An overview of equation of state constraints from EM observations of neutron stars.

Sebastien Guillot

Zamfir et al. 2018

Guillot et al. Özel et al.

Suleimanov et al. 2011

Bogdanov 2013

Sanna et al.
An open question

Which observational methods can result in reliable and precise measurements of neutron stars radii and masses?
The internal structure of neutron stars is still unknown and many theories are proposed.
Dense nuclear matter is described by an equation of state $P(\rho)$. But what is it?

Lattimer and Prakash 2001
Neutron stars with different properties and observational signatures can be useful for $R_{NS}$ measurements.
The pulsed emission caused by hot spots on a rotating neutron star can help measure the compactness.
Strong gravity permits seeing beyond the hemisphere of the neutron star.

Credits: S. Morsink
Strong gravity permits seeing beyond the hemisphere of the neutron star.

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The pulsed emission caused by hot spots on a rotating neutron star can help measure the compactness.
The pulsed emission caused by hot spots on a rotating neutron star can help measure the compactness, but this depends on the system geometry.

\[ M_{\text{NS}} = 1.4 \, M_\odot, \quad R_{\text{NS}} = 10\, \text{km} \]

(Bodganov et al. 2008)
Two main ingredients are necessary to model the lightcurve of millisecond pulsars.

**General relativity**
Schwarzschild metric + time delays, doppler boosts/aberration + oblate star

**Surface emission model**
Low-magnetic field, fully-ionized H or He atmosphere

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Miller & Lamb 1998
Morsink et al. 2007
If we can model the lightcurve, it is preferable to know the neutron star mass.

PSR J0437-4715 with XMM-Newton

$M_{NS}$ (M$_\odot$)

$R_{NS}$ (km)

$f_{\text{spin}} = 173$ Hz

$\frac{M}{R}$ extracted from lightcurve

Bogdanov (2013)
NICER’s main science goal is to measure $R_{NS}$ for four millisecond pulsars.

Launched in June 2017
NICER now routinely observes four key target millisecond pulsars.

Analysis is in progress…
There are some **difficulties** involved with the lightcurve modelling technique used by NICER.

- Atmospheric composition
- Hot spots properties (these are fitted, but there are some degeneracies)
- Difficult to precisely determine the NICER background (non X-ray and X-ray backgrounds)
What’s new?
Be patient...
X-ray observations of thermally-cooling neutron stars permit obtaining the radius.

To measure the radius, we need to:

- observe/model the surface emission,
- know the distance independently.

\[ F_X \propto \left( \frac{R_\infty}{D} \right)^2 \sigma T^4 \]
Quiescent low-mass X-ray binaries are ideal systems for radius measurements.

Surface thermal emission at $T_{\text{eff}} \sim 10^6$ K, powered by residual heat from the interior radiating outwards through the atmosphere with $L_X = 10^{32-33}$ erg/sec

$$F_X \propto \left(\frac{R_\infty}{D}\right)^2 \sigma T^4$$

$$R_\infty = R_{\text{NS}} (1 + z) = R_{\text{NS}} \left(1 - \frac{2GM_{\text{NS}}}{R_{\text{NS}} c^2}\right)^{-1/2}$$
To sum up, the X-ray spectra of thermally emitting neutron stars is used to extract their $M_{\text{NS}}$ and $R_{\text{NS}}$. 

Bogdanov et al. 2016
We want to find which equation of state is common to all these M-R measurements.

\[ R_{\infty} = R_{\text{NS}} (1 + z) = R_{\text{NS}} \left( 1 - \frac{2GM_{\text{NS}}}{R_{\text{NS}} c^2} \right)^{-1/2} \]
A solution consists in combining these observations in a statistical analysis.

Constant $R_{\text{NS}}$

i.e., the radius is the same for all neutron stars


Analytical parameterizations

Raithel et al. (2016)
Some constraints have been obtained using these analytical parameterisations of the EoS.

Özel et al. (2016)  
Updated results (2017)
There are some difficulties involved with the surface thermal emission technique.

- **NS atmospheric composition**
- **NS surface temperature distribution**
- **NS magnetic field and rotation**
- **Need to parameterise the EoS**

Multi-wavelength observations can help!
What’s new?

- New X-ray data (from Chandra X-ray Observatory)
- New distances (expected from Gaia DR3)
- New method (realistic parameterisation of the EoS)
Using a realistic parameterisation of the equation of state, we improve on previous estimates.

\[ P = f (\rho, E_{\text{sat}}, E_{\text{sym}}, L_{\text{sym}}, K_{\text{sym}}, K_{\text{sat}}, Q_{\text{sat}}, \ldots) \]

Model of Margueron et al. (2018)
X-ray observations of thermally-cooling neutron stars permit obtaining the radius.

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Neutron stars in supernova remnants

Neutron stars in quiescent low-mass X-ray binaries

Neutron stars with thermonuclear bursts

Thermally-cooling isolated neutron stars
X-ray observations of thermally-cooling neutron stars permit obtaining the radius.

To measure the radius, we need to:

- observe/model the surface emission,
- know the distance independently.

\[ F_X \propto \left( \frac{R_\infty}{D} \right)^2 \sigma T^4 \]
Some LMXBs exhibiting thermonuclear bursts with Photospheric Radius Expansion.
The peak flux correspond to the Eddington flux, and the cooling tail gives the size of the emitting area.

\[ F_{\text{Edd}, \infty} = \frac{GcM_{\text{NS}}}{\kappa D^2} \frac{1}{(1 + z)} \]

\[ A_{\infty} = \frac{R^2}{f_c^4 D^2} (1 + z)^2 \]

with \( (1 + z) = \left(1 + \frac{2GM_{\text{NS}}}{c^2 R_{\text{NS}}} \right)^{-1/2} \)
Different analyses and types of sources result in different constraints... It’s a heated debate!

Özel et al. 2016

Suleimanov et al. 2011

**Data-driven selection of (high-soft state) bursts**

**Theory-driven selection of low-hard state bursts**
There are some difficulties involved with the Type I X-ray burst technique. 

- Atmosphere modelling (i.e., the conversion to/from a blackbody with a colour correction factor $f_c$)
- Neutron star atmospheric composition
- Distance sometimes unknown
What’s new?

- New method!
- New instrument (NICER)
- New “problem”
Until recently, the thermal emission was fit with a Planck function, and a colour-correction was used.

**Solution**: fit every single spectrum with realistic model

\[ A_\infty = \frac{R^2}{f_c^4 D^2} (1 + z)^2 \]

\( f_c \) is model dependent!
A robust modeling of the cooling tail avoids using color-correction factors.

Nattila et al. 2017
The observation of type I X-ray burst with NICER showed the presence of a *un-modelled soft-E excess*.
The observation of type I X-ray burst with NICER showed the presence of a un-modelled soft-E excess.

Keek et al. (2018)
Other methods exist to constrain $R_{\text{NS}}$, or to place limits on it.
Limits on the maximum $M_{\text{NS}}$ also constrain the equation of state

$$M_{\text{J1614}} = 1.908 \pm 0.016M_\odot \quad \rightarrow \quad M_{\text{max}} \gtrsim 1.9M_\odot$$
### Summary – Current status

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<td>NICER will produce 4 precise $R_{\text{NS}}$ measurements</td>
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<td><strong>Quiescent low-mass X-ray binaries</strong></td>
<td>Some uncertainties related to assumptions. Need parameterisation of EOS (or masses).</td>
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<td><strong>Isolated neutron stars (and CCOs)</strong></td>
<td>Atmosphere modelling difficult (e.g. magnetic field and composition)</td>
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<td><strong>Type-I X-ray bursts</strong></td>
<td>Some modelling uncertainties. Understand the effects of accretion?</td>
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<td><strong>GW from NS-NS merger</strong></td>
<td>Promising technique! But only 1 event so far</td>
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<td><strong>Moment of inertia (radio timing)</strong></td>
<td>Spin-orbit coupling needs decades of data</td>
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<td><strong>Moment of inertia (bow-shock PWN)</strong></td>
<td>Two attempts: See Bejger et al. (2002); Romani et al. (2017)</td>
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<td><strong>Max. rotation of NSs</strong></td>
<td>Look for more pulsars and hope for a faster one</td>
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<td><strong>Inner extent of accretion disks</strong></td>
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<td><strong>Max. mass of NS</strong></td>
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## Summary – Current results

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<td>CAN’T TELL YOU!!!</td>
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<td>$R_{NS} \sim 10 – 13 \text{ km}$</td>
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<td>$R_{NS} \sim 10 – 14 \text{ km}$</td>
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<td>$R_{NS} \sim 11 – 13 \text{ km}$</td>
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<td><strong>GW from NS-NS merger</strong></td>
<td>$R_{NS} \sim 10.5 – 13.5 \text{ km}$</td>
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<td><strong>Moment of inertia (radio timing)</strong></td>
<td>No results yet</td>
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<td><strong>Moment of inertia (bow-shock PWN)</strong></td>
<td>$I = 2.4 \times 10^{45} \text{ g cm}^2$ – Romani et al. (2017)</td>
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<td><strong>Max. rotation of NSs</strong></td>
<td>Limits from fastest pulsar: $R_{NS} &lt; 17 \text{ km}$</td>
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<td><strong>Inner extent of accretion disks</strong></td>
<td>Limit from accreting NS: $R_{NS} &lt; 12.6 \text{ km}$</td>
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<td>Understand effects of accretion / higher collecting area (e.g. eXTP)</td>
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<td>More events from LIGO / Virgo / Karga / LIGO India</td>
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<td>More data from current observatories / SKA</td>
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<td>3D modelling of the shock</td>
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