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Pion condensation and QCD phase diagram at finite isospin density

PHAROS WG1+WG2 meeting
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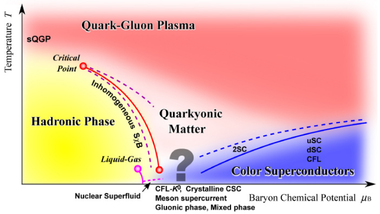
2. References: Phys. Rev. **D** 97 076005 (2018)
   Phys. Rev. **D** 95, 036017 (2017)
   e-Print: arXiv:1805.08599, arXiv:1807.08951

1. EoS relevant for early universe, heavy-ion collisions and compact stars.
2. Quark-gluon plasma at high temperature
3. CFL phase at large baryon chemical potential.



Fukushima et al. Rept. Prog. Phys. 74 (2011) 014001.

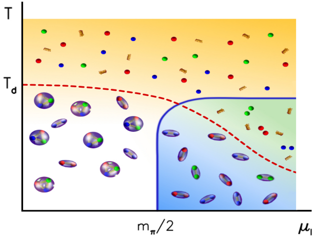
4. Only few exact results known (QGP, CFL).
5. Sign problem at finite μ_B. Monte Carlo simulations difficult.
6. No sign problem at finite μ_I and $\mu_B = 0$. Lattice simulations possible. [1]

[1] Kogut and Sinclair (2002), Brandt, Endrodi, and Schmalzbauer (2018).

Pion condensation and QCD phase diagram at finite isospin density

1. Isospin chemical potential μ_I introduces an imbalance between up and down-quarks $\mu_u = \mu + \mu_I$, $\mu_d = \mu - \mu_I$.

2. This talk [2]



B. B. Brandt, G. Endrodi, and S. Schmalzbauer, Phys.Rev. D **97**, 054514 (2018).

a) Phase diagram in $\mu - \mu_I$ plane. Inhomogeneous phases and competition with a pion condensate at $T = 0$?

b) Phase diagram in the $\mu_I - T$ plane. Chiral and deconfinement transitions?

c) Pion stars

[2] Son and Stephanov (2001).

1. Quark-meson model

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}\left[(\partial_\mu\sigma)(\partial^\mu\sigma)+(\partial_\mu\pi_3)(\partial^\mu\pi_3)\right]+(\partial_\mu+2i\mu_I\delta_\mu^0)\pi^+(\partial^\mu-2i\mu_I\delta_0^\mu)\pi^- \\ &-\frac{1}{2}m^2(\sigma^2+\pi_3^2+2\pi^+\pi^-)-\frac{\lambda}{24}(\sigma^2+\pi_3^2+2\pi^+\pi^-)^2 \\ &+h\sigma+\bar{\psi}\left[i\partial\!\!\!/+\mu_f\gamma^0-g(\sigma+i\gamma^5\boldsymbol{\tau}\cdot\boldsymbol{\pi})\right]\psi\end{aligned}$$

2. Chiral density wave and constant pion condensate

$$\sigma=\phi_0\cos(qz)\,,\qquad \pi_1=\pi_0\,,\qquad \pi_3=\phi_0\sin(qz)$$

3. Meson potential with $\Delta=g\phi_0$ and $\rho=g\pi_0$

$$V_0=\frac{1}{2}\frac{q^2}{g^2}\Delta^2+\frac{1}{2}\frac{m^2}{g^2}\Delta^2+\frac{1}{2}\frac{m^2-4\mu_I^2}{g^2}\rho^2+\frac{\lambda}{24g^4}\left(\Delta^2+\rho^2\right)^2-\frac{h}{g}\Delta\cos(qz)\delta_{q,0}$$

1. Quark energies

$$E_u^{\pm} = E(\pm q, -\mu_I)\,, E_d^{\pm} = E(\pm q, \mu_I)\,, E_{\bar{u}}^{\pm} = E(\pm q, \mu_I)\,, E_{\bar{d}}^{\pm} = E(\pm q, -\mu_I)\,,$$

$$E(q, \mu_I) = \left[\left(\sqrt{p_\perp^2 + \left(\sqrt{p_\parallel^2 + \Delta^2} + \frac{q}{2}\right)^2} + \mu_I\right)^2 + \rho^2\right]^{\frac{1}{2}}\,,$$

2. Integrate over fermions (regulator artefacts)

$$V_1 = -\frac{1}{2} N_c \int_p \left(E_u^{\pm} + E_d^{\pm} + E_{\bar{u}}^{\pm} + E_{\bar{d}}^{\pm}\right) + \text{medium contribution}$$

3. Model parameters are fixed using the on-shell renormalization scheme.

4. Isolate divergences

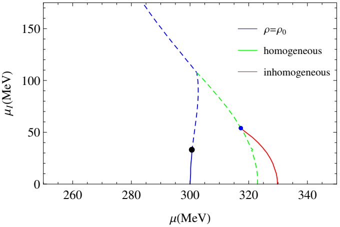
$$
\begin{aligned}
V_{\rm div} &= -4N_c \int_p \left[\sqrt{p^2+\Delta^2+\rho^2} + \frac{\mu_I^2\rho^2}{2(p^2+\Delta^2+\rho^2)^{\frac{3}{2}}} + \frac{3q^2\mu_I^2\rho^2(4\Delta^2+4p_\parallel^2-p_\perp^2-\rho^2)}{16(p^2+\Delta^2+\rho^2)^{\frac{7}{2}}} \right. \\
&\quad \left. + \frac{q^2(p_\perp^2+\rho^2)}{8(p^2+\Delta^2+\rho^2)^{\frac{3}{2}}} + \frac{q^4(p_\perp^2+\rho^2)(4\Delta^2+4p_\parallel^2-p_\perp^2-\rho^2)}{128(p^2+\Delta^2+\rho^2)^{\frac{7}{2}}} \right] \\
&= \frac{2N_c}{(4\pi)^2}\left(\frac{e^{\gamma_E}\Lambda^2}{\Delta^2+\rho^2}\right)^{\epsilon} \left\{ 2\left(\Delta^2+\rho^2\right)^2\Gamma(-2+\epsilon) + q^2\Delta^2\Gamma(\epsilon) - 4\mu_I^2\rho^2\Gamma(\epsilon) \right. \\
&\quad \left. -2q^2\mu_I^2\frac{\Delta^2\rho^2}{(\Delta^2+\rho^2)^2}\Gamma(2+\epsilon) - \frac{q^4}{12}\frac{\Delta^2}{(\Delta^2+\rho^2)^2}\left[(1-\epsilon)\Delta^2+2\rho^2\right]\Gamma(1+\epsilon) \right\} .
\end{aligned}
$$

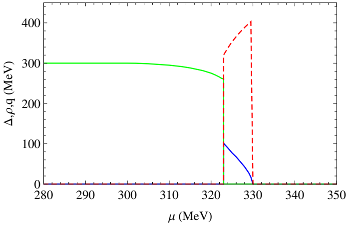
5. Remainder $V_{\rm fin}$ is finite and reads $V_{\rm fin} = V_1 - V_{\rm div}$.

6. Effective potential independent of q in the limit $\Delta \to 0$.

$$
\begin{aligned}
V_1 \quad = \quad & \frac{1}{2} f_\pi^2 q^2 \left\{ 1 - \frac{4m_q^2 N_c}{(4\pi)^2 f_\pi^2} \left[\log \frac{\Delta^2+\rho^2}{m_q^2} + H(m_\pi^2) \right] \right\} \frac{\Delta^2}{m_q^2} \\
& + \frac{3}{4} m_\pi^2 f_\pi^2 \left\{ 1 - \frac{4m_q^2 N_c}{(4\pi)^2 f_\pi^2} m_\pi^2 F'(m_\pi^2) \right\} \frac{\Delta^2 + \rho^2}{m_q^2} \\
& - \frac{1}{4} m_\sigma^2 f_\pi^2 \left\{ 1 + \frac{4m_q^2 N_c}{(4\pi)^2 f_\pi^2} \left[\left(1 - \frac{4m_q^2}{m_\sigma^2} \right) F(m_\sigma^2) + \frac{4m_q^2}{m_\sigma^2} - H(m_\pi^2) \right] \right\} \frac{\Delta^2 + \rho^2}{m_q^2} \\
& - 2\mu_I^2 f_\pi^2 \left\{ 1 - \frac{4m_q^2 N_c}{(4\pi)^2 f_\pi^2} \left[\log \frac{\Delta^2+\rho^2}{m_q^2} + H(m_\pi^2) \right] \right\} \frac{\rho^2}{m_q^2} \\
& + \frac{1}{8} m_\sigma^2 f_\pi^2 \left\{ 1 - \frac{4m_q^2 N_c}{(4\pi)^2 f_\pi^2} \left[\frac{4m_q^2}{m_\sigma^2} \left(\log \frac{\Delta^2+\rho^2}{m_q^2} - \frac{3}{2} \right) - G(m_\sigma^2) + H(m_\pi^2) \right] \right\} \frac{(\Delta^2 + \rho^2)^2}{m_q^4} \\
& - \frac{1}{8} m_\pi^2 f_\pi^2 \left[1 - \frac{4m_q^2 N_c}{(4\pi)^2 f_\pi^2} m_\pi^2 F'(m_\pi^2) \right] \frac{(\Delta^2 + \rho^2)^2}{m_q^4} - m_\pi^2 f_\pi^2 \left[1 - \frac{4m_q^2 N_c}{(4\pi)^2 f_\pi^2} m_\pi^2 F'(m_\pi^2) \right] \frac{\Delta \delta_{q,0}}{m_q} \\
& + V_{\rm fin} + N_c \int_p \left[(E_u^\pm - \mu)\theta(\mu - E_u^\pm) + (E_d^\pm - \mu)\theta(\mu - E_d^\pm) \right] .
\end{aligned}
$$

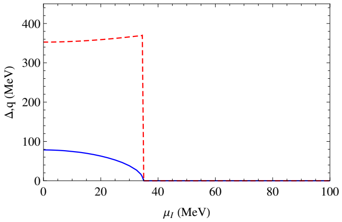
1. Phase diagram and condensates in the chiral limit.

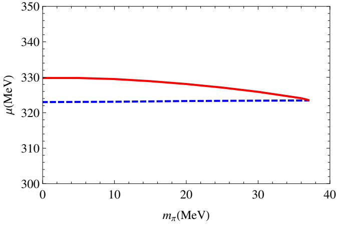




a) First - and second-order transitions with critical endpoints.

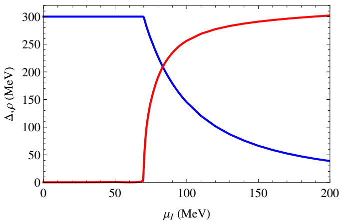
b) No coexistence of inhomogeneous chiral condensate and pion condensate.

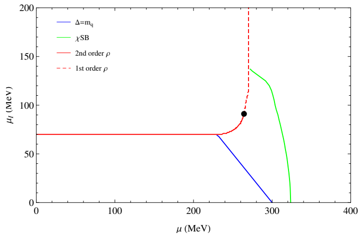




2. Chiral condensate and wave vector q as functions of μ_I and m_π-dependence of inhomogeneous condensate.

3. Rotation of condensates and homogeneous phase diagram at the physical point.





a) Critical m_π^c for existence of inhomogeneous condensate. (cf. Nickel PRD 2009)

b) Critical $\mu_I^c = \frac{1}{2} m_\pi$ at $T = 0$ and Silver Blaze property. [3]

[3] T. D. Cohen, Phys. Rev. Lett. **91**, 222001 (2003).

1. Wilson line

$$L(\mathbf{x}) \quad = \quad \mathcal{P}\exp\left[i\int_0^\beta d\tau A_4(\mathbf{x},\tau)\right] \ .$$

2. Polyakov loop order parameter for deconfinement

$$\Phi = \frac{1}{N_c}\langle \mathrm{Tr}L\rangle \ , \quad \bar{\Phi} = \frac{1}{N_c}\langle \mathrm{Tr}L^\dagger\rangle \ .$$

3. Polyakov gauge and coupling to quarks [4]

$$A_4 \quad = \quad t_3 A_4^3 + t_8 A_4^8 \ .$$

[4] K. Fukushima Phys.Lett. B **591**, 277 (2004)

4. Fermionic contribution to effective potential

$$\begin{aligned} V_T &= -2T \int \frac{d^3p}{(2\pi)^3} \Big\{ \mathrm{Tr}\log\left[1 + 3(\Phi + \bar{\Phi} e^{-\beta E_u}) e^{-\beta E_u} + e^{-3\beta E_u}\right] \\ & + \mathrm{Tr}\log\left[1 + 3(\bar{\Phi} + \Phi e^{-\beta E_{\bar{u}}}) e^{-\beta E_{\bar{u}}} + e^{-3\beta E_{\bar{u}}}\right] \\ & + \mathrm{Tr}\log\left[1 + 3(\Phi + \bar{\Phi} e^{-\beta E_d}) e^{-\beta E_d} + e^{-3\beta E_d}\right] \\ & + \mathrm{Tr}\log\left[1 + 3(\bar{\Phi} + \Phi e^{-\beta E_{\bar{d}}}) e^{-\beta E_{\bar{d}}} + e^{-3\beta E_{\bar{d}}}\right] \Big\} . \end{aligned}$$

5. $\Phi = \bar{\Phi} = 1$ one recovers the usual fermion contribution.

6. Glue potential [5]

$$\frac{\mathcal{U}}{T^4} = -\frac{1}{2}a\Phi\bar{\Phi} + b\log\left[1 - 6\Phi\bar{\Phi} + 4(\Phi^3 + \bar{\Phi}^3) - 3(\Phi\bar{\Phi})^2\right] ,$$

with

$$a = 3.51 - 2.47\left(\frac{T_0}{T}\right) + 15.2\left(\frac{T_0}{T}\right)^2 , \qquad b = -1.75\left(\frac{T_0}{T}\right)^3 .$$

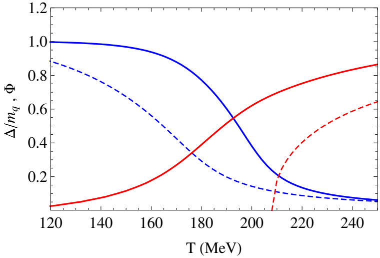
7. μ_I-dependent parameter [6]

$$T_0(N_f, \mu_I) = T_\tau e^{-1/(\alpha_0 b(\mu_I))} .$$

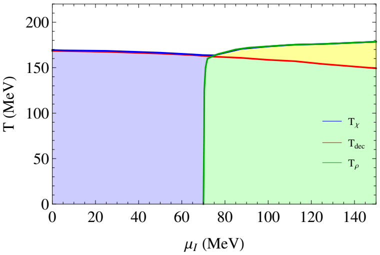
[5] S. Roessner, C. Ratti and W. Weise, Phys. Rev. D **75**, 034007 (2007).

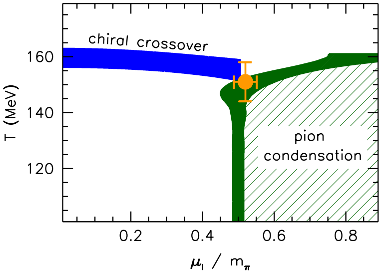
[6] B.-J. Schaefer, J. M. Pawlowski, and J. Wambach, Phys. Rev. D **76**, 074023 (2007).

8. Order parameters at vanishing μ and μ_I in the QM and PQM models.



1. Phase diagram[7]





1. BEC line always second order.
2. BEC line and chiral line merge.

[7] Endrodi B. Brandt, G. Endrodi, and S. Schmalzbauer Phys. Rev. D **97**, 054514 (2018)

Pion stars:

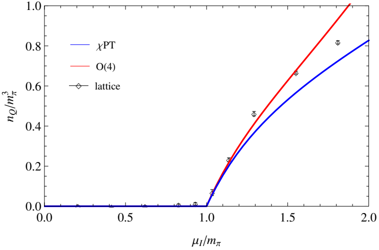
1. It has recently been suggested that pion stars form in the early universe. [8]
2. Can use EoS from the lattice to study these stars.
3. Use chiral perturbation theory or low-energy effective model, including the pions and the sigma.

$$
\begin{aligned}
\mathcal{L} &= \frac{1}{2}\left[(\partial_\mu \sigma)(\partial^\mu \sigma) + (\partial_\mu \pi_3)(\partial^\mu \pi_3)\right] + (\partial_\mu + 2i\mu_I \delta_\mu^0)\pi^+(\partial^\mu - 2i\mu_I \delta_0^\mu)\pi^- \\
&\quad -\frac{1}{2}m^2(\sigma^2 + \pi_3^2 + 2\pi^+\pi^-) - \frac{\lambda}{24}(\sigma^2 + \pi_3^2 + 2\pi^+\pi^-)^2 + h\sigma
\end{aligned}
$$

[8] Brandt et al, arXiv:1802.06685, JOA and P. Kneschke, arXiv:1807.08951.

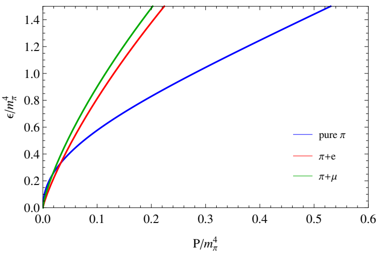
4. Electric charge density

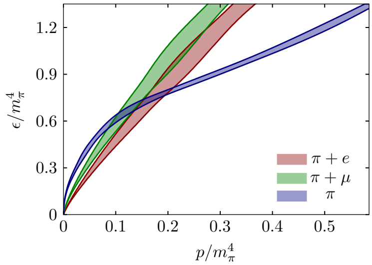
$$n_I = \mu_I f_\pi^2 \left[1 - \frac{m_\pi^4}{\mu_I^4}\right] .$$



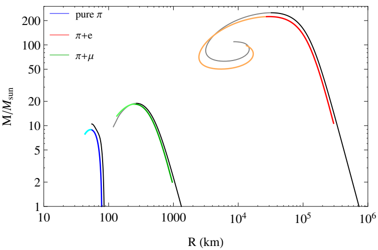
5. Equation of state

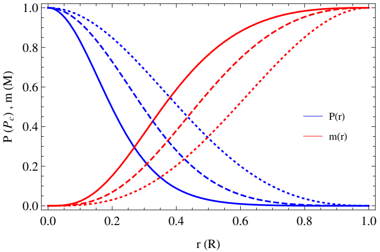
$$\frac{P}{\epsilon} = \frac{\mu_I^2 - m_\pi^2}{\mu_I^2 + 3m_\pi^2} \,.$$





6. Mass-radius relation and pressure/accumulated mass





Conclusions:

1. Rich phase diagrams. Onset of pion condensation at exactly $\mu_I^c = \frac{1}{2}m_\pi$.
2. No inhomogeneous chiral condensate for physical quark masses.
3. Good agreement between lattice simulations and model calculations.
   a) Second-order transition to a BEC state.
   b) BEC and chiral transition lines merge at large μ_I.
   c) Mass-radius relation for pion stars.
4. If pion stars exist, what are their properties?