



Neutrino mixing parameters



Neutrino Parameters

Current values

- Prior to 2006, limits and values on θ and Δm^2 were encoded based on P_{osc} using 2-neutrino formulae. Useful information was only in the reviews.
- Starting in 2006, we encoded the 3-neutrino parameters θ_{12} , θ_{13} , θ_{23} , Δm^2_{32} , Δm^2_{21} , adding δ_{CP} in 2014
- Reminder – If light sterile neutrinos (as a solution to the short-baseline anomalies) exist, all of these numbers are some mixture of wrong, meaningless, or approximations.

(B) Three-neutrino mixing parameters

$\sin^2(\theta_{12})$	0.307 ± 0.013
Δm^2_{21}	$(7.53 \pm 0.18) \times 10^{-5} \text{ eV}^2$
$\sin^2(\theta_{23})$	$0.417^{+0.025}_{-0.028} (S = 1.2) \dots$
Δm^2_{32}	$0.00251 \pm 0.00005 \text{ eV}^2 (S = 1.1)$
$\sin^2(\theta_{13})$	0.0212 ± 0.0008



Parameter errors(t)

(based on values; not %)

RPP	2006	07	09	10	11	12	13	14	15	16	2017	18
$\sin^2 2\theta_{12}$	+ .03 - .04		± 0.03		+ .026 - .022	+ .023 - .025		+ .021 - .021	+ .021 - .021			
$\sin^2 2\theta_{13}$	<.19			<.15		± 0.13	± 0.10	± 0.08	± 0.05			
$\sin^2 2\theta_{23}$	>.92					>.95		.018 N .018 I	.018 N .018 I			
$\sin^2 \theta_{12}$									+ .014 - .013		+ .013 - .012	
$\sin^2 \theta_{13}$									± 0.12		± 0.11	± 0.08
$\sin^2 \theta_{23}$									$\pm .056$	± 0.05		+ .025 - .028
Δm^2_{32} 10^{-3}eV^2	1.1		± 0.13			+ .12 - .08		$\pm .07$ N $\pm .06$ I	$\pm .06$		± 0.05	
Δm^2_{21} 10^{-5}eV^2	+ .4 - .3	$\pm .3$	+ .19 - .21		+ .20 - .21	+ .19 - .20						± 0.18

N – normal ordering, I – inverted ordering
 δ_{CP} added in 2014 but no averages quoted
 $\sin^2 2\theta$ switched to $\sin^2 \theta$ in 2015; only affecting θ_{23}



2017 review vs. listings

From: Alessandra.Re@mi.infn.it
Subject: About the PDG 2017 update - neutrino oscillation parameters
Date: July 7, 2017 at 11:49:09 AM GMT+2

we (the Borexino collaboration) are writing some new articles and we would like to take your PDG review as reference as regards the neutrino oscillation parameters.

July 2017 is time of PDG-middle-term update so I checked on the webpage: <http://pdg.lbl.gov/2017/tables/rpp2017-sum-leptons.pdf>
 The "neutrino mixing" summary table at pg. 10.

I'm a bit confused now...

The values there reported are quite different from those "officially" included in Table 14.1, pg 8 of the 2016 review.
<http://pdg.ihep.su/2017/reviews/rpp2016-rev-neutrino-mixing.pdf>

As far as I understand, the 2016 values were taken from Capozzi, Lisi et al. (<https://doi.org/10.1016/j.nuclphysb.2016.02.016>):
 where do the 2017 updated values come from? Is there any work I can read?

Thanks in advance!

Best regards,
 Alessandra

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F. Capozzi et al. / Nuclear Physics B 908 (2016) 218–234

Table 1
 Results of the global 3ν oscillation analysis, in terms of best-fit values and allowed 1, 2 and 3σ ranges for the 3ν mass-mixing parameters. See also Fig. 1 for a graphical representation of the results. We recall that Δm^2 is defined as $m_3^2 - (m_1^2 + m_2^2)/2$, with $+\Delta m^2$ for NH and $-\Delta m^2$ for IH. The CP violating phase is taken in the (cyclic) interval $\delta/\pi \in [0, 2]$. The last row reports the (statistically insignificant) overall χ^2 difference between IH and NH.

Parameter	Hierarchy	Best fit	1σ range	2σ range	3σ range
$\delta m^2/10^{-5} \text{ eV}^2$	NH or IH	7.37	7.21–7.54	7.07–7.73	6.93–7.97
$\sin^2 \theta_{12}/10^{-1}$	NH or IH	2.97	2.81–3.14	2.65–3.34	2.50–3.54
$\Delta m^2/10^{-3} \text{ eV}^2$	NH	2.50	2.46–2.54	2.41–2.58	2.37–2.63
$\Delta m^2/10^{-3} \text{ eV}^2$	IH	2.46	2.42–2.51	2.38–2.55	2.33–2.60
$\sin^2 \theta_{13}/10^{-2}$	NH	2.14	2.05–2.25	1.95–2.36	1.85–2.46
$\sin^2 \theta_{13}/10^{-2}$	IH	2.18	2.08–2.27	1.96–2.38	1.86–2.48
$\sin^2 \theta_{23}/10^{-1}$	NH	4.37	4.17–4.70	3.97–5.63	3.79–6.16
$\sin^2 \theta_{23}/10^{-1}$	IH	5.69	4.28–4.91 \oplus 5.18–5.97	4.04–6.18	3.83–6.37
δ/π	NH	1.35	1.13–1.64	0.92–1.99	0–2
δ/π	IH	1.32	1.07–1.67	0.83–1.99	0–2
$\Delta\chi^2_{\text{I-N}}$	IH–NH	+0.98			

Neutrino Mixing

The following values are obtained through data analyses based on the 3-neutrino mixing scheme described in the review “Neutrino Mass, Mixing, and Oscillations” by K. Nakamura and S.T. Petcov in this Review.

$$\begin{aligned} \sin^2(\theta_{12}) &= 0.307 \pm 0.013 \\ \Delta m_{21}^2 &= (7.53 \pm 0.18) \times 10^{-5} \text{ eV}^2 \\ \sin^2(\theta_2) &= 0.51 \pm 0.04 \quad (\text{normal mass hierarchy}) \\ \sin^2(\theta_{23}) &= 0.50 \pm 0.04 \quad (\text{inverted mass hierarchy}) \\ \Delta m_{32}^2 &= (2.45 \pm 0.05) \times 10^{-3} \text{ eV}^2 [i] \quad (\text{normal mass hierarchy}) \\ \Delta m_{32}^2 &= (2.52 \pm 0.05) \times 10^{-3} \text{ eV}^2 [i] \quad (\text{inverted mass hierarchy}) \\ \sin^2(\theta_{13}) &= (2.10 \pm 0.11) \times 10^{-2} \end{aligned}$$



Averaging θ_{23} ??

- The fit used unpublished data, NOvA at Neutrino 2016 (published in 17)
- NOvA 2016 disfavored maximal mixing at 2.6σ , while NOvA 2018 only disfavors it at 0.8σ .
- Averaging values from a bimodal distribution can lead to problems.
- I thought this was a bigger deal than it was in RPP2016 given the published NOvA errors at the time.
- Going forward, it was agreed the review should use PDG averages and in RPP2018 averages were separated by octant & mass order.

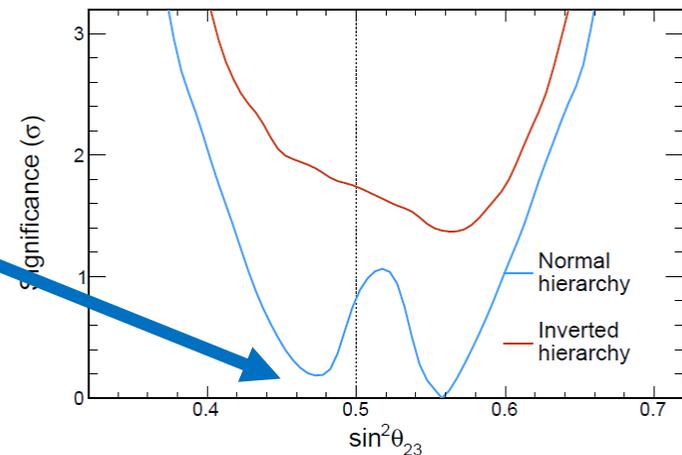


FIG. 14. Significance at which each value of $\sin^2 \theta_{23}$ is disfavored in the normal (blue, lower) or inverted (red, upper) mass hierarchy. The vertical dotted line indicates the point of maximal mixing.



Errors using the minima of χ^2

- Calculating significance for parameters using normal/inverted mass ordering, some use the global minimum, others local.
- Example
 - NOvA & T2K 2018 use global min
 - Salas global fit uses local min
 - NOvA IO quotes a best fit for δ , but no value at 90% CL.
 - T2K seems to have a much more precise value of δ for the unfavored order.
- Different people can get different answers from the same data.
- Don't want to average values taken different ways.

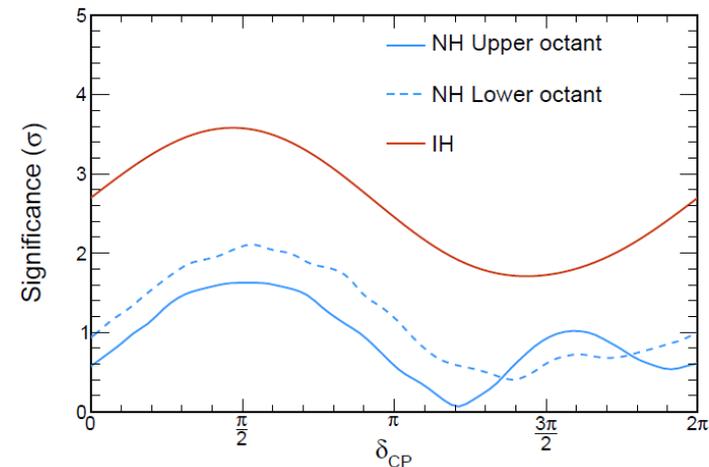


FIG. 15. Significance at which each value of δ_{CP} is disfavored in the normal (blue, lower) or inverted (red, upper) mass hierarchy. The normal mass hierarchy is divided into upper (solid) and lower (dashed) θ_{23} octants corresponding to the near degeneracy in $\sin^2 \theta_{23}$.



possible changes (1)

not related to sterile vs

I'm not suggesting we make this change but I feel this needs to be mentioned

- ✱ With 3 ν 's, There are six numbers Δm^2_{ij}
- ✱ Only two of them are independent
- ✱ In 2006 we chose to encode Δm^2_{32} and Δm^2_{21}
- ✱ (Pet peeve – people who mix up Δm^2_{32} & Δm^2_{23} since we are trying to measure if it is positive or negative.)
- ✱ Experiments actually measure a mixture, now defined $\Delta m^2_{\mu\mu}$ & Δm^2_{ee}
 - $\sin^2[1.27 \Delta m^2_{ee} L(m)/E(\text{MeV})] \cong$
 $\cos^2\theta_{12} \sin^2[1.27 \Delta m^2_{31} L(m)/E(\text{MeV})]$
 $+ \sin^2\theta_{12} \sin^2[1.27 \Delta m^2_{32} L(m)/E(\text{MeV})]$



possible changes (2)

not related to sterile vs

* Add a node for mass ordering

- Papers are calculating $P(N) + P(I) = 1$
- $P(N) > P(I)$

* Rewrite minireview regarding listings.

- It was originally written when $\Delta m^2_{32} \sim \Delta m^2_{31}$ was a good approximation. It has been tweaked since then, but is no longer useful and needs a rewrite.

* Switch averages to global fits

- Some clear advantages
- Some clear disadvantages



Averages vs. global fits

Global fit disadvantages:

- Use unpublished data (A PDG sin)
- Experimenters can better understand their own correlations

Global fit advantages

- Most oscillation measurements depend on multiple parameters, so they can treat all data with a consistent set of numbers 
- Avoids θ_{23} averaging problem
- Avoids averaging local & global minima answers

Possible idea? -- Work with a team of global fitters and PDG specifies which (published) data to include.