Temperature-dependent oscillation modes in rotating superfluid neutron stars

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Inertial modes in rotating superfluid NSs

Restoring force is the Coriolis force

eigenfrequencies $\sigma \propto \Omega + \mathcal{O}(\Omega^3)$

Normal modes (i^o–modes) – similar to the modes in a non-superfluid star; normal and superlfuid liquid components are comoving

Superfluid modes (i^s–modes) – exist only in superfluid stars; normal and superfluid components have different velocities; eigenfrequencies are very temperature-dependent; effective damping via mutual friction.

Motivation: r-mode instability

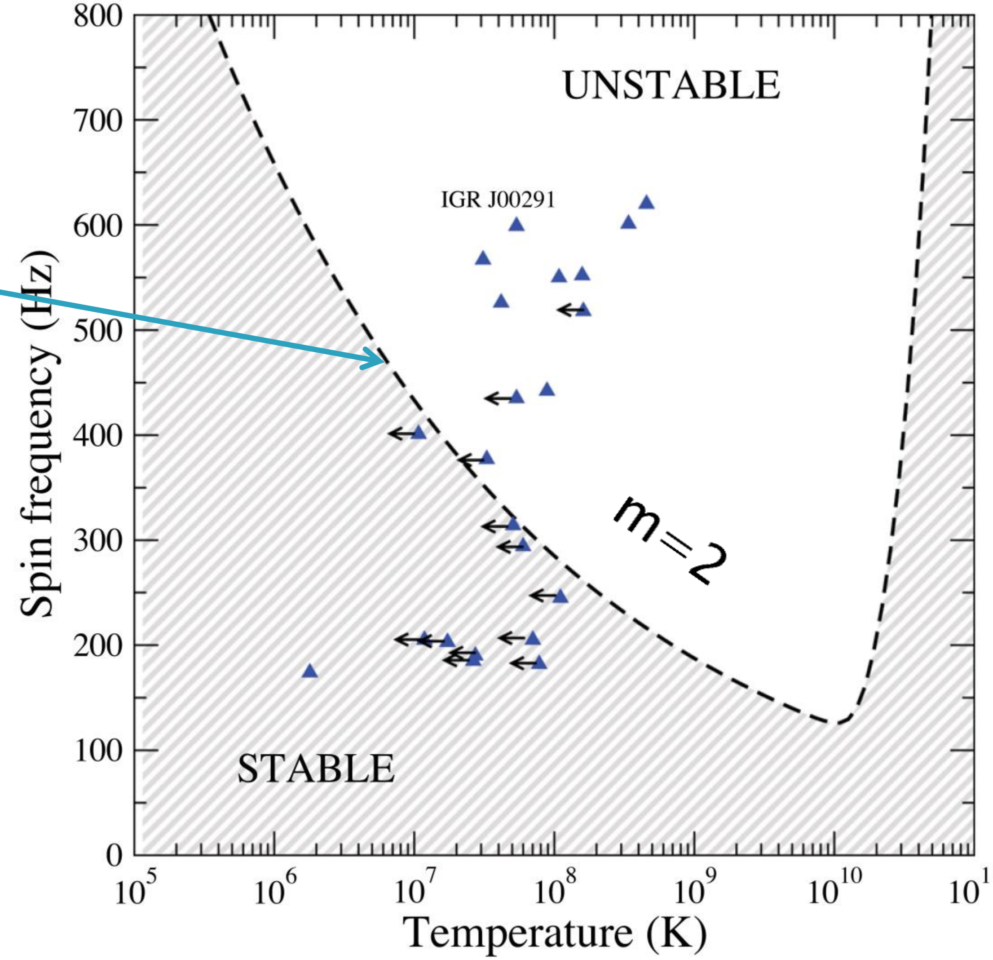
Rotating NSs support inertial oscillation modes, in particular r-modes, which are unstable with respect to gravitational waves emission (CFS instablilty). Dissipation suppresses this instability.

Instability window boundary:

$$\frac{1}{\tau_{\rm GR}} + \frac{1}{\tau_{\rm Diss}} = 0$$

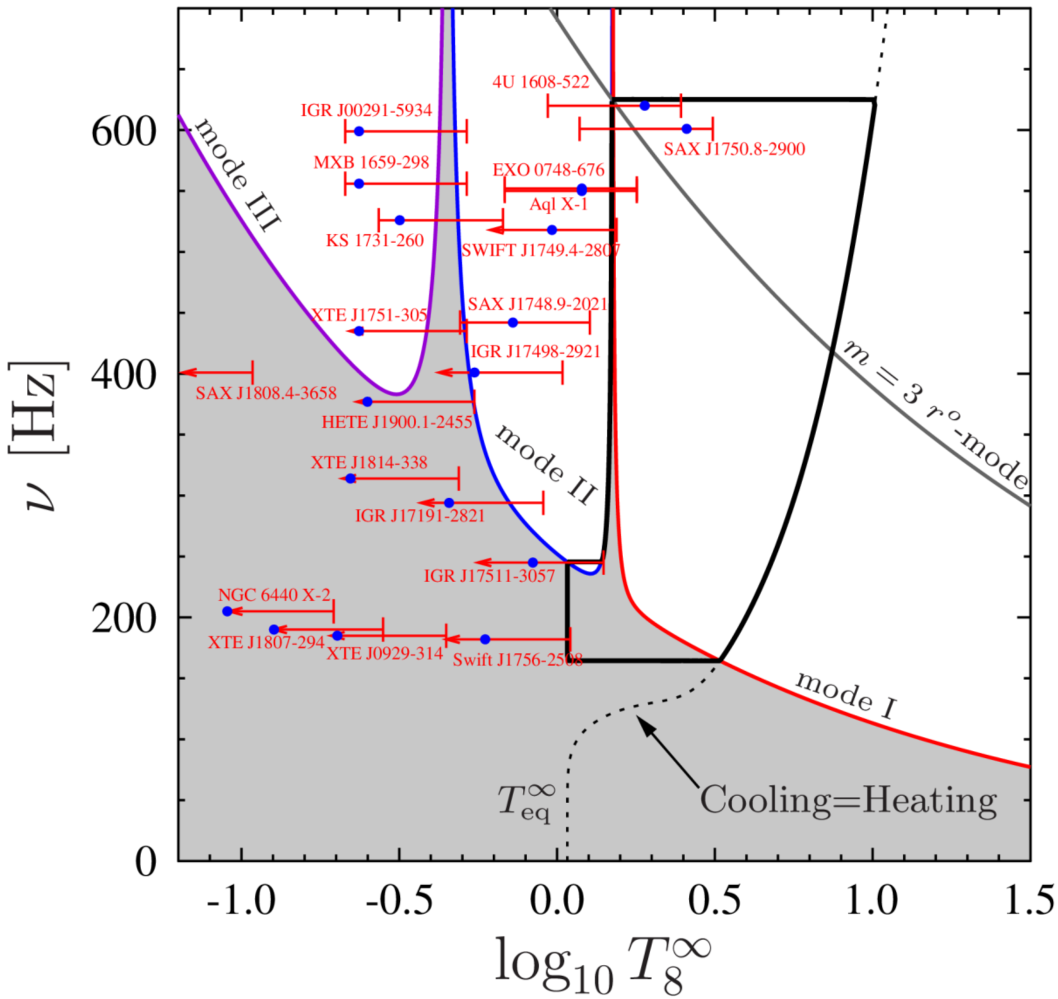
GW excitation (→ $\tau_{\rm GR}$)

dissipation (→ $\tau_{\rm Diss}$)



Haskell, Degenaar, Ho (2012)

Resonance stabilization of r-modes



Gusakov, Chugunov, Kantor,
Phys. Rev. D, 90, 063001 (2014)

At certain temperatures the r-mode experiences an avoided crossing with a superfluid mode.

=> strong damping due to mutual friction.
=> NS evolution proceeds along the stability peak

Open problem

Is the resonance stabilization scenario relevant for real NSs?

Can we use it to constrain properties of superdense matter?

Method:

1) Calculate temperature-dependent inertial modes spectrum for realistic EOSs and superfluidity models, find resonance interactions with normal r-modes

2) Calculate instability windows

3) Match instability windows against observational data (spin frequencies and temperatures for NSs in LMXBs)

Microphysics input:

EOS

thermodynamic derivatives

superfluid entrainment matrix (Andreev-Bashkin matrix)

transport coefficients: bulk viscosity, shear viscosity, mutual friction…

Previous works

In most of papers SFL inertial modes are studied only at $T = 0$.

(e.g. Lindblom & Mendell 2000; Prix et al. 2002; Lee & Yoshida 2003; Andersson et al. 2009)

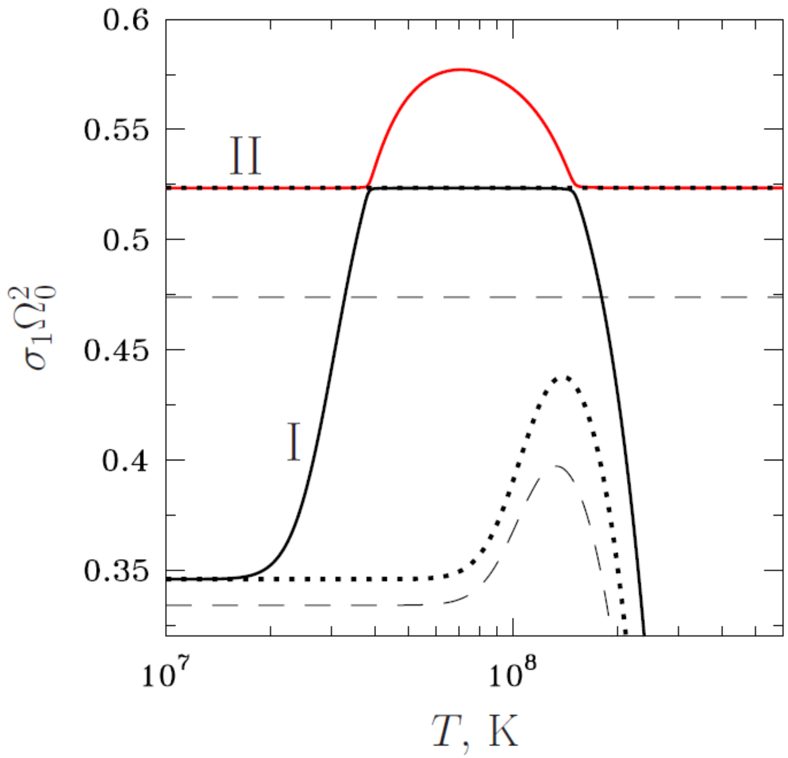
Kantor & Gusakov (2017):

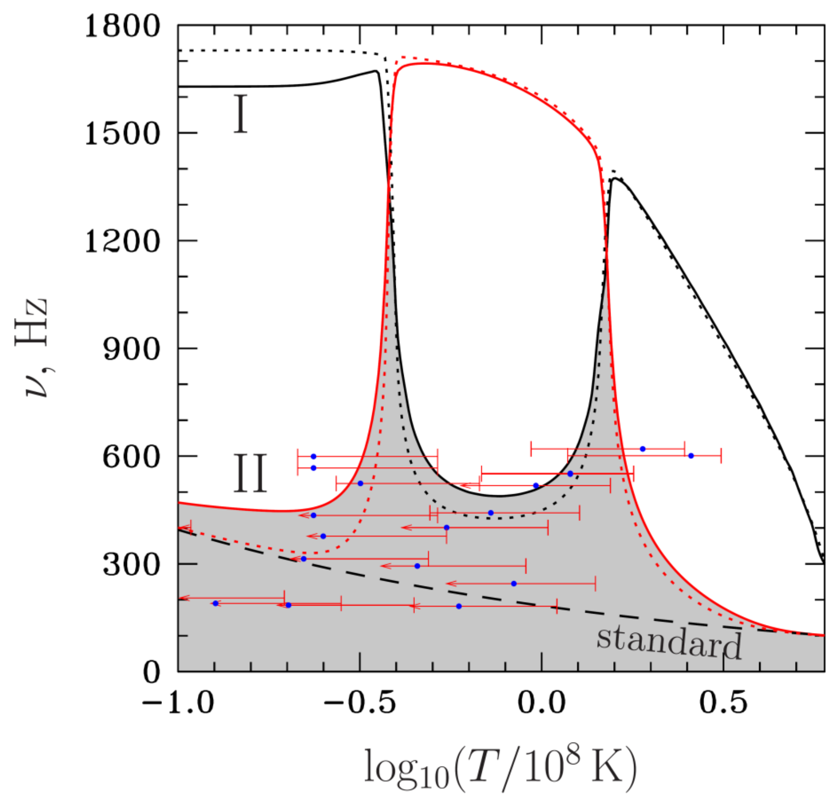
- finite temperatures
- normal and superfluid r-modes only
- second order in Ω, $\sigma = \Omega(\sigma_0 + \sigma_1\Omega^2)$
- Newtonian limit + Cowling approximation
- npe, $npe\mu$ NS core composition
- ignored entrainment between SFL neutrons and protons (it is important, see Lee & Yoshida 2003)

Previous works

Kantor & Gusakov (2017):

avoided crossings of normal and superfluid r-modes in npe and $npe\mu$ NS





This work

Normal and SFL inertial modes

- Newtonian limit + Cowling approximation
- lowest order in rotational frequency Ω
- npe NS
- entrainment between superfluid neutrons and protons
- weak drag regime
- finite-temperature effects
- constant critical temperatures T_{cn}, T_{cp}

Oscillation equations

small perturbations $\propto \mathrm{e}^{\imath\sigma t}$

Continuity equations

$$\delta n_{\rm b} + \mathrm{div}(n_{\rm b}\boldsymbol{\xi}_{\rm b}) = 0$$

$$\delta n_{\rm e} + \mathrm{div}(n_{\rm e}\boldsymbol{\xi}) = 0$$

Euler equation:

$$-\sigma^2\boldsymbol{\xi}_{\rm b} + 2\imath\sigma\boldsymbol{\Omega}\times\boldsymbol{\xi}_{\rm b} = \frac{\delta w}{w^2}\boldsymbol{\nabla}P - \frac{\boldsymbol{\nabla}\delta P}{w}$$

'superfluid' equation

$$h\sigma^2\boldsymbol{z} - 2\imath h_1\sigma\boldsymbol{\Omega}\times\boldsymbol{z} = c^2 n_{\rm e}\boldsymbol{\nabla}\Delta\mu_{\rm e}$$

$$\boldsymbol{j}_{\rm e} = n_{\rm e}\boldsymbol{u}$$

$$\boldsymbol{j}_i = n_i\boldsymbol{u} + Y_{ik}\boldsymbol{w}_k, \quad i = \mathrm{n, p}$$

$$\boldsymbol{v}_{si} = (\boldsymbol{w}_i + \mu_i\boldsymbol{u})/(m_i c^2)$$

$$\boldsymbol{\xi}_{\rm b} \equiv \boldsymbol{j}_{\rm b}/(\imath\sigma n_{\rm b}) \quad \boldsymbol{\xi} \equiv \boldsymbol{j}_e/(\imath\sigma n_{\rm e})$$

$$\boldsymbol{z} \equiv \boldsymbol{\xi}_{\rm b} - \boldsymbol{\xi}$$

$$w \equiv (P+\varepsilon)/c^2$$

$$\Delta\mu_{\rm e} \equiv \mu_{\rm n} - \mu_{\rm p} - \mu_{\rm e}$$

$$h = \mu_{\rm n} n_{\rm b}\left(\frac{n_{\rm b}}{\mu_{\rm n}(Y_{\rm nn} - Y_{\rm np}^2/Y_{\rm pp})} - 1\right)$$

$$h_1 = \mu_{\rm n} n_{\rm b}\left(\frac{n_{\rm b}}{\mu_{\rm n}Y_{\rm nn} + \mu_{\rm p}Y_{\rm np}} - 1\right)$$

Oscillation equations

Lowest-order terms in spin frequency Ω:

$$\sigma = \Omega\sigma_0 \left(1 + \Omega^2\sigma_1\right)$$

$$\boldsymbol{d} = \left(\boldsymbol{d}^0 + \Omega^2\boldsymbol{d}^1\right) \exp(\imath\sigma t + \imath m\phi)$$

$$\delta f = \Omega^2 \delta f^1 \exp(\imath\sigma t + \imath m\phi)$$

Poloidal-toroidal decomposition (Saio 1982):

$$\xi^0_{\mathrm{b}\theta} = \frac{\partial}{\partial\theta}Q(r,\theta) + \frac{\imath m T(r,\theta)}{\sin\theta}, \quad \xi^0_{\mathrm{b}\phi} = \frac{\imath m Q(r,\theta)}{\sin\theta} - \frac{\partial}{\partial\theta}T(r,\theta),$$

$$z^0_\theta = \frac{\partial}{\partial\theta}Q_z(r,\theta) + \frac{\imath m T_z(r,\theta)}{\sin\theta}, \quad z^0_\phi = \frac{\imath m Q_z(r,\theta)}{\sin\theta} - \frac{\partial}{\partial\theta}T_z(r,\theta).$$

Legendre polynomial expansion

$$\xi^0_{\mathrm{br}}(r,\theta) = \sum_{l_2} \xi^0_{\mathrm{br}\,l_2 m}(r) P^m_{l_2}(\cos\theta),$$

$$z_r(r,\theta) = \sum_{l_2} z^0_{r\,l_2 m}(r) P^m_{l_2}(\cos\theta),$$

$$Q(r,\theta) = \sum_{l_2} Q_{l_2 m}(r) P^m_{l_2}(\cos\theta),$$

$$Q_z(r,\theta) = \sum_{l_2} Q_{z\,l_2 m}(r) P^m_{l_2}(\cos\theta),$$

$$T(r,\theta) = \sum_{l_1} T_{l_1 m}(r) P^m_{l_1}(\cos\theta),$$

$$T_z(r,\theta) = \sum_{l_1} T_{z\,l_1 m}(r) P^m_{l_1}(\cos\theta),$$

$$\delta P^1(r,\theta) = \sum_{l_2} \delta P^1_{l_2 m}(r) P^m_{l_2}(\cos\theta),$$

$$\Delta\mu^1_{\mathrm{e}}(r,\theta) = \sum_{l_2} \Delta\mu^1_{\mathrm{e}\,l_2 m}(r) P^m_{l_2}(\cos\theta),$$

fixed m, summation over l

$\Rightarrow$ Infinite set of ODEs for harmonics with different l

'odd' modes:

$$l_1 = m + 2k, \quad l_2 = m + 2k + 1$$

'even' modes:

$$l_1 = m + 2k + 1, \quad l_2 = m + 2k + 2$$

Numerical results

$M = 1.4M_\odot$

npe core – APR EOS (Akmal, Pandharipande, Ravenhall 1998), Heiselberg & Hjorth-Jensen 1999 parametrization

NS crust – EOS BSk20 (Potekhin et al. 2013)

critical temperatures $T_{cn} = 6 \times 10^8$ K, $T_{cp} = 5 \times 10^9$ K

entrainment matrix $Y_{ik} = Y_{ik}(n_b, \frac{T}{T_{cn}}, \frac{T}{T_{cp}})$: Kantor & Gusakov (2011)

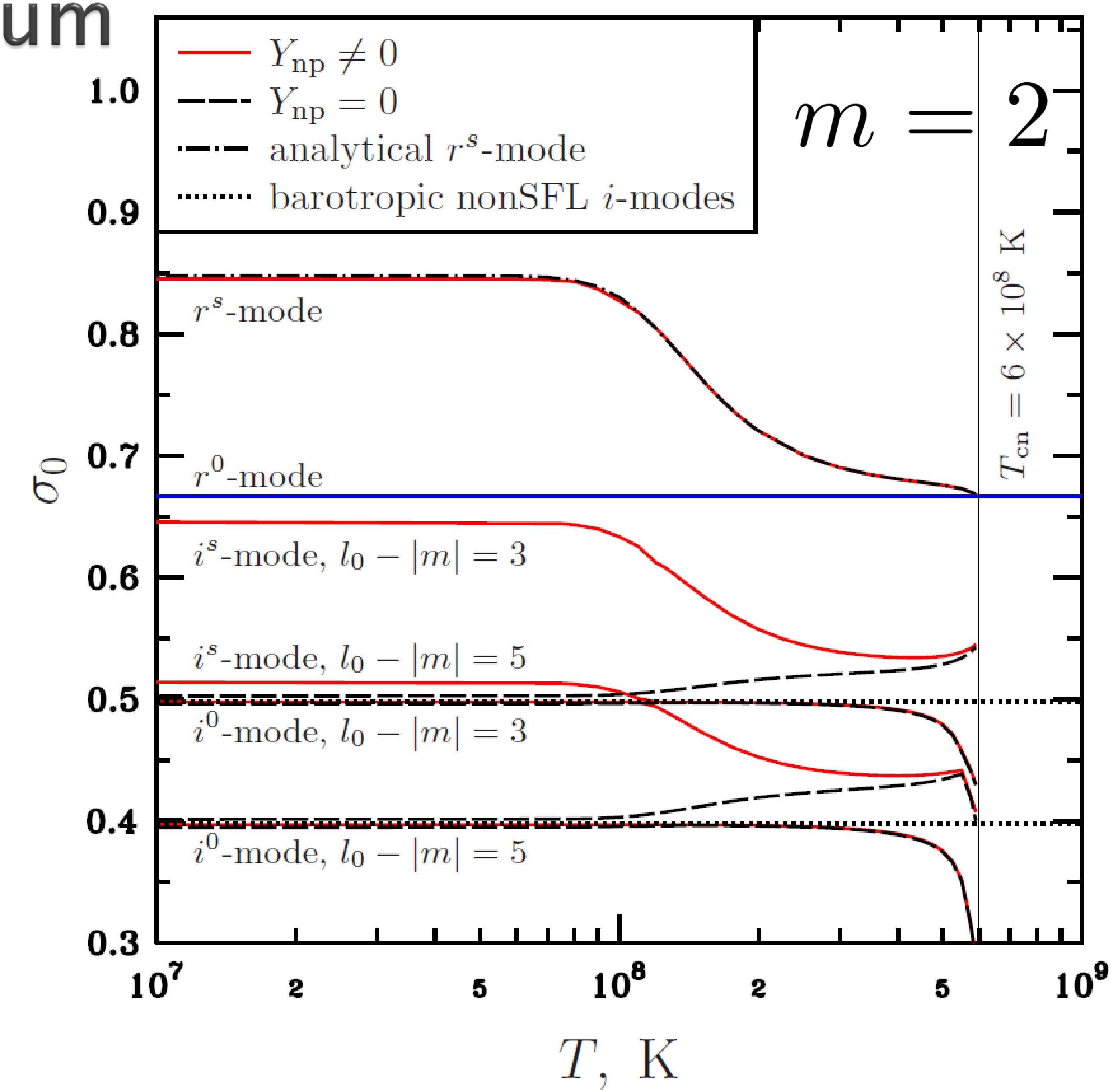
$m = 2$ inertial modes, $l_0 - m = 1,3,5$ (odd modes)

Keep only lowest harmonics, $l \leq m + 2k_{max} + 1$

$k_{max} = 3$ for $l_0 - m = 3,5$

$k_{max} = 2$ for $l_0 - m = 1$ (r-modes)

Spectrum



$Y_{np} \neq 0$
$Y_{np} = 0$
analytical r^s-mode
barotropic nonSFL i-modes
$m = 2$
$T_{cn} = 6 \times 10^8$ K
r^s-mode
r^0-mode
i^s-mode, $l_0 - |m| = 3$
i^s-mode, $l_0 - |m| = 5$
i^0-mode, $l_0 - |m| = 3$
i^0-mode, $l_0 - |m| = 5$
σ_0
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
10^7
2
5
10^8
2
5
10^9
T, K

Superfluid r^s–mode in the limit of small entrainment: analytical calculation

In case $Y_{np} = 0$
(no entrainment)
there are two purely toroidal modes:
normal r^o–mode and superfluid r^s–mode,
with the same frequency

$$\sigma_0 = \frac{2}{m+1} \qquad \sigma = \sigma_0 \Omega + \mathcal{O}\left(\Omega^3\right)$$

(Andersson & Comer 2001, Lee & Yoshida 2003, Andersson et al. 2009)

R–modes in the presence of entrainment

$$h = \mu_{\rm n} n_{\rm b} \left(\frac{n_{\rm b}}{\mu_{\rm n}(Y_{\rm nn} - Y_{\rm np}^2/Y_{\rm pp})} - 1 \right), \quad h_1 = \mu_{\rm n} n_{\rm b} \left(\frac{n_{\rm b}}{\mu_{\rm n} Y_{\rm nn} + \mu_{\rm p} Y_{\rm np}} - 1 \right)$$

In case $Y_{np} \neq 0$ ($h_1 \neq h$) one can write a perturbation theory in $\Delta h \equiv h_1/h - 1$.
First-order corrections to eigenfrequencies and eigenfunctions can be found analytically.

Eigenfrequency for the m=2 r^s–mode coincide with the numerically calculated within the accuracy <1%

even at $\Delta h \sim 0.2 - 0.25$

R–modes in the presence of entrainment

normal r–mode (not affected by Δh):

$$\sigma_0 = \sigma_{0(0)} = \frac{2}{m+1}, \quad T^{(0)}_{mm} = C_0 r^m, \quad T^{(0)}_{zmm} = 0.$$

superfluid r–mode:

$\sigma_{0(1)}$, C_0 and C_1 are found analytically from the first–order in Δh equations

$$\sigma_0 = \frac{2}{m+1} + \sigma_{0(1)}$$

$$T^{(0)}_{zmm}(r) = C_1 \frac{n_{\rm e}(r)}{h(r)} r^m$$

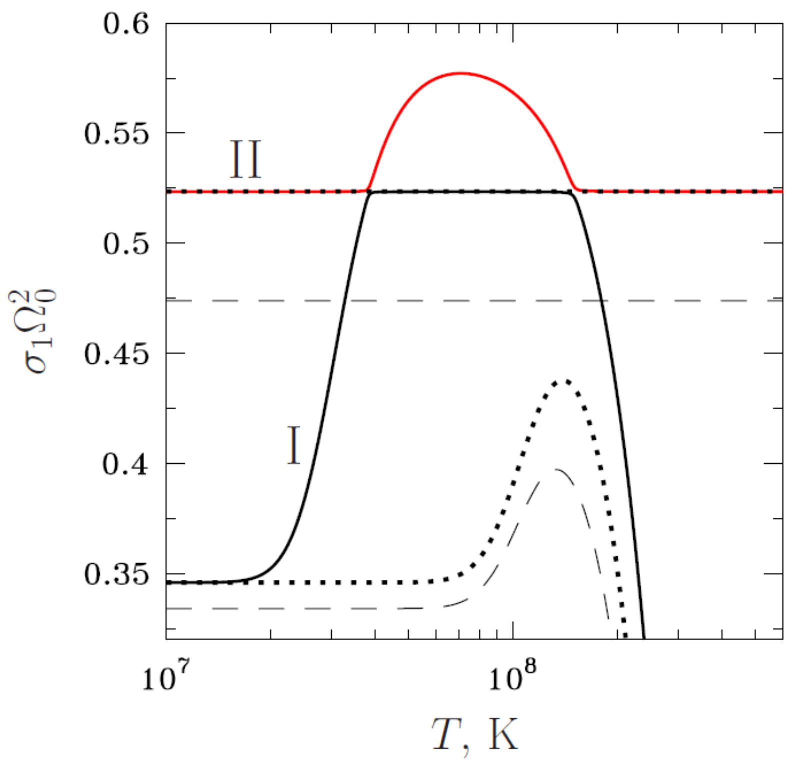
$$T^{(0)}_{mm}(r) = r^m \left(C_0 + C_1 \int_0^r \frac{\mu_{\rm n}(r_1)}{c^4 w_0(r_1)} \frac{dP_0(r_1)}{dr_1} \frac{\partial n_{\rm b}}{\partial \Delta\mu_{\rm e}}(r_1) {\rm d}r_1 \right).$$

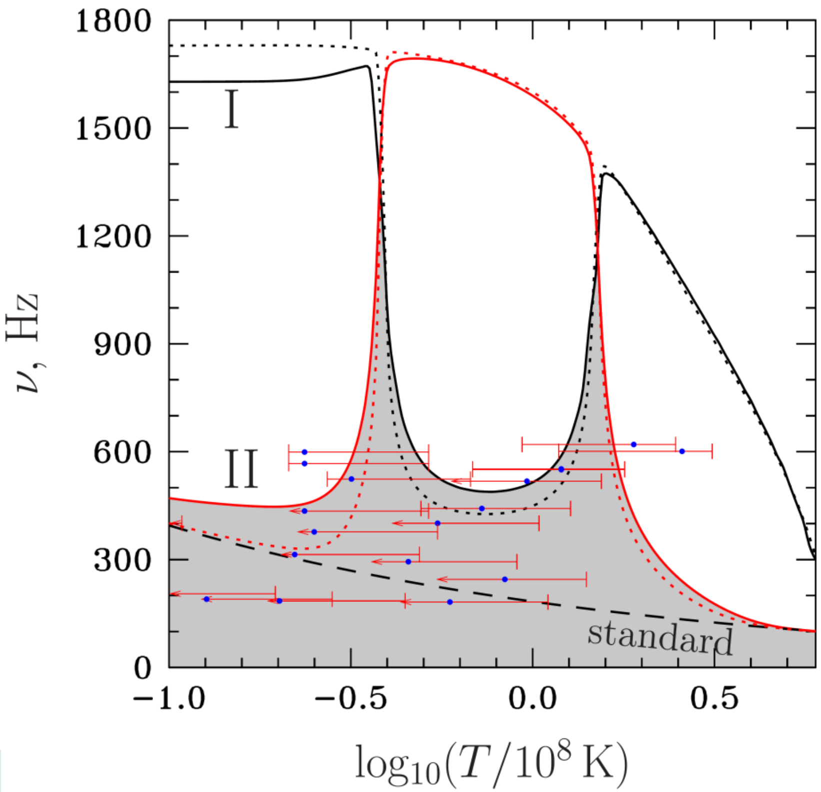
Resonance interaction of r^o– and r^s–modes

The modes are close to each other at $T \to T_c$

But in the lowest order in Ω the normal r-mode remains the same! => no resonance interaction

Further work: include next-to-leading order terms in Ω (Kantor & Gusakov 2017) to calculate the avoided crossings





Summary

We calculated the spectrum of inertial modes in slowly rotating SFL NSs, including both the entrainment and finite temperature effects for the first time.

We also developed an approximate method that allows to calculate the superfluid r-mode analytically in the limit of small entrainment.

Future plans:

- Calculate the superfluid r-mode analytically in next-to-leading order in Ω and Δh, find avoided crossings with the normal r-mode
- Calculate inertial modes for different models (EOS, T_c profiles), find more avoided crossings with the normal r-mode
- Calculate instability windows

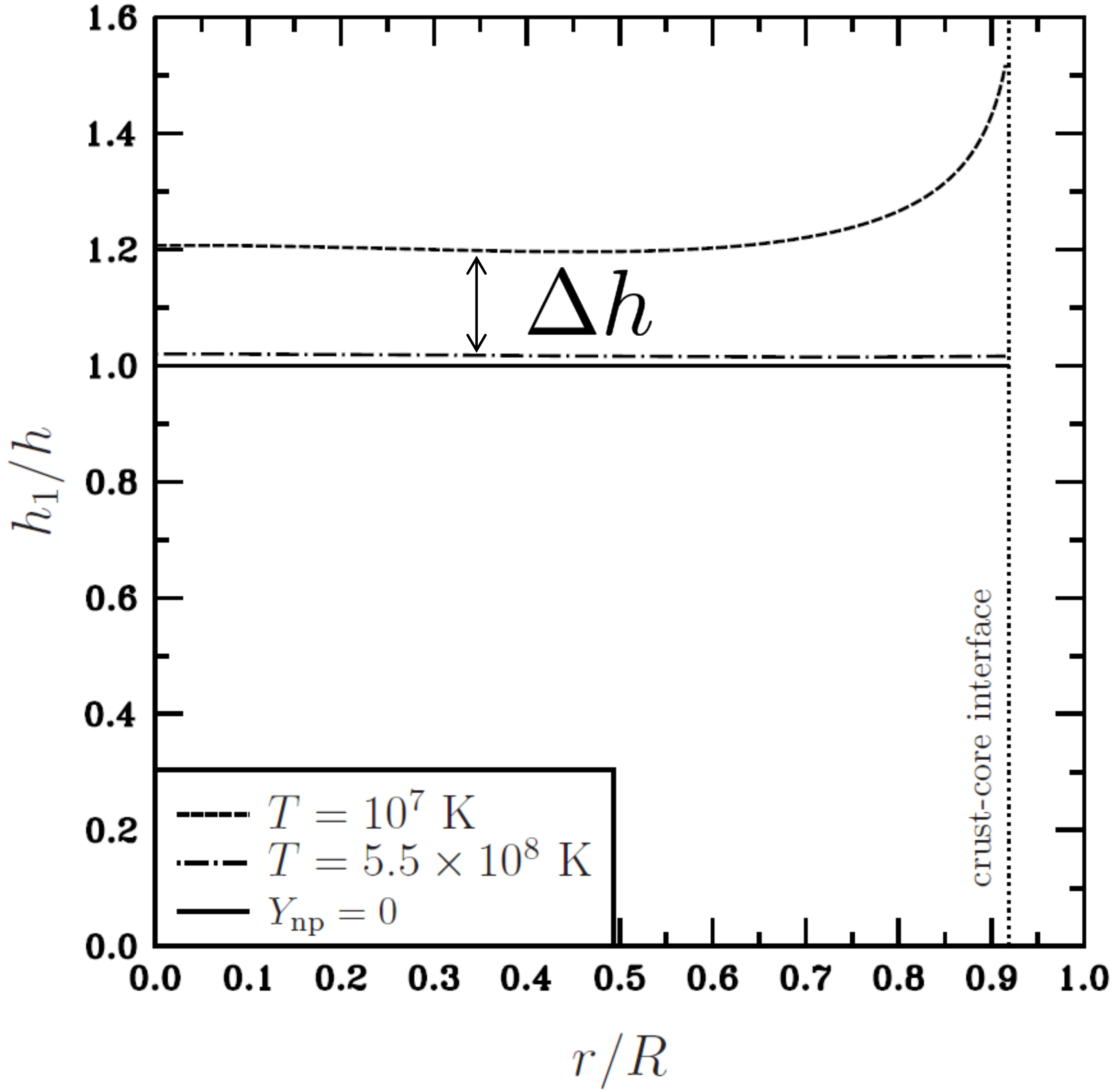
What do we need as microphysics input:

- EOS
- Thermodynamic derivatives $\partial\mu_i/\partial n_j$
- Superfluid gaps
- Superfluid entrainment matrix $Y_{ik}(n_b, T)$
- Transport coefficients: bulk viscosity, shear viscosity, mutual friction

Thank you for your attention!

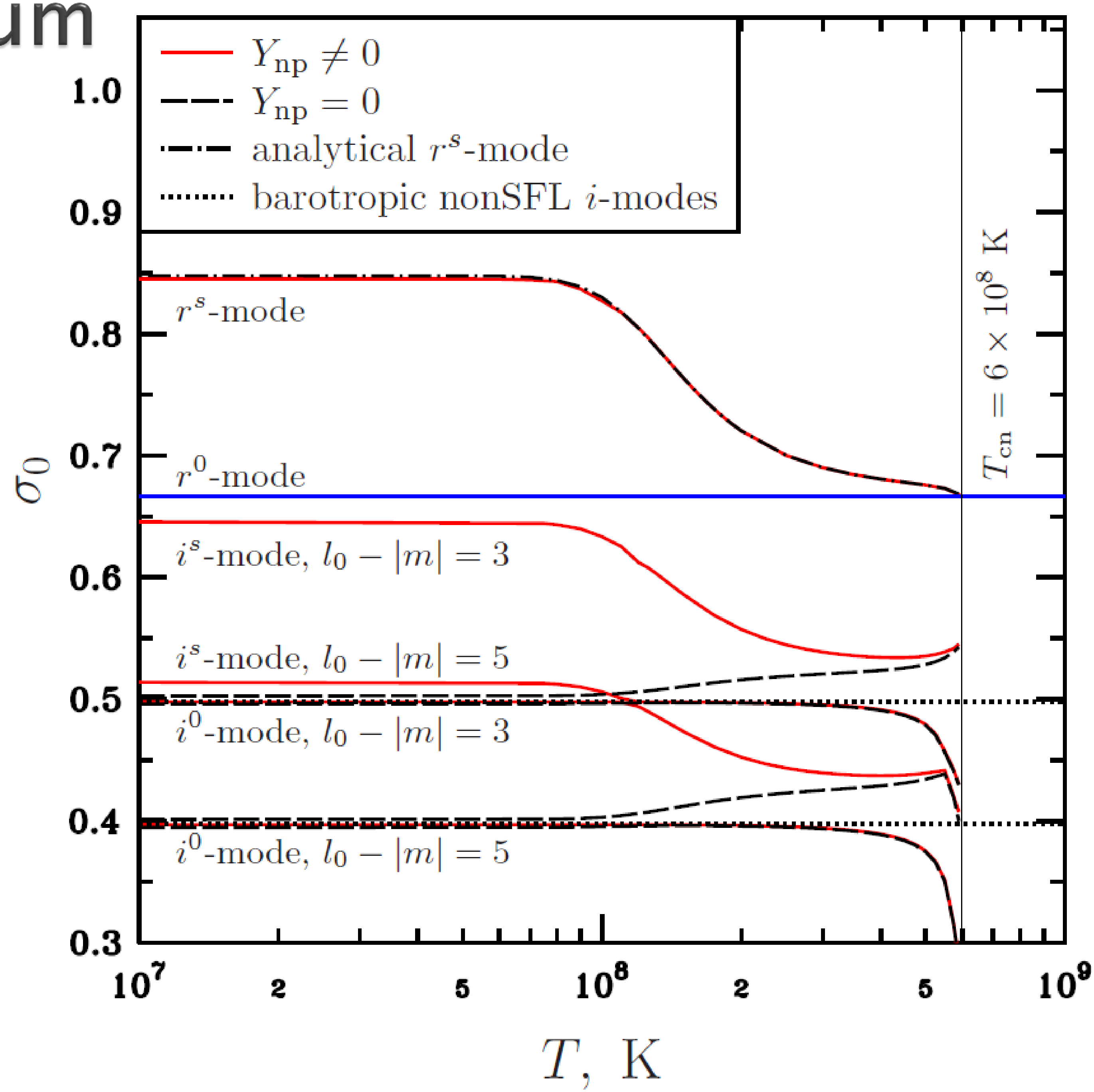
Backup slides

Plot for Δh



1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
h_1/h
Δh
crust-core interface
$T = 10^7$ K
$T = 5.5 \times 10^8$ K
$Y_{\rm np} = 0$
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
r/R

Spectrum



$Y_{np} \neq 0$
$Y_{np} = 0$
analytical r^s-mode
barotropic nonSFL i-modes
r^s-mode
r^0-mode
i^s-mode, $l_0 - |m| = 3$
i^s-mode, $l_0 - |m| = 5$
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$T_{cn} = 6 \times 10^8$ K
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0.4
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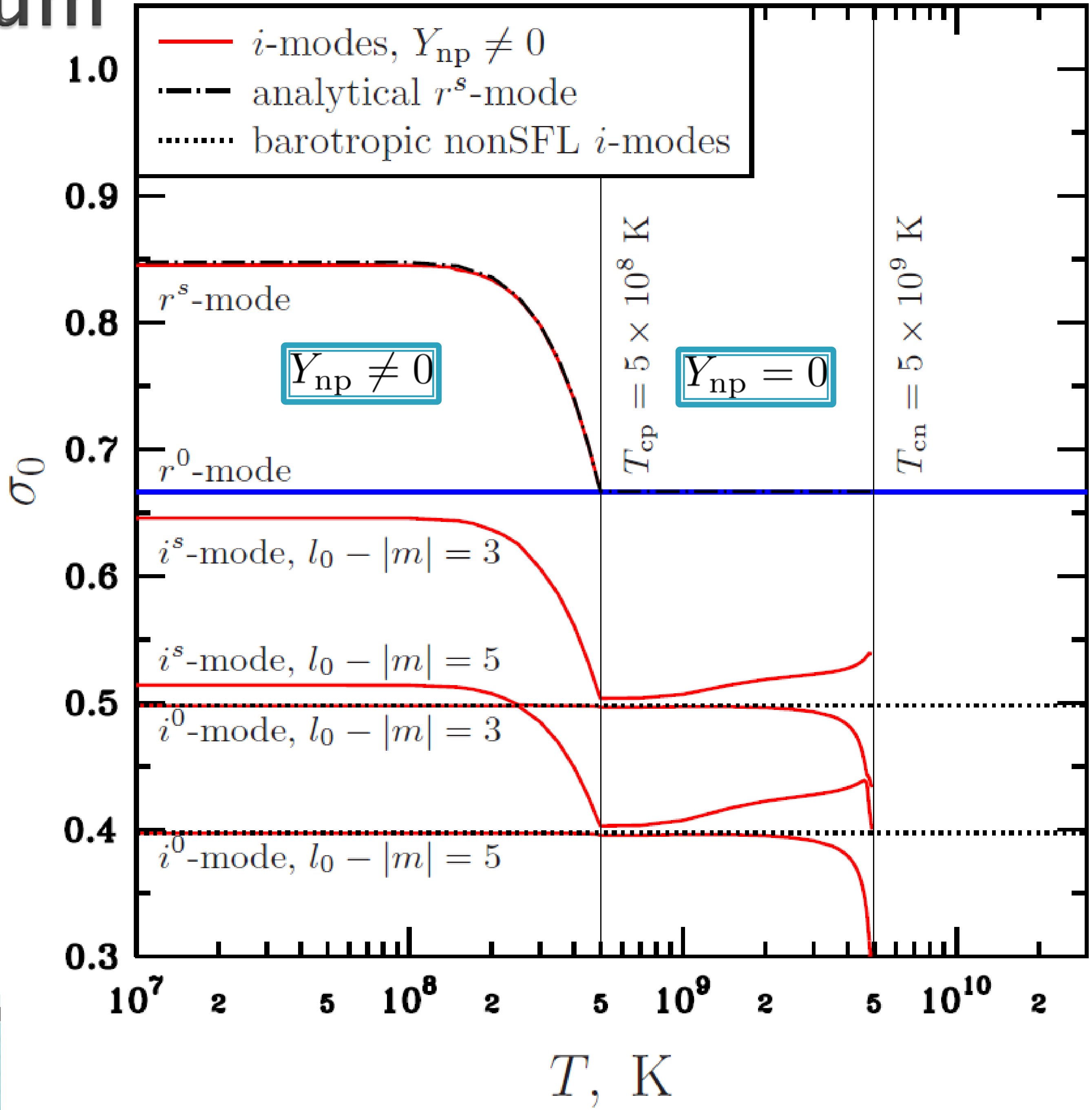
10^7
2
5
10^8
2
5
10^9
T, K

Spectrum



i-modes, $Y_{\rm np} \neq 0$
analytical r^s-mode
barotropic nonSFL i-modes
r^s-mode
$Y_{\rm np} \neq 0$
$Y_{\rm np} = 0$
$T_{\rm cp} = 5 \times 10^8$ K
$T_{\rm cn} = 5 \times 10^9$ K
r^0-mode
i^s-mode, $l_0 - |m| = 3$
i^s-mode, $l_0 - |m| = 5$
i^0-mode, $l_0 - |m| = 3$
i^0-mode, $l_0 - |m| = 5$
σ_0
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
10^7
2
5
10^8
2
5
10^9
2
5
10^{10}
2
T, K

Classification of i-modes

In barotropic nonSFL NS (Yoshida & Lee 2000):

r-mode $\sigma_0 = \dfrac{2}{m+1}$

$$
\left[\begin{array}{llll}
 & & & \cdots \\
 & & i_2 \ \ (l_0 = |m|+5) & \cdots \\
 & i_1 \ \ (l_0 = |m|+3) & i_1 \ \ (l_0 = |m|+5) & \cdots \\
i_0 \ \ (l_0 = |m|+1) & i_0 \ \ (l_0 = |m|+3) & i_0 \ \ (l_0 = |m|+5) & \cdots \\
 & i_{-1}(l_0 = |m|+3) & i_{-1}(l_0 = |m|+5) & \cdots \\
 & & i_{-2}(l_0 = |m|+5) & \cdots \\
 & & & \cdots
\end{array}\right]
$$

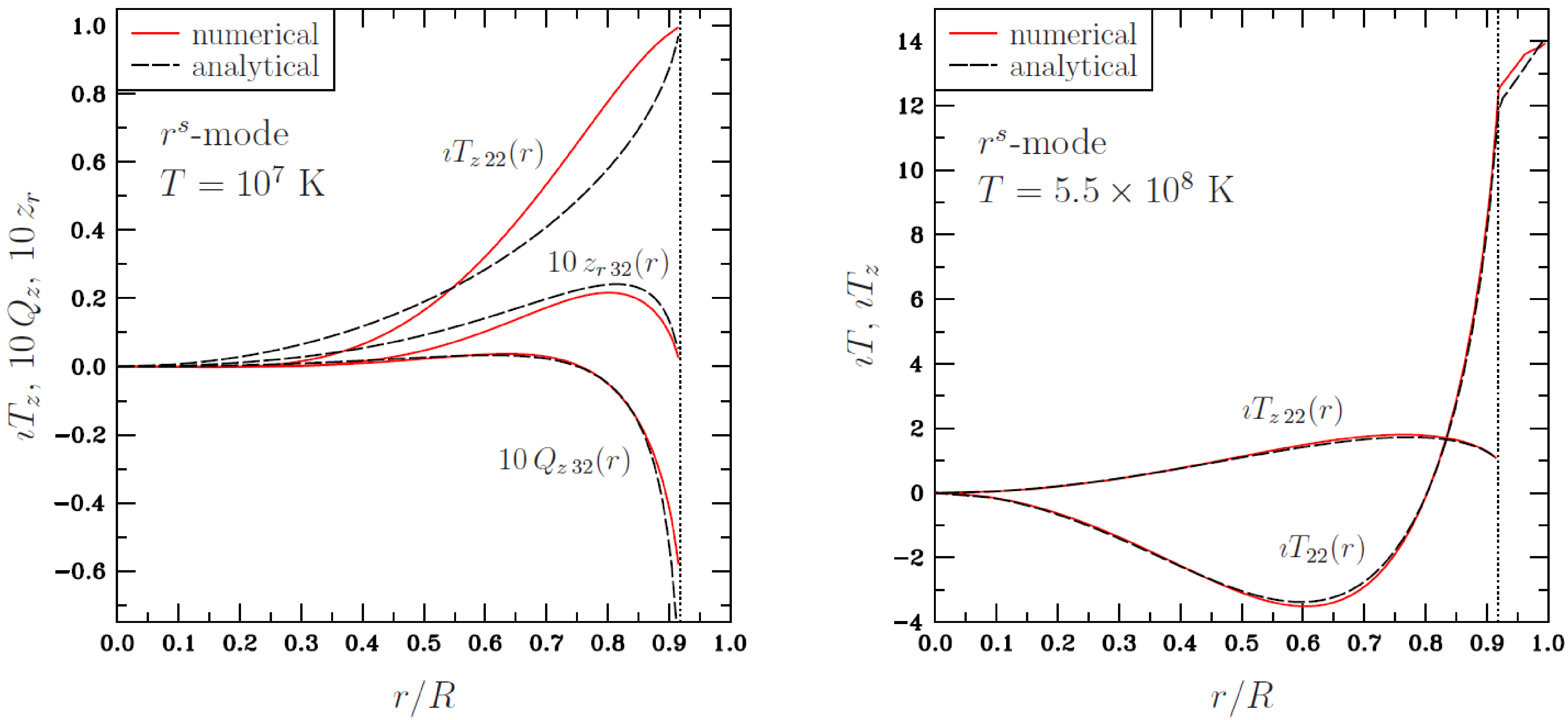
(i^0-modes)

In SFL npe NS – the same amount of SFL i^s-modes additionally

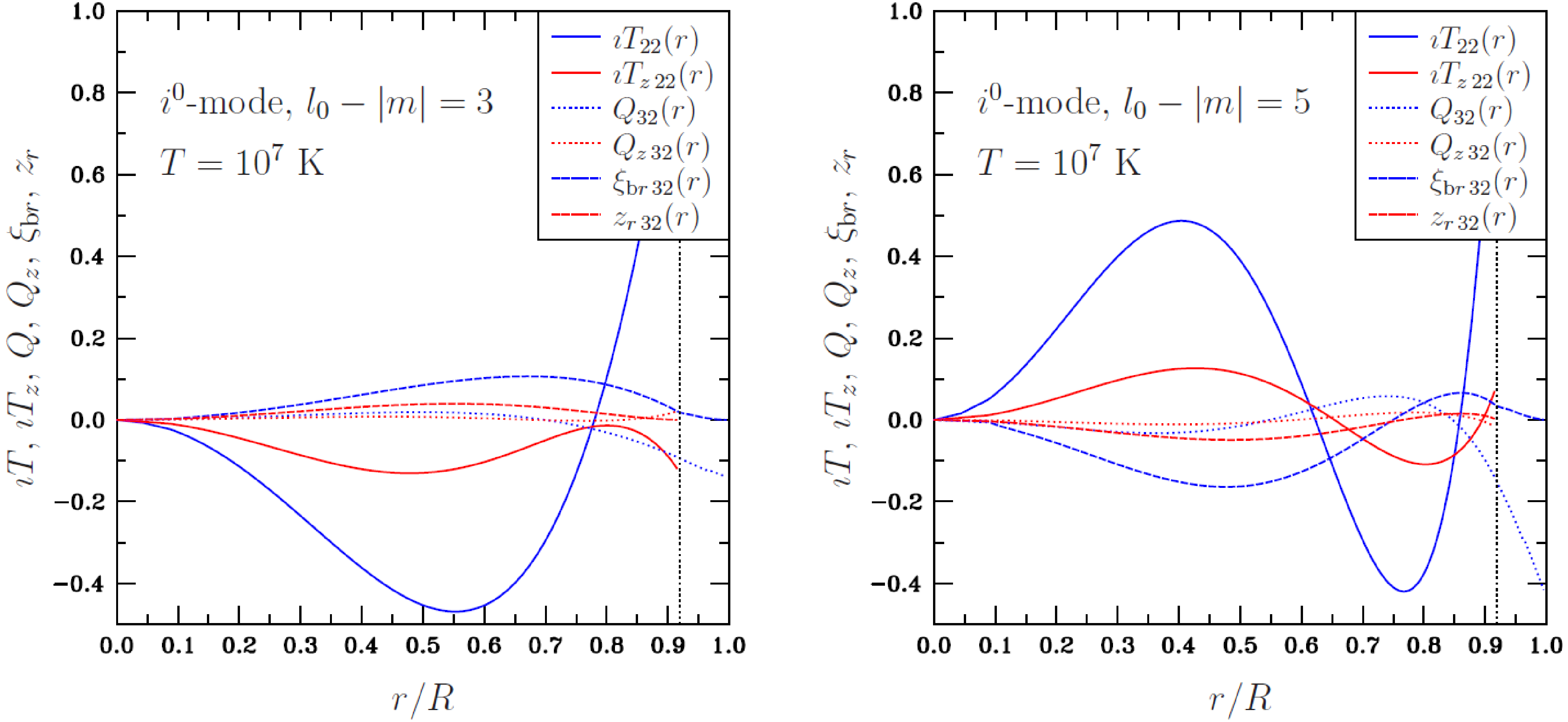
all harmonics with c $l > l_0$ are suppressed

$(l_0 - |m|)$ determines the number of nodes for eigenfunctions
(Yoshida & Lee 2000)

Eigenfunctions for superfluid r^s–mode



Eigenfunctions for i^o–modes



Eigenfunctions for i^s–modes