Statistics and Propagation Modeling of Atmospheric Lightning

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Motivation

• Lightning Protection is Imperfect
  – Buildings are at risk of being struck

• Investigation of these risks
  – Characterize risks
  – Establish bounds on potentially problematic lightning

• Related Work
  – Investigate protection and mitigation of problematic lightning

Kevin Ambrose (Weatherbook.com)
Propagation Models

• **First Principles**
  - Piolet Streamer
    • Meek ‘59
  - Bipolar Piolet with Space Leader
    • Rakotonandrasana et al. ‘08

• **Stochastic Models**
  - Random Walk Model
    • Hutzler and Hutzler-Barre ’78
  - Electrostatic Components
    • Niemeyer in ‘87
  - Charge Modelling and Branching
    • Shi et al. 2014

Gas Diffusion Models

- A stepped leader random walk model

\[ e = \frac{E}{U(R)} = \eta_0 \frac{E_0}{|E_0|} + \frac{1}{v^2} \left( \frac{r'}{l} \right)^2 \frac{r'}{|r'|} \]

**Figure 1.** (a) The local coordinate system adapted to the Laplacian background field \( E_0 \). \( E_0 \) defines the axis of a polar coordinate system \( (l, \theta, \phi) \). The tangential plane \( T \) of the field line defines the reference for the azimuth \( \phi \). (b) A leader propagation step \( l \) is decomposed into an advancement \( l \) along the field direction \( E_0 \) and a random deviation \( \Delta r \) on a surface \( S \) perpendicular to \( E_0 \).

**Figure 4.** (a) A simplified calculation of the local field distribution at the leader tip: full circle, equivalent charged sphere representing the leader tip space charge; dotted circle, streamer propagation limit. (b) The normalised voltage drop \( \Delta u \) along the streamer trajectory as a function of the angle \( \theta \).
Attachment

- Intensified electric field at lighting rod is induced
- This gives rise to an upward leader
- When the two are intercepted the stroke begins

The Lightning Stroke- II by C. F. Wagner and A. R. Hileman.
Rolling Sphere Model


Resulting Stochastic Model

- **Monte Carlo Random Walk**
  - Propagation starts to fixed height ~200 m
  - (Uniform) Random starting position (X,Y)
  - Generate Series of Random 3D Steps
    - Uniform Polar Angle (X,Y Plane)
    - Gaussian Angle from Z
  - Propagate until within striking distance
  - Strike to nearest object
  - (Record Even & Repeat)
Model Capabilities

• Monte Carlo methods of arriving at empirical probability of lightning protection failures.
  – Tolerant of cm-sized features in 100’m contexts.

• Inherently Parallelizable:
  – 100 000 runs in minutes on circa-2015 consumer desktop computers.

• Support for arbitrary CAD geometries
  – Geometry is specified by external stereolithography (STL) files.
Arriving at Total Probabilities

\[ P_{\text{PDF total}}(SD) = P(S|SD) \cdot PDF(SD) \]

Brooks et al. Investigation of Lighting Attachment Risks to Small Structures Associated with Electrogeometric Model (EGM) [unpublished manuscript]. Lubbock: Texas Tech University; 2019
Consequences of Probability Distribution

Global Current \( P = 0.0048 \)

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Expectation (E)

- **Probabilities**
  - $\text{PDF}_{\text{strike}}(Ip) \rightarrow \text{PDF}_{\text{strike}}(SD)$
  - $P_{\text{Flash Rate}}(SD) = (\text{Flash Rate})\text{PDF}_{\text{strike}}(SD)$
    - $[\text{Flash Rate}] = [\frac{\text{Flashes}}{(\text{Area} \cdot \text{Time})}]$
  - $P_{\text{total}} = \int P_{\text{strike}}(SD')P(S = SD')$

- **Expectations**
  - $E_{rate} = \int P_{\text{Flash Rate}}(SD')P(S = SD')$
    - $= (\text{Flash Rate})\int \text{PDF}_{\text{strike}}(SD')P(S = SD')$
    - $= P_{\text{total}}(Total \text{ Flash Rate})$
Collection Area

• NFPA-780
  – Rectangular Building:
    • $LW + 6H(L + W) + 9\pi H^2$
  – Estimate of Building’s Presence for use with flash density

• E.g. Unprotected Home
  – About 2700 $ft^2$ (251 $m^2$)
  – Taken as $(52 \times 52 \times 15) ft^3$
    or $(15.85 \times 15.85 \times 4.575) m^3$
  – Flash density $5/(km^2 \cdot yr)$
Count of Vaisala NLDN lightning strikes within 0.1 degree grid cells within the United States. Political boundaries based on 2017 United States Census Bureau TIGER data.
Reference Geometry

- **Parametric Sweep**
  - Building Height
  - Footprint Area
  - Inset Protection
  - Aspect Ratio

Metric for comparing graphs:

\[ P_{RMS} = \sqrt{\frac{1}{N} \sum x_i^2} \]

\[ E_{RMS} = \sqrt{\frac{(x_i - p_i)^2}{n}} \]

\[ \sigma_s = \sqrt{\frac{(x_i - p_i)^2}{n - 1}} \]

\[ E_{RMS, \ norm} = \frac{E_{RMS}}{P_{RMS}} \]

\[ \sigma_s, \ norm = \frac{\sigma_s}{P_{RMS}} \]

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Building Height

Height was Varied as $X \text{ [m]}$
Catenary Wire Height Varied as $X + 2.4$

Reference
(100 x 50) m
Building Height: 13.1 m
Catenary Height: 15.5 m
Short buildings show weak dependence on height:
• Taller buildings are at increased risk of being struck.
• For less the buildings measured effect is very slight.
Variation of Footprint Area

Risk generally increased with increasing footprint area.
- Larger buildings are at increased risk of being struck.
- The effect is much more pronounced for small structures.
Building idealized to capture **only** edge effects.
Unprotected protrusions are at significant risk.

- P is much smaller than previous values as protection is idealized.
- Inset protection of 0.15 m or less incurs limited additional risk.

2. Brooks et al. Investigation of Lighting Attachment Risks to Small Structures Associated with Electrogeometric Model (EGM) [unpublished manuscript]. Lubbock: Texas Tech University; 2019
Building Volume held constant in effort to avoid collection volume effects, if any.

Aspect taken as: Length / Width
Variation of Aspect Ratio

Extreme aspect ratio buildings were much better protected.
- This is believed to be a consequence of competition with ground.
- The reference has an aspect of 0.5 and shows good agreement with the aspect 2 case.

2. Brooks et al. Investigation of Lighting Attachment Risks to Small Structures Associated with Electrogeometric Model (EGM) [unpublished manuscript]. Lubbock: Texas Tech University; 2019
Conclusions

• A stochastic model of stepped leader propagation in downward negative lightning with inclusion of final jump process was developed.
  – Large volumes of simulated lightning strikes to complex geometries can be evaluated quickly.

• Large structures were found to produce worst case error.
  – The simulated structure was found to be vulnerable to less than 0.5% of lighting strikes.
  – Rates were found to be weakly sensitive to height.
  – Aspect ratio was found to have pronounced impact.
  – Protection inset was found to be substantial.