Modelling pulsar glitches: the hydrodynamics of superfluid vortex avalanches in neutron stars

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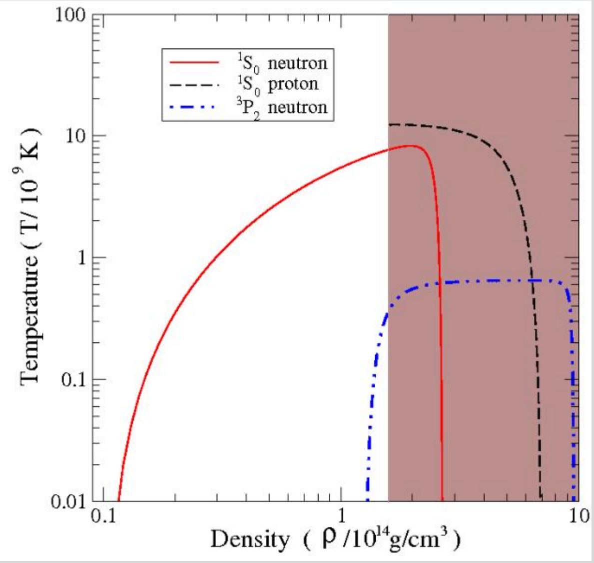
Questions:

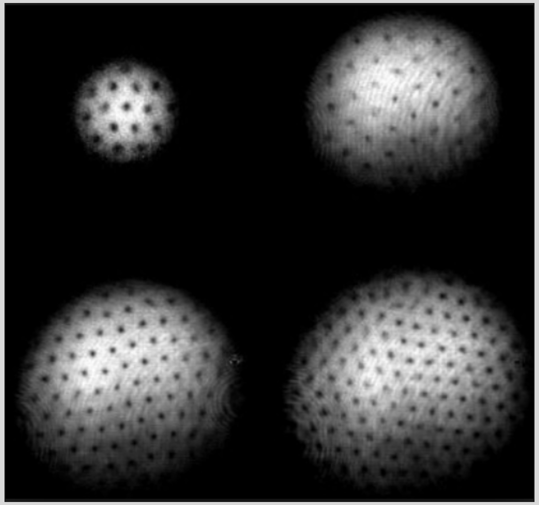
1) What are the mutual friction parameters in different regions of the star?

2) Can we develop a coarse grained hydrodynamical description of individual vortex motion?

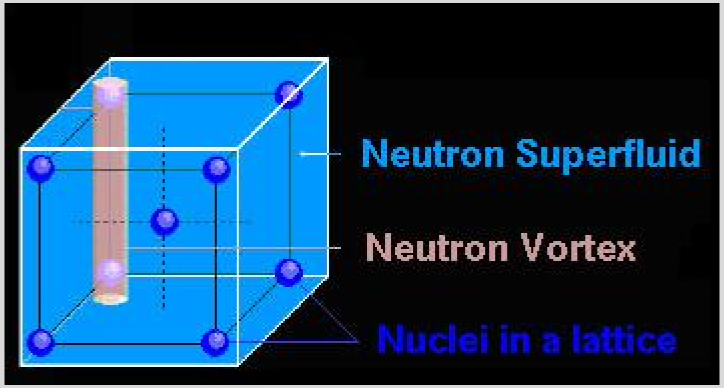
Basics of the superfluidity and pinning



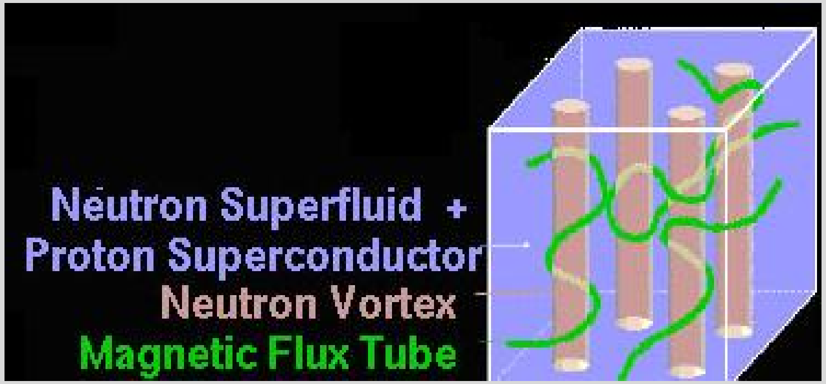
Example of calculated superfluid gaps for the neutron star [Haskell B., 2017].



Equilibrium structure of vortices in BECs



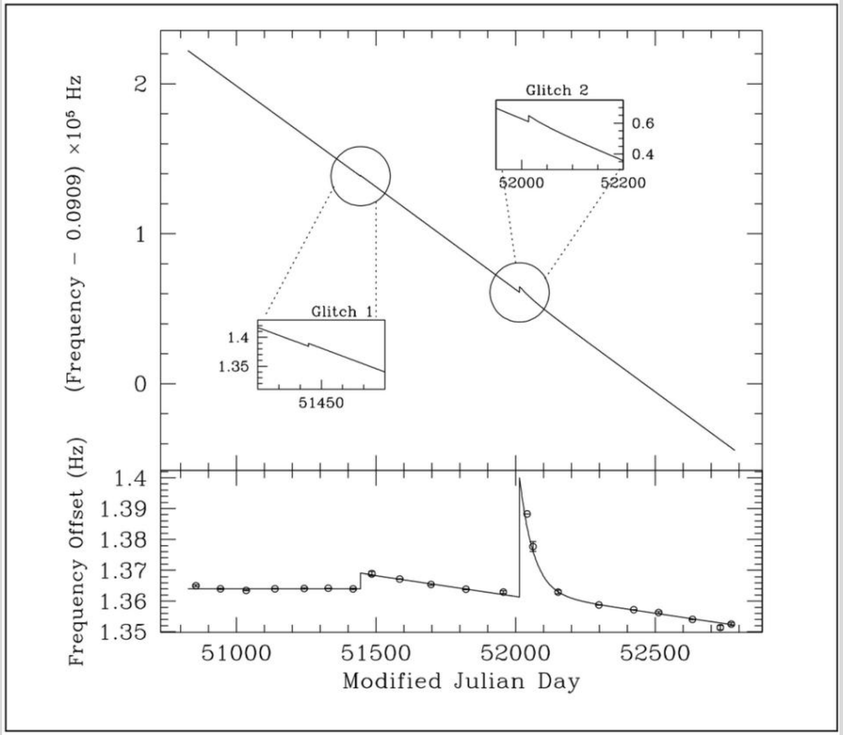
Interaction of vortices with atomic structure in a crust.

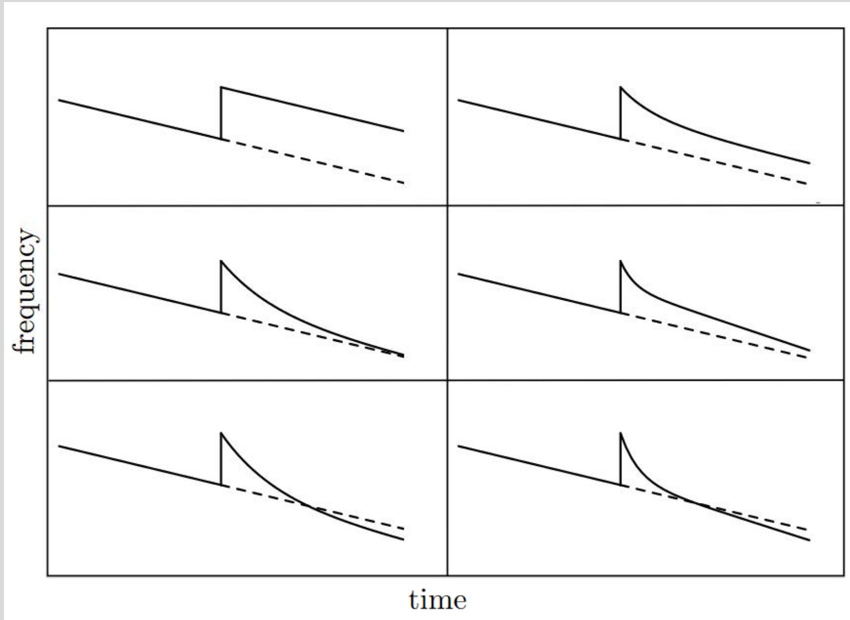


Interaction of vortices with flux tubes in a core.

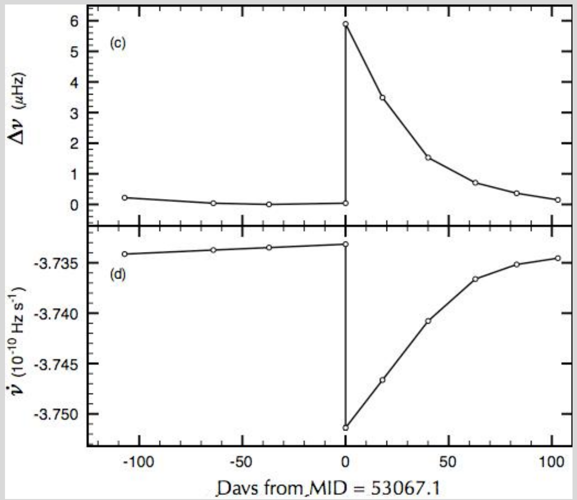
Consequences of superfluidity in NS

- Pulsar glitches





Schematic types of postglitch relaxation [Espinoza et al., 2011]



Another consequences

- Cooling
- Stability
- Energy transfer mechanisms

On magnetic fields…

Glitches as they seen on the overall trend of rotational frequency decrease.

Hydrodynamics of superfluid vortex avalanches

General multifluid framework [Andersson N., Comer G. L., 2006]:

$$\partial_t \rho_x + \nabla_i (\rho_x v_x^i) = 0 \qquad (1)$$

$$(\partial_t + v_x^j \nabla_j)(v_i^x + \varepsilon_x w_i^{yx}) + \nabla_i (\tilde{\mu}_x + \Phi) + \varepsilon_x w_{yx}^j \nabla_i v_j^x = (f_i^x + f_i^{xp}) / \rho_x \qquad (2)$$

$$\mathrm{w}_\mathrm{i}^{\mathrm{xy}} = v_i^y - v_i^x \qquad (3)$$

$$\tilde{\mu}_x = \mu_x / m_x \qquad (4)$$

Including the rotation. Feynman relation:

$$kn_v(w) = 2[\Omega_n + \varepsilon_n(\Omega_p - \Omega_n)] + w\frac{\partial}{\partial r}[\Omega_n + \varepsilon_n(\Omega_p - \Omega_n)]$$

Mutual friction contribution (straight vortices and laminar flow):

$$f_i^{\,x} = kn_v \rho_n B' \varepsilon_{ijk} \Omega_i^n w_{xy}^k + kn_v \rho_n B \varepsilon_{ijk} \Omega_j^n \varepsilon^{klm} \Omega_l^n w_m^{xy}$$

with k is a quantum of circulation, n_v vortex density per unit area, w velocity difference between constituents x,y; B, B' – mutual friction parameters.

Vortex sheet

From multifluid framework the angular velocity evolution of the two rotating components is:

$$\dot{\Omega}_n(\varpi) = \frac{Q(\varpi)}{\rho_n}\frac{1}{1-\varepsilon_n-\varepsilon_p} + \frac{\varepsilon_n}{(1-\varepsilon_n)}\dot{\Omega}_{ext} + F_p(\varpi)$$

$$\dot{\Omega}_p(\varpi) = -\frac{Q(\varpi)}{\rho_p}\frac{1}{1-\varepsilon_n-\varepsilon_p} - \dot{\Omega}_{ext} - \frac{\rho_n}{\rho_p}F_p(\varpi)$$

$$Q(\varpi) = \rho_n \gamma k n_v B(\Omega_p - \Omega_n)$$

$$\gamma = \frac{n_{free}}{n_{tot}}$$

Including the lag as: $\Delta\Omega = \Omega_p - \Omega_n$, the evolution equation is as following:

$$\frac{\partial \Delta\Omega}{\partial t} = -kn_v \frac{\gamma B}{x_p(1-\varepsilon_n-\varepsilon_p)}\Delta\Omega$$

Without taking into account dif. rotation: $\Delta\Omega \approx \Delta_0 \exp-(t/\frac{x_p(1-\varepsilon_n-\varepsilon_p)}{2\Omega_n\gamma B})$

Presence of significant differential rotation:

$$\frac{\partial \Delta^*(\varpi,t)}{\partial t} = -\Delta^* \frac{\partial \Delta^*(\varpi,t)}{\partial \varpi}, \Delta^* = \beta\Delta\Omega$$

Travelling waves solutions of Burger's equation for a lag:

1. Large number of vortices close to the maximum of the pinning force

$$\Delta\Omega(t,\varpi) = \frac{\varpi - \varpi_0}{t}$$ For $w<w_f$

$$\Delta\Omega(t,\varpi) = \Delta\Omega_M$$ For $w>w_f$

Accumulation of vortices leads to the rapid outward motion of vortices that may drive avalanches.

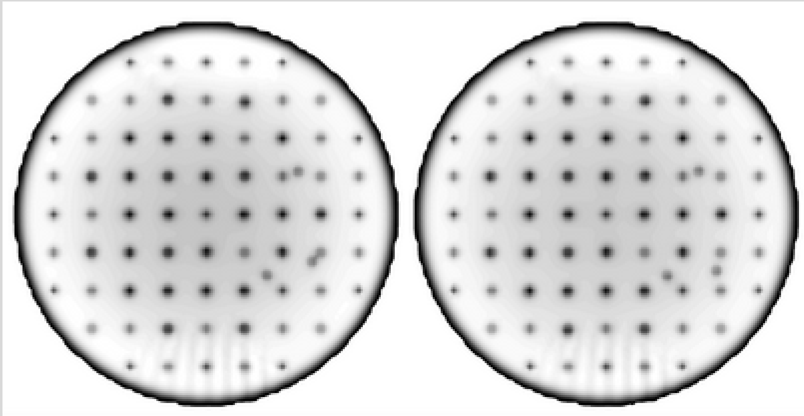
2. Upstream region from the vortex accumulation region:

$$\Delta\Omega(t,\varpi) = \Delta\Omega_M \Theta(\varpi_d - \varpi)$$

Notes on hydrodynamical applicability:

$$d_v = 1\times 10^{-3}\left(\frac{P}{10\,ms}\right)^{1/2} cm$$

Coarse graining -> Averaging over the distances much bigger then intervortex spacing



Unpinning vortex waves

In analogy with SOC systems equation for gamma can be presented as:

$$\frac{\partial \gamma}{\partial t} = \sum_n \alpha_n \gamma^n + \xi f(\gamma, \partial_r \gamma)$$

Long term evolution   Transport of vortices

case I:

$$\frac{\partial \Delta\Omega}{\partial t} = \frac{B}{x_p} \varpi_s \Delta\Omega \frac{\partial \Delta\Omega}{\partial \omega} - 2\Omega\Delta\Omega \frac{\gamma B}{x_p}$$
$$\frac{\partial \Delta\gamma}{\partial t} = \frac{B}{2x_p} \varpi_s \Delta\Omega \frac{\partial \gamma}{\partial \omega}$$

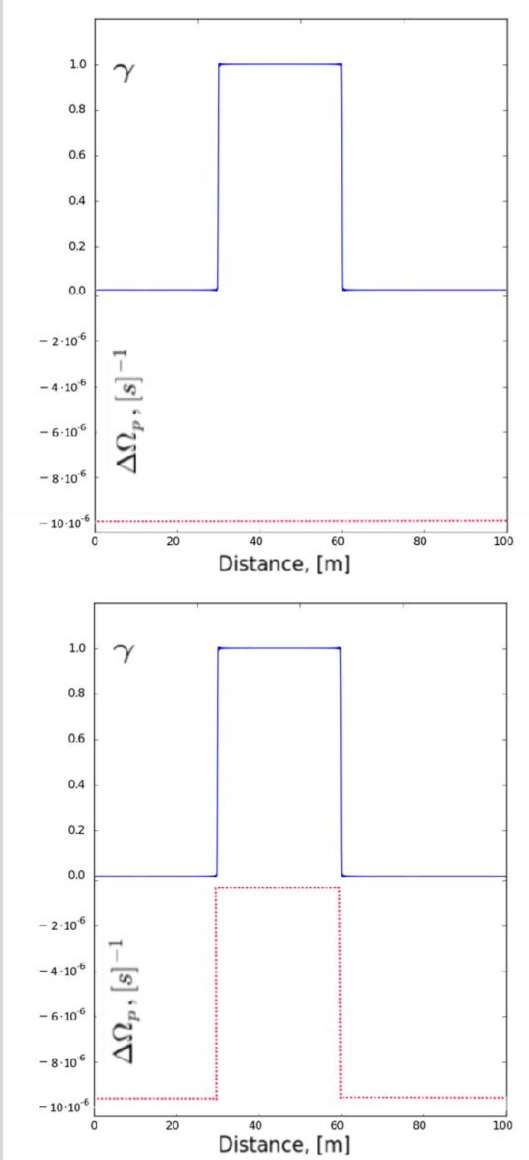
case II:

$$\frac{\partial \Delta\Omega}{\partial t} = \frac{B}{x_p} \varpi_s \Delta\Omega \frac{\partial \Delta\Omega}{\partial \omega} - 2\Omega\Delta\Omega \frac{\gamma B}{x_p}$$
$$\frac{\partial \Delta\gamma}{\partial t} = \frac{B}{2x_p} \varpi_s \Omega_{init} \frac{\partial \gamma}{\partial \omega}$$

case III:

$$\frac{\partial \Delta\Omega}{\partial t} = \frac{B}{x_p} \varpi_s \gamma \Delta\Omega \frac{\partial \Delta\Omega}{\partial \omega}$$
$$\frac{\partial \Delta\gamma}{\partial t} = \frac{B}{2x_p} \varpi_s \Omega_{init} \frac{\partial \gamma}{\partial \omega}$$

Different prescription for the vortex advection



Initial conditions

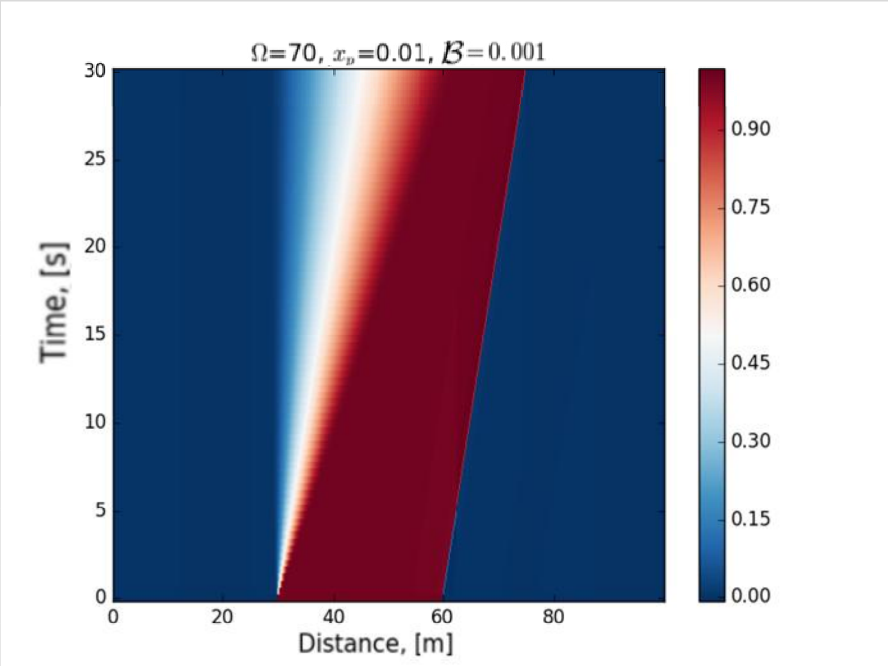
Advection of vortices:

Evaluation of changes in normal component evolution:

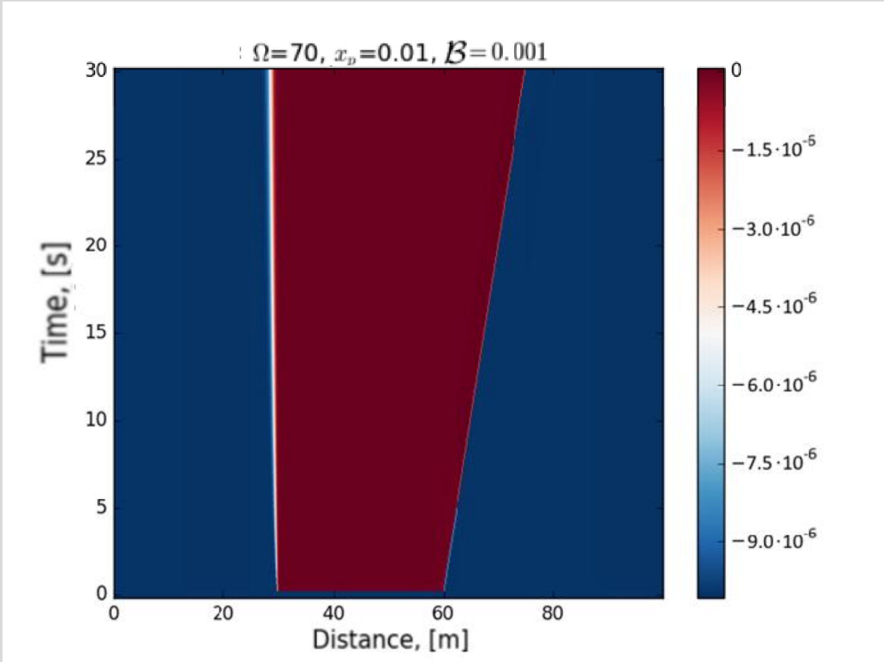
$$\Omega_p = \Omega_n - \int \left(\frac{B}{x_p}\varpi_s \Delta\Omega \frac{\partial \Delta\Omega}{d\omega} - 2\Omega\Delta\Omega \frac{B\gamma}{x_p}\right) dt$$

$$\Omega_p = \Omega_n - \int \frac{B}{x_p}\varpi_s \Delta\Omega\gamma \frac{\partial \Delta\Omega}{d\omega} dt$$

Example of numerical solution for the evolution (setup 1):

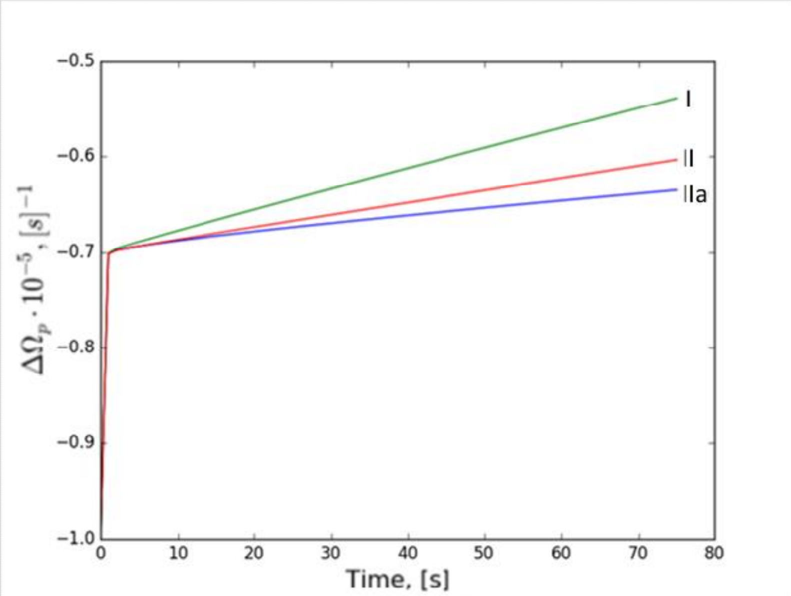


For the fraction of free vortices

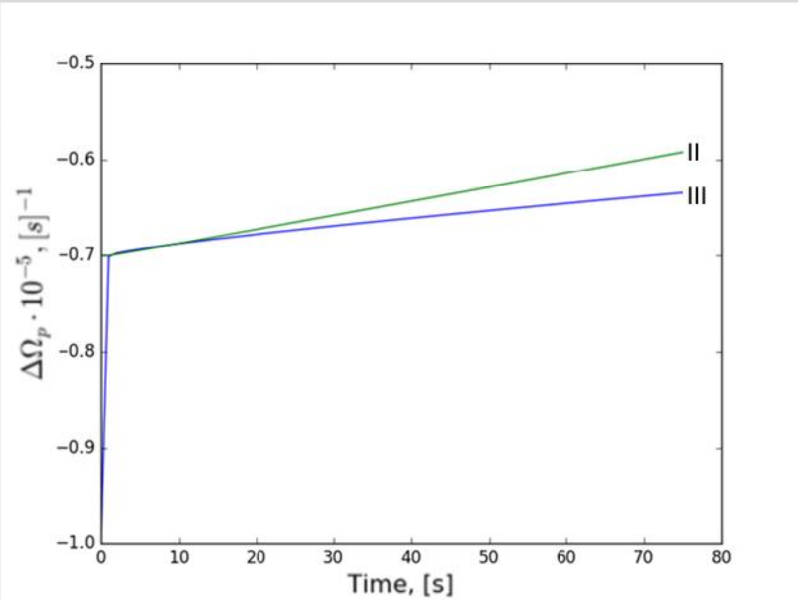


For the lag between the normal and the superfluid component

Normal component evolution in time. Results:

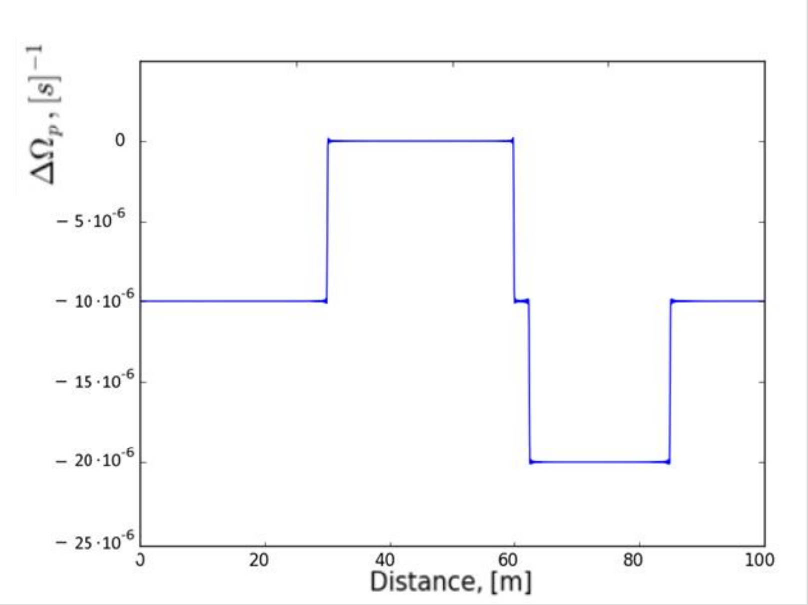


Comparison of the time evolution for I and II setups with delayed spin up.

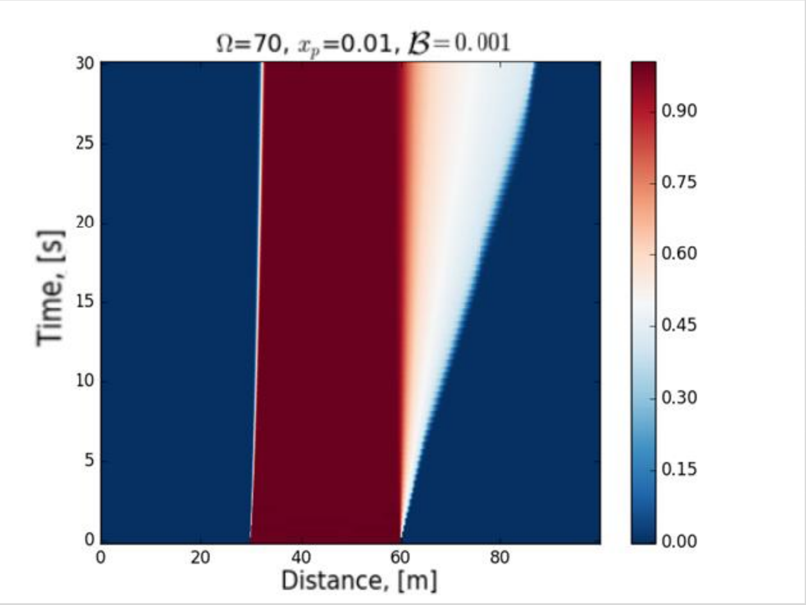


Comparison of the time evolution for setups II and III.

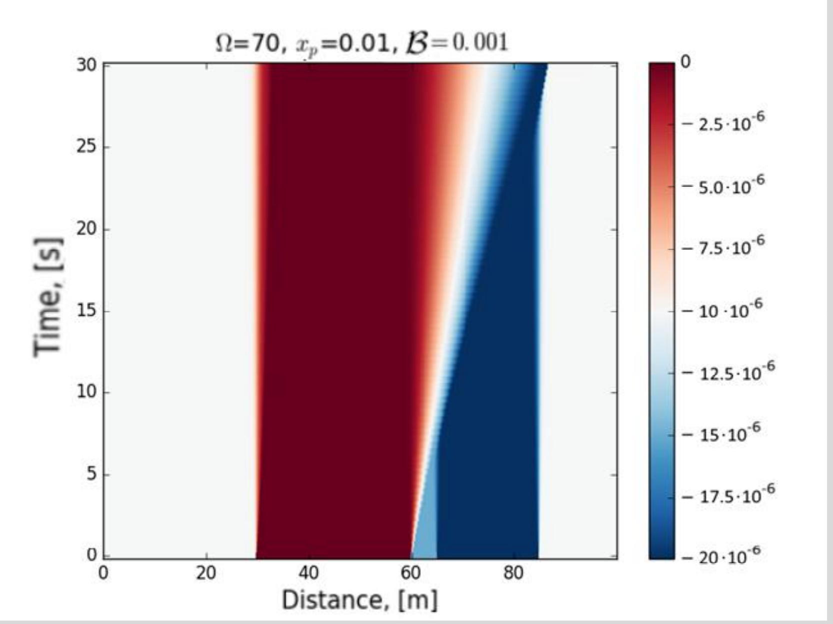
Glitch precursors:

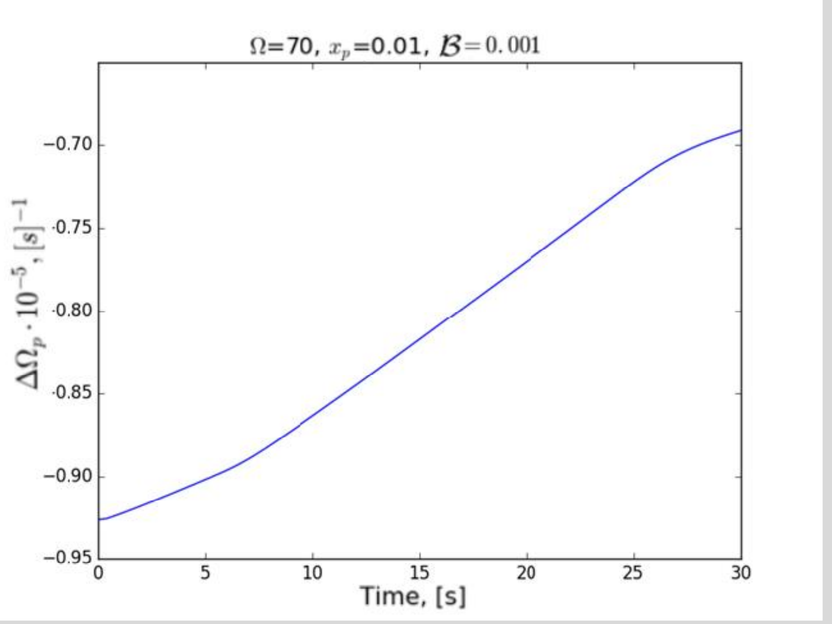


Initial condition is the lag with a sequence of steps physically corresponding to the different pinning strength.



Evolution of the fraction of free vortices parameter.



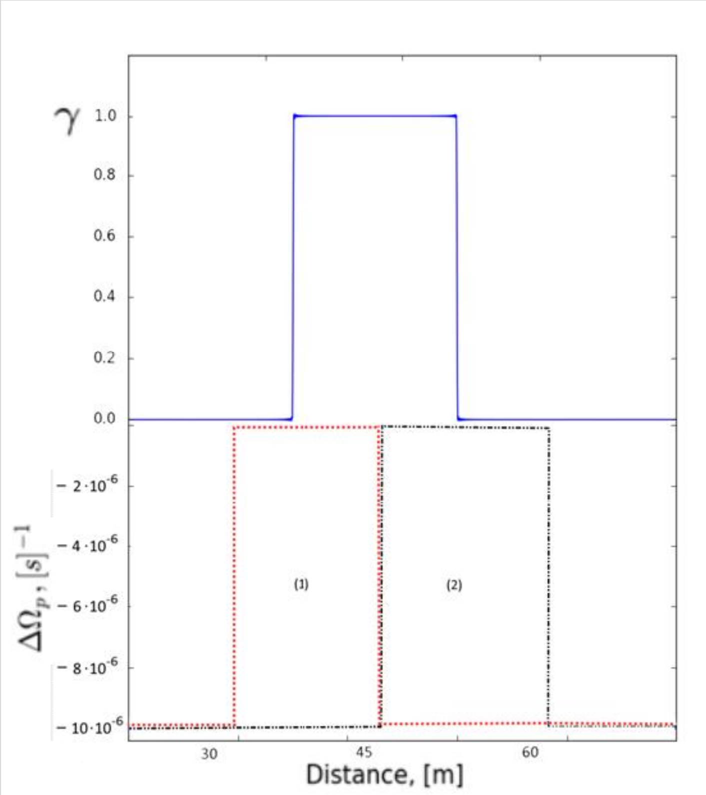


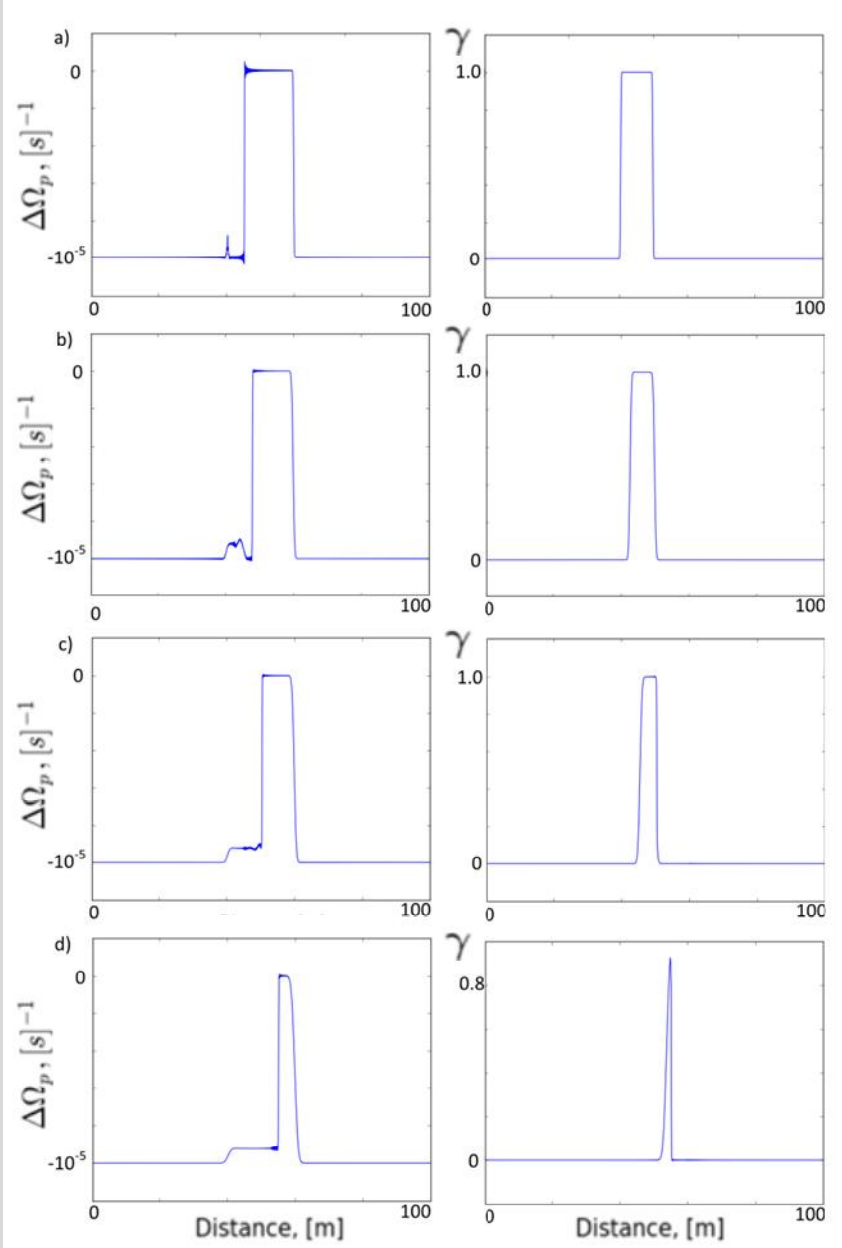
Evolution of the lag for the initial conditions with sequence of steps

Time evolution of the normal component

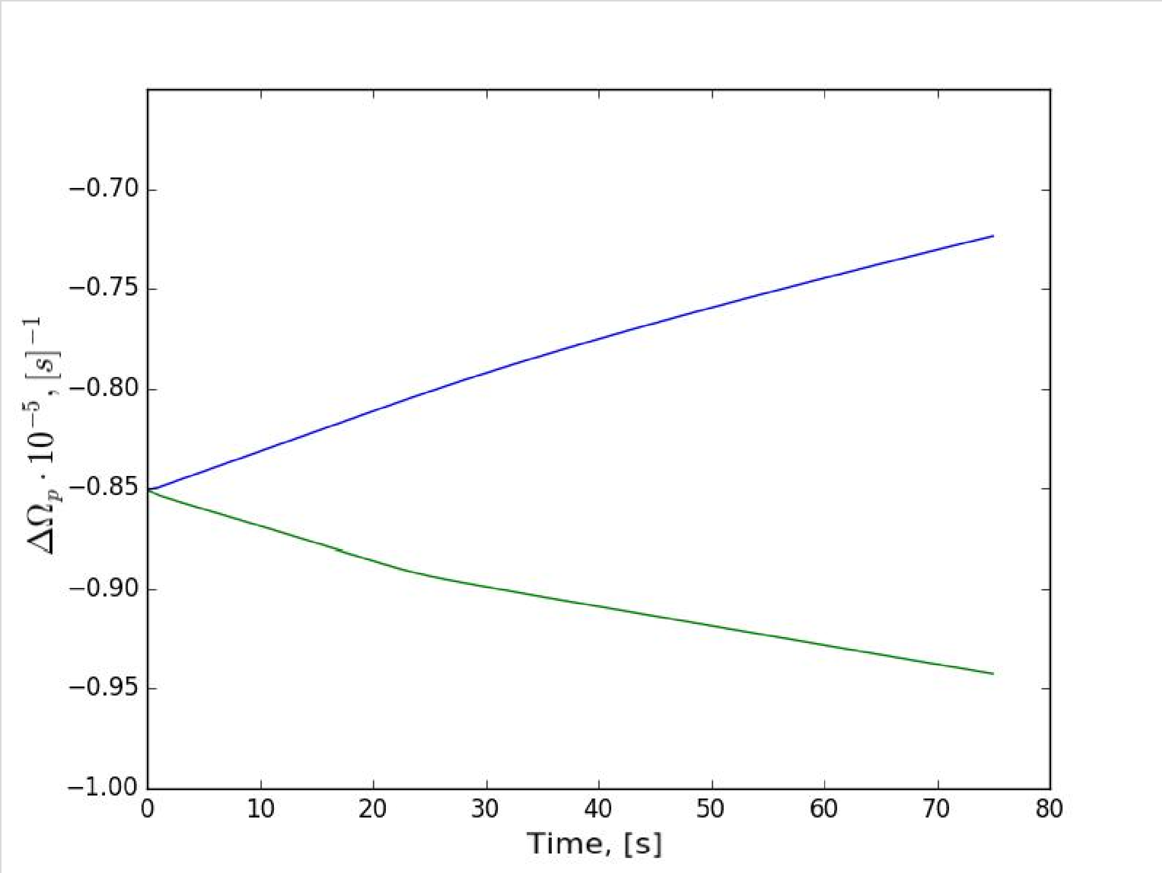
- The behavior is similar to what is seen in J0537-6910 [Middleditch, 2015]
- The interaction between normal and superfluid component may lead to the glitch itself or to the glitch precursor.

Frequency decreases and antiglitches



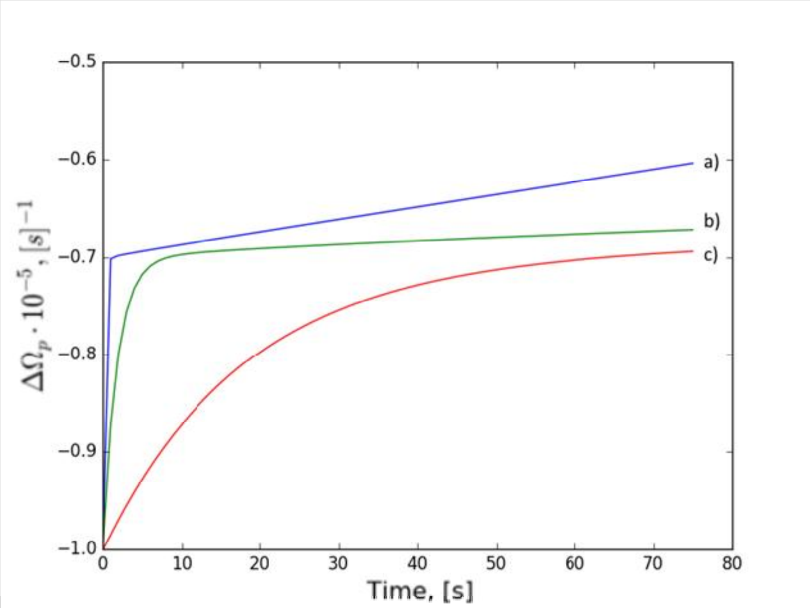


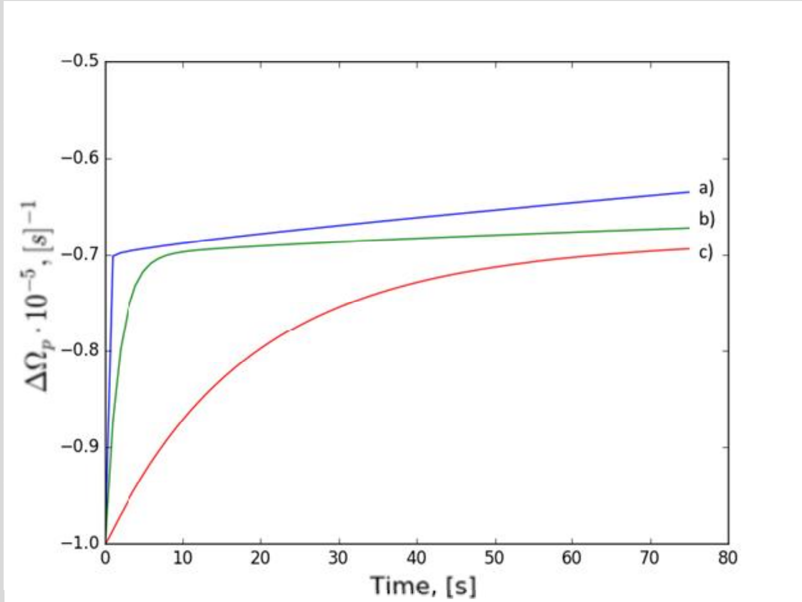
Initial conditions for glitch – antiglitch test: 1) Leading to the increase of angular velocity; 2) Leading to the decrease of the angular velocity



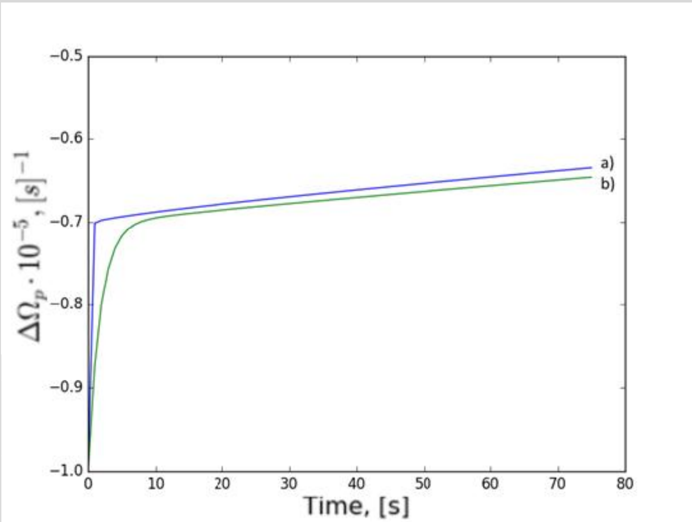
Evolution of the lag for the glitch – antiglitch test.

Parameter study





Dependence of normal component evolution on the mutual friction parameter: a) $B=10^{-3}$, b) $B=10^{-4}$, c) $B=10^{-5}$



Dependence of normal component evolution on the angular velocity of the star: a) Omega=$70s^{-1}$, b) Omega=$7s^{-1}$

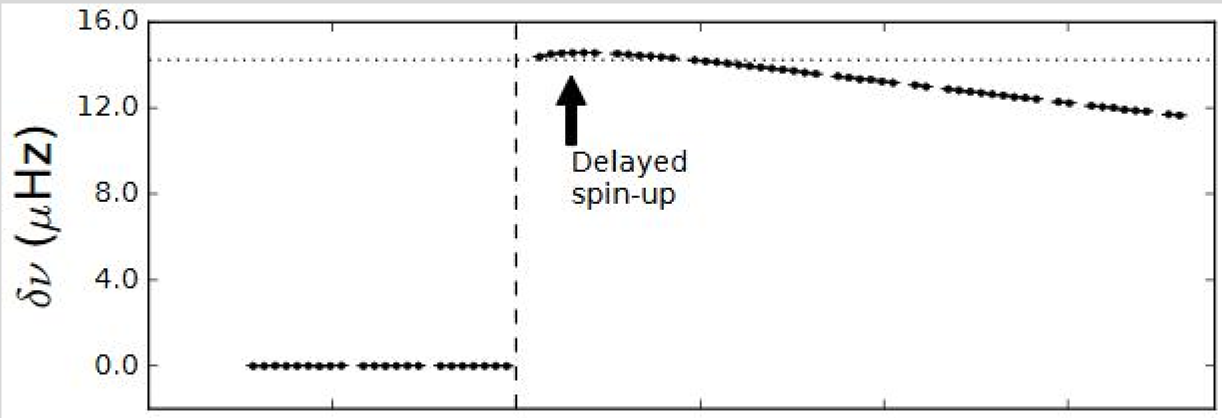
Constraining the mutual friction from the slow rise of Glitch

Glitch	$\Delta\nu_G$ (10^{-6}Hz)	Rise limit τ_r	$\Delta\nu_d$ (10^{-7}Hz)	τ_d (days)
Crab 1989	1.85	0.1 d	7	0.8
Crab 1996	0.66	0.5 d	3.1	0.5
Crab 2017	14	0.45 d	11	1.7
Vela 2000	35	40 s	–	–
Vela 2017	16	48 s	–	–

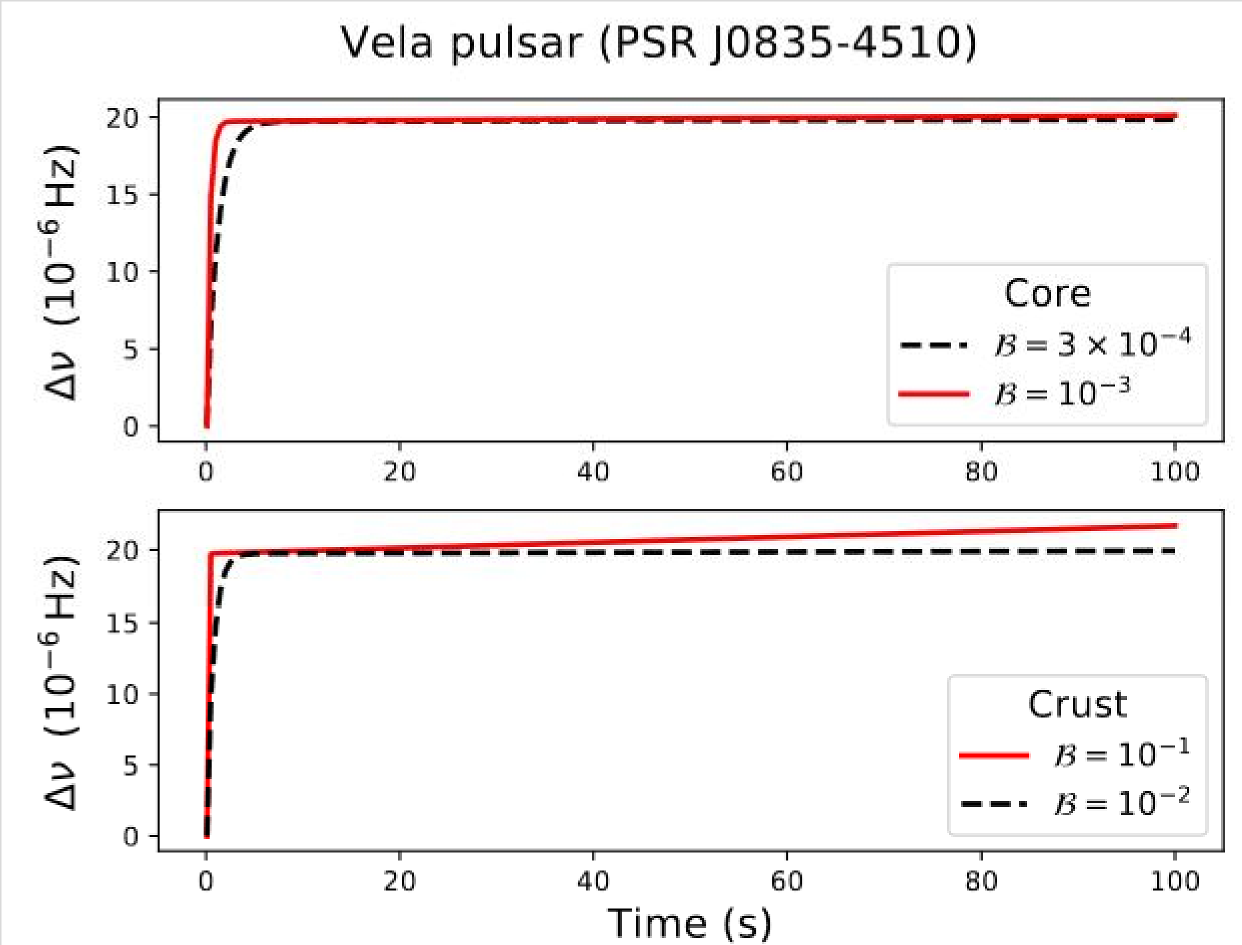
Observational data for the glitches in the Vela and the Crab pulsars

	Crab	Vela
Analytic Model:		
$\mathcal{B}$ in the core:	4×10^{-7}	10^{-3}
$\mathcal{B}$ in the crust:	3×10^{-5}	0.1
Non-linear simulations:		
$\mathcal{B}$ in the core:	$5 \times 10^{-7} - 5 \times 10^{-6}$	$3 \times 10^{-4} - 10^{-3}$
$\mathcal{B}$ in the crust:	$3 \times 10^{-4} - 10^{-5}$	$0.01 - 0.1$

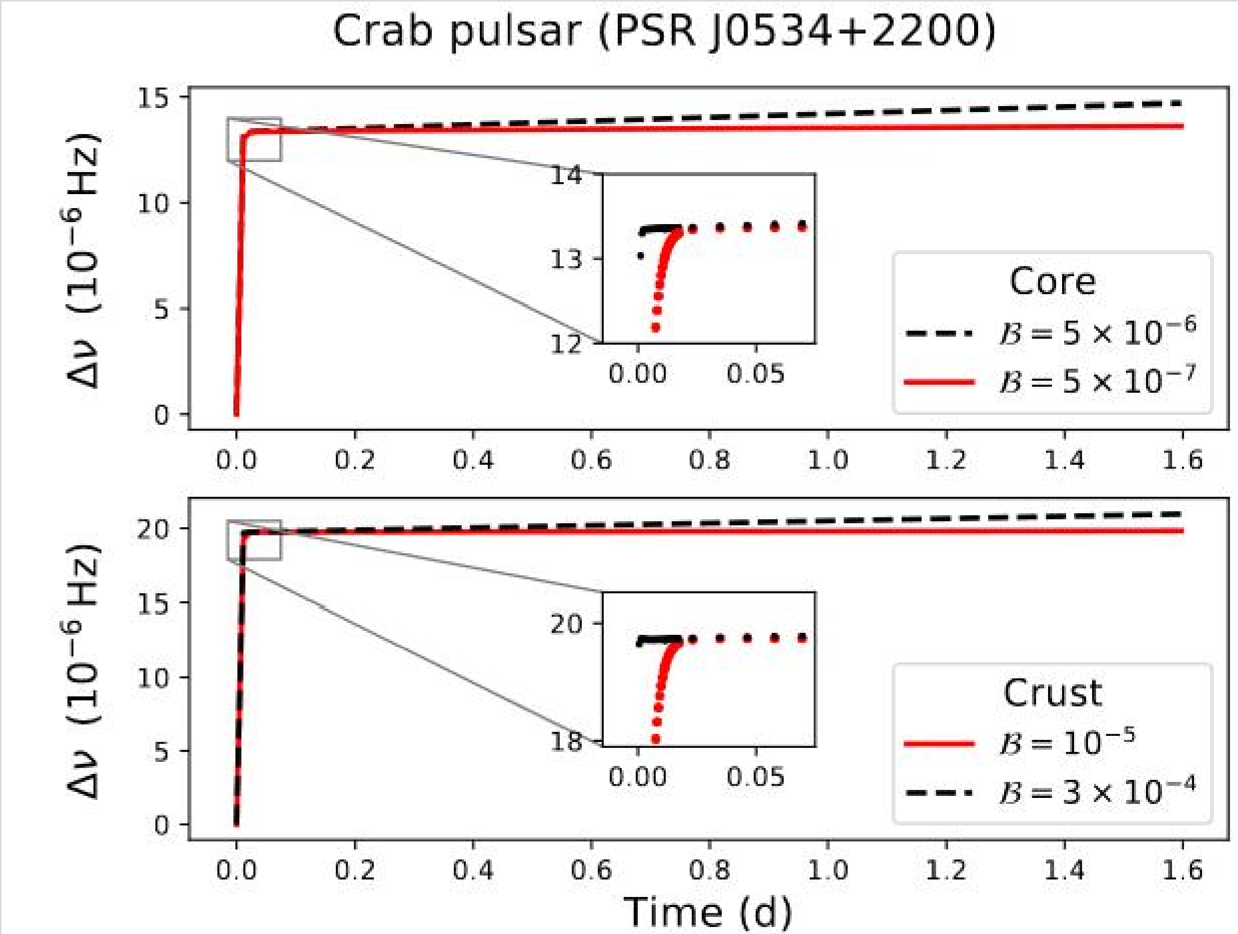
Analytical and computational constrains from the two fluid model



Spin frequency residuals for Crab’s 2017 glitch [Shaw et.al., 2018]



Example of simulation for the Vela pulsar. Short term evolution is shown. The mutual friction required to produce the linear rise differ for the Crust and Core region.



Example of simulation for the Crab pulsar. Long term evolution is shown. Mutual friction required is lower so the glitch in Crab should originate in the crust of the star.

Results

- A formalism for simulating the motion of superfluid vortex over-densities and fronts in hydrodynamical two-fluid simulations of pulsar glitches has been outlined.

- It was shown the existence of unpinning waves due to the explicit account of differential rotation.

- We find that if there are regions in which pinning decreases with density, as one expects in the deep crust (Seveso et al., 2016), then an initial unpinning event may lead to a slow change in frequency as a precursor of a larger glitch, triggered when the unpining front reaches the stronger pinning region.

-We find specific setups in which vortex motion can lead to a decrease in frequency, or an anti-glitch, such as that observed in the magnetar 1E 2259+586 (Archibald et al., 2013).

-Accumulation of vortices leads to the avalanches followed by the slow linear rise (like in Crab pulsar).