# Supergravity at Five Loops

## SAGEX Kickoff Meeting
## Queen Mary
## September 4, 2018
## Zvi Bern

UCLA The Mani L. Bhaumik Institute for Theoretical Physics

**ZB, John Joseph Carrasco, Wei-Ming Chen, Henrik Johansson, Radu Roiban, arXiv:1701.02519**

**ZB, John Joseph Carrasco, Wei-Ming Chen, Henrik Johansson, Radu Roiban, Mao Zeng, arXiv:1708.06807**

**ZB, John Joseph Carrasco, Wei-Ming Chen, Alex Edison, Henrik Johansson, Julio Parra-Martinez, Radu Roiban, Mao Zeng, arXiv:1804.09311**

# Outline

1. Duality between color and kinematics and double copy.

2. Double copy and classical solutions.

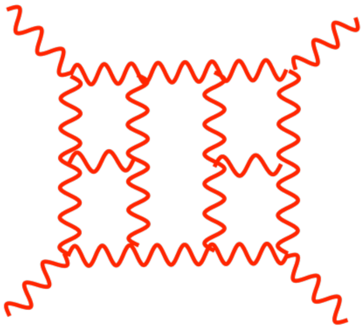
3. Applications of double copy to problem of UV divergence in quantum gravity.

4. Generalized double copy. Double copy any gauge-theory format.

5. UV properties at 5 loops in $\mathcal{N} = 8$ supergravity.

6. New UV consistency constraints.

7. Towards ever higher-loop determination of UV.



# Gravity vs Gauge Theory

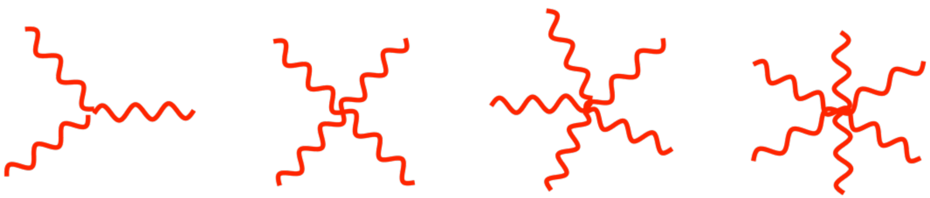
**Consider the Einstein gravity Lagrangian**

$$L_{\text{gravity}} = \frac{2}{\kappa^2}\sqrt{-g}\,R \qquad g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$$

curvature · flat-space metric · graviton field · metric

$$\kappa^2 = 32\pi\, G_{\text{Newton}}$$

**Infinite number of complicated interactions**

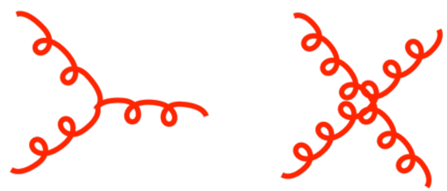


$+ \cdots$

**terrible mess**

**Compare to gauge-theory Lagrangian on which QCD is based**

$$L_{\text{YM}} = \frac{1}{g^2}F^2$$



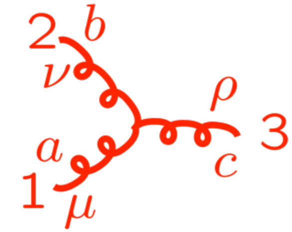
**Only three and four point interactions**

**Gravity seems so much more complicated than gauge theory.**

**Theories do not look related!**

# Three Vertices

## Standard Feynman diagram approach.




**Three-gluon vertex:**

$$V_{3\,\mu\nu\sigma}^{abc} = -g f^{abc} (\eta_{\mu\nu}(k_1 - k_2)_\rho + \eta_{\nu\rho}(k_1 - k_2)_\mu + \eta_{\rho\mu}(k_1 - k_2)_\nu)$$

**Three-graviton vertex:**

$$k_i^2 = E_i^2 - \vec{k}_i^2 \neq 0$$

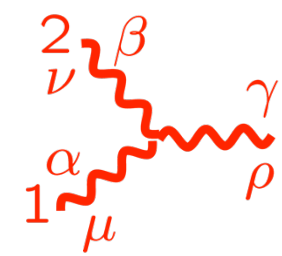
$$G_{3\mu\alpha,\nu\beta,\sigma\gamma}(k_1, k_2, k_3) =$$

$$\mathrm{sym}[-\tfrac{1}{2}P_3(k_1 \cdot k_2 \eta_{\mu\alpha}\eta_{\nu\beta}\eta_{\sigma\gamma}) - \tfrac{1}{2}P_6(k_{1\nu}k_{1\beta}\eta_{\mu\alpha}\eta_{\sigma\gamma}) + \tfrac{1}{2}P_3(k_1 \cdot k_2 \eta_{\mu\nu}\eta_{\alpha\beta}\eta_{\sigma\gamma})$$

$$+ P_6(k_1 \cdot k_2 \eta_{\mu\alpha}\eta_{\nu\sigma}\eta_{\beta\gamma}) + 2P_3(k_{1\nu}k_{1\gamma}\eta_{\mu\alpha}\eta_{\beta\sigma}) - P_3(k_{1\beta}k_{2\mu}\eta_{\alpha\nu}\eta_{\sigma\gamma})$$

$$+ P_3(k_{1\sigma}k_{2\gamma}\eta_{\mu\nu}\eta_{\alpha\beta}) + P_6(k_{1\sigma}k_{1\gamma}\eta_{\mu\nu}\eta_{\alpha\beta}) + 2P_6(k_{1\nu}k_{2\gamma}\eta_{\beta\mu}\eta_{\alpha\sigma})$$

$$+ 2P_3(k_{1\nu}k_{2\mu}\eta_{\beta\sigma}\eta_{\gamma\alpha}) - 2P_3(k_1 \cdot k_2 \eta_{\alpha\nu}\eta_{\beta\sigma}\eta_{\gamma\mu})]$$

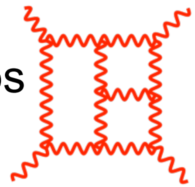



**About 100 terms in three vertex**

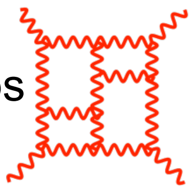
**Naïve conclusion: Gravity is a nasty mess.**

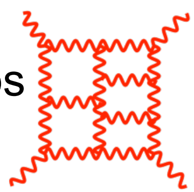
# Feynman Diagrams for Gravity

**Suppose we want to check UV properties of supergravity theories:**

3 loops  $\sim 10^{20}$ TERMS

No surprise it has never been calculated via Feynman diagrams.

4 loops  $\sim 10^{26}$ TERMS

5 loops  $\sim 10^{31}$ TERMS

More terms than atoms in your brain!

- Calculations to settle this seemed utterly hopeless!
- Seemed destined for dustbin of undecidable questions.

**Superspace helps, but not enough to make a difference.**
**Standard techniques utterly hopeless.**

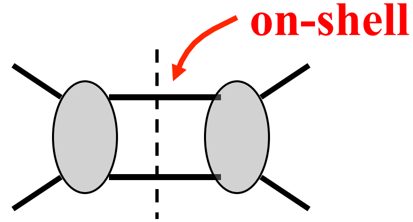
**Clearly this is the wrong way to look at it**

Bern, Dixon, Dunbar and Kosower (1994)

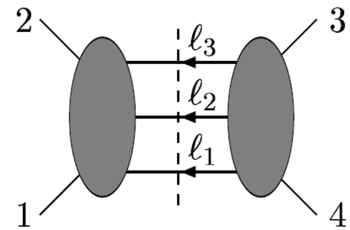
# Modern Unitarity Method

**To get KLT into loops needed new tools**

**Two-particle cut:**



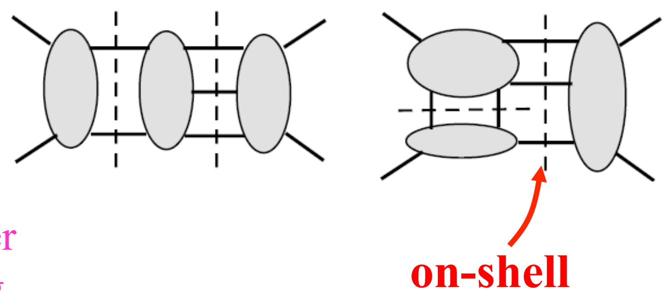

**Three-particle cut:**




**Systematic assembly of complete amplitudes from cuts for any number of particles or loops.**

**Generalized unitarity as a practical tool**

Bern, Dixon and Kosower
Britto, Cachazo and Feng
ZB, Carrasco, Johansson, Kosower




**Different cuts merged to give an expression with correct cuts in all channels.**

**Reproduces Feynman diagrams except intermediate steps of calculation based on physical quantities not unphysical ones.**

# Kawai-Lewellen-Tye String Theory Relations

**Kawai-Lewellen-Tye relations in low energy limit:** KLT (1985)

gravity — gauge theory color ordered

$$M_4^{\text{tree}}(1,2,3,4) = -is_{12}A_4^{\text{tree}}(1,2,3,4)\,A_4^{\text{tree}}(1,2,4,3)\,,$$

$$M_5^{\text{tree}}(1,2,3,4,5) = is_{12}s_{34}A_5^{\text{tree}}(1,2,3,4,5)\,A_5^{\text{tree}}(2,1,4,3,5) + is_{13}s_{24}A_5^{\text{tree}}(1,3,2,4,5)\,A_5^{\text{tree}}(3,1,4,2,5)$$

**Pattern gives explicit all-leg form** ZB, Dixon, Rozowsky Perelstein (1998)

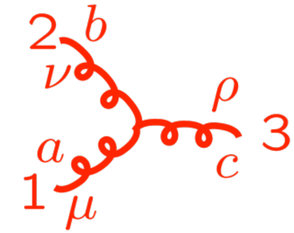

1. **Gravity is derivable from gauge theory. Standard QFT offers no hint why this is possible.**
2. **It looked very generally applicable.**
3. **It took people a while to appreciate its significance.**

# Duality Between Color and Kinematics

ZB, Carrasco, Johansson

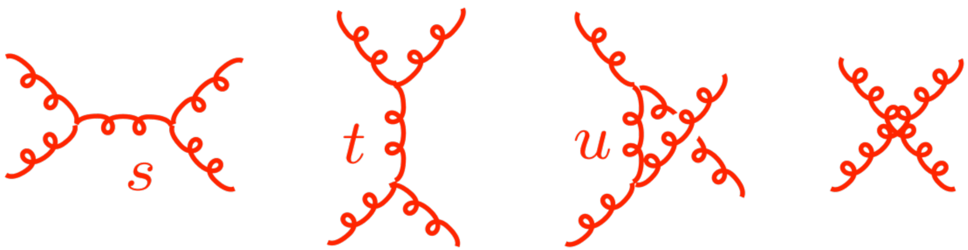
**coupling constant** → **color factor** ← **momentum dependent kinematic factor**

$$-g f^{abc}(\eta_{\mu\nu}(k_1 - k_2)_\rho + \text{cyclic})$$




**Color factors based on a Lie algebra:** $[T^a, T^b] = i f^{abc} T^c$

**Jacobi Identity** $f^{a_1 a_2 b} f^{b a_4 a_3} + f^{a_4 a_2 b} f^{b a_3 a_1} + f^{a_4 a_1 b} f^{b a_2 a_3} = 0$




**Use 1 = *s/s* = *t/t* = *u/u* to assign 4-point diagram to others.**

$s = (k_1 + k_2)^2 \quad t = (k_1 + k_4)^2$

$u = (k_1 + k_3)^2$

$$\mathcal{A}_4^{\text{tree}} = g^2 \left( \frac{n_s c_s}{s} + \frac{n_t c_t}{t} + \frac{n_u c_u}{u} \right)$$

**Color factors satisfy Jacobi identity:** $c_u = c_s - c_t$

**Numerator factors satisfy similar identity:** $n_u = n_s - n_t$

**Proven at tree level**

Zhu; Goebel, Halzen, Leveille

ZB, Carrasco, Johansson; Kiermaier; Bjerrum-Bohr, Damgaard, Sondergaard, Vanhove; Cachazo, etc

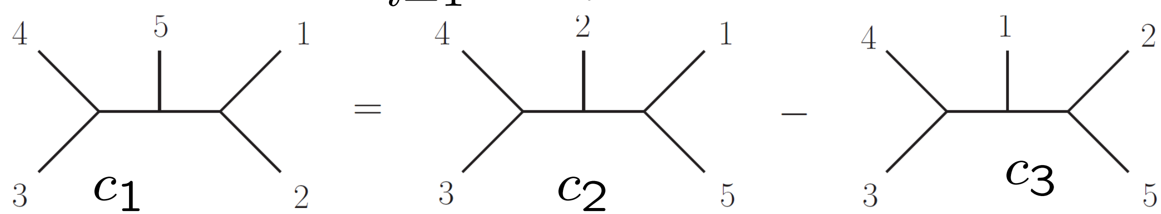
# Duality Between Color and Kinematics

**Consider five-point tree amplitude:**

ZB, Carrasco, Johansson (BCJ)

$$\mathcal{A}_5^{\text{tree}} = \sum_{i=1}^{15} \frac{c_i n_i}{\prod_{\alpha_i} p^2_{\alpha_i}}$$

- $c_i$: color factor
- $n_i$: kinematic numerator factor
- $\prod_{\alpha_i} p^2_{\alpha_i}$: Feynman propagators




$$c_1 = f^{a_3 a_4 b} f^{b a_5 c} f^{c a_1 a_2} \qquad c_2 = f^{a_3 a_4 b} f^{b a_2 c} f^{c a_1 a_5} \qquad c_3 = f^{a_3 a_4 b} f^{b a_1 c} f^{c a_2 a_5}$$

$$n_i \sim k_4 \cdot k_5 \, k_2 \cdot \varepsilon_1 \, \varepsilon_2 \cdot \varepsilon_3 \, \varepsilon_4 \cdot \varepsilon_5 + \cdots$$

See John Joseph's talk

$$c_1 + c_2 + c_3 = 0 \quad \Leftrightarrow \quad n_1 + n_2 + n_3 = 0$$

**Claim: We can always find a rearrangement so color and kinematics satisfy the *same* algebraic constraint equations.**

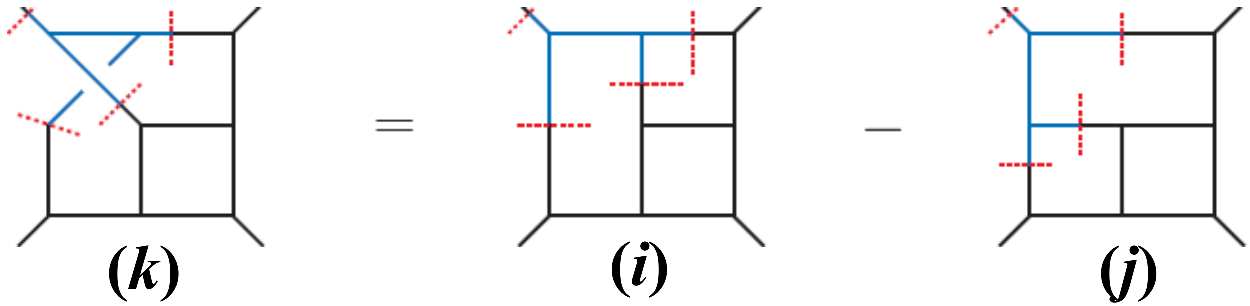
**Progress on unraveling relations.**

BCJ, Bjerrum-Bohr, Feng, Damgaard, Vanhove, ; Mafra, Stieberger, Schlotterer;
Tye and Zhang; Feng, Huang, Jia; Chen, Du, Feng; Du, Feng, Fu; Naculich, Nastase, Schnitzer
O'Connell and Montiero; Bjerrum-Bohr, Damgaard, O'Connell and Montiero; O'Connell, Montiero, White;
Du, Feng and Teng, Song and Schlotterer, etc.

# Gravity Loop Integrands from Gauge Theory

BCJ

**Ideas conjectured to generalize to loops:**




color factor

$$c_k = c_i - c_j$$

$$n_k = n_i - n_j$$

kinematic numerator

**If you have a set of duality satisfying numerators.**

**To get:**

**gauge theory → gravity theory**

**simply take**

**color factor → kinematic numerator**

$$c_k \longrightarrow n_k$$

**Gravity loop integrands follow from gauge theory!**

# Gravity From Gauge Theory

**Here we consider only simplest constructions:**

$$\mathcal{N}=8 \text{ sugra:} \quad (\mathcal{N}=4 \text{ sYM}) \times (\mathcal{N}=4 \text{ sYM})$$
$$\mathcal{N}=5 \text{ sugra:} \quad (\mathcal{N}=4 \text{ sYM}) \times (\mathcal{N}=1 \text{ sYM})$$
$$\mathcal{N}=4 \text{ sugra:} \quad (\mathcal{N}=4 \text{ sYM}) \times (\mathcal{N}=0 \text{ sYM})$$

**Spectrum controlled by simple tensor product of gauge theories.**

**More sophisticated lower-susy cases: QCD, magical supergravities, Einstein-YM with and without Higgsing, twin supergravities.**

Anastasiou, Bornsten, Duff; Duff, Hughs, Nagy; Johansson and Ochirov;
Carrasco, Chiodaroli, Günaydin and Roiban; ZB, Davies, Dennen, Huang and Nohle;
Nohle; Chiodaroli, Günaydin, Johansson, Roiban. A. Anastasiou, L. Borsten, M.J. Duff, M.J. Hughes,
Marrani, Nagy, Zoccali.

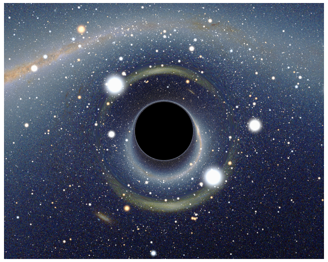
**Many other theories in double-copy story, including open and closed string theory, NLSM, Dirac Born Infeld, Galileon and Z theory.**

Cachazo, He, Yuan; Chen Du, Broedel, Schlotterer and Stieberger; Carrasco, Mafra, Schlotterer;

# Applications to Black Hole Physics

**Wouldn't it be really cool if every classical solution in gravity could be mapped to a double copy of classical solutions?**

**Where to start?** **Obviously the coolest place possible: black holes.**



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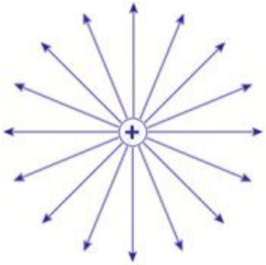


**Monteiro, O'Connell and White**

**black hole** **point charge**

**Special coordinates: Kerr-Schild coordinates:**

**Schwarzschild black hole** $$g_{\mu\nu} = \eta_{\mu\nu} + \phi k_\mu k_\nu \qquad \phi(r) = \frac{2m}{r}$$

***k* is null**

**Coulomb point charge** $$A_\mu = \phi k_\mu \qquad \phi(r) = \frac{Q}{r}$$

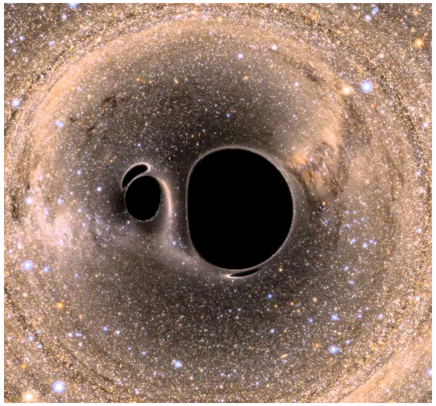
**Schwarzschild ~ (Coulomb)$^2$**

# Examples of Classical Solutions

Luna, Monteiro, O'Connell and White;
Luna, Monteiro, Nicholsen, O'Connell and White;
Ridgway and Wise; Goldberger and Ridgway
Carrillo Gonzalez,Penco, and Trodden;
Adamo, Casali, Mason, Nekovar;
Bahjat-Abbas, Luna, White

## A variety of other cases:

- **Kerr (rotating) black hole.**
- **Taub-NUT space.**
- **Various maximally symmetric spacetimes.**
- **Solutions with nontrivial backgrounds.**
- **Radiation from accelerating black hole.**



**At least in special cases, double-copy constructions for classical solutions work.**

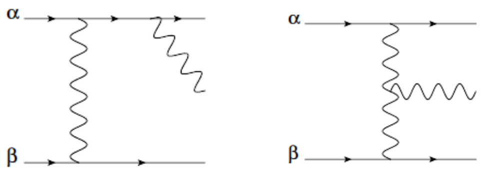
**Most promising direction is classical gravitation radiation: strong similarity to scattering problem.**

# Double Copy and Gravitational Radiation

**Can we simplify the types of calculations needed for LIGO?**

**A small industry has developed to study this:**

- **Connection to scattering amplitudes.**
  Bjerrum-Bohr, Donoghue, Holstein, Plante, Pierre Vanhove; Luna, Nicholson, O'Connell, White
  Bjerrum-Bohr, Damgaard, Festuccia, Planté, Vanhove.
- **First quantized approach for LO radiation.**
  Goldberger and Ridgway; Goldberger, Li, Prabhu, Thompson; Chester



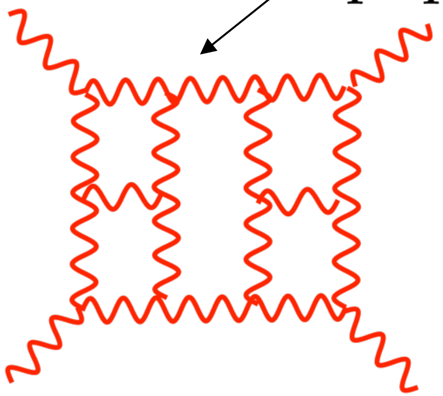

- **BCJ duality and double copy works at NLO in grav. coupling. Enormous simplification.** Chia-Hsien Shen
- **Dilaton contamination still a problem, (but don't worry about it).**
  Luna, Nicholson, O'Connell, White; Chester

- **Double copy appears to work!**
- **Challenge is to apply it to a problem of experimental interest.**
- **In the coming years you will hear a lot more about this!**

# Application of Double Copy: UV Behavior of Gravity.

# UV Behavior of Gravity?

$$\kappa = \sqrt{32\pi G_N}$$ ⟵ Dimensionful coupling




**Gravity:**
$$\int \prod_{i=1}^{L} \frac{d^D p_i}{(2\pi)^D} \frac{\cdots \kappa p_j^\mu p_j^\nu \cdots}{\text{propagators}}$$

**Gauge theory:**
$$\int \prod_{i=1}^{L} \frac{d^D p_i}{(2\pi)^D} \frac{\cdots g p_j^\nu \cdots}{\text{propagators}}$$

- **Extra powers of loop momenta in numerator means integrals are badly behaved in the UV and must diverge at some loop order.**
- **Much more sophisticated power counting in supersymmetric theories but this is basic idea.**

- **With more supersymmetry expect better UV properties.**
- **Need to worry about "hidden cancellations".**
- **$N = 8$ supergravity *best* theory to study.**

# $N = 8$ supergravity: Where is First $D = 4$ UV Divergence?

| | | |
|---|---|---|
| 3 loops $N = 8$ | Green, Schwarz, Brink (1982); Howe and Stelle (1989); Marcus and Sagnotti (1985) | ✗ |
| 5 loops $N = 8$ | Bern, Dixon, Dunbar, Perelstein, Rozowsky (1998); Howe and Stelle (2003,2009) | ✗ |
| 6 loops $N = 8$ | Howe and Stelle (2003) | ✗ |
| 7 loops $N = 8$ | Grisaru and Siegel (1982); Bossard, Howe, Stelle (2009);Vanhove; Björnsson, Green (2010); Kiermaier, Elvang, Freedman(2010); Ramond, Kallosh (2010); Biesert et al (2010); Bossard, Howe, Stelle, Vanhove (2011) | ? |
| 3 loops $N = 4$ | Bossard, Howe, Stelle, Vanhove (2011) | ✗ |
| 4 loops $N = 5$ | Bossard, Howe, Stelle, Vanhove (2011) | ✗ |
| 4 loops $N = 4$ | Vanhove and Tourkine (2012) | ✓ |
| 9 loops $N = 8$ | Berkovits, Green, Russo, Vanhove (2009) | ✗ |

ZB, Kosower, Carrasco, Dixon, Johansson, Roiban; ZB, Davies, Dennen, A. Smirnov, V. Smirnov; series of calculations.

**This is what we are most interested in and will answer here.** (← 7 loops $N = 8$)

**Weird structure. Anomaly-like behavior of divergence.** (← 4 loops $N = 4$)

**Retracted, but perhaps to be unretracted.** (← 9 loops $N = 8$)

- **Conventional wisdom holds that it will diverge sooner or later.**
- **Track record of predictions from symmetry not great.**

# Supersymmetry and Ultraviolet Divergences

Bossard, Howe, Stelle; Elvang, Freedman, Kiermaier; Green, Russo, Vanhove ; Green and Björnsson ; Bossard , Hillmann and Nicolai; Ramond and Kallosh; Broedel and Dixon; Elvang and Kiermaier; Beisert, Elvang, Freedman, Kiermaier, Morales, Stieberger; Bossard, Howe, Stelle, Vanhove, etc

- **First quantized formulation of Berkovits' pure-spinor formalism.**

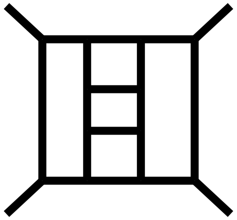
Bjornsson and Green

- **Key point: *all* supersymmetry cancellations are exposed.**

**Poor UV behavior, unless new types of cancellations between diagrams exist that are "not consequences of supersymmetry in any conventional sense".**

Bjornsson and Green

- **$N = 8$ sugra should diverge at 5 loops in $D = 24/5$.**
- **$N = 8$ sugra should diverge at 7 loops in $D = 4$.**



**Consensus agreement from all power-counting methods.**

# Scorecard on Symmetry Predictions

- $N = 4$ sugra should diverge at 3 loops in $D = 4$. ✗
- $N = 5$ sugra should diverge at 4 loops in $D = 4$. ✗
- Half maximal sugra diverges at 2 loops in $D = 5$. ✗
- $N = 8$ sugra should diverge at 5 loops in $D = 24/5$. ? ← will answer this here
- $N = 8$ sugra should diverge at 7 loops in $D = 4$. ?

ZB, Davies, Dennen (2012, 2014); ZB, Davies, Dennen, Huang(2012)

**String theory arguments against 3 loop divergence in $N = 4$.**
**Not symmetry arguments. Calculations coupled with extrapolations.**

Tourkine and Vanhove (2012); Green and Rudra (2016)

**$N = 4$ sugra has an anomaly that confuses the situation. It does diverge at 4 loops.**

Marcus; Carrasco, Kallosh, Roiban, Tseytlin;
ZB, Davies, Dennen, Smirnov, Smirnov; ZB, Parra-Martinez, Roiban

**UV cancellation of $N = 5$ supergravity at 4 loops in $D = 4$ remains a mystery, showing clear problem with standard symmetry arguments.**

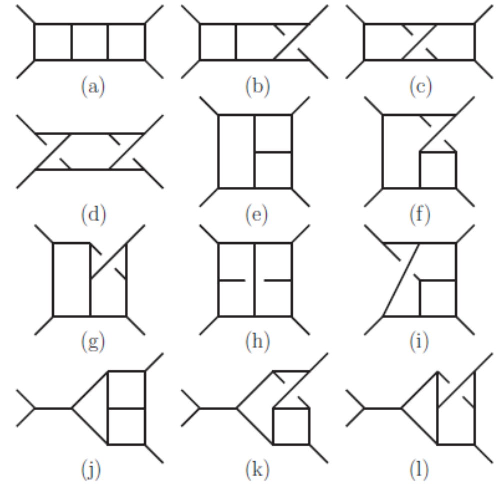
Freedman, Kallosh and Yamada (2018)

**Our goal is to provide definitive answers.**

# Enhanced UV Cancellations

ZB, Davies, Dennen

**Suppose diagrams in *all* possible Lorentz covariant representations are UV divergent, but the amplitude is well behaved.**

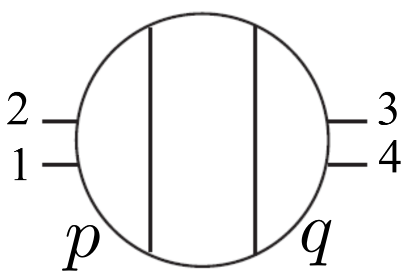



- **By definition this is an enhanced cancellation.**
- **Not the way nonabelian gauge theory works.**

**already log divergent**

***N* = 4 sugra: pure YM x *N* = 4 sYM**

***N* = 4 sugra**




$$n_i \sim s^3 t A_4^{\text{tree}} \,(p\cdot q)^2\, \varepsilon_1\cdot p\, \varepsilon_2\cdot p\, \varepsilon_3\cdot q\, \varepsilon_4\cdot q + \ldots$$

**This diagram is log divergent**

- **3 loop UV finiteness of *N* = 4 supergravity is example of "enhanced cancellation" in supergravity theories.**
- **No known standard symmetry explanation.**

# $\mathcal{N} = 5$ Supergravity Four-Loop Cancellations

ZB, Davies and Dennen

**We calculated four-loop divergence in $\mathcal{N} = 5$ supergravity.**

**Industrial strength software needed: FIRE5 and special purpose C++**

**$\mathcal{N} = 5$ sugra: ($\mathcal{N} = 4$ sYM) x ($\mathcal{N} = 1$ sYM)**

**Crucial help from (Smirnov)$^2$**

**$\mathcal{N} = 4$ sYM**

**$\mathcal{N} = 1$ sYM**

**Diagrams necessarily UV divergent.**

**$\mathcal{N} = 5$ supergravity has no divergence at four loops in $D = 4$.**

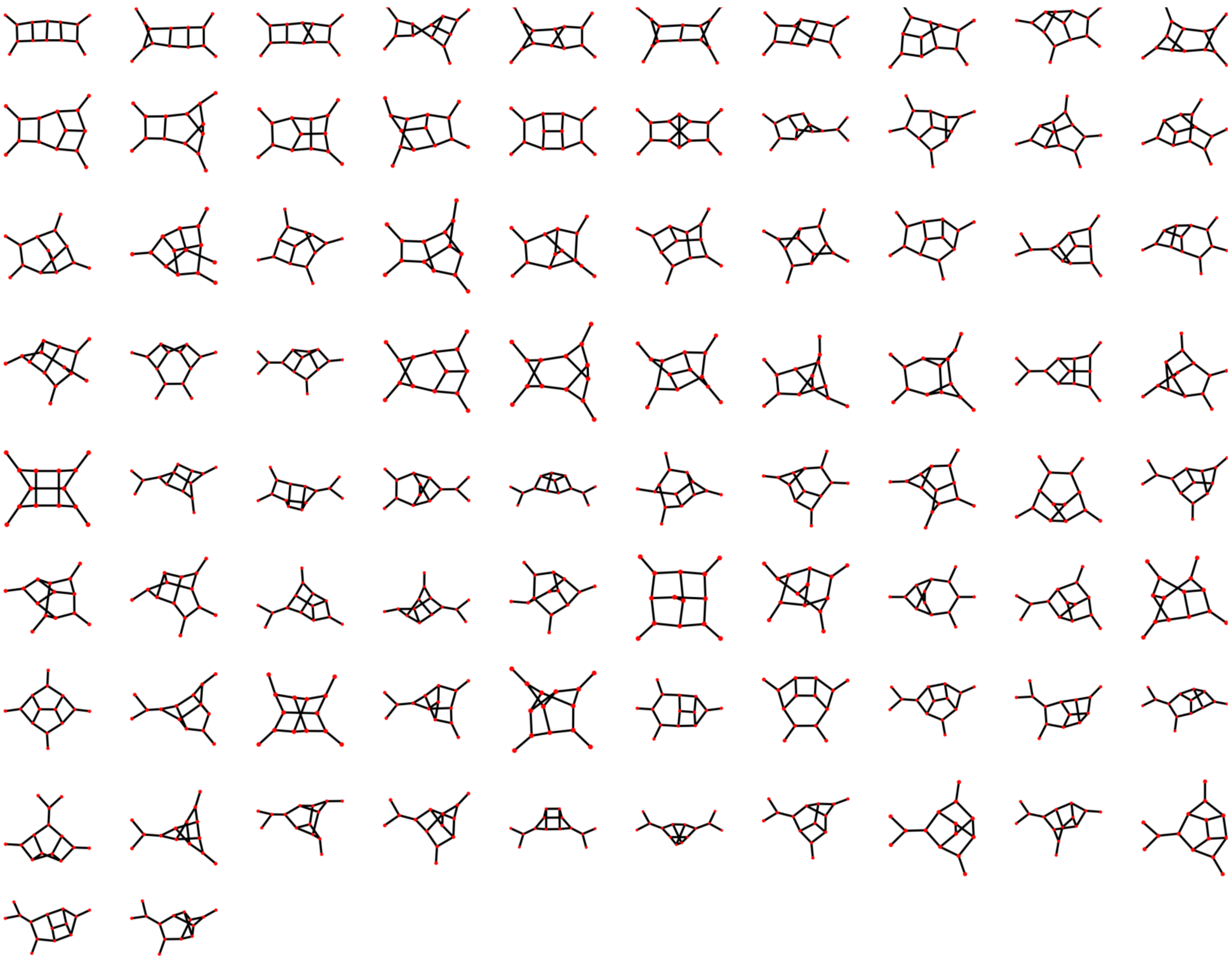
**Nontrivial example of an "enhanced cancellation".**

**No standard-symmetry explanation known!**

see recent paper from Freedman, Kallosh and Yamada

# 82 nonvanishing numerators in BCJ representation

ZB, Carrasco, Dixon, Johansson, Roiban ($\mathcal{N}$ = 4 sYM)



# $\mathcal{N}=5$ supergravity at Four Loops

## Special purpose C++ and FIRE5

ZB, Davies and Dennen

| graphs | (divergence) $\times\, u/(-i/(4\pi)^8\langle 12\rangle^2[34]^2 st A^{\rm tree}(\frac{\kappa}{2})^{10})$ |
|---|---|
| 1–30 | $\frac{1}{\epsilon^4}[\frac{7358585}{7962624}s^2+\frac{2561447}{2654208}st-\frac{872683}{1990656}t^2]+\frac{1}{\epsilon^3}[\frac{75972559}{35389440}s^2+\frac{240984061}{26542080}st+\frac{1302037}{1310720}t^2] + \frac{1}{\epsilon^2}[\zeta_3(-\frac{369234283}{11059200}s^2-\frac{257792411}{4915200}st-\frac{101847769}{14745600}t^2)+\zeta_2(\frac{7358585}{3981312}s^2+\frac{2561447}{1327104}st-\frac{872683}{995328}t^2) - \mathrm{S2}(\frac{1223621}{49152}s^2+\frac{46816475}{442368}st+\frac{2639903}{221184}t^2)+\frac{206093335871}{11466178560}s^2+\frac{320983191023}{3822059520}st+\frac{53309416589}{2866544640}t^2] + \frac{1}{\epsilon}[\zeta_5(-\frac{84777347}{368640}s^2+\frac{382194721}{1474560}st+\frac{417476581}{1474560}t^2)-\zeta_4(\frac{3062401}{2457600}s^2+\frac{3881051}{3276800}st-\frac{112081813}{29491200}t^2) + \zeta_3(\frac{28162691399797}{53747712000}s^2+\frac{19354492750651}{35831808000}st-\frac{22092683352811}{107495424000}t^2)-\zeta_2(\frac{70861961}{17694720}s^2+\frac{227180689}{13271040}st + \frac{105727243}{53084160}t^2)+\mathrm{T1ep}(-\frac{1223621}{663552}s^2-\frac{46816475}{5971968}st-\frac{2639903}{2985984}t^2)-\mathrm{S2}(\frac{11916028151}{5898240}s^2 + \frac{72637733971}{13271040}st+\frac{17223563447}{53084160}t^2)+\mathrm{D6}(-\frac{9001177}{552960}s^2-\frac{264491}{10240}st-\frac{2610157}{552960}t^2) + \frac{110945914744727}{1146617856000}s^2+\frac{16989492195991}{127401984000}st-\frac{21362122998269}{573308928000}t^2]$ |
| 31–60 | $\frac{1}{\epsilon^4}[-\frac{5502451}{2654208}s^2-\frac{3675877}{884736}st+\frac{11269}{497664}t^2]+\frac{1}{\epsilon^3}[\frac{38102993}{26542080}s^2-\frac{291607201}{106168320}st-\frac{565798829}{318504960}t^2] + \frac{1}{\epsilon^2}[\zeta_3(\frac{108955183}{2211840}s^2+\frac{653019571}{8847360}st+\frac{9453043}{1769472}t^2)+\zeta_2(-\frac{5502451}{1327104}s^2-\frac{3675877}{442368}st+\frac{11269}{248832}t^2) + \mathrm{S2}(\frac{16797481}{1327104}s^2+\frac{1172969}{16384}st+\frac{978427}{82944}t^2)-\frac{304243754383}{19110297600}s^2-\frac{2032063711381}{19110297600}st-\frac{257798086613}{7166361600}t^2] + \frac{1}{\epsilon}[\zeta_5(\frac{33327659}{122880}s^2+\frac{13276219}{24576}st+\frac{22251887}{184320}t^2)+\zeta_4(\frac{12299887}{1474560}s^2+\frac{258056147}{5898240}st+\frac{46913759}{5898240}t^2) + \zeta_3(-\frac{26846001990157}{42998169600}s^2-\frac{337106527201}{265420800}st-\frac{5298324906787}{42998169600}t^2)+\zeta_2(\frac{282283789}{39813120}s^2+\frac{975199319}{53084160}st + \frac{60394451}{159252480}t^2)+\mathrm{T1ep}(\frac{16797481}{17915904}s^2+\frac{1172969}{221184}st+\frac{978427}{1119744}t^2)+\mathrm{S2}(\frac{10516980893}{4976640}s^2 + \frac{389045625329}{53084160}st+\frac{216032337589}{159252480}t^2)+\mathrm{D6}(\frac{503413}{23040}s^2+\frac{12342607}{552960}st+\frac{3661}{184320}t^2) - \frac{166777358259461}{1146617856000}s^2-\frac{565137511429117}{1146617856000}st-\frac{21629055712141}{191102976000}t^2]$ |
| 61–82 | $\frac{1}{\epsilon^4}[\frac{285899}{248832}s^2+\frac{1058273}{331776}st+\frac{275869}{663552}t^2]+\frac{1}{\epsilon^3}[-\frac{380329649}{106168320}s^2-\frac{74703227}{11796480}st+\frac{124701919}{159252480}t^2] + \frac{1}{\epsilon^2}[\zeta_3(-\frac{1371419}{86400}s^2-\frac{236241539}{11059200}st+\frac{4326077}{2764800}t^2)+\zeta_2(\frac{285899}{124416}s^2+\frac{1058273}{165888}st+\frac{275869}{331776}t^2) + \mathrm{S2}(\frac{8120143}{663552}s^2+\frac{1893289}{55296}st+\frac{92293}{663552}t^2)-\frac{58867708103}{28665446400}s^2+\frac{71191292711}{3185049600}st+\frac{83016363427}{4777574400}t^2] + \frac{1}{\epsilon}[\zeta_5(-\frac{1520563}{36864}s^2-\frac{1178767861}{1474560}st-\frac{595491677}{1474560}t^2)-\zeta_4(\frac{6539029}{921600}s^2+\frac{313837819}{7372800}st+\frac{21665663}{1843200}t^2) + \zeta_3(\frac{20790944575597}{214990848000}s^2+\frac{6505876281371}{8957952000}st+\frac{70676991239557}{214990848000}t^2)+\zeta_2(-\frac{491377507}{159252480}s^2-\frac{66476563}{53084160}st + \frac{128393639}{79626240}t^2)+\mathrm{T1ep}(\frac{8120143}{8957952}s^2+\frac{1893289}{746496}st+\frac{92293}{8957952}t^2)+\mathrm{S2}(-\frac{14810628499}{159252480}s^2 - \frac{19698937889}{10616832}st-\frac{10272602953}{9953280}t^2)+\mathrm{D6}(-\frac{616147}{110592}s^2+\frac{1939907}{552960}st+\frac{1299587}{276480}t^2) + \frac{9307894793789}{191102976000}s^2+\frac{206124003456599}{573308928000}st+\frac{21562322533673}{143327232000}t^2]$ |

| graphs | (divergence) $\times\, u/(-i/(4\pi)^8\langle 12\rangle^2[34]^2 st A^{\rm tree}(\frac{\kappa}{2})^{10})$ |
|---|---|
| 1–30 | $\frac{1}{\epsilon^4}[\frac{1052159}{995328}s^2+\frac{509789}{331776}st-\frac{121001}{497664}t^2]+\frac{1}{\epsilon^3}[\frac{9042569}{1474560}s^2+\frac{34360945}{1327104}st+\frac{73518401}{13271040}t^2] + \frac{1}{\epsilon^2}[\zeta_3(-\frac{11443919}{2764800}s^2+\frac{32520079}{552960}st+\frac{5836531}{230400}t^2)+\zeta_2(\frac{1052159}{497664}s^2+\frac{509789}{165888}st-\frac{121001}{248832}t^2) - \mathrm{S2}(\frac{637991}{6144}s^2+\frac{10978729}{27648}st+\frac{5080825}{55296}t^2)+(\frac{270806866183}{7166361600}s^2+\frac{89848068067}{597196800}st+\frac{218093645149}{7166361600}t^2)] + \frac{1}{\epsilon}[\zeta_5(\frac{100843}{360}s^2+\frac{17118043}{30720}st-\frac{30266471}{92160}t^2)+\zeta_4(\frac{11435323}{614400}s^2+\frac{232002227}{1843200}st+\frac{22211783}{460800}t^2) + \zeta_3(\frac{223300432349}{3359232000}s^2-\frac{178732984847}{716636160}st+\frac{951659436383}{53747712000}t^2) - \zeta_2(\frac{5492357}{245760}s^2+\frac{53468887}{663552}st+\frac{129714599}{6635520}t^2)+\mathrm{T1ep}(-\frac{637991}{82944}s^2-\frac{10978729}{373248}st-\frac{5080825}{746496}t^2) + \mathrm{S2}(-\frac{5700088747}{3686400}s^2-\frac{69470348491}{16588800}st-\frac{713512871}{6635520}t^2)+\mathrm{D6}(-\frac{357421}{43200}s^2-\frac{2891743}{230400}st-\frac{470219}{138240}t^2) - \frac{3571506237341}{28665446400}s^2-\frac{1611591325291}{5971968000}st+\frac{2301084608777}{143327232000}t^2]$ |
| 31–60 | $\frac{1}{\epsilon^4}[-\frac{150715}{82944}s^2-\frac{668333}{221184}st-\frac{7213}{1990656}t^2]+\frac{1}{\epsilon^3}[-\frac{68021833}{13271040}s^2-\frac{36852103}{1327104}st-\frac{298377299}{39813120}t^2] + \frac{1}{\epsilon^2}[\zeta_3(-\frac{36448033}{2764800}s^2-\frac{455889533}{2764800}st-\frac{82059281}{1382400}t^2)+\zeta_2(-\frac{150715}{41472}s^2-\frac{668333}{110592}st-\frac{7213}{995328}t^2) + \mathrm{S2}(\frac{13910839}{165888}s^2+\frac{1340033}{4096}st+\frac{26303855}{331776}t^2)-\frac{68286245653}{2388787200}s^2-\frac{20649690431}{119439360}st-\frac{351701043553}{7166361600}t^2] + \frac{1}{\epsilon}[\zeta_5(-\frac{2362679}{9216}s^2-\frac{178668311}{92160}st-\frac{1268313}{10240}t^2)+\zeta_4(-\frac{124344121}{1843200}s^2-\frac{491722333}{1843200}st-\frac{68141309}{921600}t^2) - \zeta_3(\frac{630084012997}{53747712000}s^2-\frac{1250670277213}{663552000}st-\frac{6913218302303}{13436928000}t^2) + \zeta_2(\frac{352368061}{19906560}s^2+\frac{35509679}{663552}st+\frac{227699801}{19906560}t^2)+\mathrm{T1ep}(\frac{13910839}{2239488}s^2+\frac{1340033}{55296}st+\frac{26303855}{4478976}t^2) + \mathrm{S2}(\frac{188312318729}{99532800}s^2+\frac{110749829741}{16588800}st+\frac{5056299197}{3981312}t^2)+\mathrm{D6}(\frac{1220779}{76800}s^2+\frac{44791}{6912}st-\frac{1159831}{230400}t^2) + \frac{2755666297013}{28665446400}s^2+\frac{5622513975899}{35831808000}st-\frac{196197363193}{1769472000}t^2]$ |
| 61–82 | $\frac{1}{\epsilon^4}[\frac{756421}{995328}s^2+\frac{985421}{663552}st+\frac{163739}{663552}t^2]+\frac{1}{\epsilon^3}[-\frac{1670161}{1658880}s^2+\frac{415193}{221184}st+\frac{4863881}{2488320}t^2] + \frac{1}{\epsilon^2}[\zeta_3(\frac{110861}{6400}s^2+\frac{16293841}{153600}st+\frac{9408019}{276480}t^2)+\zeta_2(\frac{756421}{497664}s^2+\frac{985421}{331776}st+\frac{163739}{331776}t^2) + \mathrm{S2}(\frac{1657459}{82944}s^2+\frac{7734025}{110592}st+\frac{4181095}{331776}t^2)-\frac{8243516153}{895795200}s^2+\frac{558349337}{24883200}st+\frac{11133949867}{597196800}t^2] + \frac{1}{\epsilon}[\zeta_5(-\frac{1094509}{46080}s^2+\frac{63657091}{46080}st+\frac{5210161}{11520}t^2)+\zeta_4(\frac{11254769}{230400}s^2+\frac{129860053}{921600}st+\frac{23717743}{921600}t^2) - \zeta_3(\frac{2745647960587}{53747712000}s^2+\frac{3654260151947}{2239488000}st+\frac{5720906529119}{10749542400}t^2) + \zeta_2(\frac{11564107}{2488320}s^2+\frac{2244901}{82944}st+\frac{40360999}{4976640}t^2)+\mathrm{T1ep}(\frac{1657459}{1119744}s^2+\frac{7734025}{1492992}st+\frac{4181095}{4478976}t^2) + \mathrm{S2}(-\frac{420043}{1215}s^2-\frac{825589625}{331776}st-\frac{5785239343}{4976640}t^2)+\mathrm{D6}(-\frac{210731}{27648}s^2+\frac{4196129}{691200}st+\frac{1457647}{172800}t^2) + \frac{33976742047}{1194393600}s^2+\frac{4046536311847}{35831808000}st+\frac{212357840779}{2239488000}t^2]$ |

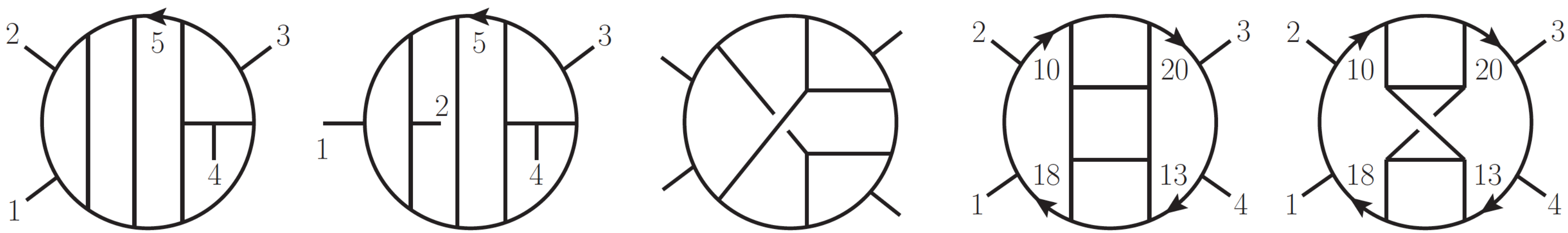
**Adds up to zero: no divergence. Enhanced cancellations!**

**No standard (super)symmetry explanation exists.**

see paper a few weeks ago from Freedman, Kallosh and Yamada

# $N = 8$ Sugra 5 Loop Calculation

**What is the true UV behavior of $N = 8$ sugra.**




**Place your bets:**

- **At 5 loops in $D = 24/5$ does $N = 8$ supergravity diverge?**
- **At 7 loops in $D = 4$ does $N = 8$ supergravity diverge?**

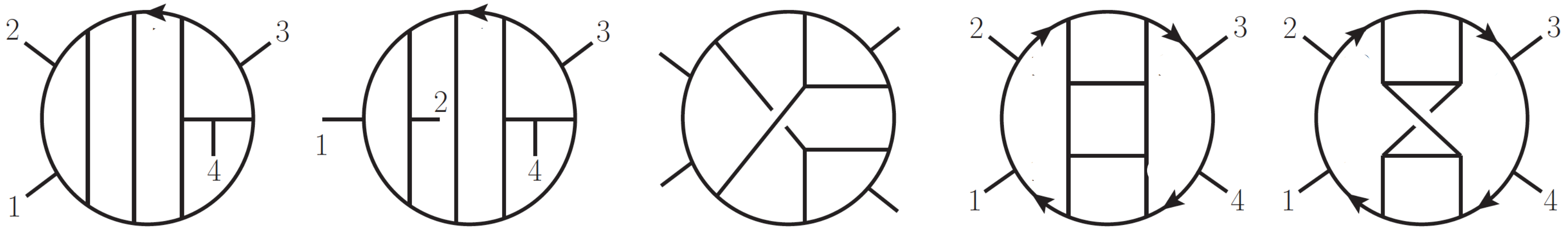
**5 loops**

**Kelly Stelle:**
**English wine**
**"It will diverge"**

**Zvi Bern:**
**California wine**
**"It won't diverge"**

# *N* = 8 Sugra 5 Loop Calculation

**What is the true UV behavior of *N* = 8 sugra.**




**Place your bets:**

- **At 5 loops in *D* = 24/5 does *N* = 8 supergravity diverge?**
- **At 7 loops in *D* = 4 does *N* = 8 supergravity diverge?**

**7 loops**

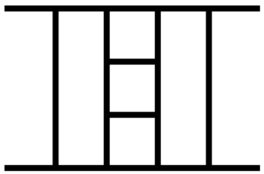
**David Gross: California wine**
**"It will diverge"**

**Zvi Bern: California wine**
**"It won't diverge"**

# Finding BCJ Forms Nontrivial

**Gravity integrands might be "free", but gauge-theory ones are not. Trouble beyond four loops.**

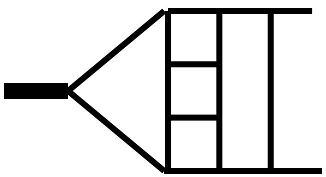
**5-loop 4-pt $\mathcal{N} = 4$ sYM amplitude:**



**Despite considerable effort no one has succeeded in finding a BCJ form.**

Besides us at least two other groups tried

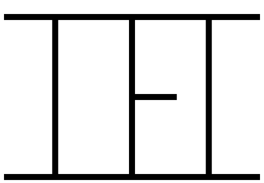
**$\mathcal{N} = 4$ sYM 5 loop form factor:**



**On other hand, no trouble with form factor.**

Gang Yang (2016)

**Two-loop five-point QCD identical helicity:**



**This required an ansatz with curiously high power counting.**

O'Connell and Mogull (2015)

**It can be difficult to find BCJ representations.**

# New Contact Term Method

ZB, Carrasco, Chen, Johansson, Roiban, Zeng (2017)

**Task is to convert $\mathcal{N}=4$ sYM 5-loop integrand into $\mathcal{N}=8$ sugra.**

**BCJ representation hard to find.**

**New method:**

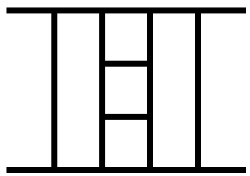
**Start with "naïve double copy" of *any* correct sYM integrand:**

$$c_i \rightarrow n_i$$ ***Not* a BCJ representation**

**Without BCJ duality, *not* the correct $\mathcal{N}=8$ integrand**

**$\mathcal{N}=8$ cuts:**

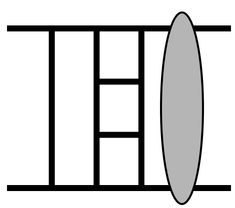
**Max cuts:**



**Automatic**

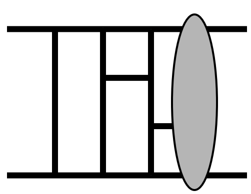
**Generalized Unitarity**
**All exposed legs on shell**

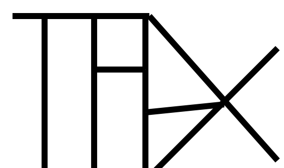
**Nmax cuts:**



**Automatic via BCJ, 4pt trees always work**

**$N^2$max cuts:**

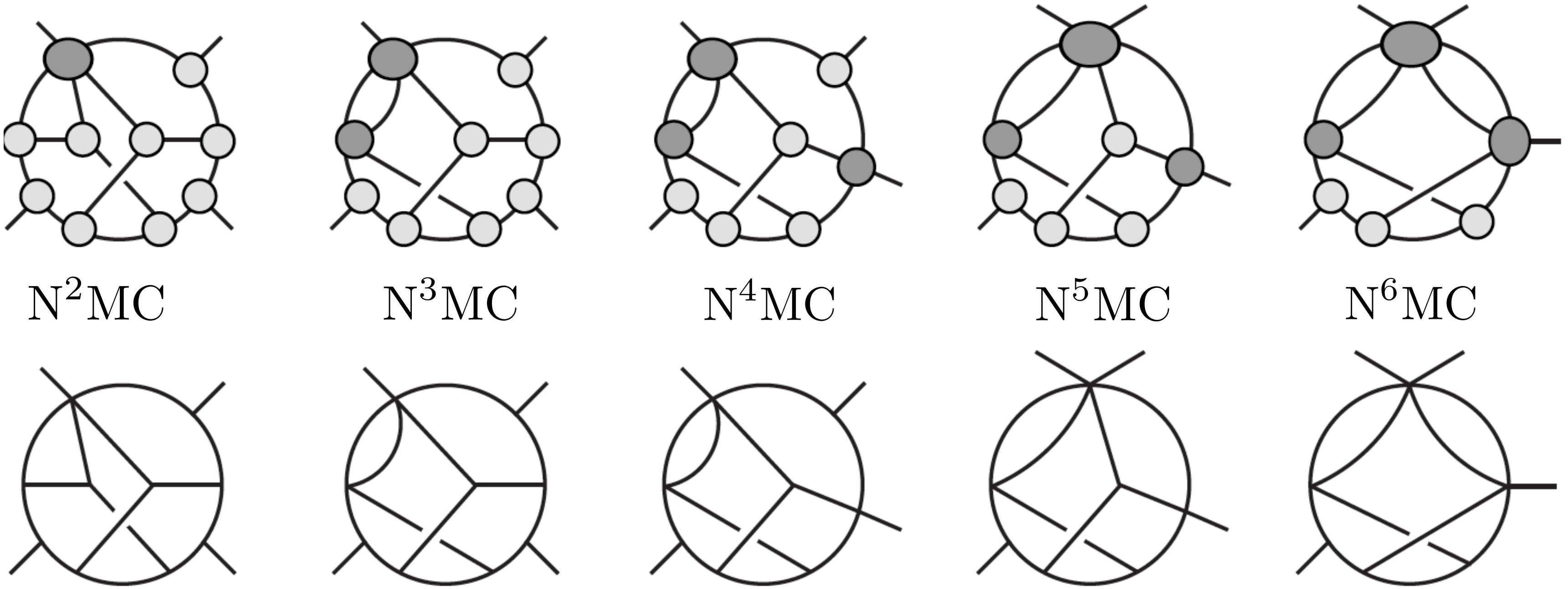




**Add contact term to make it work**

# Contact Term Method

**contact = (gravity cut) – (cut of incomplete amplitude)**




- **Contact each associated with each cut directly giving missing piece of amplitude.**
- **75K cuts need to be evaluated.**
- **Sounds daunting. Not for faint of heart!**

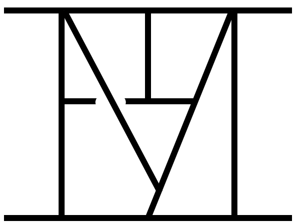
**Game for optimists: "Simplifying miracle is around the corner"**

# A Simplifying Miracle

ZB, Carrasco, Chen, Johansson, Roiban, Zeng (2017)

**contact = (gravity cut) – (cut of incomplete amplitude)**

1. **Most contact terms vanish!**
2. **Gravity contacts far simpler than expected.**
3. **Four-point double-contacts factorize. Extremely striking.**



double 4pt contact

$$\left[2s^3 - s^2 u + 4s^2(2k_1\cdot l_6) + \cdots\right]$$
$$\times\left[s^2 u + 2su^2 - s^2(2k_1\cdot l_6) + \cdots\right]$$

**Each factor looks like gauge theory**

**Reminds us of KLT factorization:**

$$M^{\rm tree}(1,2,3,4) = s_{12}A^{\rm tree}(1,2,3,4)\times A^{\rm tree}(1,2,4,3)$$

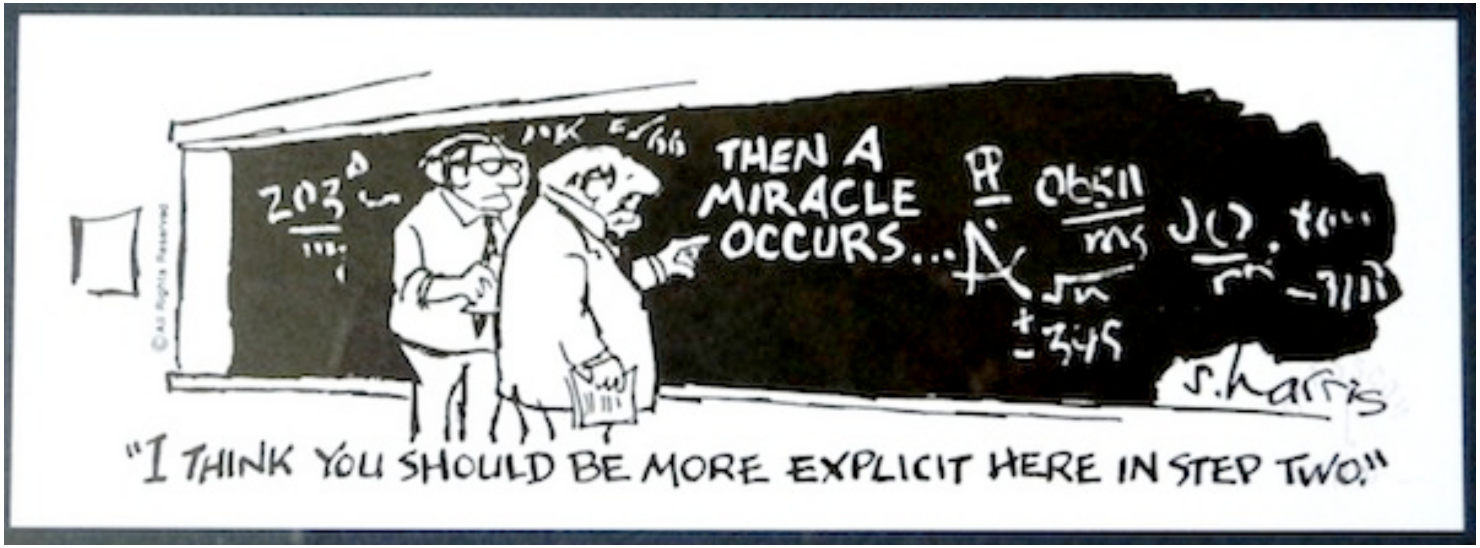
**For 5 or higher-point contacts no overall factorization, as with KLT.**

**Can we write down formulas that give missing gravity pieces directly from gauge theory, bypassing complicated gravity cuts?**

# A Miracle

1. Start from gauge-theory loop amplitude.
2. Construct naïve double copy.
3. Compute cut of naïve double copy.
4. Compute gravity cut from gauge-theory cuts via KLT.
5. Subtract and shake hard (nontrivial).
6. Extract surprisingly simple gravity contact terms.

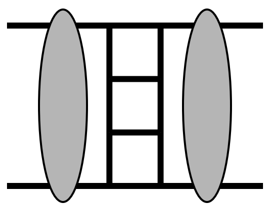
**Miracle: The contact terms are so simple we should be able write down missing gravity contacts directly from gauge theory.**

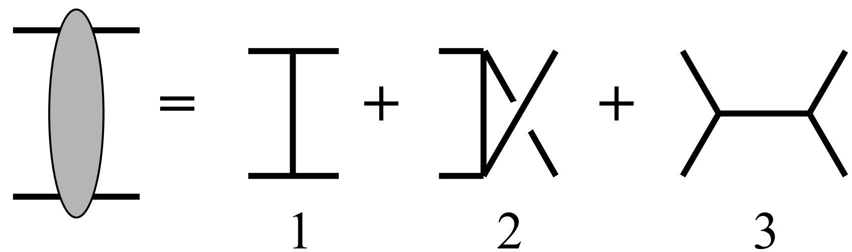


# BCJ Discrepancy Functions

**Need a function defined purely in gauge theory as building block for missing gravity pieces.**

**Inside multiloop diagram**






**kinematic numerators**

**BCJ discrepancy function:** $$J \equiv \sum_{i=1}^{3} n_i$$

**Vanishes if we have BCJ form of gauge theory.**

**Obvious guess is these are building blocks for missing gravity pieces.**

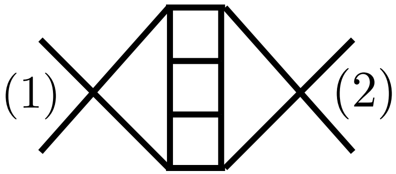
**Missing pieces:** $$\sim \sum J \times J$$

# Gravity from Gauge Theory

ZB, Carrasco, Chen, Johansson, Roiban

## Missing gravity from any gauge theory representation

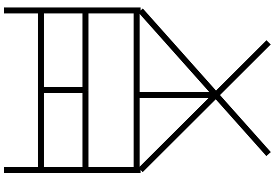
BCJ discrepancy functions



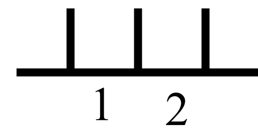

$$\mathcal{E}_{\rm GR}^{4\times 4} = -\frac{1}{d_1^{(1)} d_1^{(2)}} \left( J_{\bullet,1} \tilde{J}_{1,\bullet} + J_{1,\bullet} \tilde{J}_{\bullet,1} \right)$$

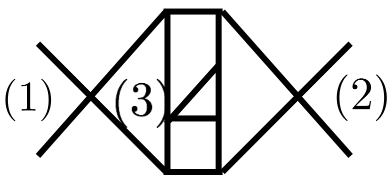
propagators cancel trivially

Expand into 15 diagrams



$$\mathcal{E}_{\rm GR}^{5} = -\frac{1}{6} \sum_{i=1}^{15} \frac{J_{\{i,1\}} \tilde{J}_{\{i,2\}} + J_{\{i,2\}} \tilde{J}_{\{i,1\}}}{d_i^{(1)} d_i^{(2)}}$$



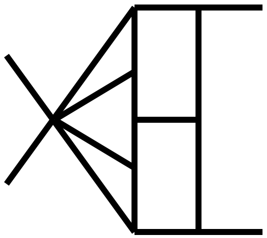




$$\mathcal{E}_{\rm GR}^{4\times 4\times 4} = -\sum_{i_3=1}^{3} \frac{J_{\bullet,1,i_3} \tilde{J}_{1,\bullet,i_3}}{d_1^{(1)} d_1^{(2)} d_{i_3}^{(3)}} - \sum_{i_2=1}^{3} \frac{J_{\bullet,i_2,1} \tilde{J}_{1,i_2,\bullet}}{d_1^{(1)} d_{i_2}^{(2)} d_1^{(3)}} - \sum_{i_1=1}^{3} \frac{J_{i_1,\bullet,1} \tilde{J}_{i_1,1,\bullet}}{d_{i_1}^{(1)} d_1^{(2)} d_1^{(3)}} + \frac{J_{\bullet,1,1} \tilde{J}_{1,\bullet,\bullet}}{d_1^{(1)} d_1^{(2)} d_1^{(3)}} + \frac{J_{1,\bullet,1} \tilde{J}_{\bullet,1,\bullet}}{d_1^{(1)} d_1^{(2)} d_1^{(3)}} + \frac{J_{1,1,\bullet} \tilde{J}_{\bullet,\bullet,1}}{d_1^{(1)} d_1^{(2)} d_1^{(3)}} + \{J \leftrightarrow \tilde{J}\}$$

**Etc.**

- **Applies to *any* adjoint gauge theory, not just $N = 4$ sYM.**
- **Holds for asymmetric double copies.**
- **Same constructions work at tree level. Five-point formula similar to known tree formula.** Bjerrum-Bohr, Damgaard, Søndergaad, Vanhove

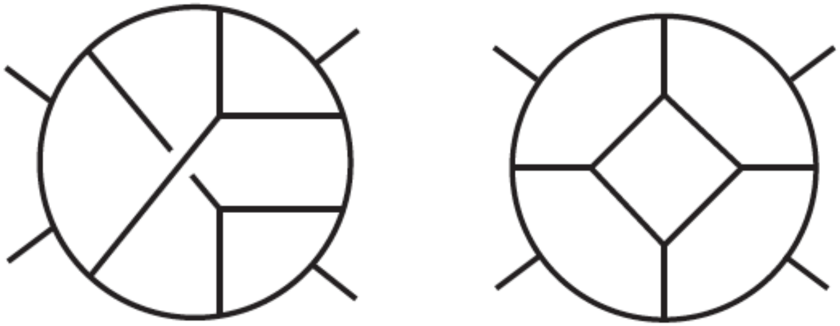
# 5 Loop $N = 8$ supergravity



**Generalized double copy enormously simplifies the computation of missing gravity contact terms. The impossible becomes doable!**

**We have constructed the five-loop integrand!**

ZB, Carrasco, Chen, Johansson, Roiban, Zeng (2017)

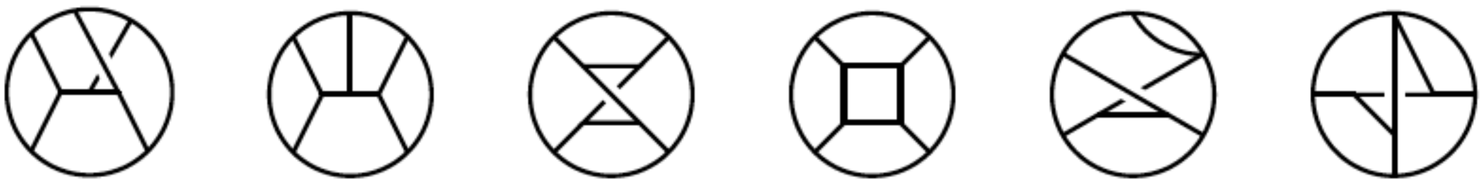


**See mathematica attachment of paper for integrand.**

# Large Loop Momentum Expansion

ZB, Carrasco, Chen, Edison, Johansson, Roiban, Parra-Martinez, Zeng (2018)

- **Series expand and large loop momentum to extract log divergent terms.**
- **Obtain diagrams with no external momenta, analogous to vacuum diagrams.**



**Things can spiral out of control at this step.**

**To deal with this:**

- **Constructed a new integrand with simpler series expansion.**
- **Avoid problematic quartic diverges.**
- **Applied efficient modern algorithms to integrate.**

# Integrating $N = 8$ supergravity

Cheterkin and Tkachov

$$\int \prod_{k=1}^{5} \frac{d^D \ell_k}{(2\pi)^D} \frac{\partial}{\partial \ell_i^\mu} \frac{v_i^\mu}{\prod_j \ell_j^2} = 0$$

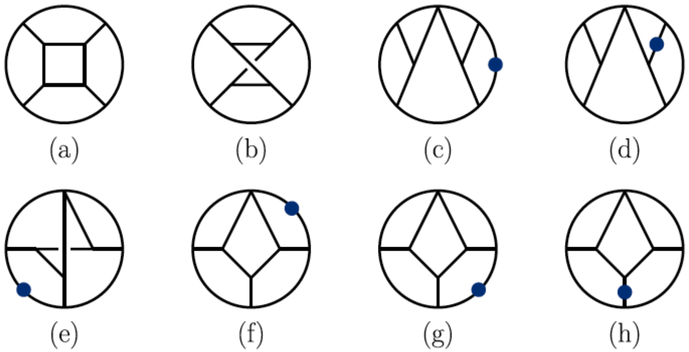
$v_i^\mu$ **ibp vector**

**Smart choices make huge difference (don't mix UV and IR!).**

- **Lorentz invariance.** ZB, Enciso, Parra-Martinez, Zeng
- **Generators of SL(5) relabeling symmetry.**
- **Modern finite field equation solvers.** Schabinger and von Manteuffel
- **Modern unitarity based IBPs.**

Gluze, Kajda, Kosower; Kosower & Larsen; Caron-Huot & Larsen; Johansson, Kosower, Larsen; Sogaard and Zhang; Schabinger; Ita; Zhang; Abreu, Febres Cordero, Ita, Page, Zeng; etc.

**Integrals reduce to 8 master integrals.**



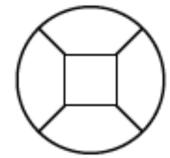

ZB, Carrasco, Chen, Edison, Johansson, Roiban, Parra-Martinez, Zeng (2018)

# The result!

ZB, Carrasco, Chen, Edison, Johansson, Roiban, Parra-Martinez, Zeng (2018)

**In *D* = 24/5 we obtain a divergence:**

$$\mathcal{M}_4^{(5)}\Big|_{\text{leading}} = -\frac{16 \times 629}{25}\left(\frac{\kappa}{2}\right)^{12}(s^2+t^2+u^2)^2 stu M_4^{\text{tree}} \left( \frac{1}{48} \right.$$



$$+ \frac{1}{16}$$

$$\left. \right)$$

**Integrals are positive definite**

**No “enhanced cancellations”**

**I lost 5 loop bet**

- ***N*** **= 8 sugra at** ***L*** **= 5 in** ***D*** **= 24/5 has no enhanced cancellation.**
- ***N*** **= 5 sugra at** ***L*** **= 4 in** ***D*** **= 4 has enhanced cancellation.**

**What is the difference?** ***D*** **= 4?**

**Analysis of unitarity cuts relevant for UV suggests** ***D*** **= 4 indeed plays crucial role.**

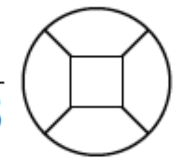
Paper yesterday from Herrmann and Trnka

# $N = 8$ UV at Five Loops

ZB, Carrasco, Chen, Edison, Johansson, Roiban, Parra-Martinez, Zeng (2018)

**In $D = 24/5$ we obtain a divergence:**

$$\mathcal{M}_4^{(5)}\Big|_{\text{leading}} = -\frac{16 \times 629}{25}\left(\frac{\kappa}{2}\right)^{12}(s^2+t^2+u^2)^2 stu M_4^{\text{tree}} \left(\frac{1}{48}\right.$$



$$+ \frac{1}{16}$$

$$\left.\right)$$

**With hindsight up to overall coefficient, it is *easy* to understand this result, as I will show you.**

**But our purpose:**

- **Determine the answer with complete certainty.**
  **No "arguments". Only proven facts and calculations.**
- **Understand the structures so we can get to 7 loops and beyond in $D = 4$.**
- **To build a firm foundation to be able to get to 7 and higher loops.**

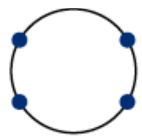
# Higher-loop Structure.

Green Schwarz, Brink; ZB, Dixon, Dunbar, Perelstein, Rozowsky;Carrasco, Dixon, Johansson, Roiban; ZB, Carrasco, Chen, Edison, Johansson, Roiban, Parra-Martinez, Zeng

**Over the years we've obtained results for *N* = 8 sugra through five loops.**

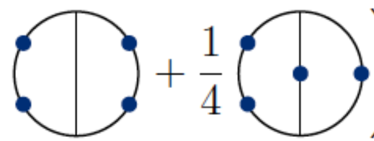
**dots represent extra propagators**

$$\mathcal{M}_4^{(1)}\Big|_{\text{leading}} = -3\,\mathcal{K}_{\rm G}\left(\frac{\kappa}{2}\right)^4$$



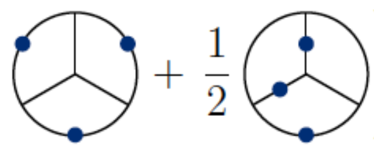
, $D_c = 8$

$$\mathcal{M}_4^{(2)}\Big|_{\text{leading}} = -8\,\mathcal{K}_{\rm G}\left(\frac{\kappa}{2}\right)^6 (s^2+t^2+u^2)\left(\frac{1}{4}\;[\ldots] + \frac{1}{4}\;[\ldots]\right)$$



, $D_c = 7$

$$\mathcal{M}_4^{(3)}\Big|_{\text{leading}} = -60\,\mathcal{K}_{\rm G}\left(\frac{\kappa}{2}\right)^8 stu\left(\frac{1}{6}\;[\ldots] + \frac{1}{2}\;[\ldots]\right)$$

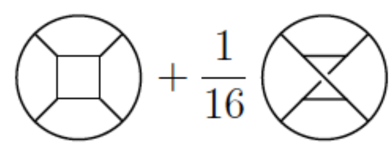


, $D_c = 6$

$$\mathcal{M}_4^{(4)}\Big|_{\text{leading}} = -\frac{23}{2}\,\mathcal{K}_{\rm G}\left(\frac{\kappa}{2}\right)^{10} (s^2+t^2+u^2)^2\left(\frac{1}{4}\;[\ldots] + \frac{1}{2}\;[\ldots] + \frac{1}{4}\;[\ldots]\right)$$

, $D_c = \frac{11}{2}$

$$\mathcal{M}_4^{(5)}\Big|_{\text{leading}} = -\frac{16\times 629}{25}\,\mathcal{K}_{\rm G}\left(\frac{\kappa}{2}\right)^{12} (s^2+t^2+u^2)^2\left(\frac{1}{48}\;[\ldots] + \frac{1}{16}\;[\ldots]\right)$$



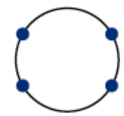
, $D_c = \frac{24}{5}$

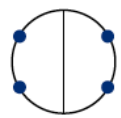
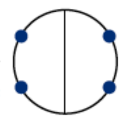
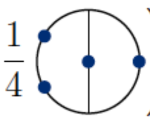
**We now have a lot of theoretical "data" to guide us.**

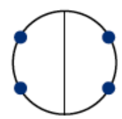
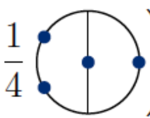
# Higher-loop Structure.

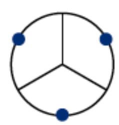
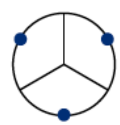
Green Schwarz, Brink; ZB, Rozowsky, Yan; ZB, Dixon, Dunbar, Perelstein, Rozowsky; Carrasco, Dixon, Johansson, Roiban; ZB, Carraco, Dixon, Douglas, von Hippel, Johansson

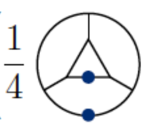
**Have up to six loop results for *N* = 4 sYM UV behavior:**

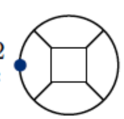
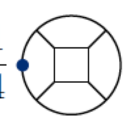
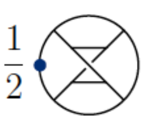
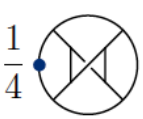
$\mathcal{A}_4^{(1)}\Big|_{\text{leading}} = g^4 \mathcal{K}_{\text{YM}} \left( N_c (\tilde{f}^{a_1 a_2 b} \tilde{f}^{b a_3 a_4} + \tilde{f}^{a_2 a_3 b} \tilde{f}^{b a_4 a_1}) - 3 B^{a_1 a_2 a_3 a_4} \right)$  ,  $D_c = 8$

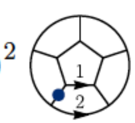
$\mathcal{A}_4^{(2)}\Big|_{\text{leading}} = -g^6 \mathcal{K}_{\text{YM}} \Big[ F^{a_1 a_2 a_3 a_4} \Big( N_c^2$  $+ 48 \Big( \frac{1}{4}$  $+ \frac{1}{4}$  $\Big)\Big)$

$+ 48 N_c G^{a_1 a_2 a_3 a_4} \Big( \frac{1}{4}$  $+ \frac{1}{4}$  $\Big)\Big]$ ,  $D_c = 7$

$\mathcal{A}_4^{(3)}\Big|_{\text{leading}} = 2 g^8 \mathcal{K}_{\text{YM}} N_c F^{a_1 a_2 a_3 a_4} \Big( N_c^2$  $+ 72 \Big( \frac{1}{6}$  $+ \frac{1}{2}$  $\Big)\Big)$ ,  $D_c = 6$

$\mathcal{A}_4^{(4)}\Big|_{\text{leading}} = -6 g^{10} \mathcal{K}_{\text{YM}} N_c^2 F^{a_1 a_2 a_3 a_4} \Big( N_c^2$  $+ 48 \Big( \frac{1}{4}$  $+ \frac{1}{2}$  $+ \frac{1}{4}$  $\Big)\Big)$ ,  $D_c = \frac{11}{2}$

$\mathcal{A}_4^{(5)}\Big|_{\text{leading}} = \frac{144}{5} g^{12} \mathcal{K}_{\text{YM}} N_c^3 F^{a_1 a_2 a_3 a_4} \Big( N_c^2$  $+ 48 \Big( \frac{1}{4}$  $+ \frac{1}{2}$  $+ \frac{1}{4}$  $\Big)\Big)$ ,  $D_c = \frac{24}{5}$

$A_4^{(6)}\Big|_{\text{leading}} = -120 g^{14} \mathcal{K}_{\text{YM}} F^{a_1 a_2 a_3 a_4} N_c^6 \Big( \frac{1}{2}$  $+ \frac{1}{4} (\ell_1 + \ell_2)^2$ 

$- \frac{1}{20}$  $\Big)$

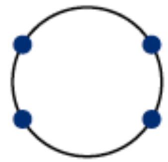
$+ \mathcal{O}(N_c^4)$ ,  $D_c = 5$

**We now have a lot of theoretical "data" to guide us.**

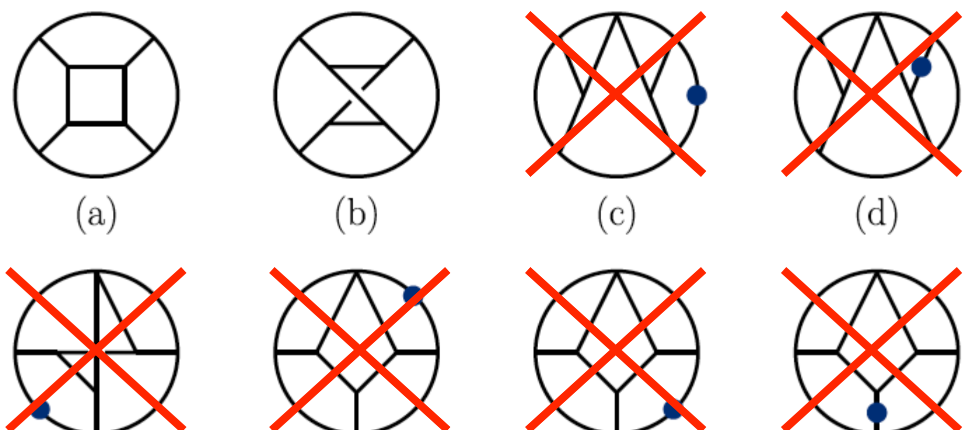
# Simple Consistency Conditions

ZB, Carrasco, Chen, Edison, Johansson, Roiban, Parra-Martinez, Zeng (2018)

$$\mathcal{M}_4^{(1)}\Big|_{\text{leading}} = -3\,\mathcal{K}_{\text{G}}\left(\frac{\kappa}{2}\right)^4$$



**Might expect all one-loop subdiagram to have 4 propagators.**




**No one-loop triangle diagrams. Similar structure well known to hold for amplitudes.**

$$\mathcal{M}_4^{(5)}\Big|_{\text{leading}} = -\frac{16\times 629}{25}\left(\frac{\kappa}{2}\right)^{12}(s^2+t^2+u^2)^2 stu M_4^{\text{tree}} \left(\frac{1}{48}\right.$$

$$+\,\frac{1}{16}$$

$$\left.\vphantom{\frac{1}{16}}\right)$$

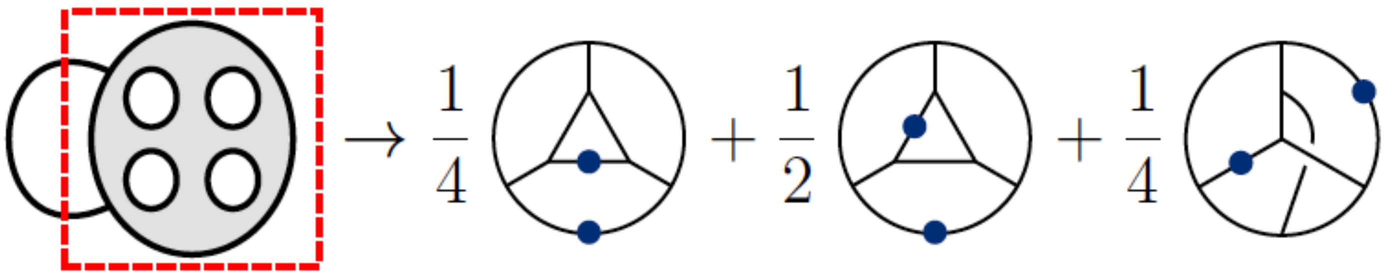
symmetry factors

**Relative coefficients are symmetry factors. Actually, easy to understand why it has this form.**

# Simple consistency conditions

ZB, Carrasco, Chen, Edison, Johansson, Roiban, Parra-Martinez, Zeng (2018)

**In fact, the different loop orders are all consistent with each other even after all the nontrvial processing!**



**Get correct 4-loop vacuum diagrams starting from 5-loop vacuum diagrams, even though in different dimensions!**

**Nontrivial processing makes it surprising that it is this simple! Should be possible to develop proof of structure.**

- **Pattern gives strong confidence we computed correctly.**
- **More importantly, points to way to compute higher loops.**

# Vacuum diagram consistency

**Helps in two key ways:**

1. **By demanding lower-loop consistency we should be able to figure out relative coefficients of vacuum integrals.**

2. **By limiting focus to certain integrals only need small part of 6 or 7 loop integrand. Apply unitarity compatible IBP methods.**

Gluze, Kajda, Kosower; Kosower & Larsen; Caron-Huot & Larsen; Johansson, Kosower, Larsen; Sogaard and Zhang; Schabinger; Ita; Zhang; ; Abreu, Febres Cordero, Ita, Page, Zeng; etc.

**To test prediction of $D = 4$ divergence need 7 loops**

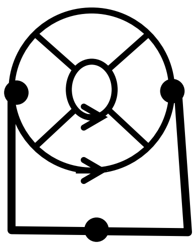
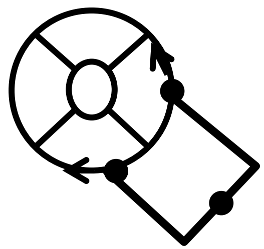
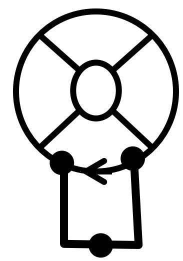
**Structure gives overwhelmingly more powerful new ways to analyze higher loops.**
**$D = 4$ and 7 loops looks within reach!**

# Nontrivial test: Bootstrap $N = 4$ sYM at 6 Loops

**Obvious first place to test is planar $N = 4$ sYM.**

**5 to 6 loop constraints:**

**Using IBPs rewrite 5 loop expression in form natural for bootstrap**

$\frac{\ell_1 \cdot \ell_2}{4}$  $\frac{\ell_1 \cdot \ell_2}{2}$  $-\frac{\ell_1^2}{2}$ 

**5 loop expression rearranged ready for bootstrap**

**Clean up with 6 loop IBPs and lower-loop consistency.**
**Unitarity based IBP up to task. Get:**

$\frac{1}{2}$  $+\frac{1}{4}(\ell_1 + \ell_2)^2$ 


**Matches 6 loop contributions!**

$-\frac{1}{20}$ 

**Missing. Maximal cuts. 5 loop subdiagrams subleading power count**

- **Bootstrap should help us greatly to go to higher loops.**
- **Note: same idea seems promising for less susy.**

# Status

**On one hand:**

- **For $N = 8$ supergravity in $D = 24/5$ no enhanced cancellation Standard symmetry bounds give correct prediction.**
  ZB, Carrasco, Chen, Edison, Johansson, Parra-Martinez, Roiban, Zeng (2018)

**On the other hand:**

- **Predicted 4-loop divergence in $N = 5$ supergravity not present: enhanced cancellation!** ZB, Davies and Dennen (2014)

- **Recent reaffirmation of *no* standard symmetry explanation for 4-loop $N = 5$ finiteness.** Kallosh, Nicolai, Roiban ,Yamada (2018); Freedman, Kallosh, Yamada (2018)

- **4-loop divergences of $N = 4$ supergravity appear in anomalous amplitudes that should be removable via local counterterm.**
  ZB, Parra-Martinez, Roiban (2018) + to appear

- **Remarkable multiloop UV cancellations identified in nontrivial unitarity cuts in $D = 4$.** Herrmann and Trnka (this week)

**Clearly there is much more to explore, especially in $D = 4$**

# Summary

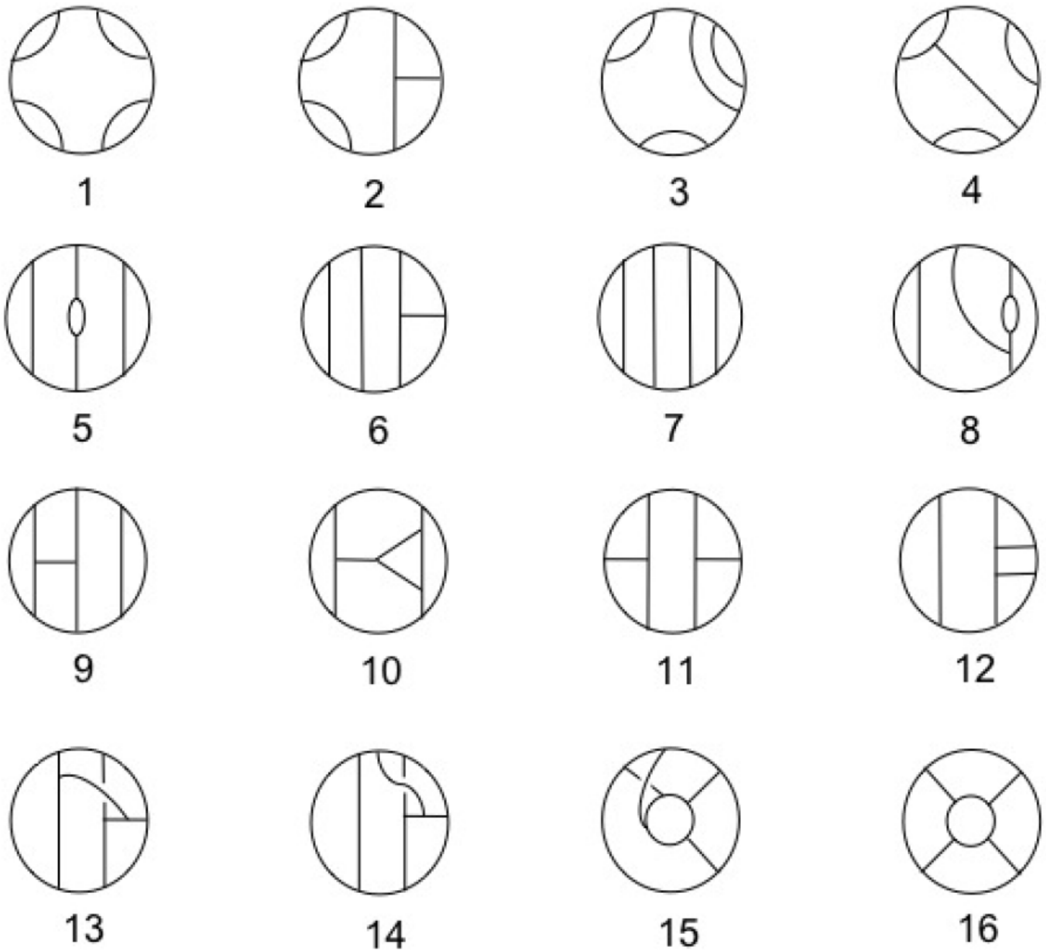
1. Duality between color and kinematics and double copy.
2. Double-copy offers remarkable insight into gravity:
   — Gravity loops from gauge theory loops.
   — Classical solutions. Gravitational radiation problem.
3. Generalized double copy: convert any representation of gauge-theory amplitude to gravity one.
4. 5-loop 4-point integrand of $\mathcal{N} = 8$ supergravity constructed.
5. $\mathcal{N} = 8$ sugra in $D = 24/5$ at $L = 5$ has no enhanced cancellations but $\mathcal{N} = 5$ sugra in $D = 4$ does. Why?
6. Simple pattern for higher-loop UV uncovered.
7. Even $D = 4$, $L = 7$, now looks within reach for $\mathcal{N} = 8$ supergravity.

**Duality between color and kinematics offer powerful tools for studying gravity at high perturbative orders.**

# Extra Slides

# First Quantized Approach

Bjornsson and Green

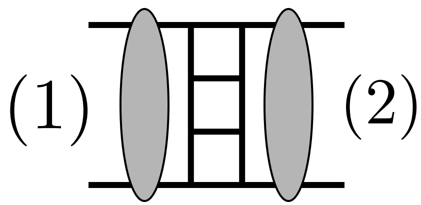


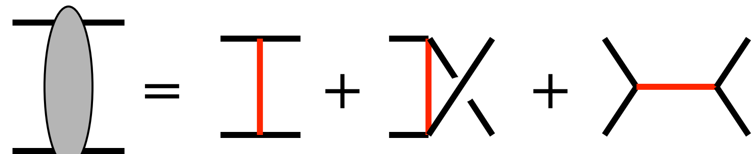

- **Contributions 15 and 16 are the crucial ones.**
- **Pure spinors have regularization issues at 5 loops and beyond**

**"Since we have not evaluated the precise values of the coefficients the possibility of terms vanishing or cancellations between different contributions to the amplitude cannot be ruled out."**

Bjornsson and Green

# Deriving Gravity Contact Formulas



color numerator

$$\mathcal{C}^{4\times 4}_{\rm YM} = \sum_{i_1,i_2} \frac{c_{i_1 i_2} n_{i_1 i_2}}{d^{(1)}_{i_1} d^{(2)}_{i_2}}$$

propagator

**Generalized gauge transformation**

$$\delta_{i1,i2} \equiv n_{i1i2} - n^{\rm BCJ}_{i1,i2} = d^{(1)}_{i1} k^{(2)}(i_2) + d^{(2)}_{i_2} k^{(1)}(i1)$$

**Generalized gauge invariance:**

$$\sum_{i_1,i_2} \frac{c_{i_1 i_2}\delta_{i_1 i_2}}{d^{(1)}_{i_1} d^{(2)}_{i_2}} = 0 = \sum_{i_1,i_2} \frac{n^{\rm BCJ}_{i_1 i_2}\delta_{i_1 i_2}}{d^{(1)}_{i_1} d^{(2)}_{i_2}}$$

**BCJ discrepancy function:**

$$J^{(1)}_{i_2} \equiv \sum_{i_1}^{3} n_{i_1 i_2} = d^{(1)}_{i_2} \sum_{i_1}^{3} k^{(1)}(i_1) \qquad J^{(2)}_{i_1} \equiv \sum_{i_2}^{3} n_{i_1 i_2} = d^{(2)}_{i_1} \sum_{i_2}^{3} k^{(2)}(i_2)$$

$$\mathcal{C}^{4\times 4}_{\rm SG} = \sum_{i_1,i_2} \frac{n^{\rm BCJ}_{i_1 i_2} n^{\rm BCJ}_{i_1 i_2}}{d^{(1)}_{i_1} d^{(2)}_{i_2}}$$

**cross term between numerators and discrepancy vanishes.**

**Formula for missing contact:**

$$\mathcal{C}^{4\times 4}_{\rm SG} = \sum_{i_1,i_2} \frac{n_{i_1 i_2} n_{i_1 i_2}}{d^{(1)}_{i_1} d^{(2)}_{i_2}} - \frac{2}{d^{(1)}_{1} d^{(2)}_{1}} J^{(1)}_{1} J^{(2)}_{1}$$

# Some Related Recent Activities

- **Examples of exact classical solutions, including black holes.**
  Monteiro, O'Connell, White; Luna,Monteiro, O'Connell, White (2015); Bahjat-Abbas, Luna, White (2017)
- **Perturbative constructions of general classical solutions, including gravitational radiation problems (LIGO)**
  Goldberger, Ridgway (2016); Luna, Monterio, Nicholson, O'Connell, Ochirov, Westerberg, White (2016)
- **Loop level KLT and BCJ: using CHY, ambitwistor string, Q-cuts**
  Song He, Oliver Schlotterer (2016), Tourkine, Vanhove (2016,2017); Hohenegger, S. Stieberger (2017); Y. Geyer, L. Mason, R. Monteiro, P. Tourkine (2016) K. A. Roehrig, D. Skinner (2017)
- **Analytic properties of gravity integrands.** Herrmann and Trnka (2016)
- **Simplified gravity Lagrangian.**
  Cheung and Remmen (2016,2017)
- **Double copy as consequence of gauge invariance.**
  Chiodaroli; Boels, Medina (2016), Arkani-Hamed, Rodina, Trnka (2016), Feng et al (2016)
- **Applications in string theory.** Steiberger; Vahhove
  Carrasco, Mafra, Schlotterer, (2016); Mafra and Schlotterer (2015, 2016)