolution $1 < x \ll x_{\mathrm{Gauss}}$

Analysis: BS $\Leftrightarrow$ α-stable

Goals: stability parameter, α, and scale c (half width at 1/e height)

Landau distribution, $\alpha = 1,\ \beta = 1.$



$c = 1,\ \mu = 1, 2, 3$

CF of α-stable distribution

$$\hat{\phi}(s) = \exp\left\{ \mathrm{i} s\mu - |cs|^{\alpha}\left[1 - \mathrm{i}\beta\,\mathrm{sgn}(s)\Phi\right]\right\}$$

$$\Phi = \begin{cases} \tan\left(\frac{\pi\alpha}{2}\right), & \alpha \neq 1, \\ -\frac{2}{\pi}\log|s|, & \alpha = 1, \end{cases}$$

$\alpha \in (0, 2]$ stability parameter

$\beta \in [-1, 1]$ skewness parameter

$c \in (0, \infty)$ scale parameter

$\mu \in (-\infty, \infty)$ location parameter

CF of BS distribution

$$\hat{S}_x(s) = \hat{w}(s)\exp\left[x(\hat{w}(s) - 1)\right]$$

Physical reasoning

$\alpha \in (1, 2]$ $= 2$ Gaussian $x \to \infty$

$c \sim \sqrt{x\overline{\omega^2}}$ FP $x \to \infty$

$\mu = x\overline{\omega}$ energy conservation

Analysis

BS is of nonlinear logarithmic dependence of s. "Linearising" it in s_*, the root of

$$\Re[x\,(1-\hat{w}(s_*))] = 1 = |\pi c s_*|^{\alpha}\ ,$$

we deduce

$$\alpha = \left.\frac{sD_s\hat{w}}{1-\hat{w}}\right|_{s=s_*}\ ,$$

where $D_s\cdot = \partial\cdot/\partial s$
Scaling parameter

$$c(x) = [x m_\alpha[w]]^{1/\alpha} = \frac{1}{\pi s_*}$$

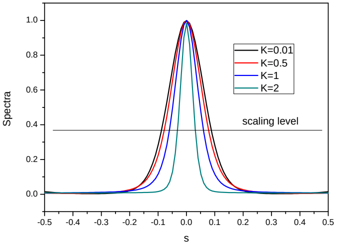
with α–moment of the recoil

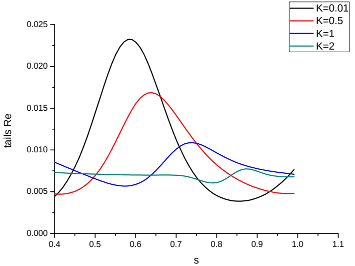
$$m_\alpha[w] = \int \omega^\alpha w(\omega)\,\mathrm{d}\omega\ , \qquad \omega > 0$$

Validation at limits: $\alpha \to 2$ when $x \to \infty$ (CLT); $\alpha = 1$ for $w(\omega) \sim 1/\omega^2$ (L.D.Landau, 1944)

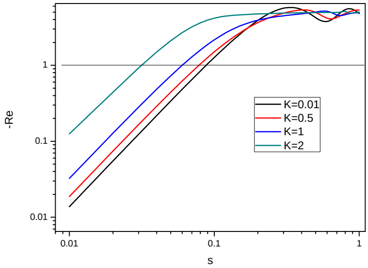
Illustrations

Characteristic function for undulator's straggling, $X = 5$





Minus real part of characteristic exponent

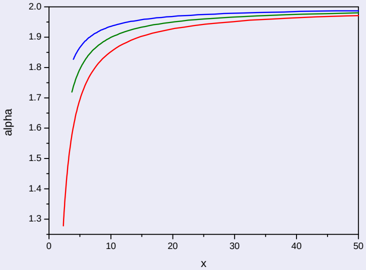


Procedure

Logarithmic derivative at $s = s_*$

Undulators Stability Parameter

$K = 0.01, 0.3, 1$



when K increases

- later the Gaussian distribution attracted
- thicker the tail
- faster the scaling parameter increases

Applications to PosiPol

- Ionization losses of positrons
  - width of spectrum proportional to the target thickness
  - yield proportional to the initial positron energy
  - the undulator with smaller $K \approx 0.5$ instead of $K \approx 1$ produces the same positrons flux at much smaller deterioration of the electron beam (Bulyak, Shul'ga, 2015)
- Compton gamma sources
  - $\alpha < 2$ distributions inherent heavy tail dramatically reduces the beam lifetime in Compton rings
  - the effect can be substantially mitigated by the asymmetric laser cooling method (proposed and studied by Bulyak, Urakawa, Zimmermann)

Backup slide. Asymmetric cooling in Compton rings.

E. Bulyak, J. Urakawa, F. Zimmermann

Storage rings specifics

- Losses of energy compensated by RF voltage, $x \to \infty$.
- Oscillations of energy (synchrotron oscillations).
- Dependence of recoil magnitude on the particle energy $\to$ cooling.
- Balance between diffusion and cooling $\to$ steady distribution.

Fast cooling: correlation between the particle energy and laser position

- More intensive cooling $\to$ smaller the bunch length
- Mitigation the ‘thick tail’ in the vicinity of the separatrix lower branch

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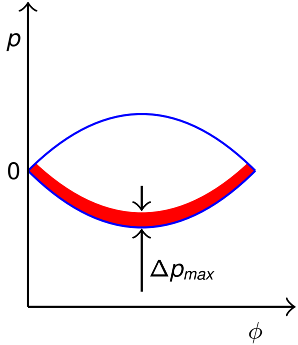
Where tails matter: Quantum lifetime in Compton rings

longitudinal phase space



- electrons escape the separatrix downward, in the 'tail direction'
  separatrix cuts out the tail
- rate of losses $\propto$ bunch density $\times$ laser density
- red band width $\propto$ tail length $\times$ synchrotron period

We proposed to mitigate quantum losses via *the asymmetric cooling*

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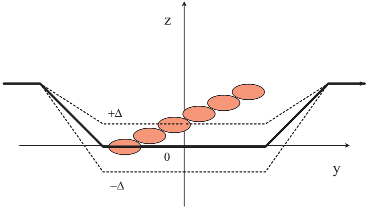
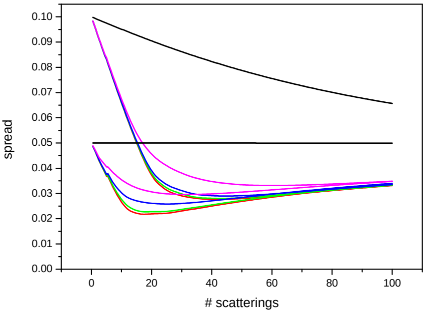
Asymmetric (Fast) Cooling. E.Bulyak, J.Urakawa, F.Zimmermann 2011–2013



Model setup

Laser radiation field exists at $z \geq 0$



Spread vs. # scatterings