From quantum theory of nuclear matter to hydrodynamics of neutron stars

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Introduction
Towards mesoscopic model
Neutron Star crust modelling
Future work objectives

Colaboration

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Presentation plan

Open question: How to perform transition from quantum to macroscopic regime preserving “the physics”? 

1 Introduction
   - Star internal structure
   - Description on different length scales

2 Towards mesoscopic model
   - Methods
   - Reduction of DoFs
   - Vortex Filament Model

3 Neutron Star crust modelling
   - Vortex self-interaction
   - Phenomenological dissipation
   - Applications

4 Future work objectives
Correction to M. Antonelli: Transition from superfluid coherence length to intervortex distance scale.

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3. Neutron Star crust modelling
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4. Future work objectives
Plan of presentation

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**Star internal structure**

### Macroscopic - hydrodynamics

- Density (kg/m³)
  - Outer Crust: $10^9$
  - Inner Crust: $4.3 \times 10^{14}$
  - Outer Core: $2 \times 10^{17}$
  - Inner Core: $1.3 \times 10^{18}$

### Microscopic - quantum

- Radius (km)
  - Outer Crust: 10.6
  - Inner Crust: 10.3
  - Outer Core: 9.7

- avg. nuclear spacing – tens of fm

Source: https://apatruno.wordpress.com/neutron-stars/

**Macroscopic - hydrodynamics**

- **Density (kg/m³)**
  - 10³
  - 4.3 × 10¹⁴
  - 2 × 10¹⁷
  - 1.3 × 10¹⁸

**Microscopic - quantum**

- **Radius (km)**
  - 10.6
  - 10.3
  - 9.7

- **avg. nuclear spacing – tens of fm**

Source: https://apatruno.wordpress.com/neutron-stars/

Description on different length scales

**Microscopic**
- Quantum many-body description
- Dynamics of neutrons and protons

**Mesoscopic**
- Semi-classical model
- **Distinguishable vortices and nuclei**

**Macroscopic**
- Relativistic hydrodynamics?
- Continuous vorticity

Method: TDDFT
DoF: neutrons and protons.
Scale: ~10^{-15} m

Method: Vortex Filament Model
DoF: impurities and vortices
Scale: ~10^{-9} m

Method: Hydrodynamics
DoF: fluid elements
Scale: ~size of star

Introduction Towards mesoscopic model Neutron Star crust modelling Future work objectives Star structure Constraints on models
Introduction

Towards mesoscopic model

Neutron Star crust modelling

Future work objectives

Description on different length scales

Microscopic

- Quantum many-body description
- Dynamics of neutrons and protons

Mesoscopic

- Semi-classical model
- Distinguishable vortices and nuclei

Macroscopic

- Relativistic hydrodynamics?
- Continuous vorticity

Collective effort required!

Method: TDDFT
DoF: neutrons and protons
Scale: $\sim 10^{-13}$ m

Method: Vortex Filament Model
DoF: impurities and vortices
Scale: $\sim 10^{-9}$ m

Method: Hydrodynamics
DoF: fluid elements
Scale: $\sim$ size of star
Our aim

Model based on QM

Parameters estimation

Mesoscopic model

Simulations

Simulations

Same dynamics?
Plan of presentation

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Methods
Reduction of DoFs
Vortex Filament Model
Methods - our proposal

**Microscopic**
- ASLDA framework
- Explicit s-wave superfluidity
- Equivalent of Schrödinger eq.

**Mesoscopic**
- Relativistic?
- Vortex Filament Model
- Explicit dynamics of nuclei and vortices

**Macroscopic**
- Relativistic
- Fluid of clusters
- Fluid of neutrons
- Continuous vorticity

- At macroscopic level we want to describe fluid of nuclei and neutronic superfluid
- How to estimate friction between them?
- Pinning force between nuclei and vortices in neutron superfluid

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A. Melatos, C. Peralta
Introduction 

Towards mesoscopic model

Neutron Star crust modelling

Future work objectives

Methods

Microscopic

- ASLDA framework
- Explicit s-wave superfluidity
- Equivalent of Schrödinger eq.

Mesoscopic

- Relativistic?
- Vortex Filament Model
- Explicit dynamics of nuclei and vortices

Macroscopic

- Relativistic
- Fluid of nuclei
- Fluid of neutrons
- Continuous vorticity

\[ \rho \kappa \times (\dot{s} - v_{\text{ind}}(s) - v_{\text{ext}}) + f^{VN}(s, r) + f^{D}(\dot{s}, \dot{r}) = 0 \]

\[ s = s(\xi) \]

where \( \xi \) is a parameter describing position of line element along curve
## Methods

**Microscopic**
- ASLDA framework
- Explicit s-wave superfluidity
- Equivalent of Schrödinger eq.

**Mesoscopic**
- Relativistic?
- Vortex Filament Model
- Explicit dynamics of nuclei and vortices

**Macroscopic**
- Relativistic
- Fluid of nuclei
- Fluid of neutrons
- Continuous vorticity

Two sets HFB equations with density functional for protons and neutrons

\[
\begin{align*}
\begin{pmatrix}
  u_n^{\uparrow}(\mathbf{r}, t) \\
  u_n^{\downarrow}(\mathbf{r}, t) \\
  v_n^{\uparrow}(\mathbf{r}, t) \\
  v_n^{\downarrow}(\mathbf{r}, t)
\end{pmatrix}
&= i\hbar \frac{\partial}{\partial t}
\begin{pmatrix}
  h_{\uparrow,\uparrow}(\mathbf{r}, t) & h_{\downarrow,\downarrow}(\mathbf{r}, t) & 0 & \Delta(\mathbf{r}, t) \\
  h_{\downarrow,\uparrow}(\mathbf{r}, t) & h_{\uparrow,\downarrow}(\mathbf{r}, t) & -\Delta(\mathbf{r}, t) & 0 \\
  0 & -\Delta^*(\mathbf{r}, t) & -h_{\uparrow,\uparrow}^*(\mathbf{r}, t) & -h_{\downarrow,\downarrow}^*(\mathbf{r}, t) \\
  \Delta^*(\mathbf{r}, t) & 0 & -h_{\downarrow,\uparrow}^*(\mathbf{r}, t) & -h_{\uparrow,\downarrow}^*(\mathbf{r}, t)
\end{pmatrix}
\begin{pmatrix}
  u_n^{\uparrow}(\mathbf{r}, t) \\
  u_n^{\downarrow}(\mathbf{r}, t) \\
  v_n^{\uparrow}(\mathbf{r}, t) \\
  v_n^{\downarrow}(\mathbf{r}, t)
\end{pmatrix}
\end{align*}
\]

SLDA method capabilities

Two sets HFB equations with density functional for protons and neutrons

\[
\begin{aligned}
\frac{i \hbar}{\partial t} & \begin{pmatrix} u_{n\uparrow}(r, t) \\ u_{n\downarrow}(r, t) \\ v_{n\uparrow}(r, t) \\ v_{n\downarrow}(r, t) \end{pmatrix} = \\
& \begin{pmatrix} h_{\uparrow,\uparrow}(r, t) & h_{\uparrow,\downarrow}(r, t) & 0 & \Delta(r, t) \\ h_{\downarrow,\uparrow}(r, t) & h_{\downarrow,\downarrow}(r, t) & -\Delta(r, t) & 0 \\ 0 & -\Delta^*(r, t) & -h^*_{\uparrow,\uparrow}(r, t) & -h^*_{\uparrow,\downarrow}(r, t) \\ \Delta^*(r, t) & 0 & -h^*_{\downarrow,\uparrow}(r, t) & -h^*_{\downarrow,\downarrow}(r, t) \end{pmatrix} \begin{pmatrix} u_{n\uparrow}(r, t) \\ u_{n\downarrow}(r, t) \\ v_{n\uparrow}(r, t) \\ v_{n\downarrow}(r, t) \end{pmatrix}
\end{aligned}
\]

Present computing capabilities:
(for nuclear systems)

- full 3D (unconstrained) dynamics
- volumes up to 100³ fm³
- number of particles of order 10⁴
- up to 10⁶ time steps (for nuclear systems it gives trajectory length 10⁻¹⁹ sec)
Aim to construct mesoscopic model preserving low-energy physics

When exact dynamics of quantum system is known a semi-classical model preserving low-energy excitations dynamics can be constructed.

Exact dynamics of all fermionic DoFs
Computational cost limits size of system
Example of transition between micro- and mesoscopic regime
Quantum mechanics vs hydrodynamics

\[ i\hbar \partial_t \Psi = \left[ \hat{H}_{\text{one-body}} + \hat{H}_{\text{int}} + V_{\text{ext}}(x,t) \right] \Psi, \quad \Psi = \sqrt{n} e^{i\hbar \theta} \]

\[ \frac{d\mathbf{v}}{dt} = \partial_t \mathbf{v} + \mathbf{v} \cdot \nabla \mathbf{v} = -\frac{1}{m} \nabla \left( -\frac{\hbar^2}{2m} \frac{\nabla^2 \sqrt{n}}{\sqrt{n}} + V_{\text{ext}}(x,t) \right) \partial_t n + \nabla (n\mathbf{v}) = 0 \]

probability current

\[ n\mathbf{v} = \mathbf{j} = \frac{\hbar}{2mi} \left[ \Psi^*(\nabla \Psi) - \Psi (\nabla \Psi^*) \right] \]

Helmholtz-Hodge decomposition (HHD)

Compressible and incompressible velocity fields

\[ \mathbf{v} = \mathbf{v}_{\text{ext}} + \mathbf{v}_{\text{ind}} + \mathbf{v}_{\text{nucl}} + \mathbf{v}_{\text{boundary}} \]

How to estimate \( \nabla \phi_N \)?
Vortex line motion

Description with fluid velocities

\[ \dot{s} = v_{\text{ind}} + v_b + v_{\text{ext}} + \nabla \phi_N + \text{dissipative terms} \]

Description with forces acting on a vortex line

\[ \rho_s \kappa \times \dot{s} = \rho_s \kappa \times [v_{\text{ind}} + v_b + v_{\text{ext}} + \nabla \phi_N] + f^D \]

\[ \rho_s \kappa \times \dot{s} = \rho_s \kappa \times [v_b + v_{\text{ext}}] - f^T - f^V - f^D \]

Vortex-nucleus interaction

Nucleus is dragged by force adjusted to preserve uniform motion. Interaction has to equilibrate system.
$m_{\text{nucl.}} \frac{d}{dt} v_{\text{nucl.}} = 0 = F_{\text{ext}} + \hat{r} \int_{\mathcal{L}} f(r) \sin \alpha dl$

Methods
Reduction of DoFs
Vortex Filament Model

At close separation the force is not a function of distance $R$ only
Plan of presentation

1. Introduction
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Setup of DFT simulations

Periodic boundary conditions

External potential trapping neutron fluid
Introduction Towards mesoscopic model Neutron Star crust modelling Future work objectives

Setup of realistic system for NS crust simulation

Periodic boundary conditions - only horizontal direction

Periodic lattice of nuclei

No boundaries for fluid flow!

(Constrained by intervortex distance)

Vortex self-interaction Dissipation Applications
Self-induced velocity

\[
\rho_s \kappa \times \dot{s} = \rho_s \kappa \times \left[ v_b + v_{\text{ext}} + \mathbf{v}_{\text{ind}} \right] - f^V N - f^D
\]

\[
v_{\text{ind}}(r, t) = \frac{\kappa}{4\pi} \int_{\mathcal{L}} d\mathbf{z} \frac{(s(z,t)-r) \times \mathbf{\hat{t}}(z,t)}{|s(z,t)-r|^3}
\]

is divergent when \( r \to s(z) \)

over vortex line

Local Induction Approximation (LIA) - a canonical regularization

\[
v_{\text{ind}}(s, t) = v_{\text{LIA}}(s, t) + \frac{\kappa}{4\pi} \int_{\mathcal{L}} d\mathbf{z} \frac{(s(z,t)-s) \times \mathbf{\hat{t}}(z,t)}{|s(z,t)-s|^3} \approx v_{\text{LIA}}(r, t)
\]

ommiting divergent point

\[
v_{\text{LIA}}(s(z), t) = \frac{\kappa}{4\pi R(z)} \ln \left( \frac{R(z)}{a_0} \right) \mathbf{\hat{b}}
\]

\( a_0 \) - vortex core radius
\( R(r) \) - radius of curvature

Vortex self-interaction  Dissipation  Applications
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(Constrained by intervortex distance)

Vortex self-interaction
Dissipation
Applications
Self-induced velocity

\[ \rho_s \kappa \times \dot{s} = \rho_s \kappa \times \left[ v_b + v_{\text{ext}} + v_{\text{ind}} \right] - f^{VN} - f^{D} \]

\[ \omega[s] = \nabla \times v[s] \text{ - vorticity field, } s \text{ - vortex line element position} \]

\[ \partial_t \omega[s] = \nabla \times [v[s] \times \omega[s]] \implies \nabla^2 A = -\omega[s] \]

**Effect of different domain**

\[ v_{\text{ind}} = \nabla \times A \]

\[ A = \sum_{n=-\infty}^{+\infty} e^{i k_n (z-z')} \int_0^L \frac{\kappa(z')}{2\pi} K_0 \left( |k_n| \sqrt{(x-s_x(z'))^2 + (y-s_y(z'))^2} \right) dz' \]

Also divergent when \( r \to s(z) \)
**Vortex self-interaction**

\[ \rho_s \kappa \times \dot{s} = \rho_s \kappa \times [v_b + v_{ext}] - f^T - f^{VN} - f^D \]

\[ v_{ind} (r, t) = \frac{\kappa}{4\pi} \int_{\mathcal{L}} dz \frac{(s(z, t) - r) \times \hat{t}(z, t)}{|s(z, t) - r|^3} \] is divergent when \( r \to s(z) \)

Energy flow can be rewritten as

\[ E_V = \sum_n \omega(k_n) \hat{u}(k_n) \cdot \hat{u}^*(k_n) \]

Leading to

\[ f^T = \mathcal{F}^{-1} \left[ \frac{\delta E_V(k_n)}{\delta \hat{u}^*(k_n)} \right] \]

\[ f^T = \mathcal{F}^{-1} \left[ \omega(k_n) \hat{u}(k_n) \right] \]

A. Fetter, Phys. Rev. 162.143 (1967)
sir W. Thompson, Phil. Mag. 10, 155 (1880)
Dissipation force

\[ \rho_s \kappa \times \dot{s} = \rho_s \kappa \times [v_b + v_{ext}] - f^T - f^{VN} - f^D \]

- Dissipation through vortex-phonon interactions
  \[ f^D_1 = -\eta_1 (\dot{u} - v_{ext}) \]
- Dissipation through nuclei deformation
  \[ f^D_2 = -\eta_2 (\dot{u} - \dot{r}_N) \]

Estimation methodology

drag impurity with various velocities...
look at excitation energy...

\[ \Delta E(t) = \Delta E_{\text{vortex}}(t) + \Delta E_{\text{compressible}}(t) + \Delta E_{\text{internal}}(t) \]
Dissipation force

\[ \rho_s \kappa \times \dot{s} = \rho_s \kappa \times [v_b + v_{ext}] - f^T - f^{VN} - f^D \]

- Dissipation through vortex-phonon interactions: \[ f_1^D = -\eta_1 (\dot{u} - v_{ext}) \]
- Dissipation through nuclei deformation: \[ f_2^D = -\eta_2 (\dot{u} - \dot{r}_N) \]

Estimation methodology

Use “ansatz” here for the dissipation force, and try to reproduce \( W_d(t) \)...

\[ W_d(t) = \int_L \left[ \int_0^t f^D(t') \cdot \dot{s}_l(t') dt' \right] dl \]

\[ \Delta E(t) = \Delta E_{vortex}(t) + \Delta E_{compressible}(t) + \Delta E_{internal}(t) \]

Reflected by the vortex line deformation in VFM

\( W_d(t) = \text{Dissipated energy (from point of view of VFM)} \)

Known from simulations...
Glitch size estimation


Test for crustal vs flux tubes pinning!
Glitch size estimation

\[
\Delta \Omega_{\text{max}} = \frac{\pi^2}{\kappa I} \int_0^{R_d} dr \, r^3 \, f_P(r)
\]

Quantum of circulation
Moment of inertia
Pinning force (force needed to move a vortex through the medium with “impurities”)

Observed relative sizes of glitches:
\[ \Delta v/v \sim 10^{-11} - 10^{-5} \]


Could provide test for microscopic models?
Glitch size estimation

\[ \Delta \Omega_{\text{max}} = \frac{\pi^2}{\kappa I} \int_0^{R_d} dr \, r^3 \, f_P(r) \]

- maximum glitch amplitude
- quantum of circulation
- moment of inertia
- pinning force
  (force needed to move a vortex through the medium with “impurities”)

Observed relative sizes of glitches:
\[ \Delta v/v \sim 10^{-11} - 10^{-5} \]

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Future work objectives

- Development of nuclear SLDA
  1. More accurate density functionals - BSk
  2. Nuclei lattice - structure and dynamics

- Vortex Filament Model validation
  1. Comparison between LIA and small deflections approach
  2. Extraction for Kelvin Waves dispersion relation
  3. Inclusion of nuclei dynamics
  4. Extraction of dissipation coefficients

- Large-scale VFM simulations
  1. Extraction of pinning force
  2. Connection to relativistic hydrodynamics of NS

- Rigorous testing of density functionals?
## High performance computing

<table>
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<th>Rank</th>
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<th>System</th>
<th>Cores</th>
<th>Rmax  (TFlop/s)</th>
<th>Reak  (TFlop/s)</th>
<th>Power  (kW)</th>
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<td>DOE/SPIRAL-IUC National Laboratory</td>
<td>Summit - IBM Power System AC/912, IBM POWER9 22C 3.07 Ghz, NVIDIA Volta (P100), Dual rail Mellanix EDR Infiniband IRM</td>
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<td>Sunway TaihuLight - Sunway MPP, Sunway SW26010 2600 1.45 Ghz, Sunway NRPC</td>
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<td>DOE/NSF/LLNL United States</td>
<td>Sierra - IBM Power System S922LC, IBM POWER9 22C 3.10 Ghz, NVIDIA Volta (P100), Dual rail Mellanix EDR Infiniband IRM</td>
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<td>National Super Computer Center in Guangzhou</td>
<td>Tianhe-2A - TH-MB-PfF Cluster, Intel Xeon E5-2692v4 12C 2.2GHz, TH Express-2, Matrix-2000 NUDT</td>
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<td>National Institute of Advanced Industrial Science and Technology (AIST)</td>
<td>Al Bridging Cloud Infrastructure (ABC) - PRIMEVRY CX259 M6, Xeon Gold 6148 20C 2.6GHz, NVIDIA Tesla V100 SXM2, Infiniband IRM Fujitsu</td>
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<td>Swiss National Supercomputing Centre (CSCS)</td>
<td>Piz Daint - Cray XK60, Xeon E3-2690v4 12C 2.65GHz, Aries interconnect, NVIDIA Tesla P100 Cray Inc.</td>
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</tr>
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</table>
Thank you!
Effects of boundaries

\[
\rho_s \kappa \times \dot{s} = \rho_s \kappa \times \left[ \mathbf{v}_b + \mathbf{v}_{\text{ext}} \right] - \mathbf{f}^T - \mathbf{f}^{V_N} - \mathbf{f}^D
\]

Flow past boundaries have to be suppressed \( \implies \mathbf{v}^\infty \cdot \mathbf{n} = \mathbf{v}_b \cdot \mathbf{n} \)