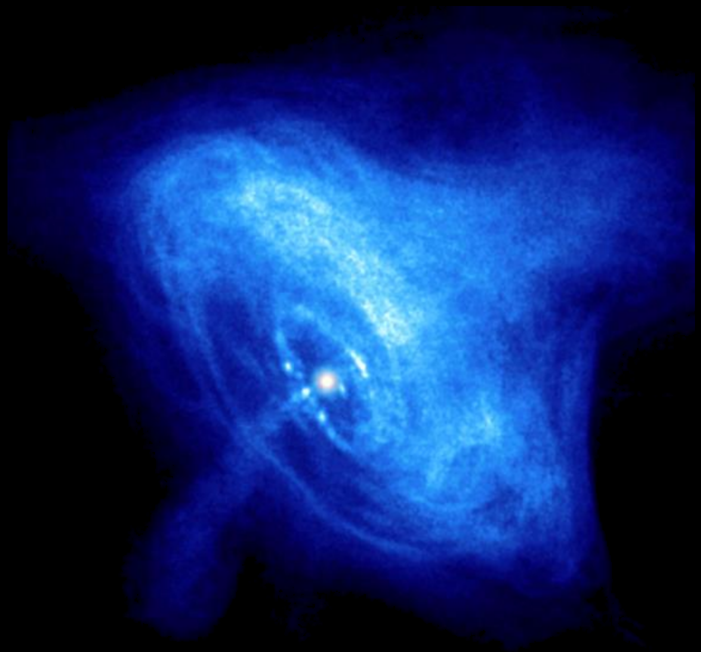
Strains and stresses in rotating neutron stars



Elia Giliberti
G. Cambiotti, M. Antonelli, P.M. Pizzochero
School in Physics of Università degli Studi di Milano
Pharos Workhop, 27 Sep. 2018

Rotating neutron stars

What is the dependence of stellar deformations on the star's mass?

What is the impact of the adiabatic index on stellar deformations?

https://arxiv.org/abs/1809.08542

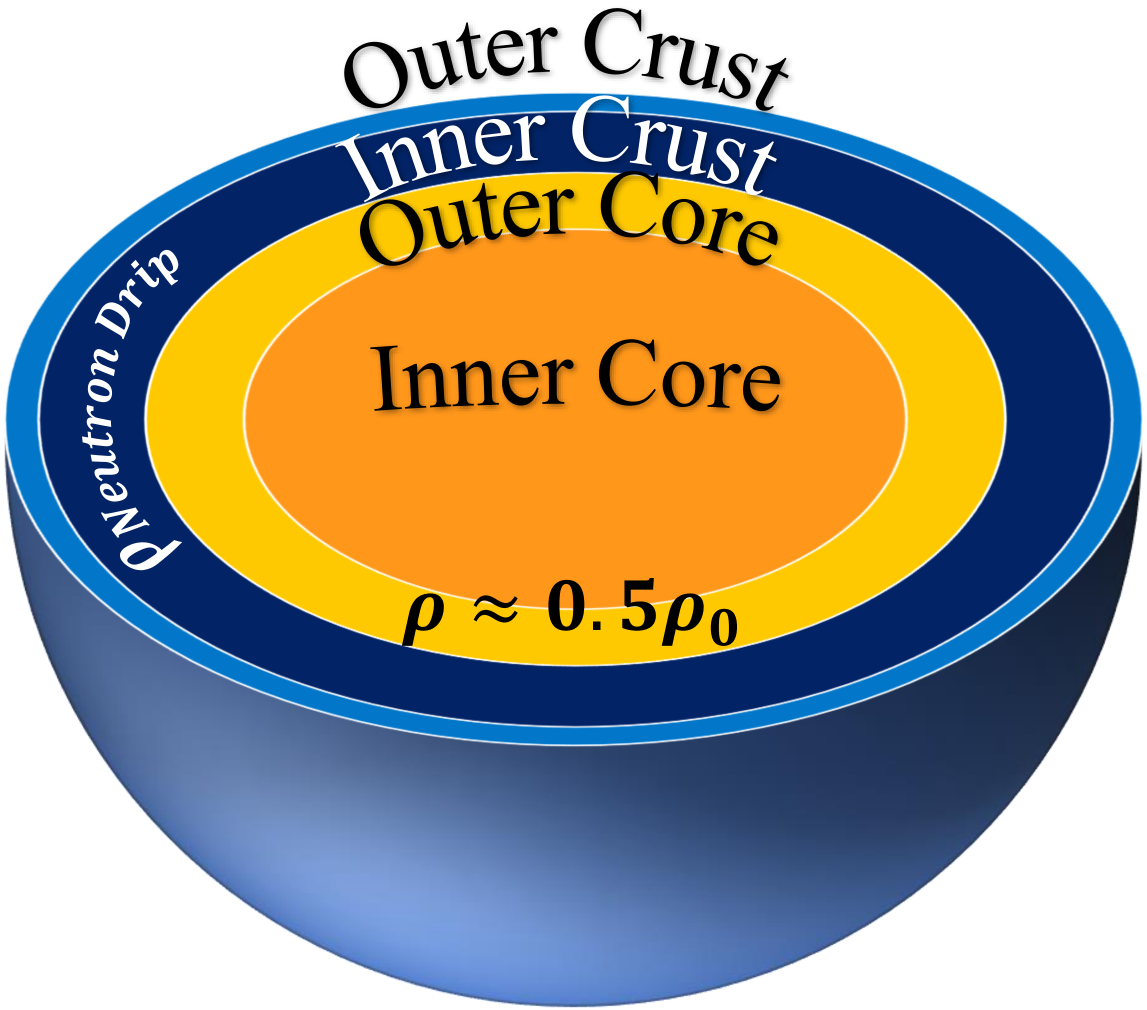
"The importance of the adiabatic index in modeling strains and stresses in spinning-down pulsars"

Internal structure

OUTER CORE
Neutrons Superfluid
Protons, electrons
and muons

INNER CORE
Hyperons?
Kaons? Quarks?

CORE

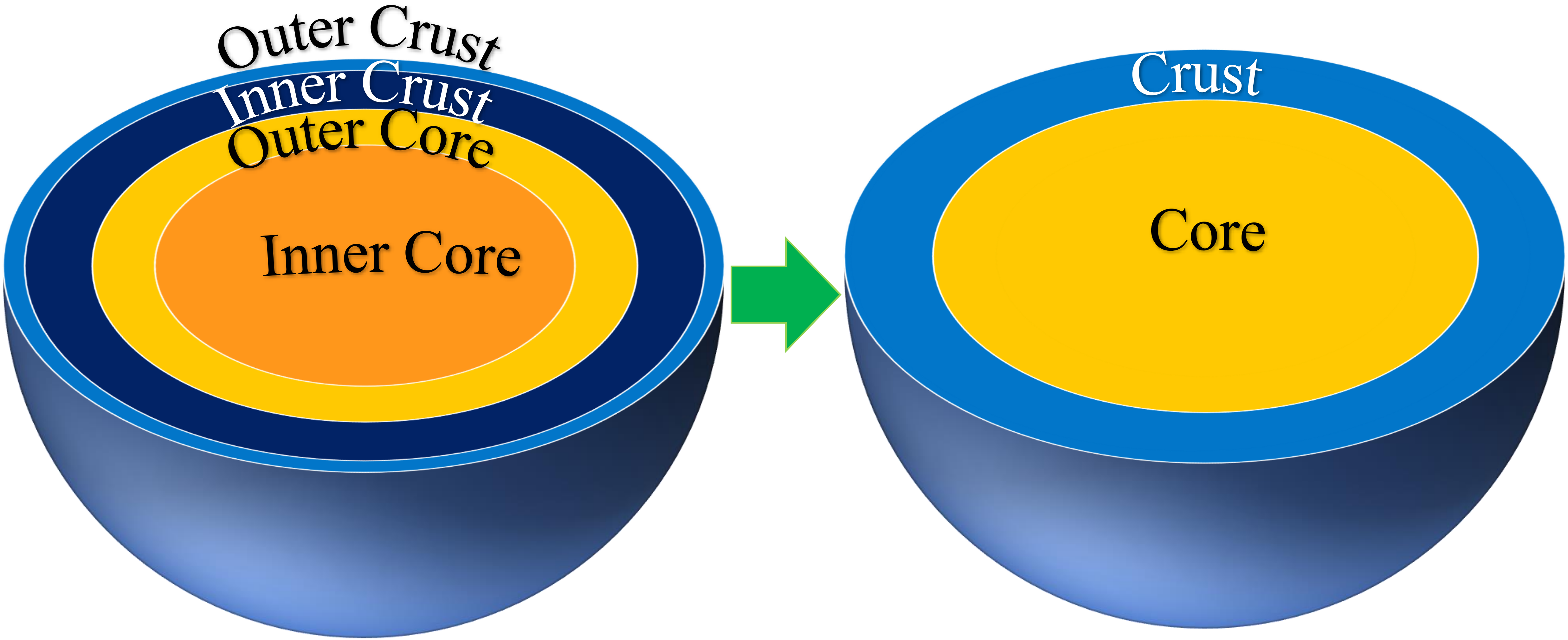


OUTER CRUST
$\sim 0.1 - 0.5$ km
Crystal lattice of
nuclei + electrons

INNER CRUST
$\sim 1\ km$
Crystal lattice of
nuclei + electrons
Free neutrons

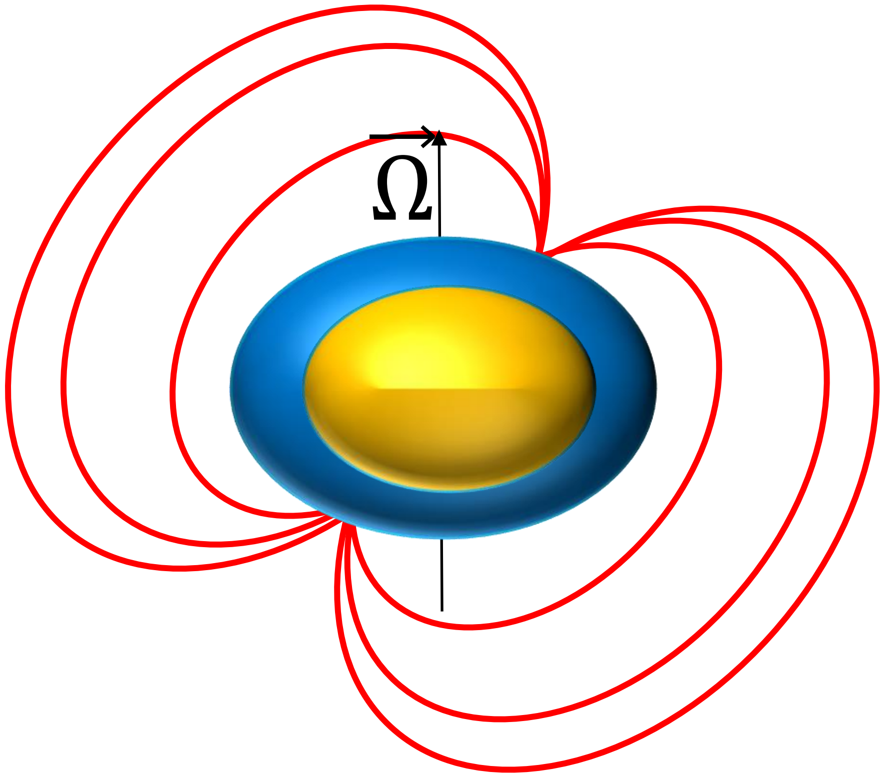
CRUST

Work scheme



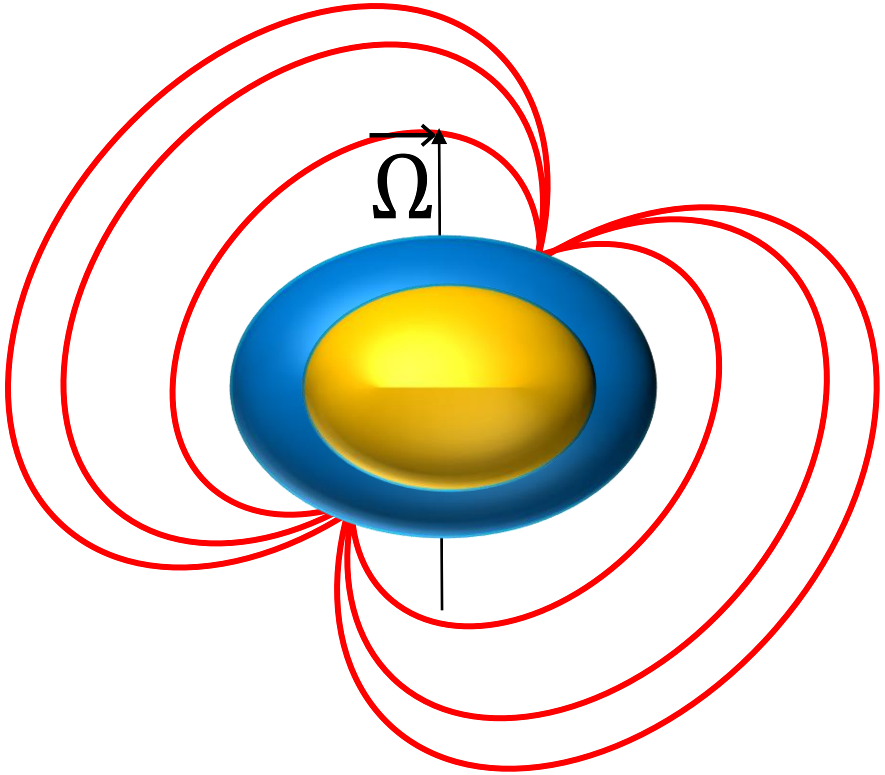
Model's hypothesis summary

- Newtonian Gravity;
- Slow rotation; $$\frac{\Omega^2 R^2}{v_K^2} \cong 10^{-4} \ll 1$$
- Elastic crust;



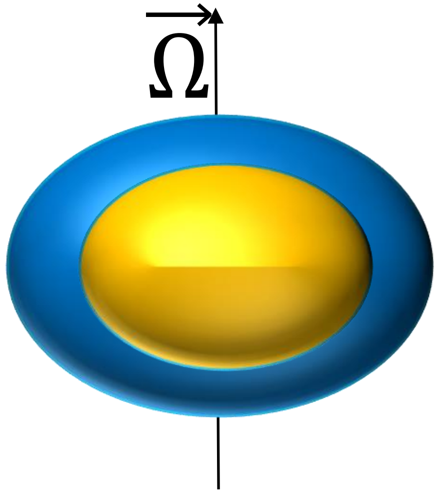
Model's hypothesis summary

- Newtonian Gravity;
- Slow rotation; $\frac{\Omega^2 R^2}{v_K^2} \cong 10^{-4} \ll 1$
- Elastic crust;
- Initial unstressed configuration;



Model's hypothesis summary

- Newtonian Gravity;
- Slow rotation; $$\frac{\Omega^2 R^2}{v_K^2} \cong 10^{-4} \ll 1$$
- Elastic crust;
- Initial unstressed configuration;
- No magnetic field;



Equations

$$\vec{\nabla} \cdot \bar{\bar{T}} - \rho \vec{\nabla} \Phi + \vec{h} = 0$$

EQUILIBRIUM

Equations

$$\vec{\nabla} \cdot \bar{\bar{T}} - \rho \vec{\nabla} \Phi + \vec{h} = 0$$

EQUILIBRIUM

non-conservative forces
e.g. pinning

Equations

$$\vec{\nabla} \cdot \overline{\overline{T}} - \rho \vec{\nabla} \Phi + \vec{h} = 0$$

EQUILIBRIUM

$$\nabla^2 \Phi = 4\pi G \rho + 2\Omega^2$$

POISSON (RIGID ROTATION)

Equations

EQUILIBRIUM

POISSON

PERTURBATION

EQUILIBRIUM

POISSON (RIGID ROTATION)

SPHERICAL HARMONICS ESPANSION

$$A(r,\theta,\varphi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} A_{lm}(r) Y_{2m}(\theta,\varphi)$$

Equations

EQUILIBRIUM

POISSON

PERTURBATION →

EQUILIBRIUM

POISSON (RIGID ROTATION)

\+ SPHERICAL HARMONICS ESPANSION

$$\dot{\vec{y}}_{lm}(r) = \bar{\bar{A}}_l(r)\vec{y}_{lm}(r)$$

The elastic properties of matter $(\mu(r), \gamma(r))$ are in the matrix $\bar{\bar{A}}_l(r)$

Equations

$$\dot{\vec{y}}_{lm}(r) = \bar{\bar{A}}_l(r)\vec{y}_{lm}(r)$$

$$\vec{y}_{lm}(r) = \begin{pmatrix} U_{lm}(r) \\ V_{lm}(r) \\ R_{lm}(r) \\ S_{lm}(r) \\ \Phi_{lm}(r) \\ Q_{lm}(r) \end{pmatrix}$$

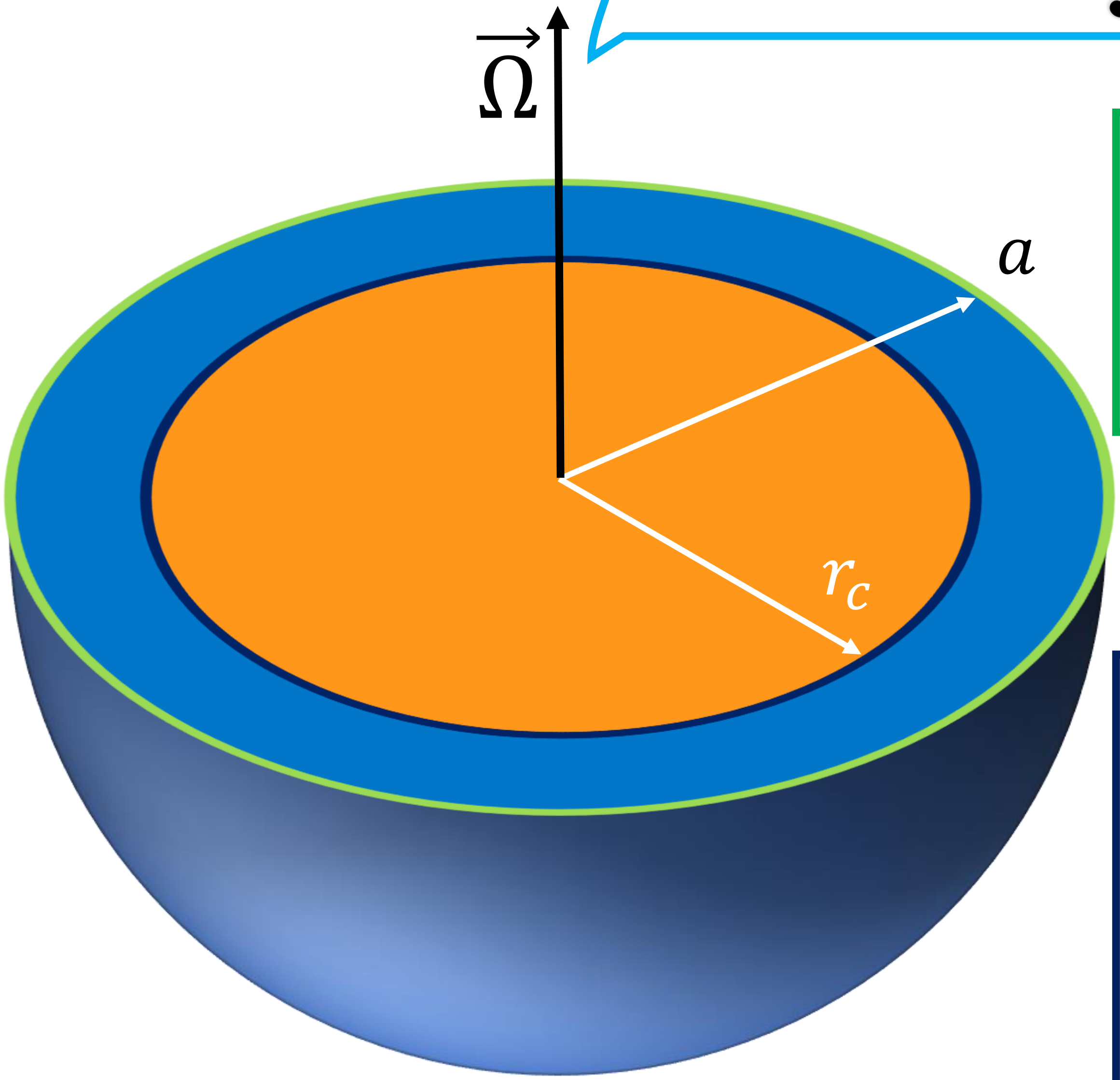
RADIAL DISPLACEMENT
TANGENTIAL DISPLACEMENT

RADIAL STRESS
TANGENTIAL STRESS

POTENTIAL
POTENTIAL STRESS

$$Q_{lm} = \partial_r\Phi + \frac{l+1}{r}\Phi + 4\pi G\rho_0 U_{lm}(r)$$

Boundary conditions



$S = 0$ (*no shear stress in vacuum*)

$T_{rr} = 0$ a $r = a$

Q *continuous at* $r = a$

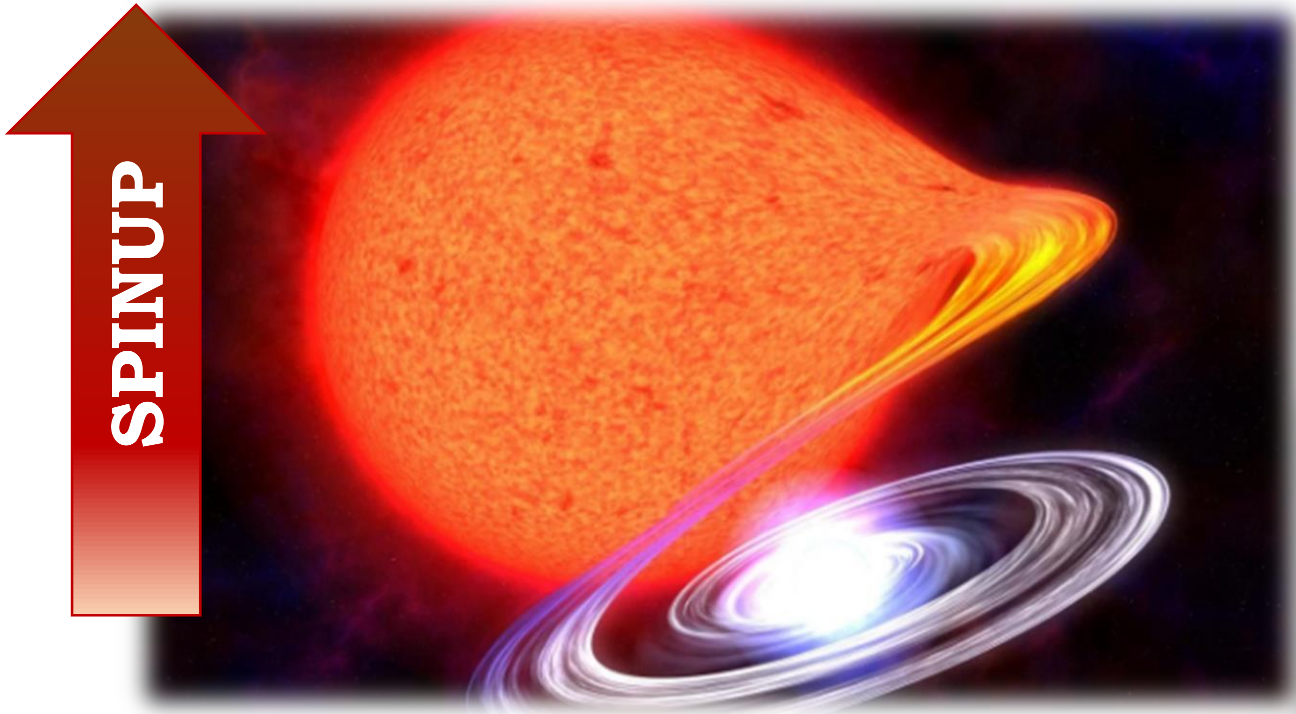
$S = 0$

(*fluid cannot support shear stress*)

T_{rr} continuous at $r = r_c$

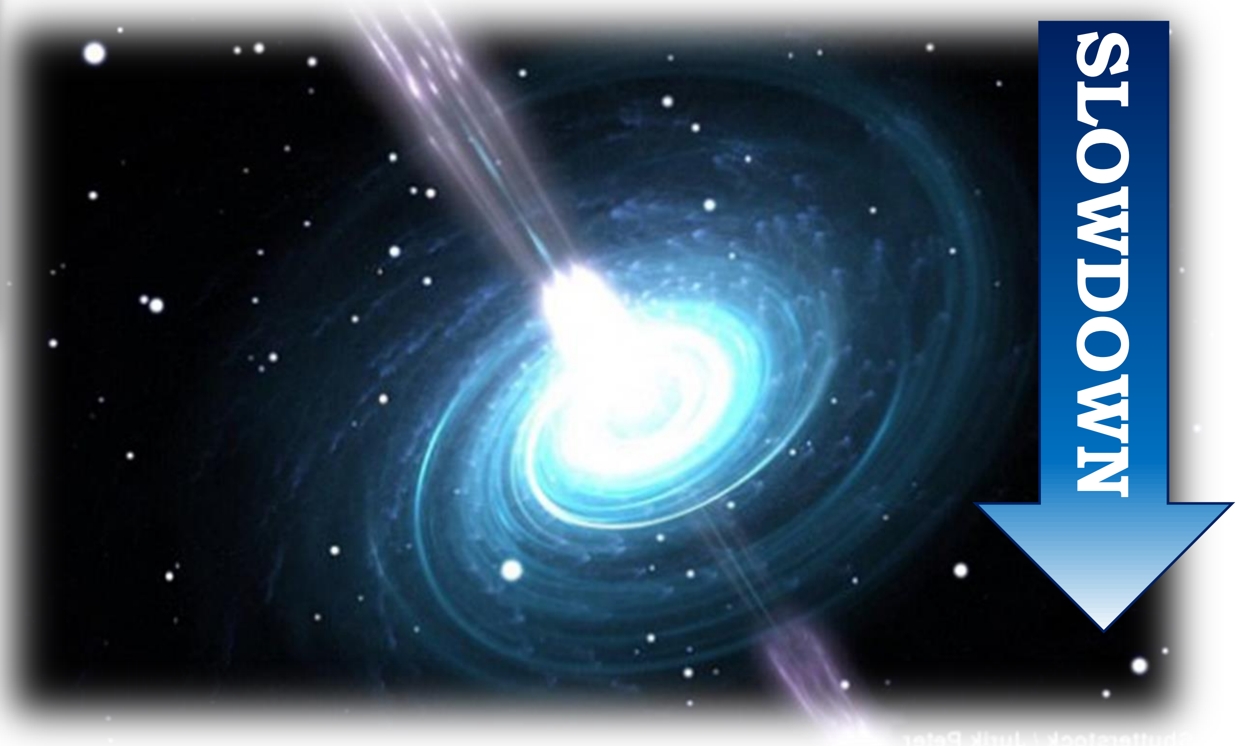
Q *continuous at* $r = r_c$

Rotation



Accreting Neutron Star

Isolated Pulsar



Adiabatic Index

SLOW

$$\tau_{dynamical} \gg \tau_{reactions}$$

$$\gamma_e = \frac{n_b}{P}\frac{\partial P(n_b)}{\partial n_b}$$

FAST

$$\tau_{dynamical} \ll \tau_{reactions}$$

$$\gamma_{frozen} = \frac{n_b}{P}\frac{\partial P(n_b, x_e, x_p, x_n, \ldots)}{\partial x_i}$$

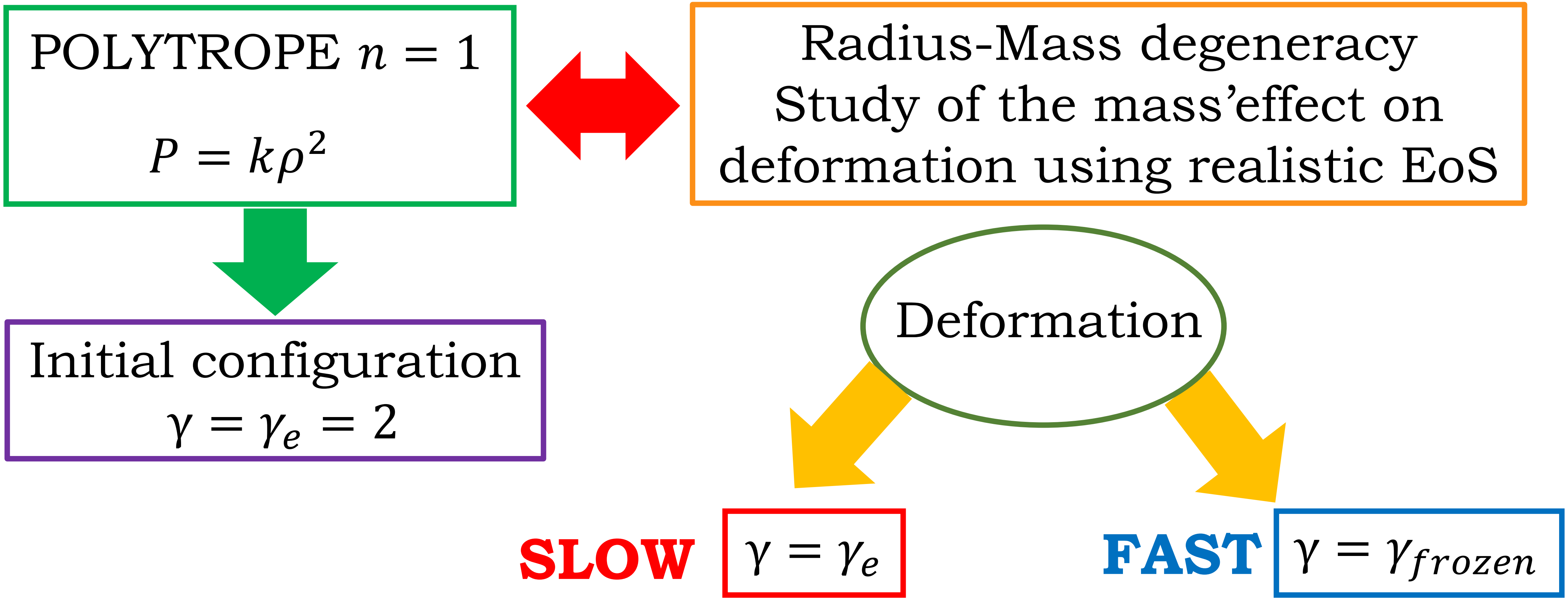
POLYTROPE $n = 1$

$$P = k\rho^2$$

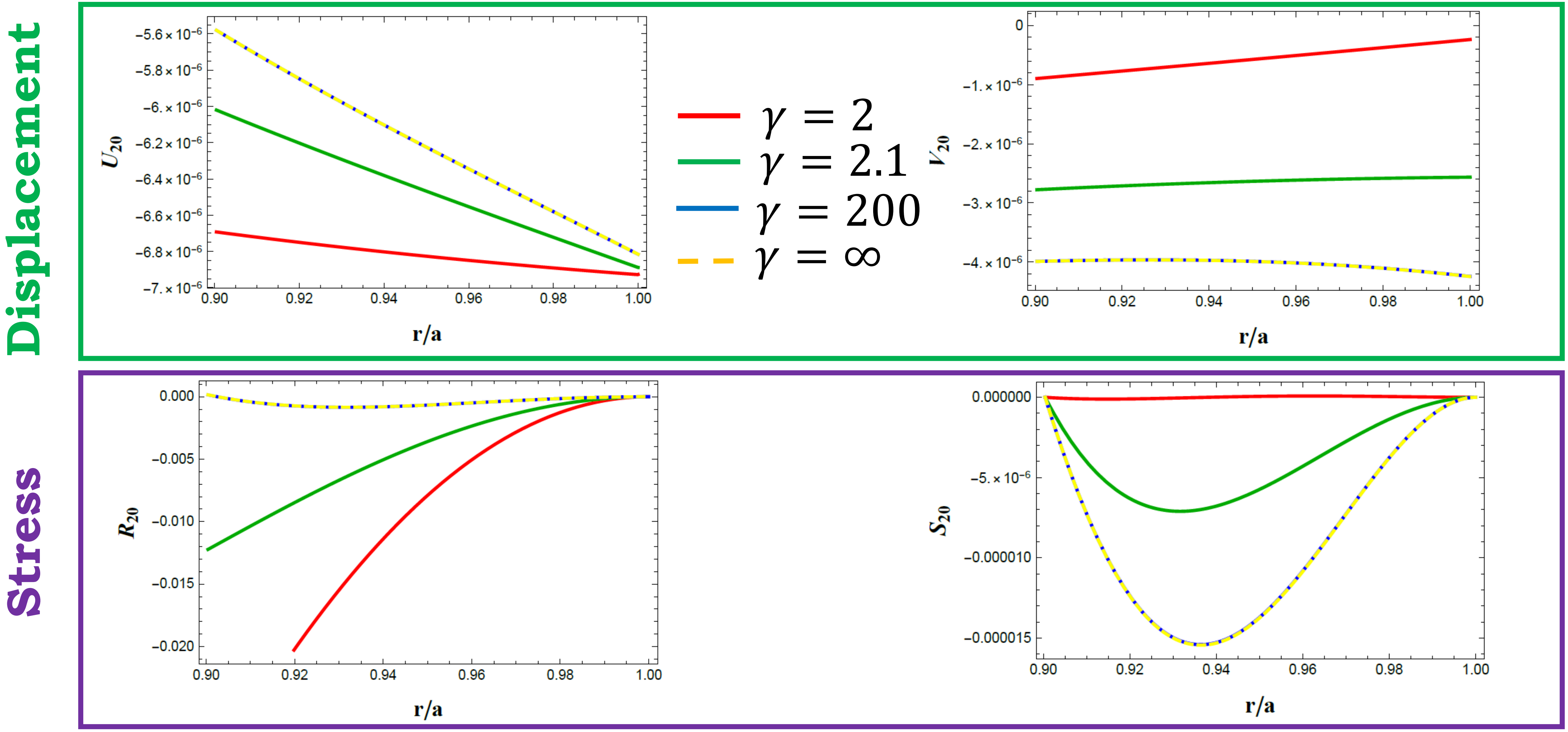
$$\gamma_e = 2$$

$$\gamma_{frozen} = 2.1, 200, \infty$$

Our model


POLYTROPE $n = 1$
$P = k\rho^2$
Radius-Mass degeneracy
Study of the mass'effect on deformation using realistic EoS
Initial configuration
$\gamma = \gamma_e = 2$
Deformation
SLOW
$\gamma = \gamma_e$
FAST
$\gamma = \gamma_{frozen}$

Rotation: $l = 2, m = 0$ harmonic



Physical explanation

$\kappa = \gamma P$

$$\frac{\mu}{\kappa} \approx 10^{-3}$$

FLUID LIMIT $\mu = 0$

$$\gamma = \gamma_e + \delta\gamma$$

$$\gamma_e = \frac{\rho_0}{P_0}\frac{\partial P}{\partial \rho}$$

FLUID EQUILIBRIUM

$$\frac{P_0}{\rho_0^2}\partial_r \rho_0 (\gamma - \gamma_e)\chi_{lm} = 0$$

VOLUME CHANGE χ_{lm}

Failure criterion

σ_{ij} Strain tensor

TRESCA CRITERION

$$\varepsilon_{Max} - \varepsilon_{Min} = \alpha \geq \frac{\sigma_{max}}{2}$$

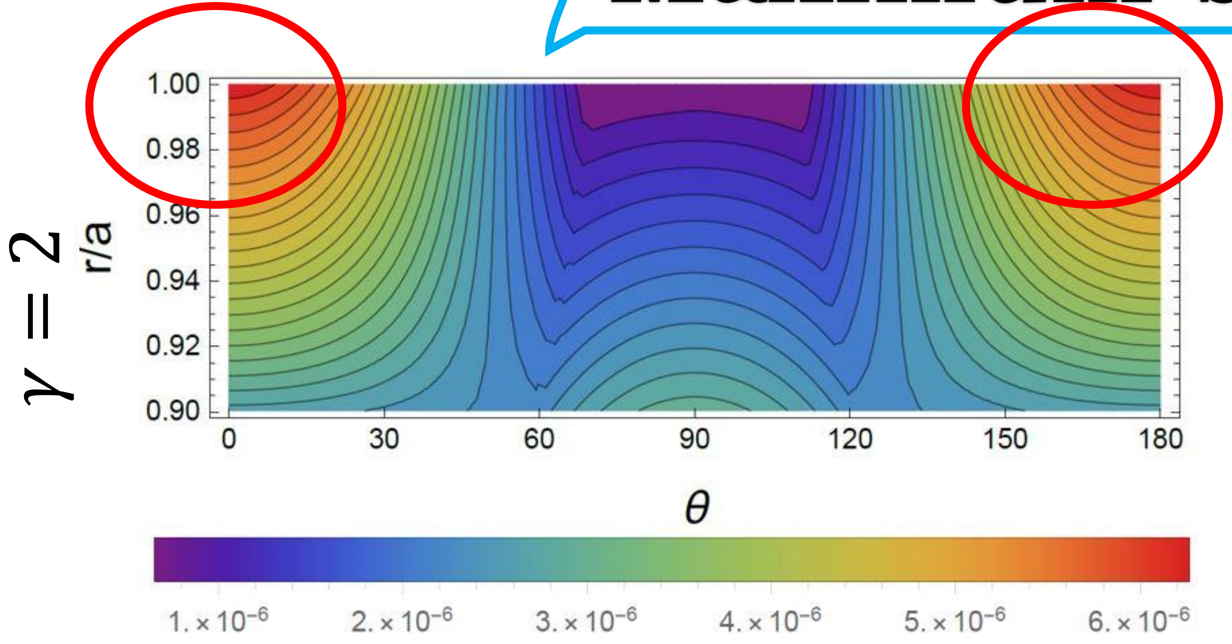
ε Eigenvalues

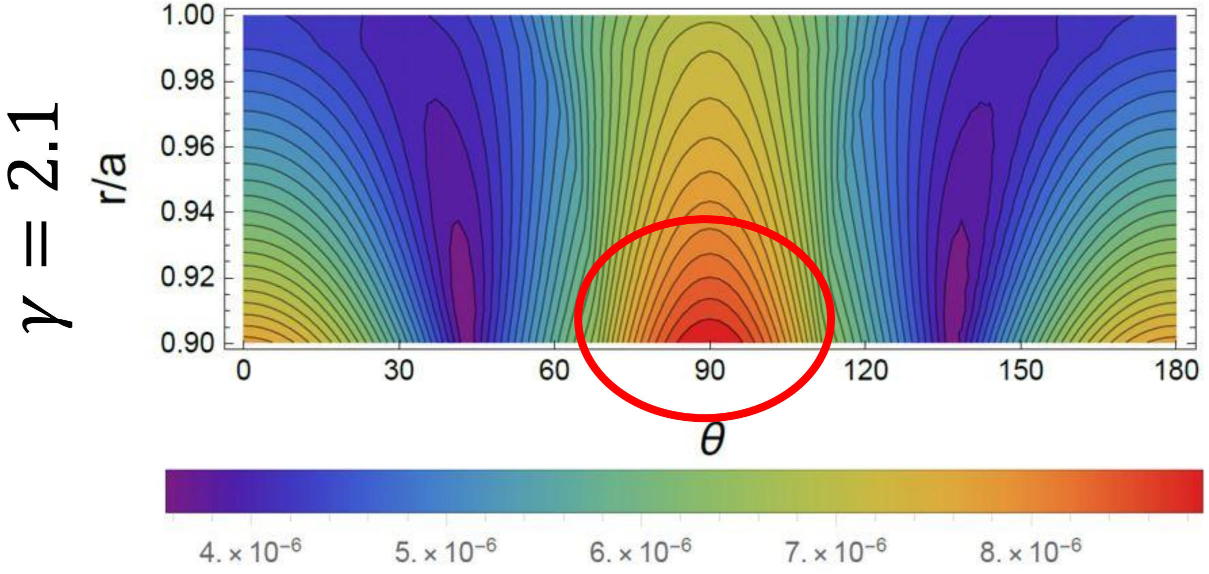
$\vec{\varepsilon}$ Eigenvectors

VON MISES CRITERION

$$\bar{\sigma} = \sqrt{\sigma_{ij}\sigma^{ij}} \geq \sigma_{max}$$

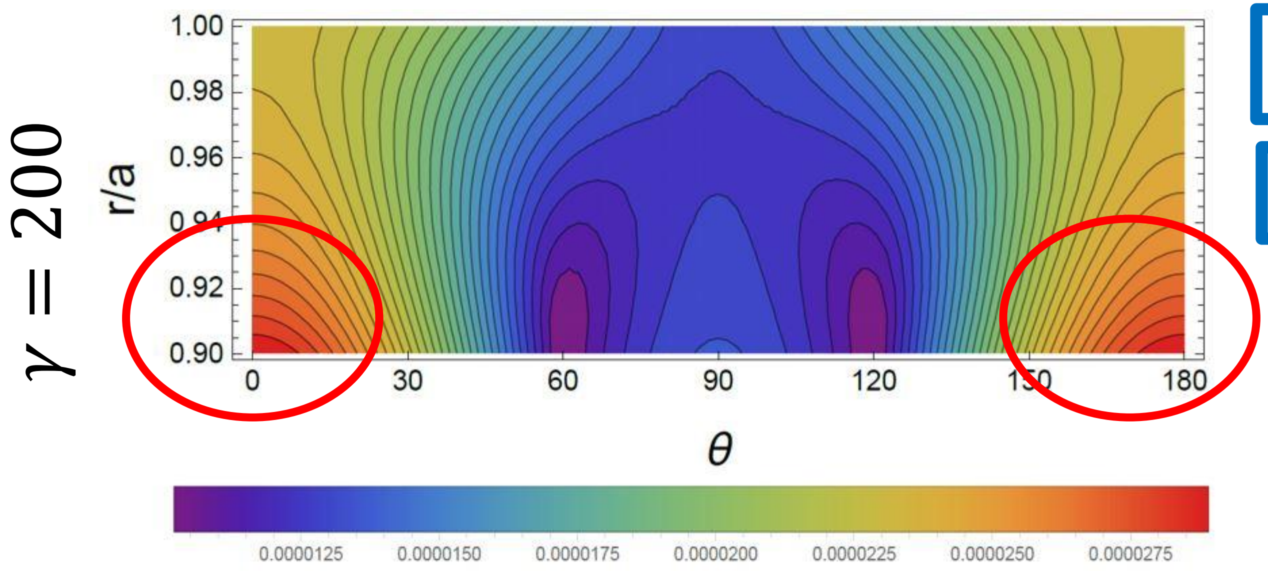
Maximum strain angle





Inter-glitch

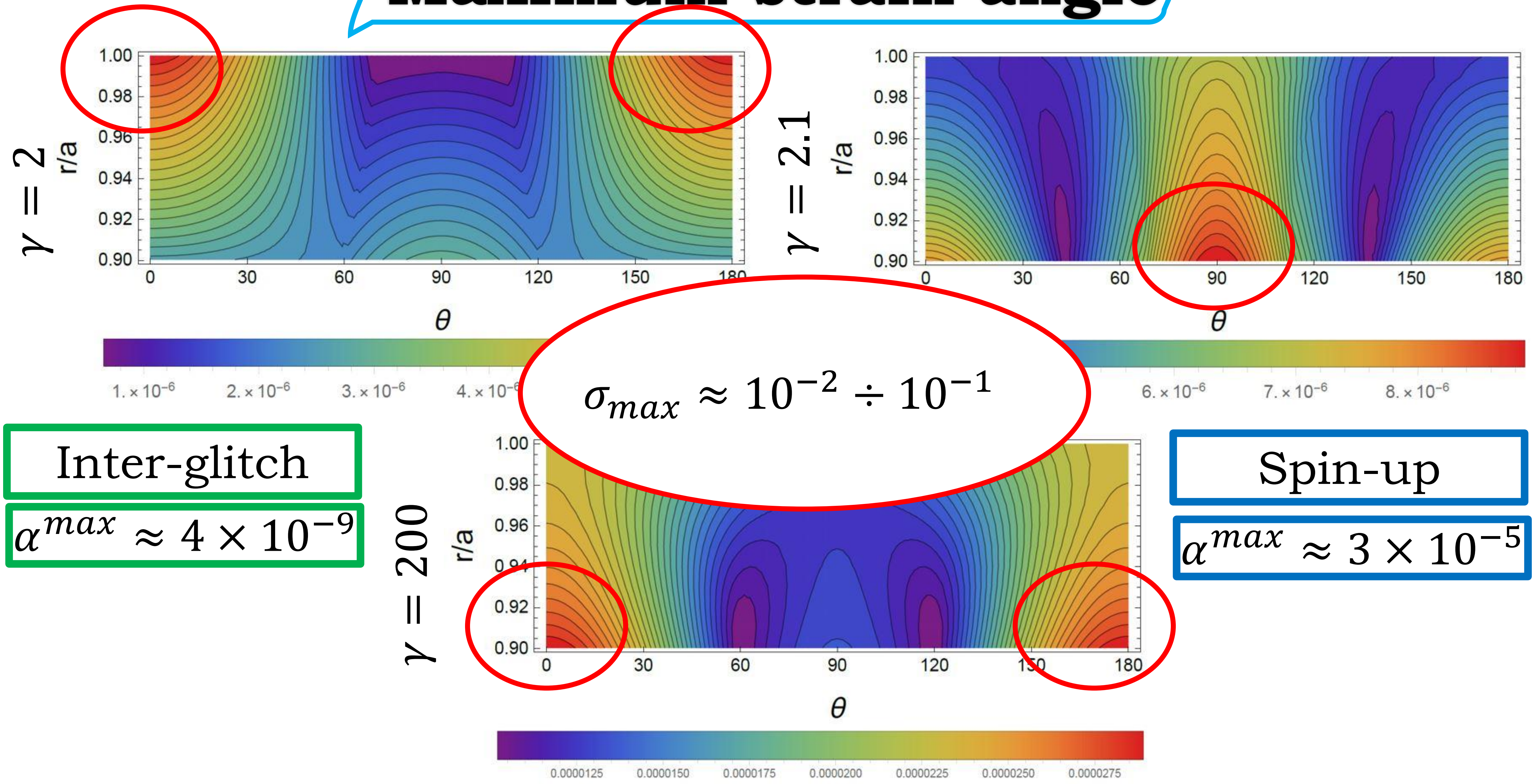
$\alpha^{max} \approx 4 \times 10^{-9}$



Spin-up

$\alpha^{max} \approx 3 \times 10^{-5}$

Maximum strain angle



Inter-glitch

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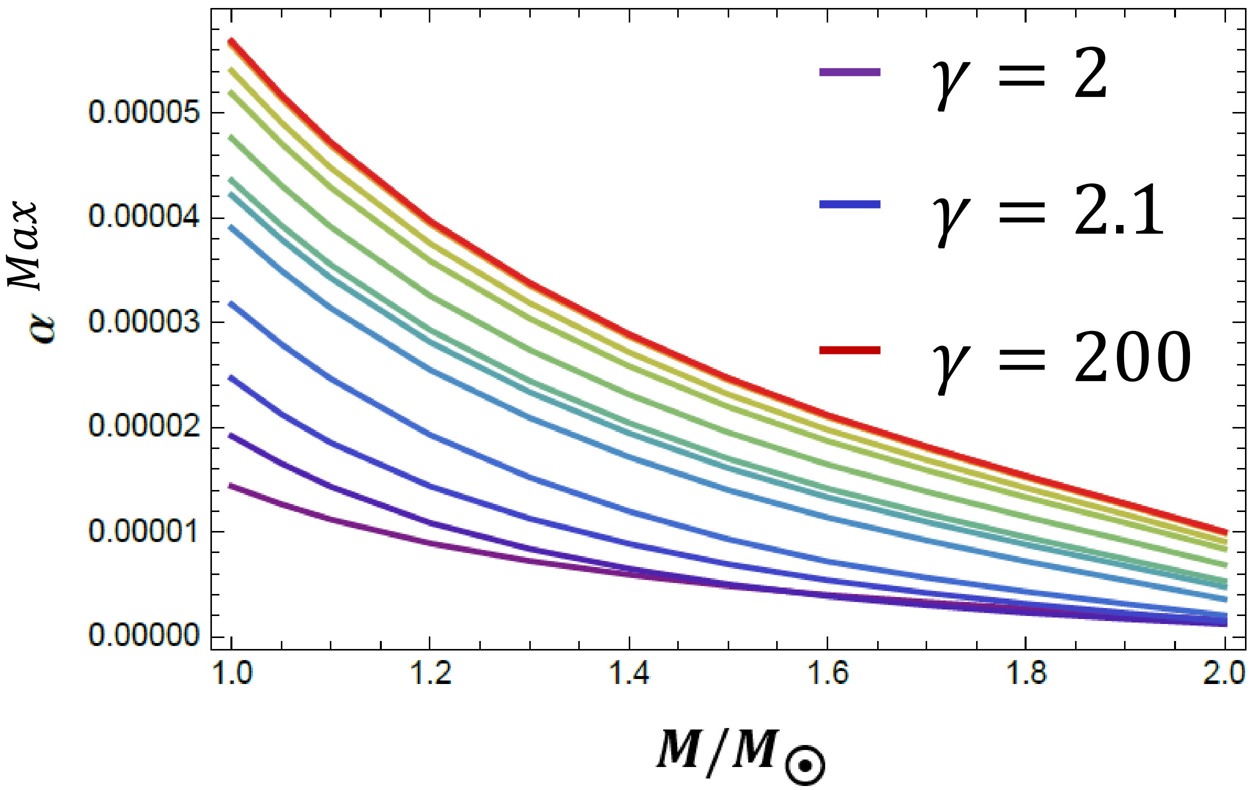
Spin-up

$\alpha^{max} \approx 3 \times 10^{-5}$

Effect of the mass



Maximum strain angle is a decreasing function of the stellar mass

Conclusion

Construction of a Newtonian model for the study of different loads on neutron stars.

Big impact of the adiabatic index on star's response: the reason lies in the small ratio $\frac{\mu}{\kappa} \approx 10^{-3}$.

Strain angle is a decreasing function of the stellar mass.

Challenge of the idea of crust breaking due to rigid rotation, used as example as possible trigger of glitches.

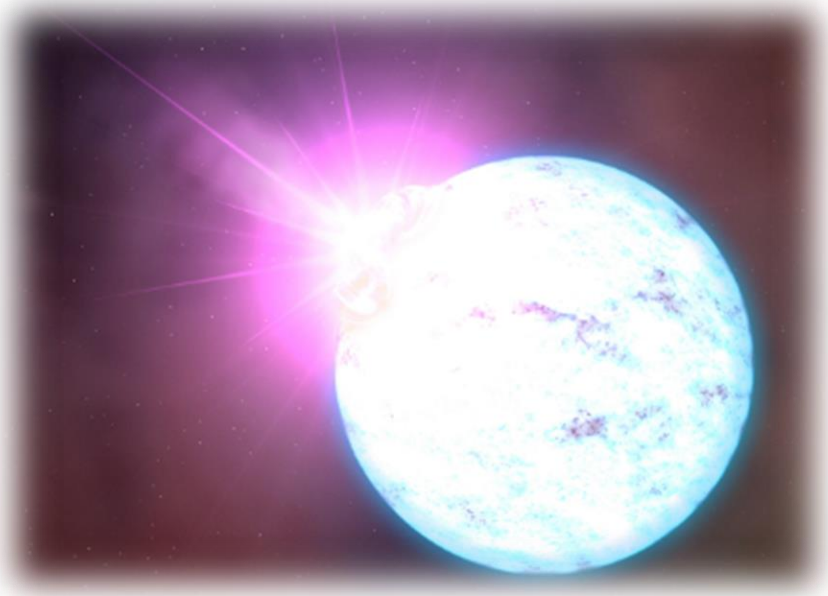
Open questions

What are the elastic properties of the inner crust? What is the impact of the superfluid?

Development of elastic models with two components: solid elastic layer and superfluid in the same point.

MOUNTAINS

Study of non-axial perturbation due to quakes on star. Evaluation of the emitted gravitational waves.




Thank you for your attention

Physical explanation

$$\boldsymbol{\nabla}\rho_0 = \frac{\partial\rho}{\partial P}\Big|_{s_0,c_0}\boldsymbol{\nabla}P + \frac{\partial\rho}{\partial s}\Big|_{P_0,c_0}\boldsymbol{\nabla}s + \frac{\partial\rho}{\partial c}\Big|_{P_0,s_0}\boldsymbol{\nabla}c =$$

$$= -\frac{\rho_0^2 g}{P_0}\left(\frac{1}{\gamma_e} + \frac{1}{\gamma^*}\right)\hat{\boldsymbol{e}}_r = -\frac{\rho_0^2 g}{P_0}\frac{1}{\gamma}\hat{\boldsymbol{e}}_r$$

$$\gamma_e = \frac{P_0}{\rho_0}\frac{\partial P}{\partial\rho}$$

$$\kappa = \gamma P$$

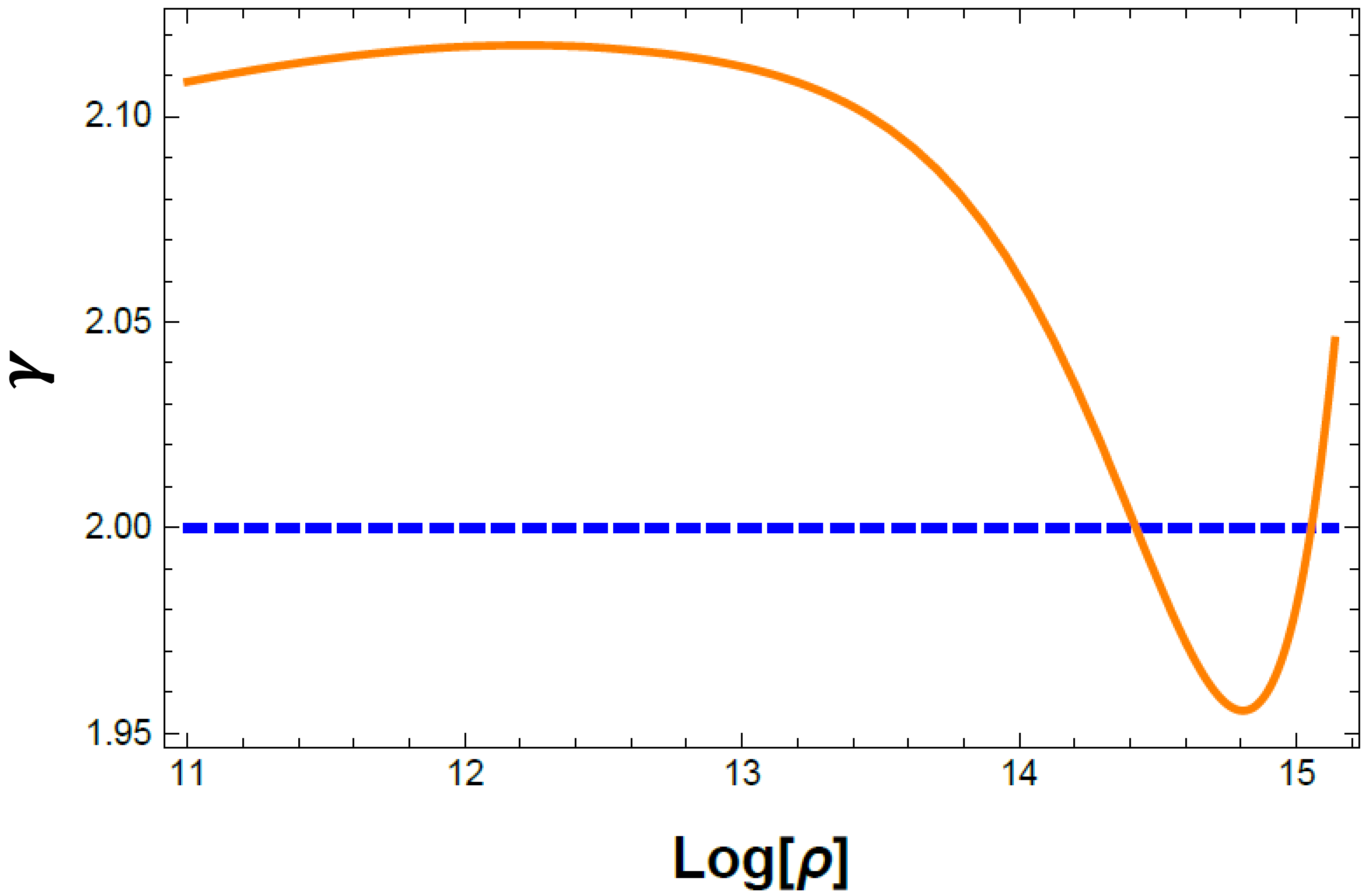
$$\frac{\mu}{\kappa} \approx 10^{-3}$$

FLUID LIMIT $\mu = 0$

$$\frac{\gamma P_0}{\rho_0^2}\left(\partial_r\rho_0 + \frac{\rho_0^2 g}{\kappa}\right)\chi_l = 0$$

$$\frac{P_0}{\rho_0^2}\partial_r\rho_0(\gamma - \gamma_e)\chi_l = 0$$

Warning



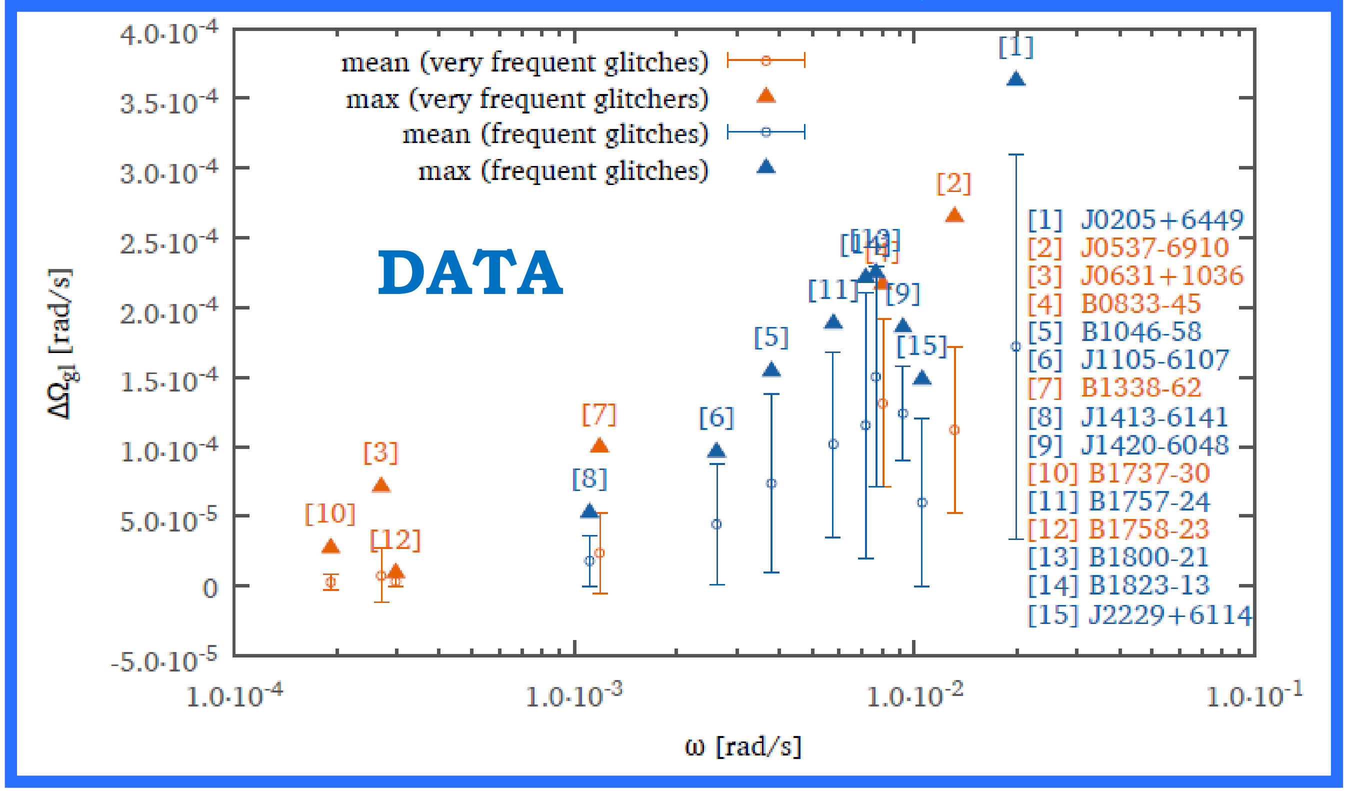
$$\frac{\gamma P_0}{\rho_0^2}\left(\partial_r \rho_0 + \frac{\rho_0^2 g}{\gamma P}\right)\chi_l = 0$$

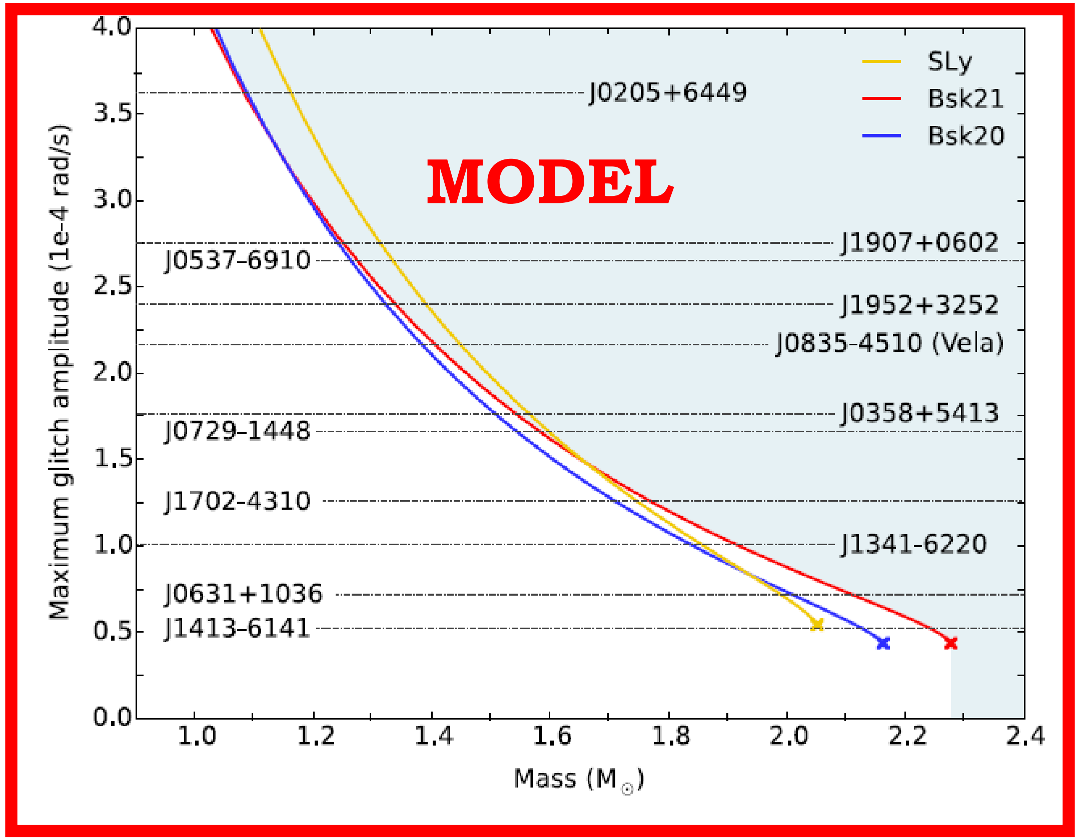
$$\gamma = -\frac{\rho_0^2 g}{P \partial_r \rho_0}$$

General Relativity

Newtonian Gravity

Trigger?





$$\omega = |\dot{\Omega}| \times \langle T_{interglitches} \rangle$$

DATA+MODEL

Large ω ⟷ Large $\frac{\Delta\Omega}{\Omega}$ ⟷ Small mass

Trigger?

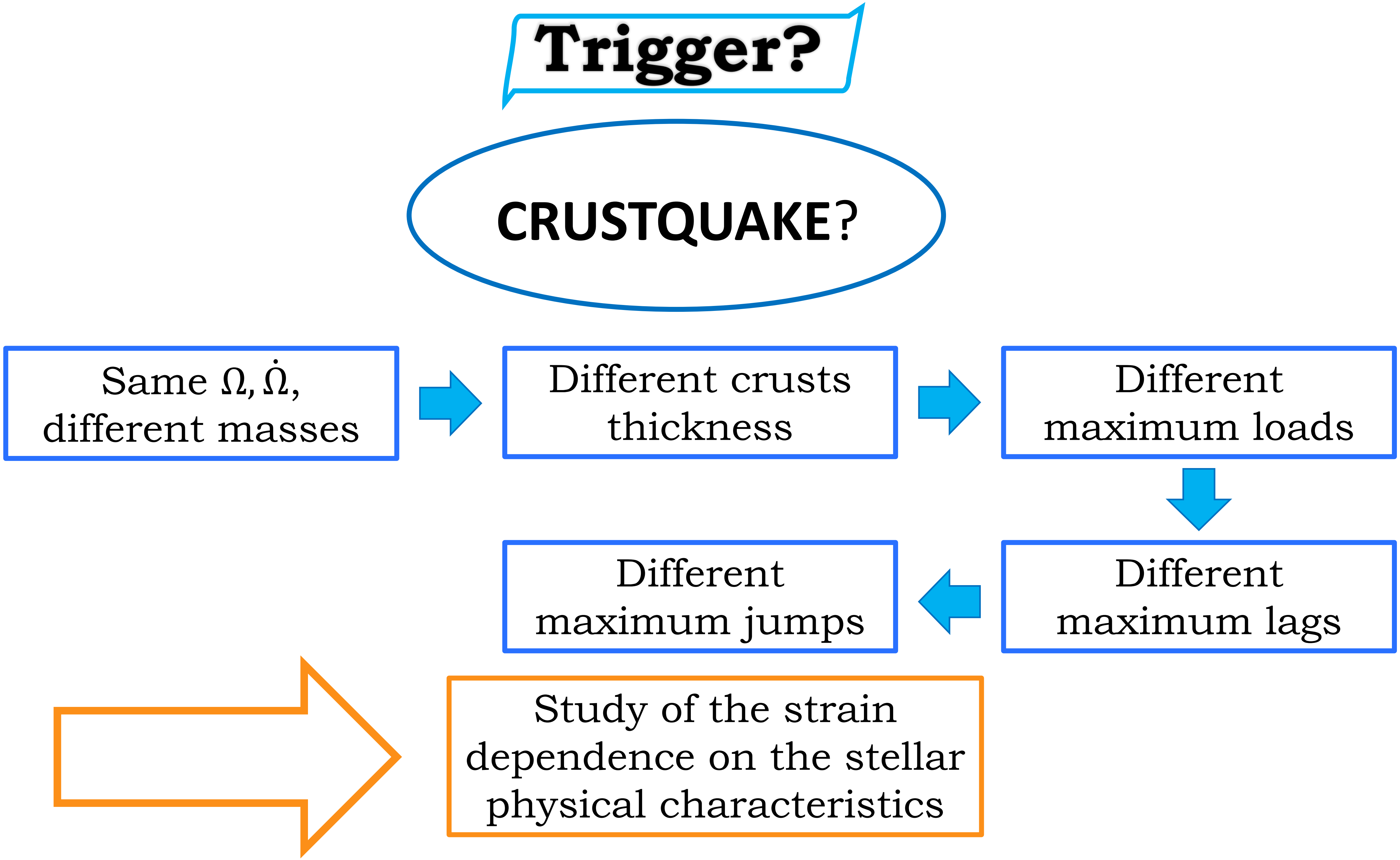
DATA+MODEL

Large ω ⟷ Large $\frac{\Delta\Omega}{\Omega}$ ⟷ Small mass

TOV EQUATIONS+EOS

Heavier stars have thinner crusts and smaller radii

CRUSTQUAKE?


Trigger?
CRUSTQUAKE?
Same $\Omega, \dot{\Omega}$, different masses
Different crusts thickness
Different maximum loads
Different maximum lags
Different maximum jumps
Study of the strain dependence on the stellar physical characteristics

Curt Cutler, Greg Ushomirsky and Bennett Link

The crustal rigidity of a neutron star and implication for PSR B182811 and other precession candidates

The Astrophysical Journal, 588:975–991, 2003 May 10

Incompressible

Incompressible medium

$\vec{\nabla} \cdot \vec{u} = 0$

$\frac{\mu}{\kappa} \approx 10^{-3}$

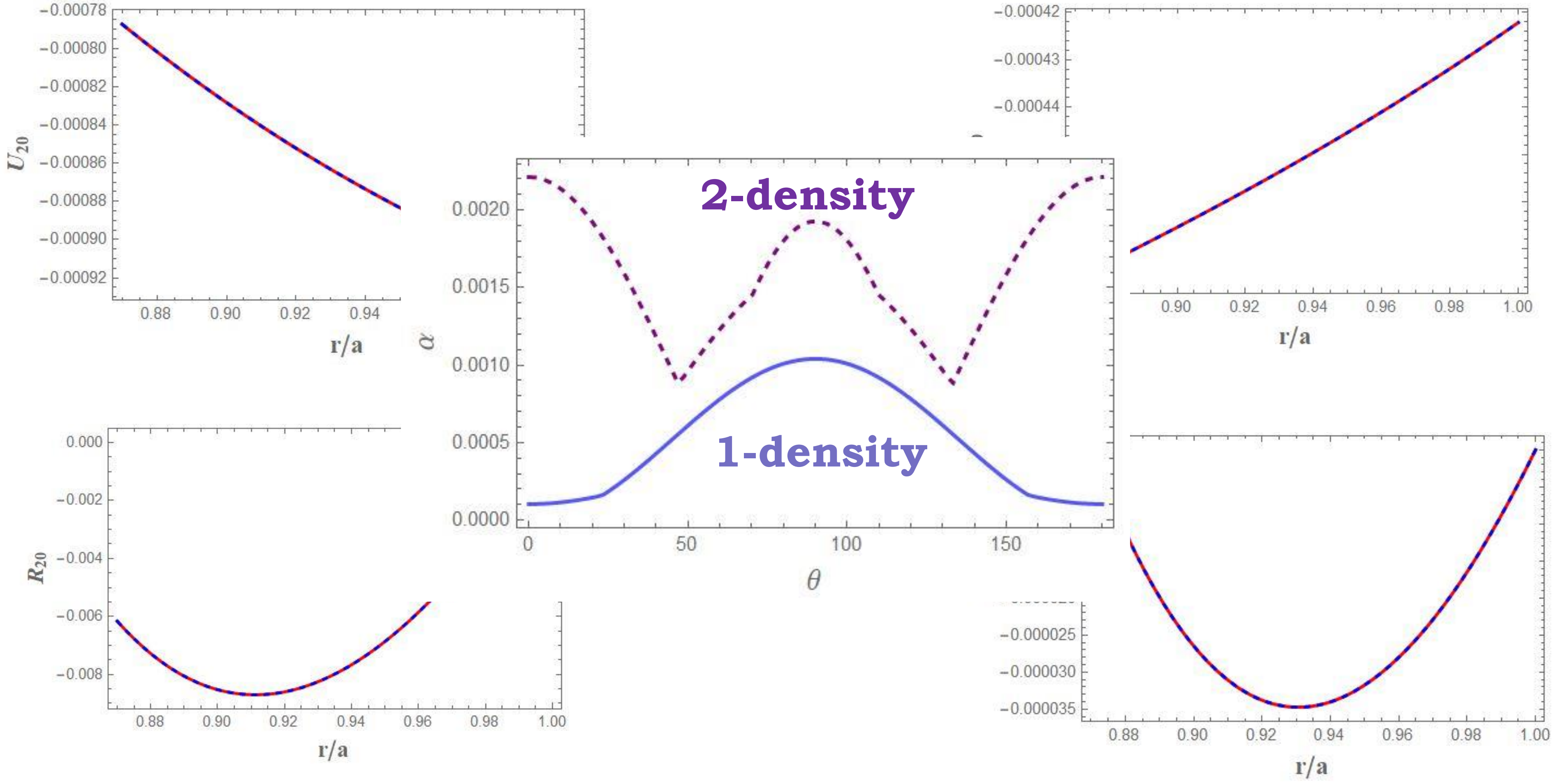
Homogeneous crust
Homogeneous core

$\rho_{crust} = cost$
$\rho_{core} = cost$

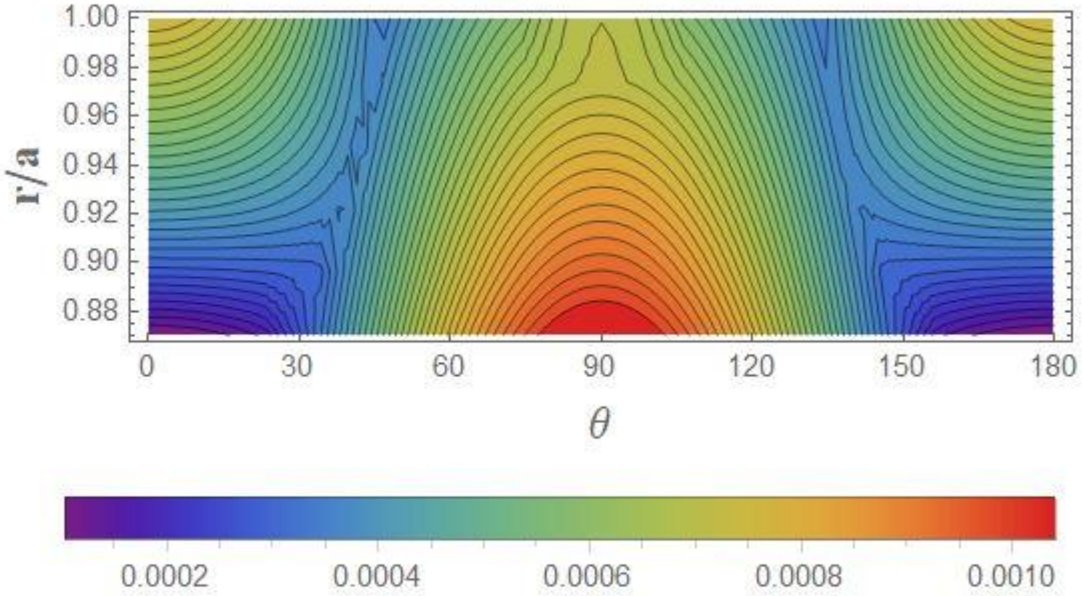
$\gamma, \mu_{crust} = cost$
$\gamma, \mu_{core} = cost$
$\mu_{core} = 0$

Analytical solution
First code check

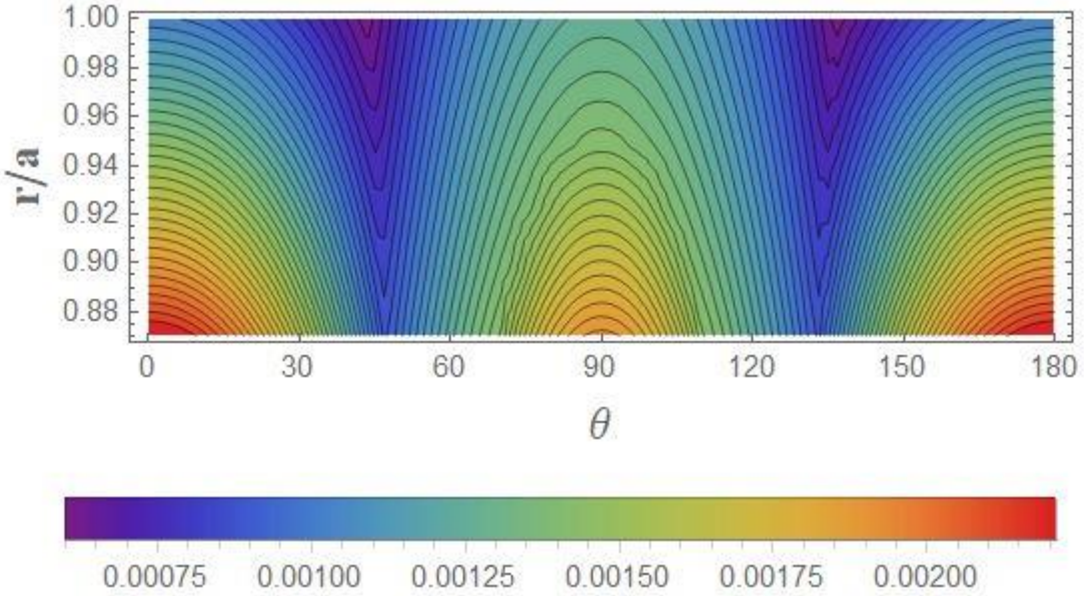
Code check



Incompressible

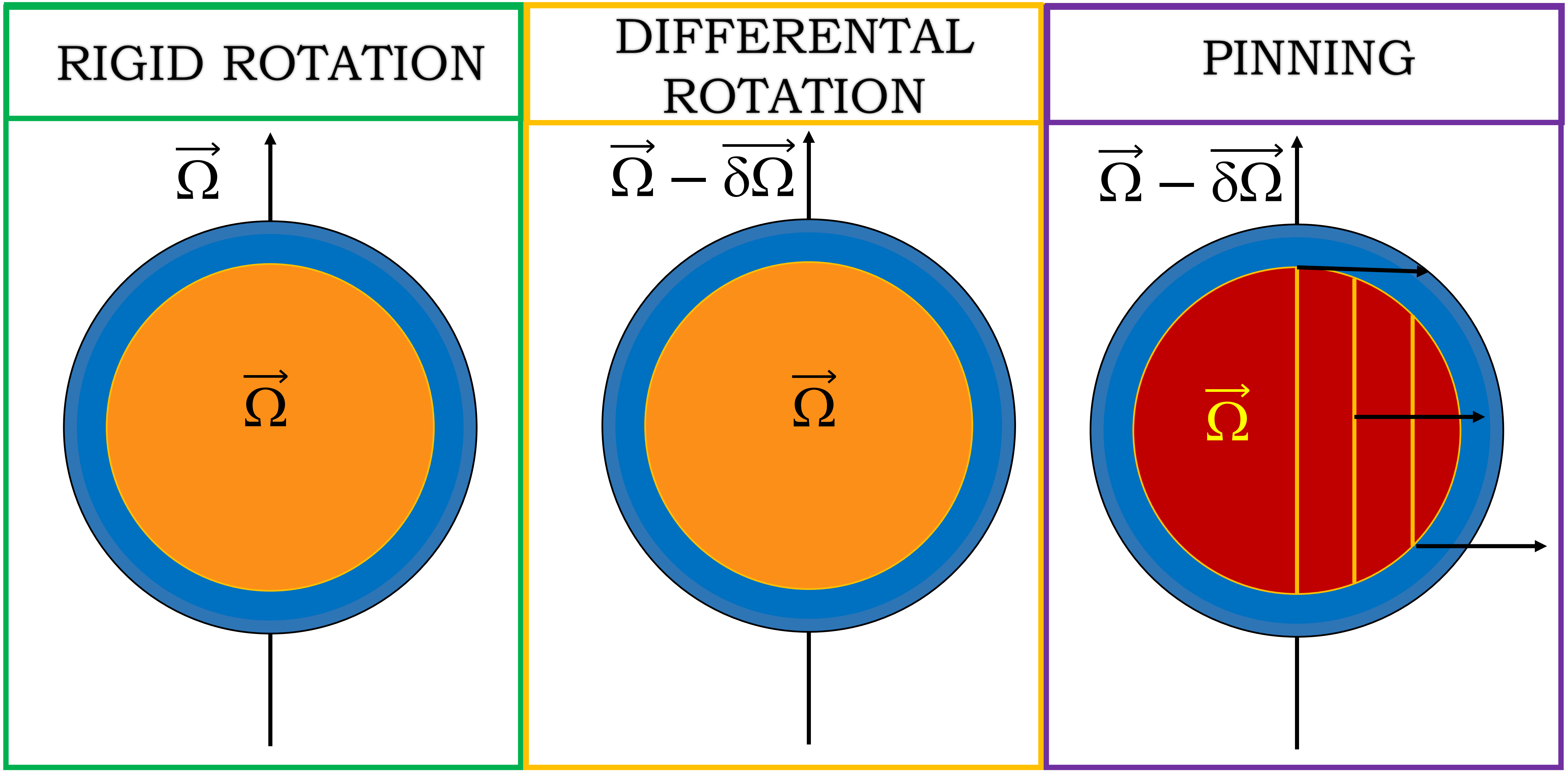


Incompressible, homogeneous

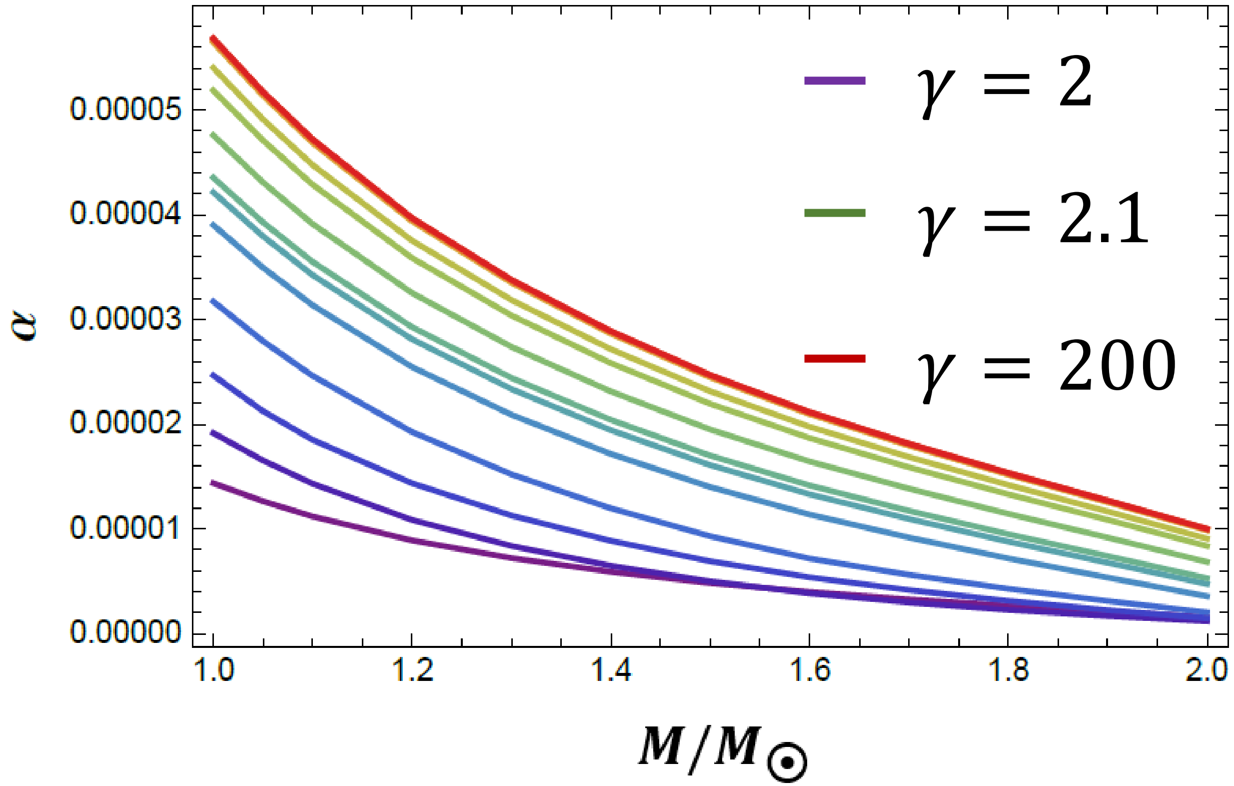


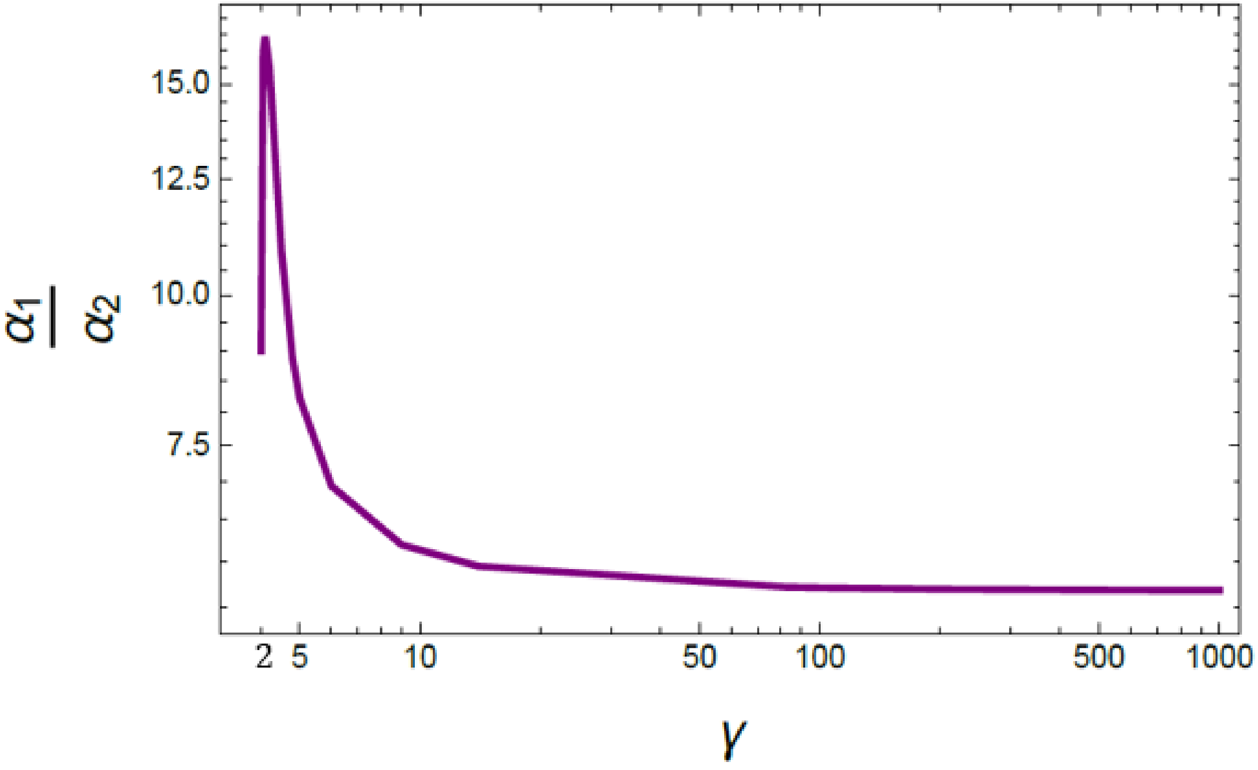
Incompressible, 2 different densities

Types of loads



Effect of the mass

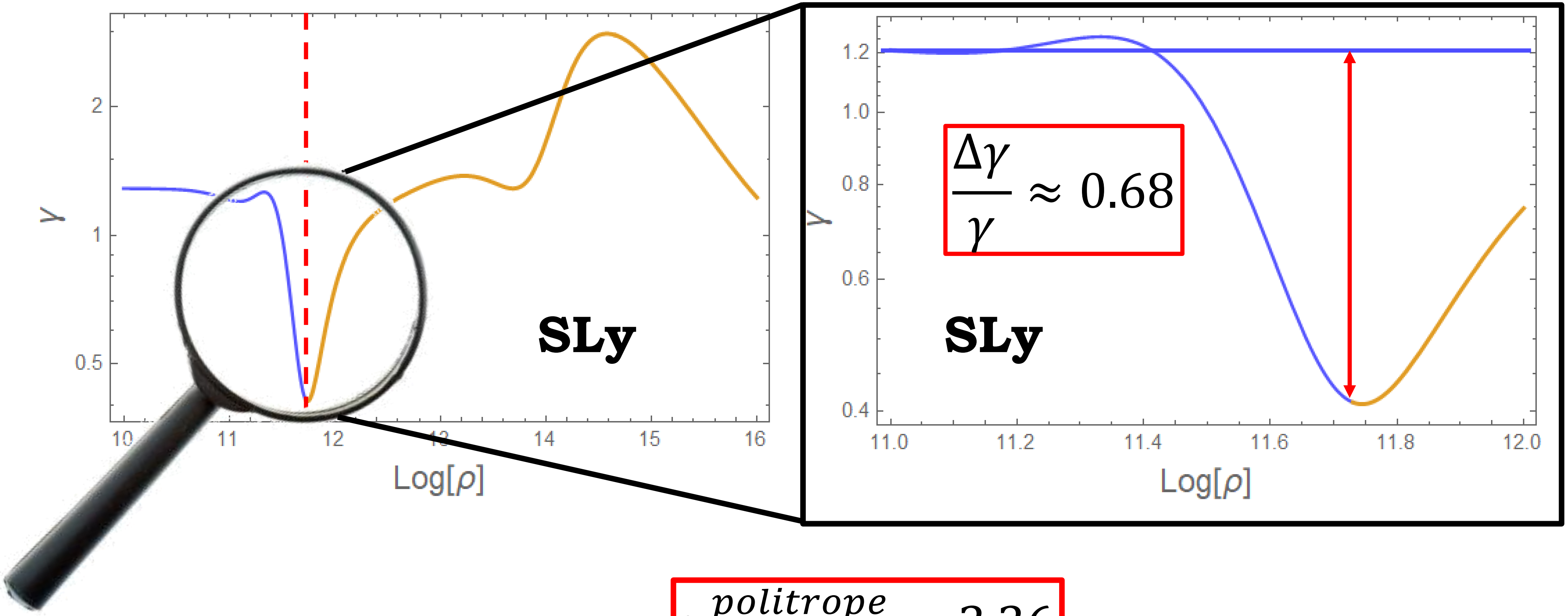




Maximum strain angle is a decreasing function of the stellar mass

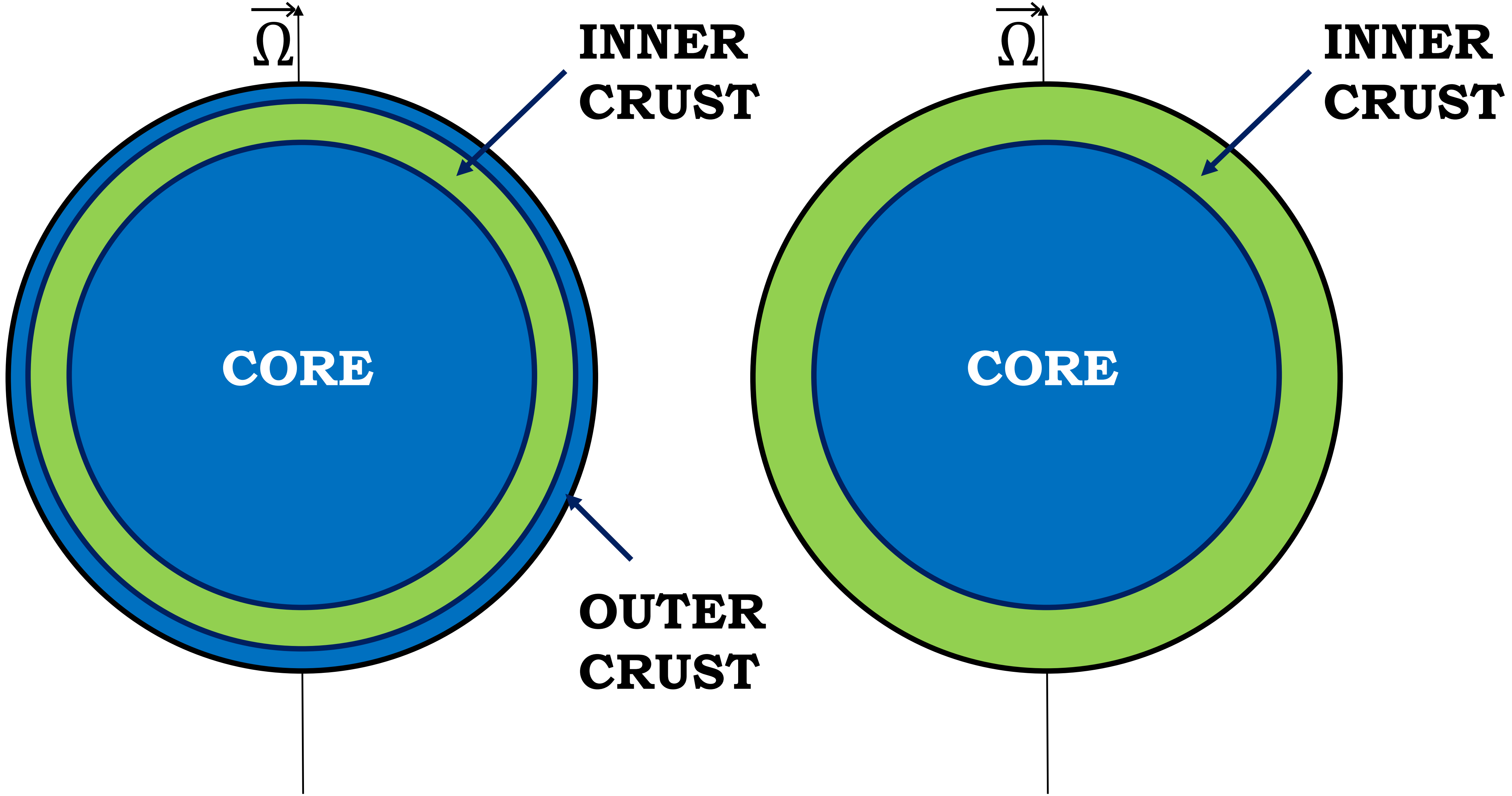
Warning

$$\gamma_{frozen} > \gamma_e$$

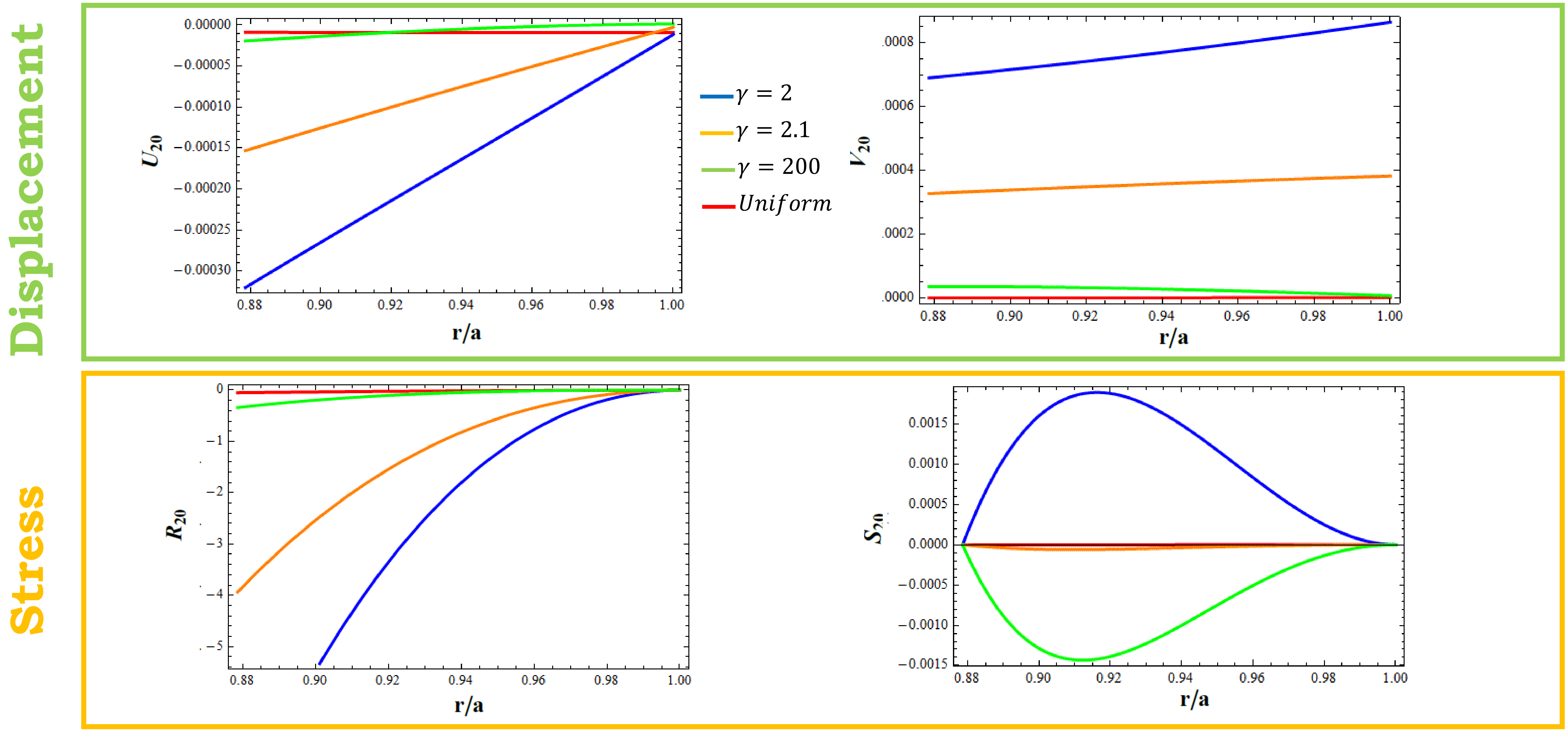


$$\gamma_{frozen}^{politrope} \approx 3.36$$

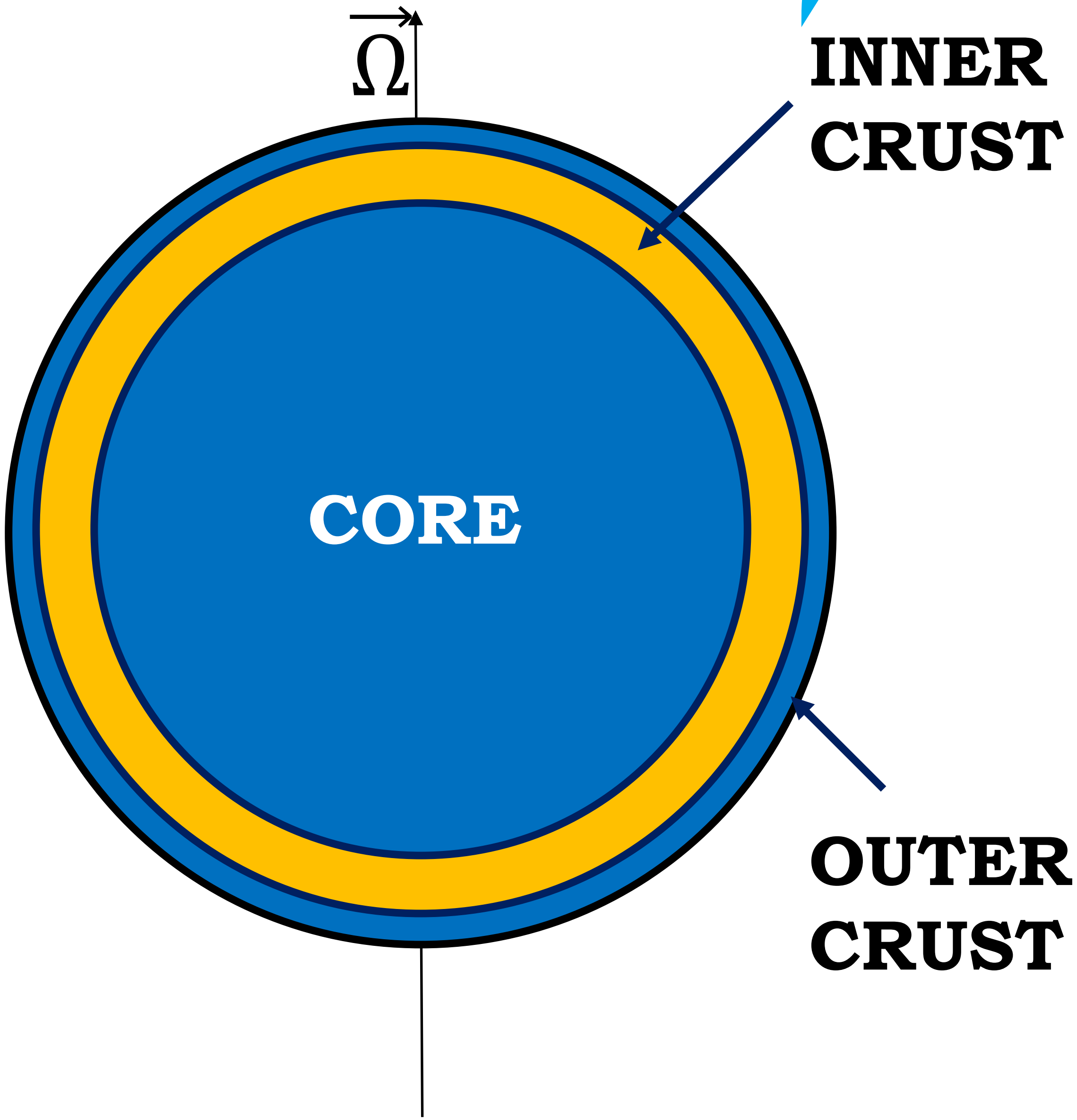
Differential rotation



Differential rotation: $l = 2, m = 0$ harmonic

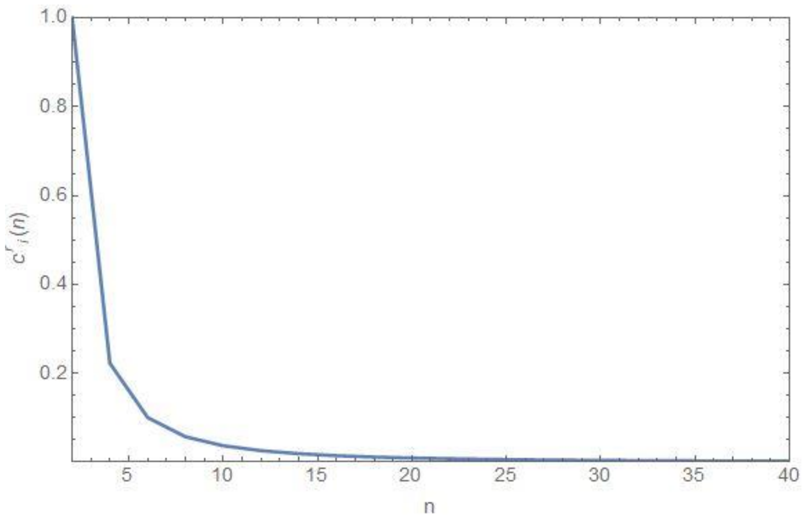


Pinning

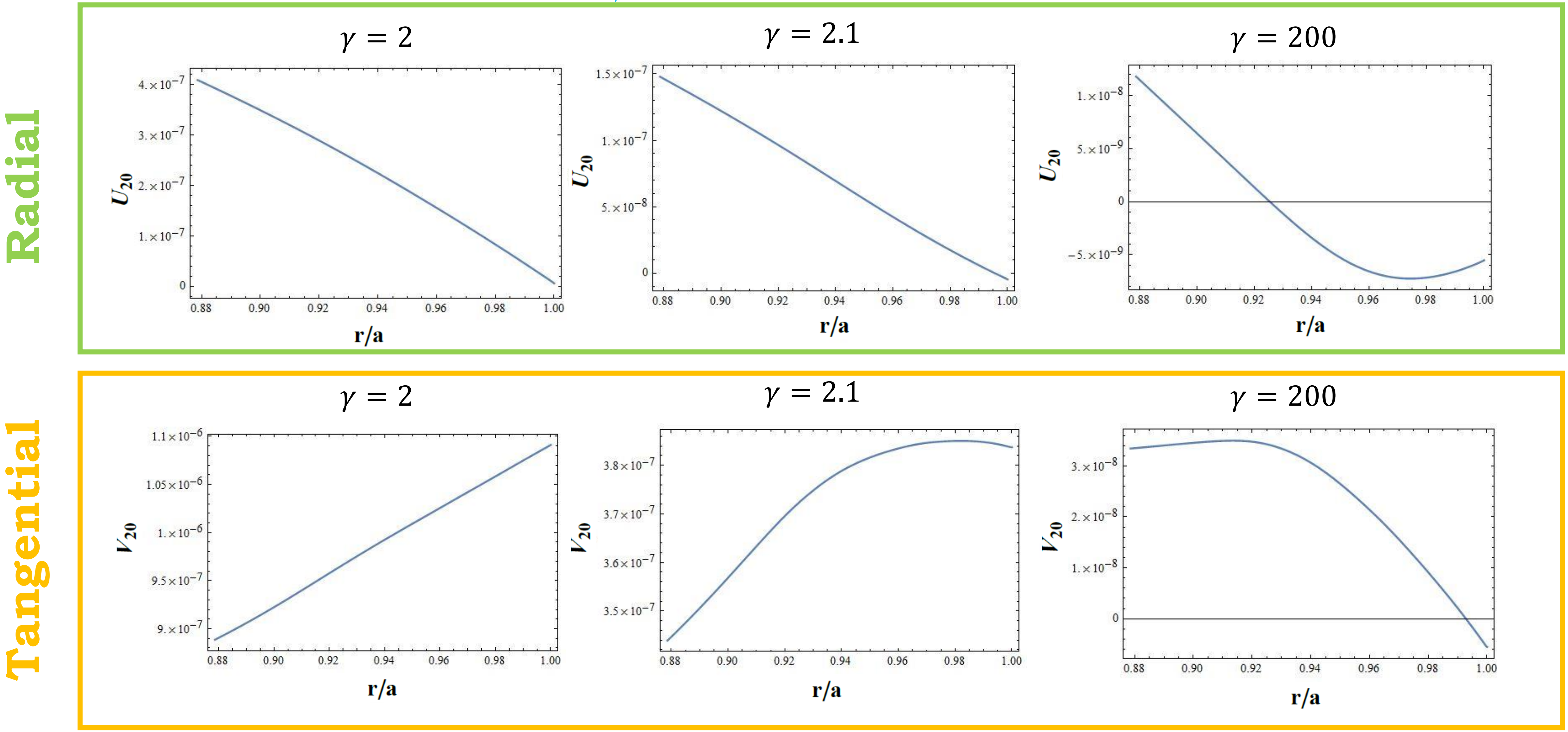


Slack pinning

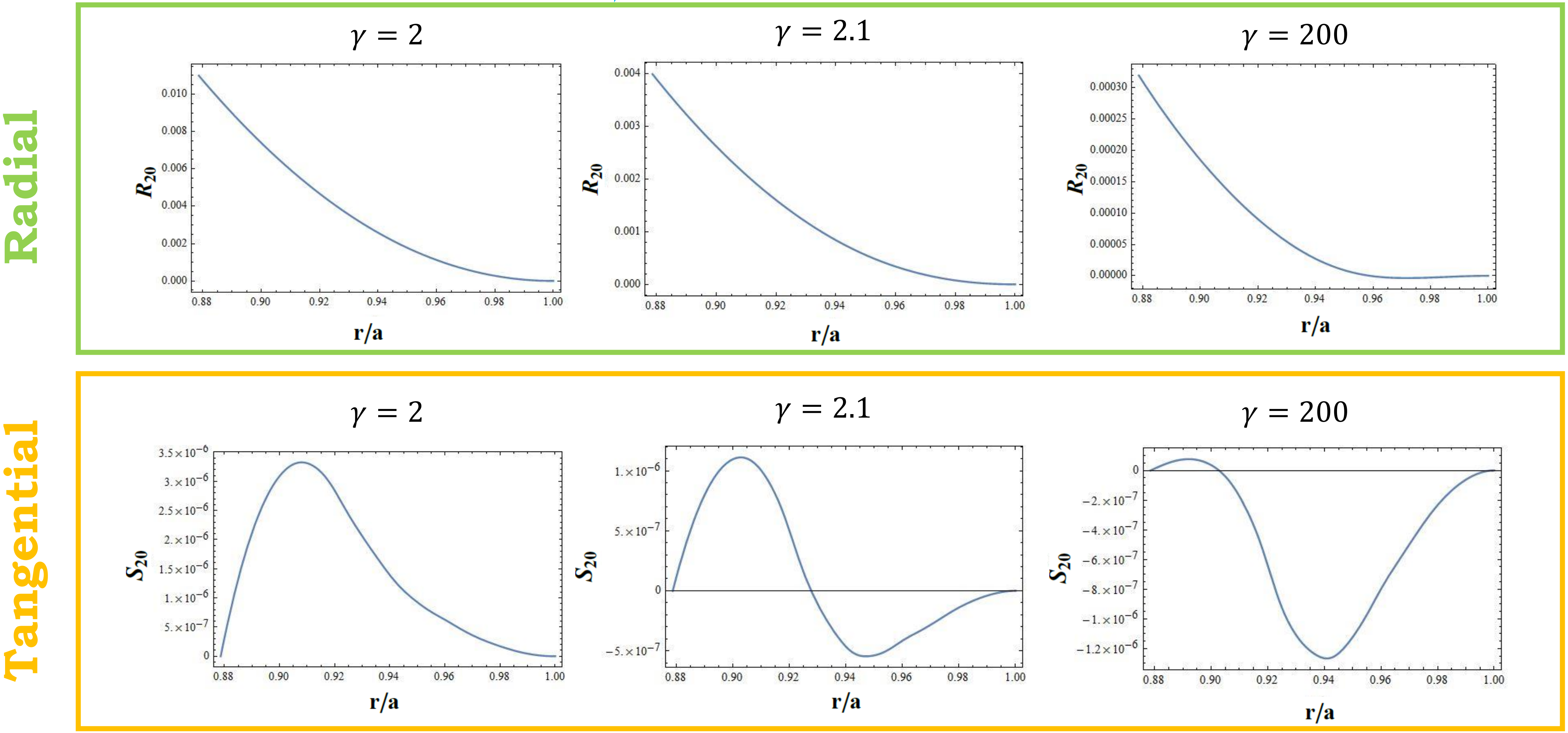
Few spherical harmonics needed



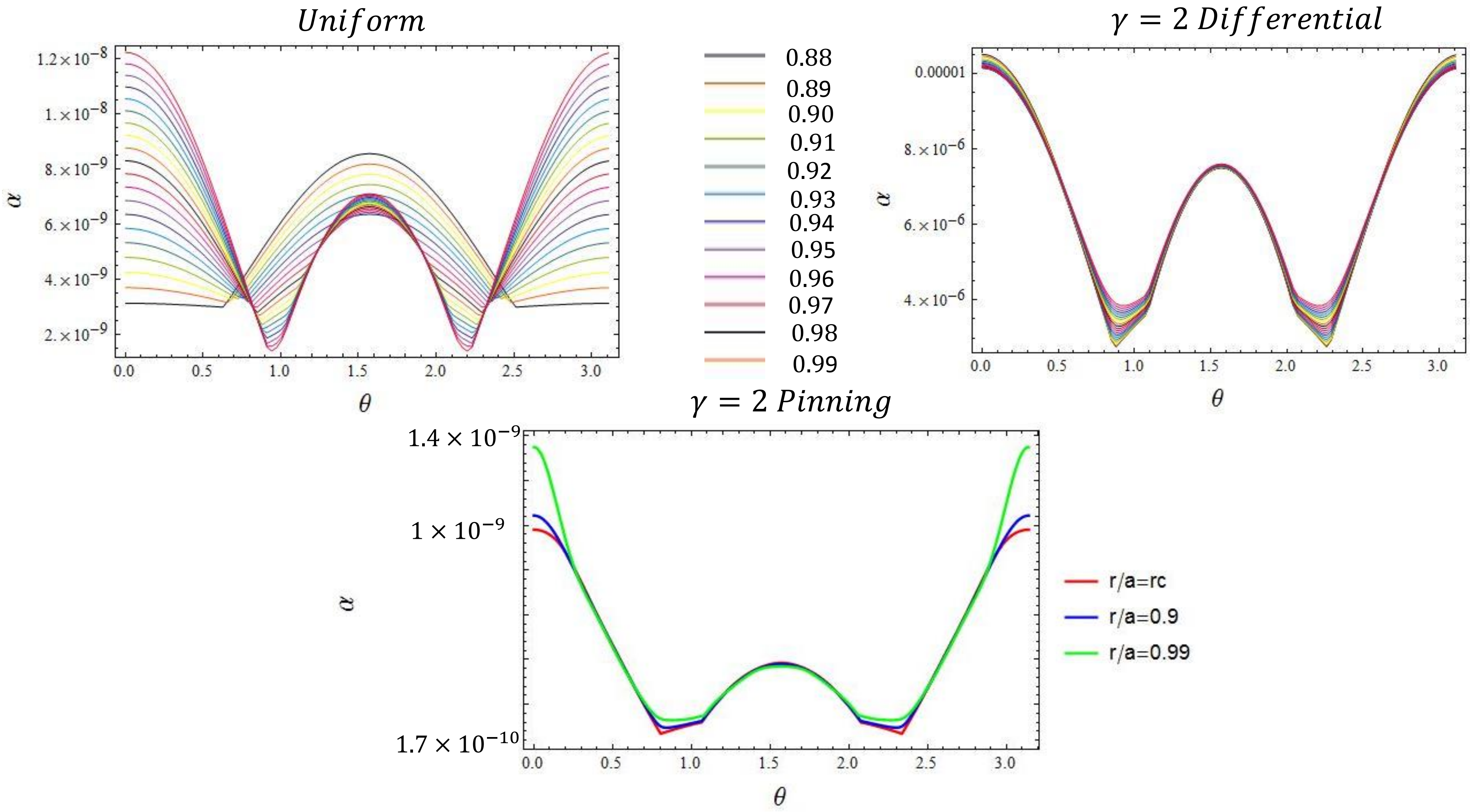
Pinning



Pinning



Maximum strain comparison



Incompressible

Incompressible medium

$\vec{\nabla} \cdot \vec{u} = 0$

$\frac{\mu}{\kappa} \approx 10^{-3}$

Homogeneous crust
Homogeneous core

$\rho_{crust} = cost$

$\rho_{core} = cost$

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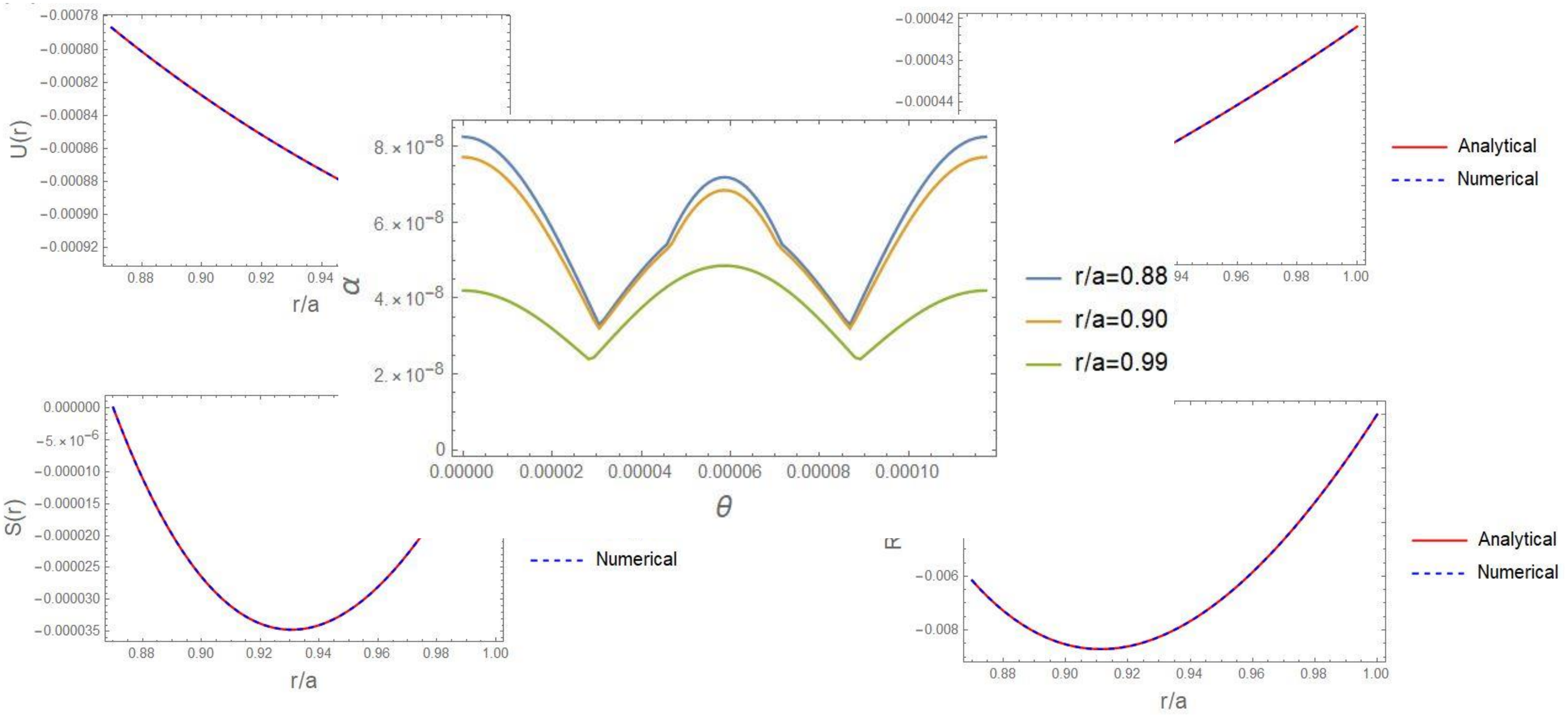
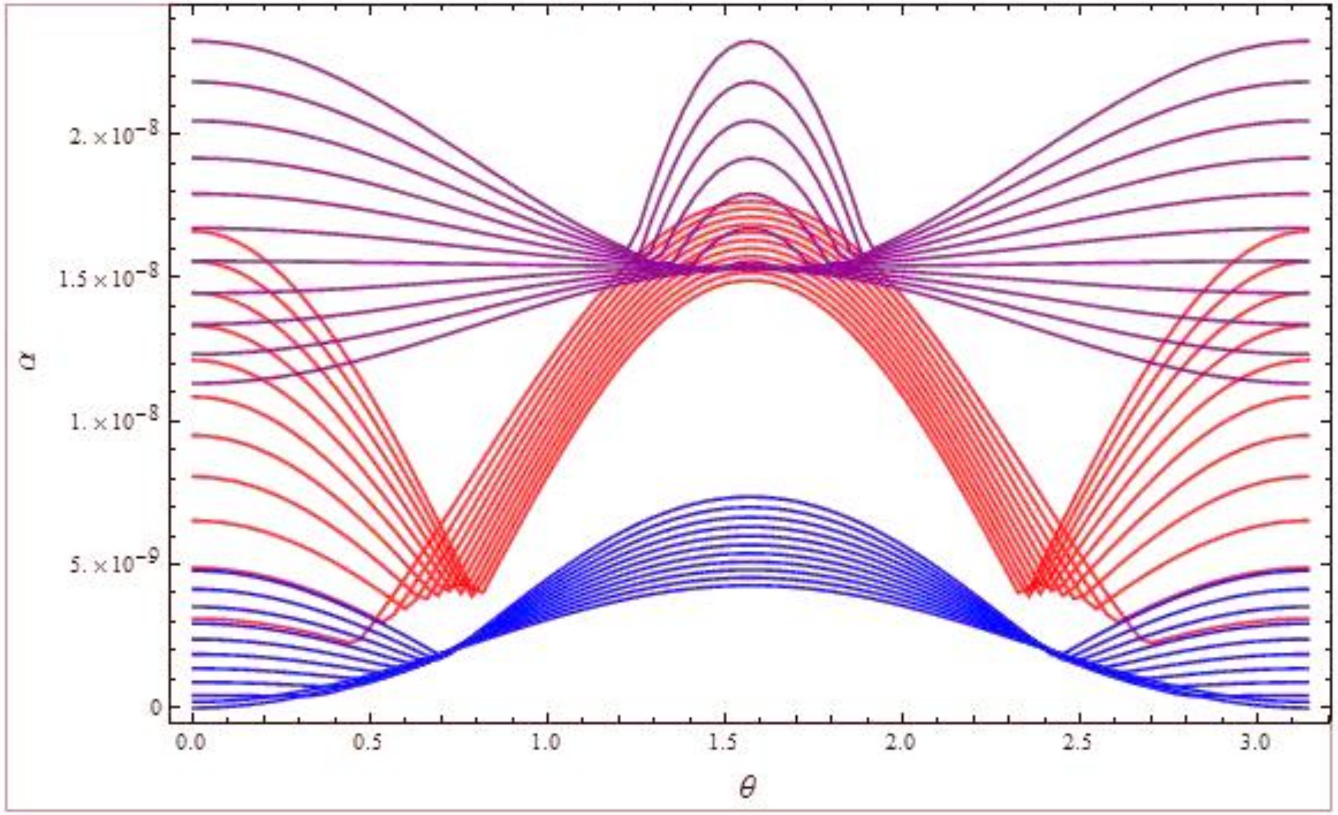
Analytical solution
First code check

Code check



U(r)
r/a
S(r)
r/a
Numerical
α
θ
r/a=0.88
r/a=0.90
r/a=0.99
Analytical
Numerical
Analytical
Numerical
r/a

Incompressible

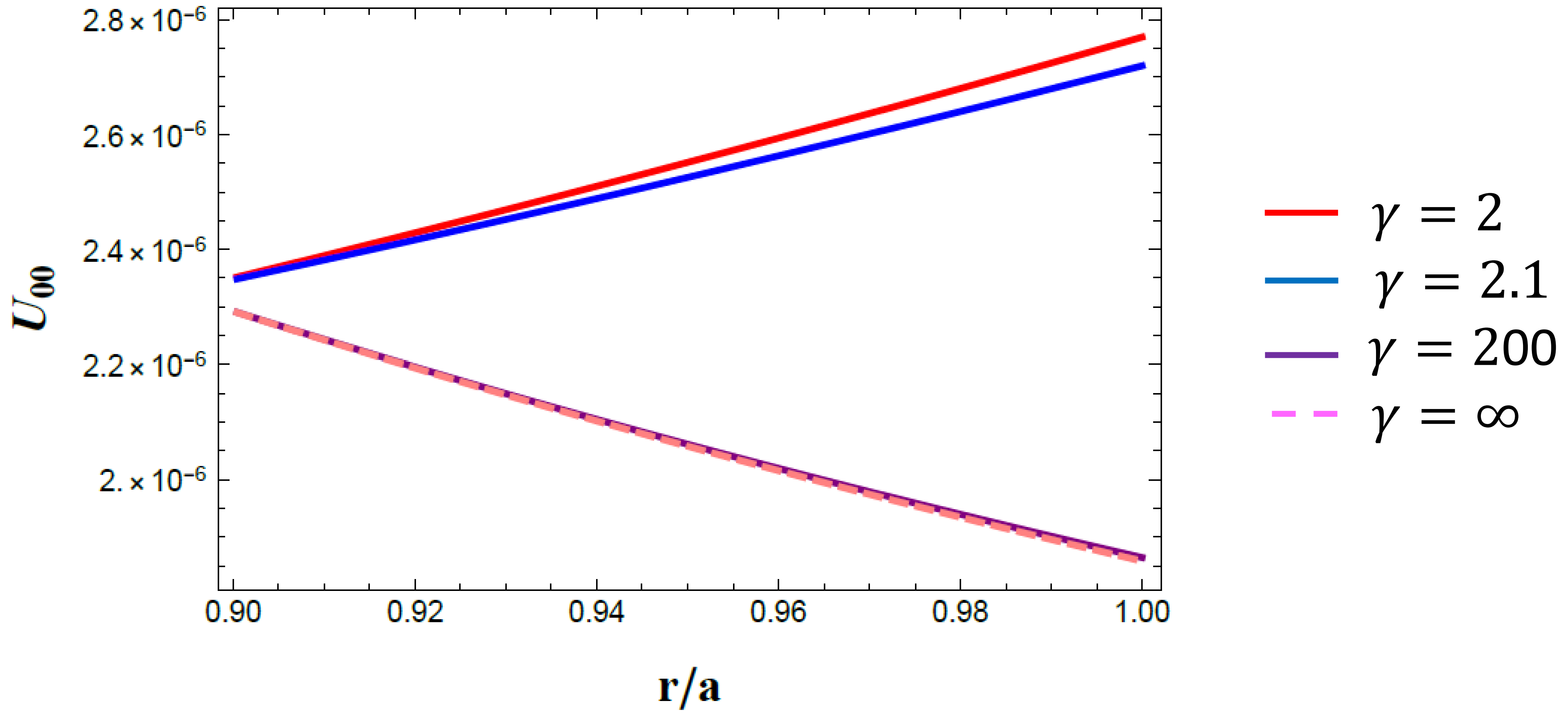


LFE

Incompressible, homogeneous

Incompressible, 2 different densities

Rotation: $l = 0, m = 0$ harmonic



Rotation: $l = 0, m = 0$ harmonic

$$\chi_{lm} = \partial_r U_{lm} + \frac{2}{r} U_{lm} - \frac{l(l+1)}{r} V_{lm}$$

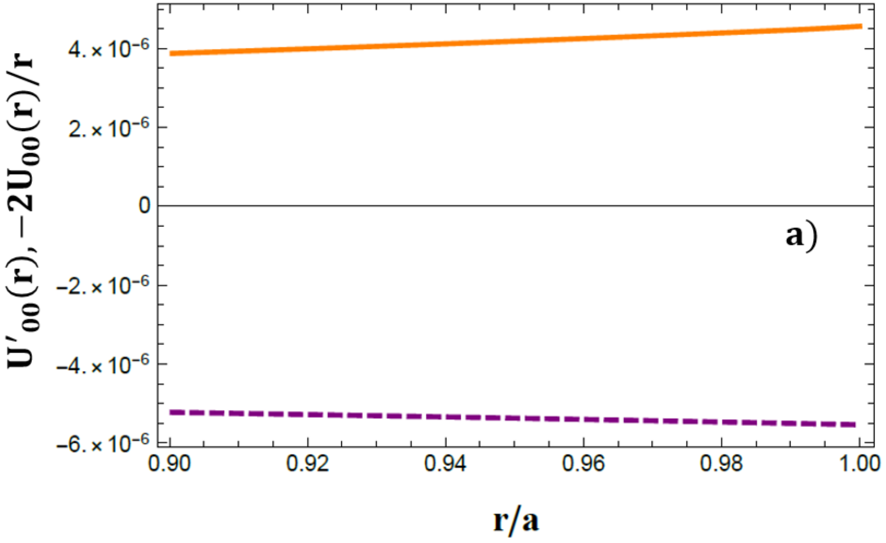
Larger γ → Incompressibility → $\chi_{lm} = 0$

Rotation: $l = 0, m = 0$ harmonic

$$\partial_r U_{lm} = -\frac{2}{r} U_{lm}$$

Larger γ → Incompressibility → $\chi_{lm} = 0$

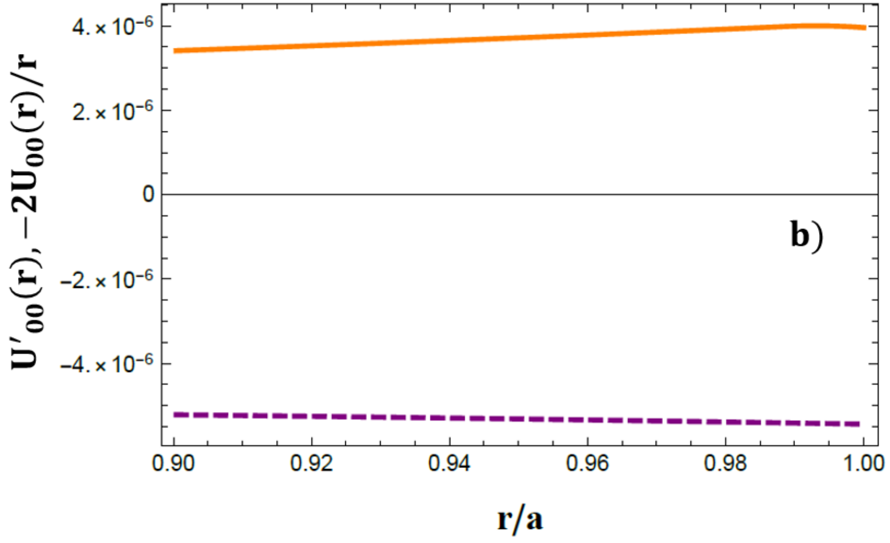
Rotation: $l = 0, m = 0$ harmonic



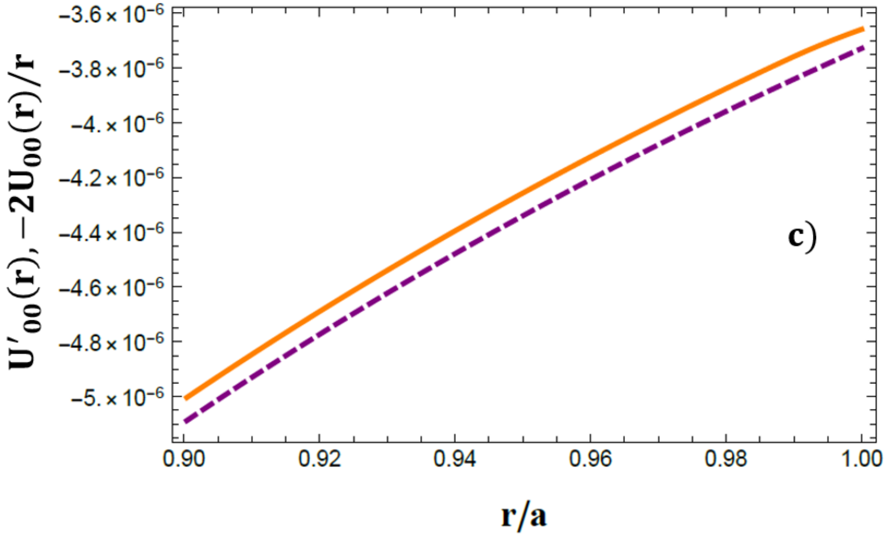
$\gamma = 2$

$$\partial_r U_{lm} = -\frac{2}{r} U_{lm}$$

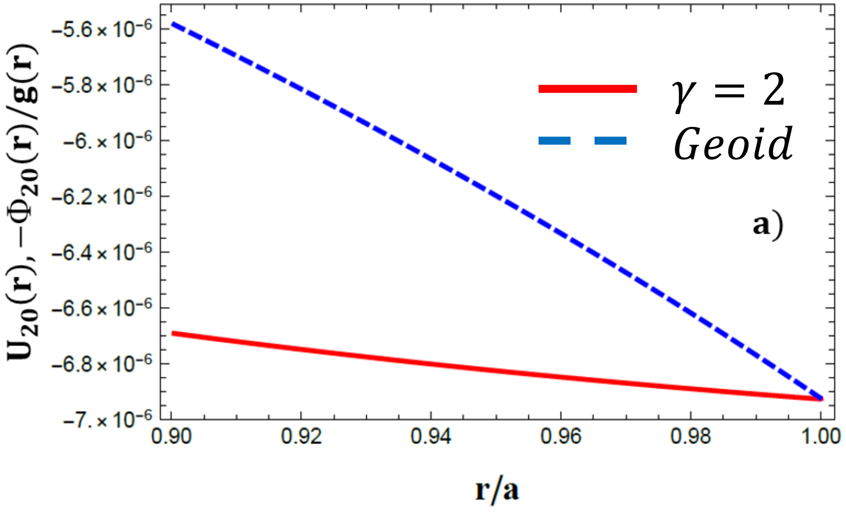
$\gamma = 200$

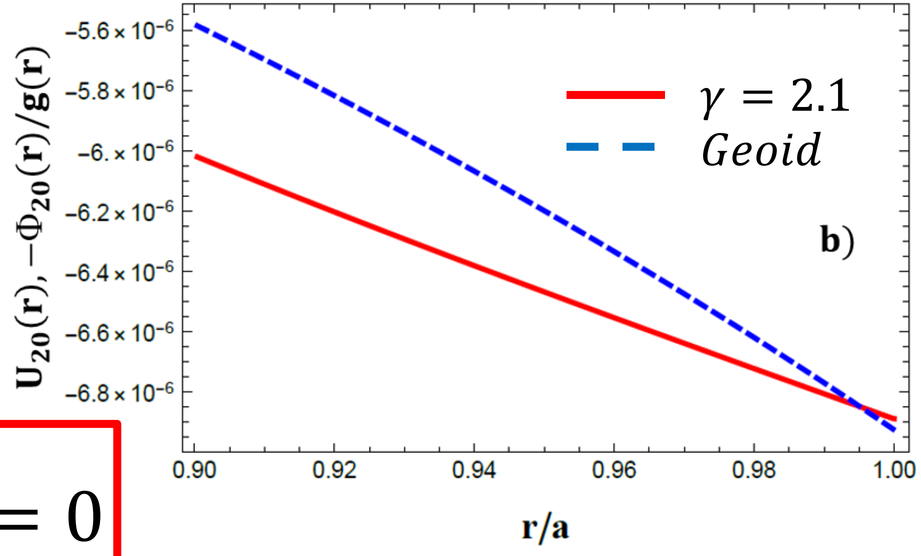


$\gamma = 2.1$

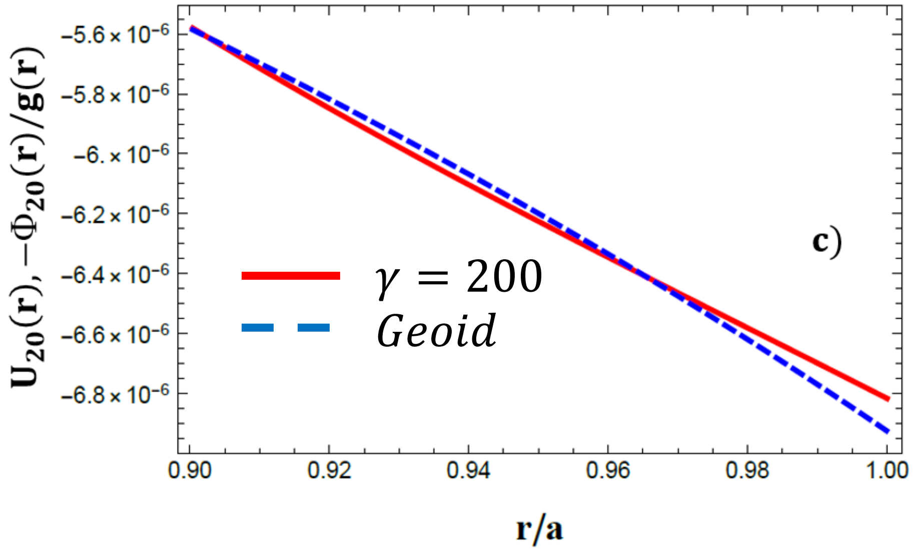


Physical explanation





$$\frac{P_0}{\rho_0^2}\partial_r \rho_0 (\gamma - \gamma_e)\chi_l = 0$$



Open questions

Estimations of the elastic properties of the inner crust (superfluid impact).

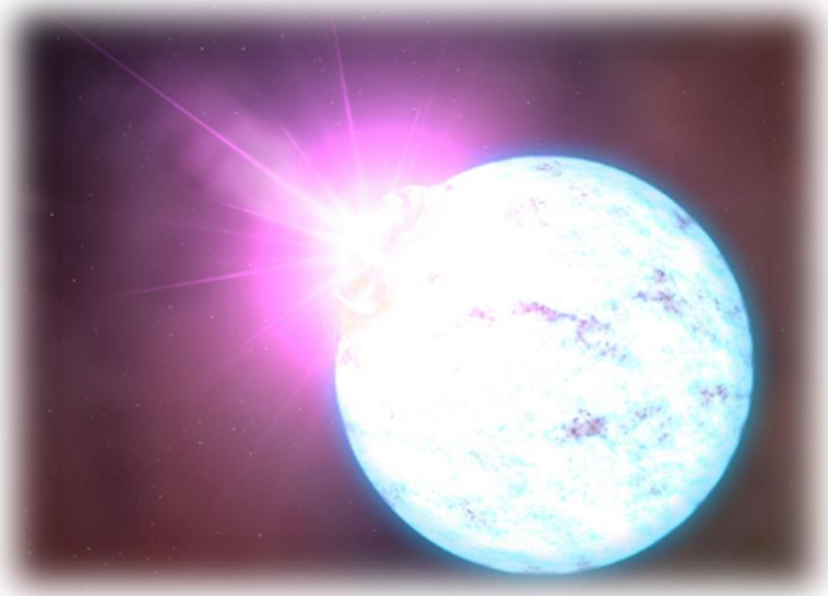
Development of elastic models with two components: solid elastic layer and superfluid in the same point.

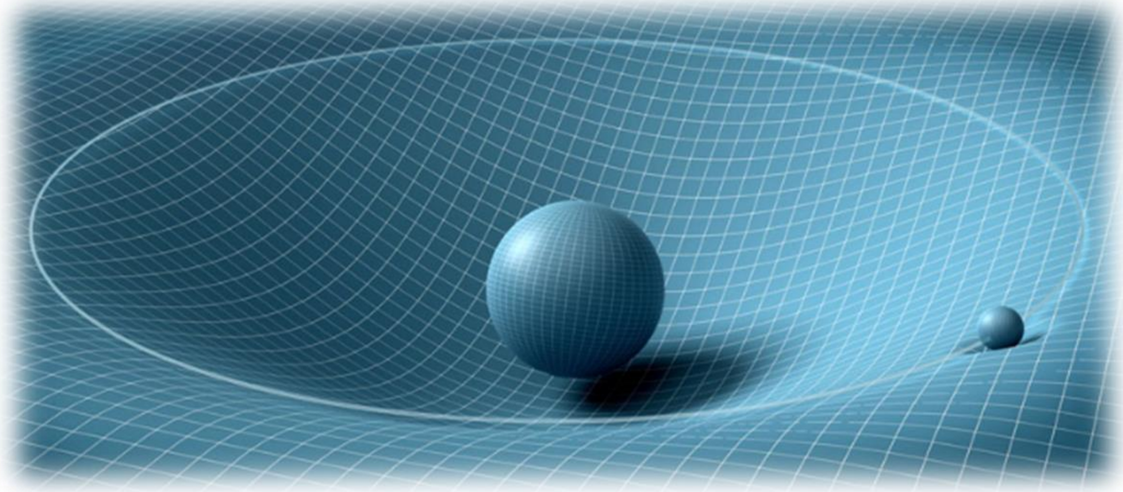
?

Future steps

MOUNTAINS

Study of non-axial perturbation due to quakes on star. Evaluation of the emitted gravitational waves.





GR

Development of a full Relativistic approach for the study of NS deformations.