Form Factors

from N=4 SYM to Higgs+gluon amplitudes

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with

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(and earlier work with E. Hughes, R. Panerai, B. Spence, C. Wen, G. Yang & D. Young)

SAGEX KICK-OFF MEETING, 5-8/9/2018

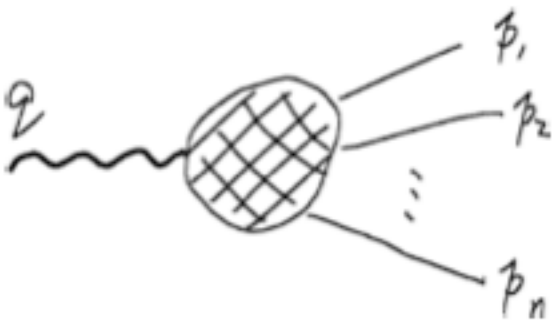
Beyond amplitudes

- Long-term goal: extend success of on-shell methods to off-shell quantities
  - ✦ Main focus today: Form Factors (partially off-shell)
    - MHV diagrams, BCFW, generalised unitarity, remainders, symbols
      AB, Kostacinska, Penante, Spence, Travaglini, Wen, Yang, Young; Bork, Kazakov, Vartanov; Gehrmann, Henn, Huber; Loebbert, Nandan, Sieg, Wilhelm, Yang;…
    - Remarkable simplicities/regularities but no dual conformal symmetry
    - Grassmannian and Twistor Formulations Frassek, Meidinger, Nandan, Wilhelm; Koster, Mitev, Staudacher, Wilhelm; Nandan, Meidinger, Penante, Wen; Chicherin, Sokatchev
    - (Ambi-)Twistor Strings, Scattering Equations: He, Zhang, Liu; AB, Hughes, Panerai, Spence, Travaglini; Bork, Onishenko
    - Dilatation operator/Integrability/Yangian Zwiebel; Koster, Mitev, Staudacher; Wilhelm; AB, Penante, Travaglini, Young

Form Factors: "going partially off-shell"

- More general objects than correlation functions, Wilson loops, amplitudes: e.g. Wilson loops with operator insertions, correlators of Wilson loops ...
- Form Factors: interpolate between correlators and amplitudes, partially off-shell

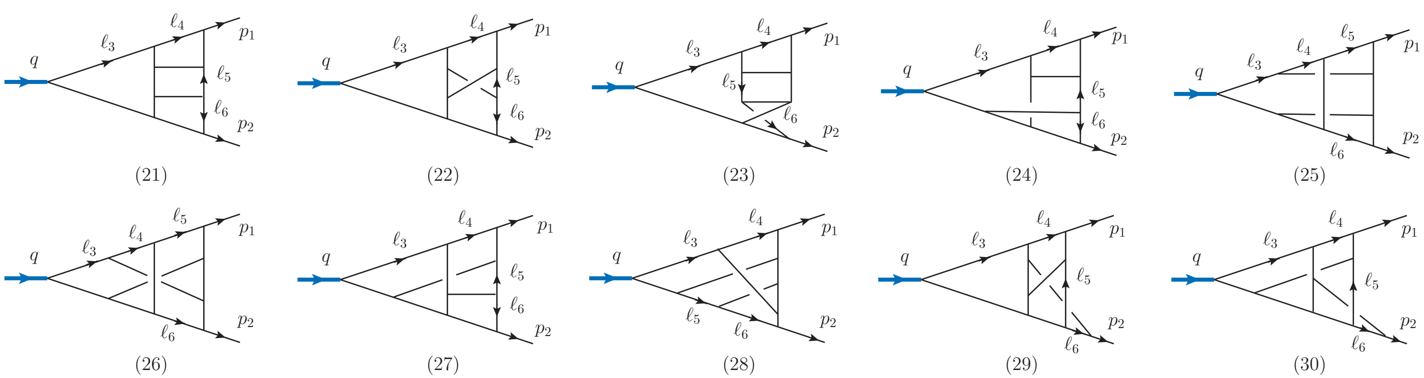
$$\int d^4x \, e^{-iqx} \, \langle 1 \cdots n | \mathcal{O}(x) | 0 \rangle \;=\; \delta^{(4)}(q - \sum_{i=1}^{n} p_i) \, \langle 1 \cdots n | \mathcal{O}(0) | 0 \rangle$$



$$q = \sum_{i=1}^{n} p_i$$

$$q^2 \neq 0 \quad , \quad \text{off} - \text{shell!}$$

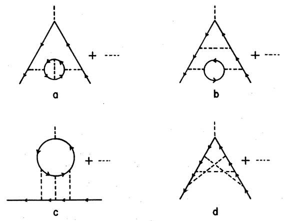
- Simplest case (QCD) Sudakov FF (n=2): IR divergences
  - In N=4: 1 & 2-Loop Sudakov FF first studied by Van Neerven in 1986
  - 3 Loops: (Gehrmann, Henn, Huber)
  - 4 & 5 Loops (Boels, Huber, Yang):
    - Color-Kinematics duality (Bern-Carrasco-Johansson)
    - Cusp anomalous dim, Casimir scaling violated at four loops



FFs appear in many physics contexts

- Five-loop correction to electron g–2

72 diagrams like



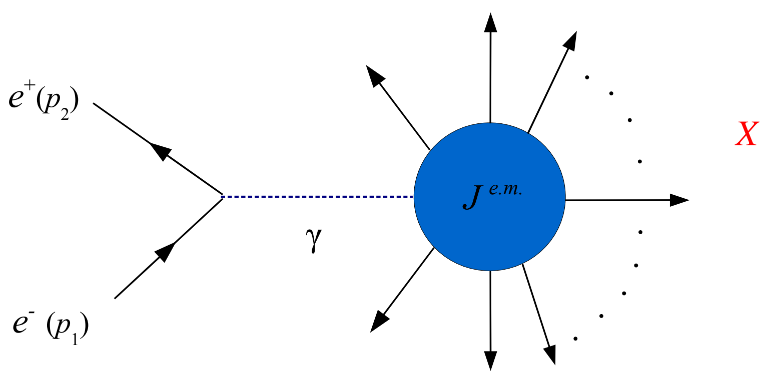
$$= (1.181241456...)\,(\alpha_{\rm e.m.}/\pi)^3$$

(Cvitanovic & Kinoshita '74)

(Laporta & Remiddi '96, …)

  - wild oscillations between individual diagram
  - result is O(1) => mysterious cancellations

- $e^+\, e^- \rightarrow$ hadrons (LEP):



$$e\,\bar{v}(p_2)\gamma_\mu u(p_1)\ \frac{\eta^{\mu\nu}}{(p_1+p_2)^2}\ (-e)\langle X|\,J_\nu^{e.m.}(0)\,|0\rangle$$

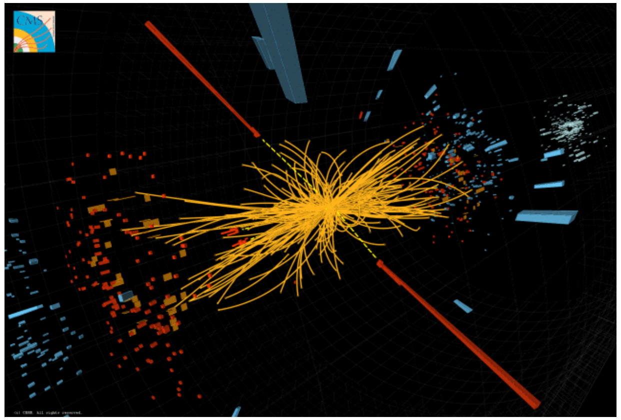
hadronic electromagnetic current

$e^+\, e^- \rightarrow$ hadrons (X)

all orders in $\alpha_{\rm strong}$, first order in $\alpha_{\rm e.m.}$

Effective Lagrangians

(Wilczek '77; Shifman, Vainshtein, Voloshin & Zakharov 79; Dawson '91; Djouadi, Graudenz, Spira, Zerwas '95)



- Higgs + multi-gluon amplitudes
  - at low M_H , dominant Higgs production at the LHC through gluon fusion
  - coupling to gluons through a quark loop
  - for $M_H < 2\, m_t$ integrate out top quark

- Effective Lagrangian description: leading

$$\mathcal{L}_{\text{eff}} \sim H\,\mathrm{Tr}\,F^2 \qquad \mathrm{Tr}F^2 = \mathrm{Tr}F^2_{\mathrm{SD}} + \mathrm{Tr}F^2_{\mathrm{ASD}}$$

  - coupling $\dfrac{\alpha_S}{12\pi v}$, $v = 246GeV$ independent of m_t

- subleading: $$\mathcal{L}_{\text{sub}} \sim \frac{C_1}{v m_t^2} H\,\mathrm{tr}F^3 + \frac{C_2}{v m_t^2} H\,\mathrm{tr}DFDF + \ldots$$

FFs = amplitudes in effective theories

- Higgs + Parton amplitudes are form factors of $\mathrm{Tr}\, F^2$
  - bring down one interaction, and Wick-contract the Higgs field

$$F_{F^2_{\mathrm{ASD}}} = \int d^4x\, e^{-iqx} \langle state|\mathrm{Tr}\, F^2_{\mathrm{ASD}}(x)|0\rangle \quad \text{with} \quad q^2 = M^2_{\mathrm{H}}$$

- Can we look at the same quantity, but in N=4 SYM?
  - Highly symmetric theory, easier to identify any structure
  - Find appropriate translation of the matrix element to N=4 SYM
    - What operator? What state?

Higgs + gluon amplitudes

- Leading order $\mathcal{L}_{\rm eff} \sim H\,{\rm Tr}\,F^2$
  - Early application of on-shell techniques to tree- and one-loop amplitudes (Badger, Dixon, Glover, Khoze; Badger, Glover, Risager, Mastrolia, Williams)

$$F^{\rm tree}_{{\rm tr}F^2}(1^-,2^-,3^+) = \frac{\langle 12\rangle^3}{\langle 23\rangle\langle 31\rangle} \quad , \quad F^{\rm tree}_{{\rm tr}F^2}(1^+,2^+,3^+) = \frac{q^4}{[12][23][31]} \quad , \quad q^2 = m_H^2$$

  - Has been pushed in QCD to 3-loop order for 2 gluons (Anastasiou, Melnikov; Harlander, Kilgore; Anastasiou, Duhr, Buehler, Herzog, Dulat, Furlan, Mistlberger),

    and to 2 loops for 3 partons (Glover, Gehrmann, Jaquier & Koukoutsakis)

- Subleading, finite top-mass corrections have been studied as well (e.g. Neill; Dawson, Lewis, Zeng….)
- Integrating out the top-quark or stringy effects induce new interaction terms such as: ${\rm tr}(F^3)$ ($q \to 0$ limit of FFs) (Dixon, Shadmi; Dixon, Glover, Khoze; Broedel, Dixon; Neill)

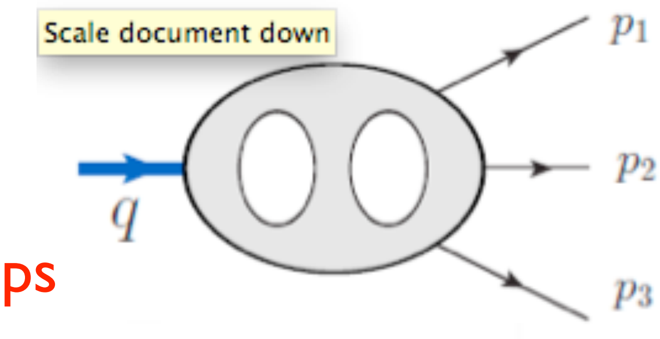
Higgs + gluon amplitudes: from QCD to N=4

- In N=4 SYM operators are organised in multiplets and are related by SUSY transformations
- A) Protected operators (zero anomalous dimension): eg. stress tensor multiplet

$$\mathrm{tr}(X^2) = \mathrm{tr}(\phi_{12}^2) \xrightarrow{Q^4} \mathcal{L}_{\text{on-shell}} \sim \mathrm{tr}(F_{\mathrm{SD}}^2) + \ldots$$

- B) Non-protected: $\mathrm{tr}(F^3)$, $\mathrm{tr}(DFDF)$, ...
  - In N=4 related to Konishi operator, $K \sim \mathrm{tr}(\bar{X}X + \bar{Y}Y + \bar{Z}Z)$
- Q: are there unexpected similarities between QCD & N=4?
  - Translate operator $\mathrm{Tr}\,(F_{\mathrm{ASD}})^2$ in QCD to $\mathcal{L}_{\mathrm{on-shell}}$ in N=4 SYM

3-point 2-loop MHV FF in N=4



- Start with 3-point FF at 2-loops

$$F_3(1,2,3) \;=\; \langle X(p_1)\, X(p_2)\, g^+(p_3)\,|\mathrm{Tr}X^2\,|0\,\rangle$$

- This is how we mimic $\langle g^\pm(p_1)\, g^\pm(p_2)\, g^+(p_3)\,|\mathrm{Tr}F_{\mathrm{ASD}}^2\,|0\,\rangle$ in QCD (Higgs into 3 gluons)

- At loop level tree FF can be stripped off

$$F_3^{(L)} \;=\; F_3^{\mathrm{tree}}\, \mathcal{G}_3^{(L)}(1,2,3)$$

- $\mathcal{G}_3^{(2)}$ is helicity-blind, scalar function, permutation symmetric
  - UV finite in N=4
  - IR divergences exponentiate

Finite remainders

(Catani; Anastasiou, Bern, Dixon, Kosower; Bern, Dixon, Smirnov)

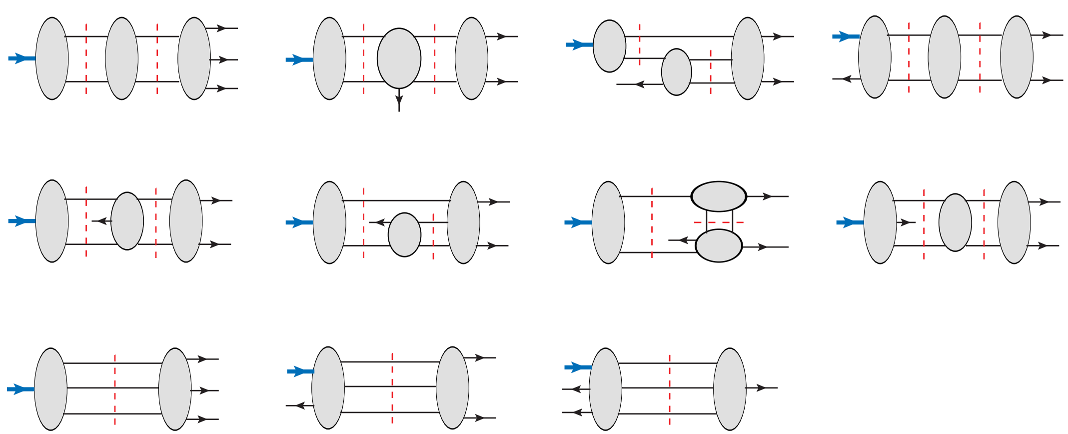
- Subtract off universal IR divergences from the (renormalised) L-loop answer

- All loops (N=4 SYM): $\mathcal{A}_{n,\mathrm{MHV}} = \mathcal{A}^{\mathrm{tree}}_{n,\mathrm{MHV}}\, \mathcal{M}_n$

$$\mathcal{M}_n := 1 + \sum_{L=1}^{\infty} a^L \mathcal{M}_n^{(L)} \sim \exp\left[\mathrm{BDS} + \mathcal{R}\right] \qquad a \sim g^2 N/(8\pi^2)$$

  - $\mathrm{BDS} \sim \mathrm{div} + \gamma_K\, \mathrm{Finite}^{(1)}(p_1, \ldots, p_n)$ BDS Ansatz, completely known
    - div = universal infrared-divergent part, exponentiation is expected
    - ☞ $\mathrm{Finite}^{(1)}(p_1, \ldots, p_n)$ = finite part of one-loop amplitude
    - γ_K = cusp anomalous dimension $\rightarrow$ integrability
    - R is the so-called remainder function the most interesting part!

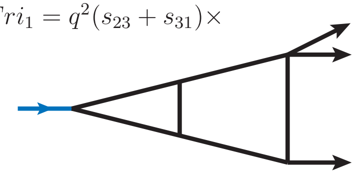
- Exponentiation of finite parts for one-loop amplitude due to dual conformal symmetry (Drummond, Henn, Korchemsky, Sokatchev)
    - Non-trivial remainder R appears from six points on (Drummond, Henn, Korchemsky, Sokatchev; Bern, Dixon, Kosower, Roiban, Spradlin, Vergu, Volovich)
- No dual conformal symmetry for form factors?
    - Still, exponentiating finite parts leads to much simpler remainders
- Generalized unitarity (Bern, Dixon, Dunbar, Kosower; BDK; Britto, Cachazo, Feng)
    - 2- and 3-particle cuts



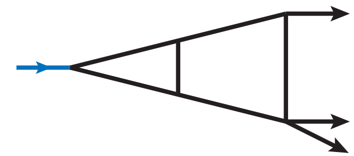
• Result of 2-loop calculation: (AB, Travaglini, Yang)

$$\frac{F_3^{(2)}}{F_3^{\rm tree}} = \sum_{i=1}^{2}(DTri_i + DBox_i) + TriPent + NBox + NTri + \text{cyclic}$$

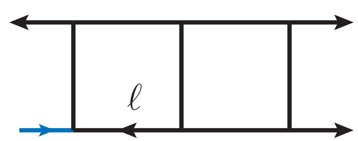
$DTri_1 = q^2(s_{23}+s_{31})\times$



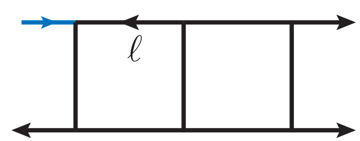
$DTri_2 = q^2(s_{12}+s_{31})\times$



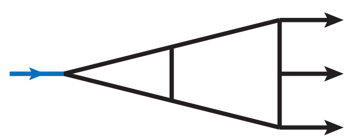
$DBox_1 = s_{23}\,(s_{31}\ell\cdot p_3 - s_{12}\ell\cdot p_2)\times$



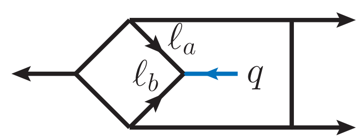
$DBox_2 = s_{12}\,(s_{31}\ell\cdot p_1 - s_{23}\ell\cdot p_2)\times$



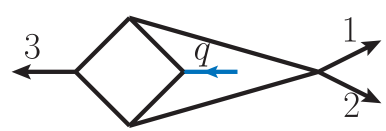
$TriPent = q^2 s_{12}s_{23}\times$



$NBox = s_{23}\left(\frac{1}{2}s_{12}s_{31} - s_{12}\ell_a\cdot p_2 - s_{31}\ell_b\cdot p_3\right)\times$



$NTri = \frac{1}{2}q^2(s_{23}+s_{31})\times$



BCJ-Dual Numerators

(Boels, Kniehl, Tarasov, Yang)

result expressed as rational coefficients × two-loop planar and non-planar integrals

- Final answer (using the symbol of transcendental functions) (AB, Travaglini, Yang)

$$\mathcal{R}_3^{(2)} = -2\left[J_4\left(-\frac{uv}{w}\right) + J_4\left(-\frac{vw}{u}\right) + J_4\left(-\frac{wu}{v}\right)\right] - 8\sum_{i=1}^{3}\left[\mathrm{Li}_4\left(1-u_i^{-1}\right) + \frac{\log^4 u_i}{4!}\right]$$
$$-2\left[\sum_{i=1}^{3}\mathrm{Li}_2(1-u_i^{-1})\right]^2 + \frac{1}{2}\left[\sum_{i=1}^{3}\log^2 u_i\right]^2 - \frac{\log^4(uvw)}{4!} - \frac{23}{2}\zeta_4$$

- $u_1 = u = s_{12}/q^2$, $u_2 = v = s_{23}/q^2$, $u_3 = w = s_{31}/q^2$ kinematic invariants
- $J_4(z) := \mathrm{Li}_4(z) - \log(-z)\mathrm{Li}_3(z) + \frac{\log^2(-z)}{2!}\mathrm{Li}_2(z) - \frac{\log^3(-z)}{3!}\mathrm{Li}_1(z) - \frac{\log^4(-z)}{48}$.
- Bloch-Wigner-Ramakrishnan(-Zagier) polylogarithmic function

- Result: extremely compact, homogeneous degree of transcendentality $= 4$

Next: compare with QCD

Higgs + parton amplitudes in QCD

- Higgs + 3 partons (Koukoutsakis 2003; Gehrmann, Glover, Jaquier & Koukoutsakis 2011)
  - $H\, g^+\, g^-\, g^-$ MHV
  - $H\, g^+\, g^+\, g^+$ maximally non-MHV
  - $H\, q\ \ \bar{q}\ \ g$ fundamental quarks

$$F^{\rm tree}(H, g_1^-, g_2^-, g_3^+) = \frac{\langle 1\, 2\rangle^2}{\langle 2\, 3\rangle\, \langle 3\, 1\rangle}$$

$$F^{\rm tree}(H, g_1^+, g_2^+, g_3^+) = \frac{q^4}{[1\, 2]\, [2\, 3]\, [3\, 1]}$$

$$q^2 = M_H^2$$

- In N=4 SYM:
  - $(H\, g^+\, g^-\, g^-)$ and $(H\, g^+\, g^+\, g^+)$ both derived from super form factor
  - from supersymmetric Ward identities:

$$\frac{F^{(L)}(g_1^-, g_2^-, g_3^+)}{F^{\rm tree}(g_1^-, g_2^-, g_3^+)} = \frac{F^{(L)}(g_1^+, g_2^+, g_3^+)}{F^{\rm tree}(g_1^+, g_2^+, g_3^+)} = \mathcal{G}^{(L)}(u, v, w)$$

$\leftarrow$ what we computed

- 2-loop QCD answer from Gehrmann, Glover, Jaquier & Koukoutsakis
  - very different looking than N=4 SYM result!
  - transcendentality 4,3,2,1 and 0 (rational). In N=4, only degree 4
  - expressed in terms of several pages of multiple polylogarithms
  - expected because of expansion as $\sum$ (coefficient x integral) !
    - each integral is separately quite complicated
- Comparing the two quantities reveals a surprising relation:

$$\mathcal{R}^{(2)}_{H\,g^- g^- g^+}\Big|_{\text{MAX TRANS}} = \mathcal{R}^{(2)}_{H\,g^+ g^+ g^+}\Big|_{\text{MAX TRANS}} = \mathcal{R}^{(2)}_{\mathcal{N}=4\,\text{SYM}}$$

- Principle of maximal transcendentality:
  - First example with kinematic dependence
  - Discovered by Kotikov, Lipatov, Onishchenko and Velizhanin in the context of anomalous dimensions of twist-2 operators (Moch-Vermaseren-Vogt)
  - several counter-examples in amplitudes, e.g. broken for one-loop amplitudes in pure Yang-Mills
- Next testing ground: form factors of higher-dimensional operators describing Higgs + multigluon scattering

From N=4 to QCD

(AB, Kostacińska, Penante, Travaglini, Young '16; AB, Kostacińska, Penante, Travaglini '17 + 18)

- Effective field theory description for finite m_{top} corrections
  - Beyond leading-order term $\mathcal{L}^{(0)}_{\text{eff}} \sim H\,\text{Tr}F^2$ (infinite m_{top})
  - Next corrections: 4 dimension-7 operators in QCD
  - Two particular operators also present in N=4 SYM:

$$\mathcal{L}^{(1)}_{\text{eff}} \sim H\,\text{Tr}F^3 \qquad \mathcal{L}^{(2)}_{\text{eff}} \sim H\,\text{Tr}(D_\mu F_{\rho\sigma})(D^\mu F^{\rho\sigma})$$

  - Goal: compute in N=4 SYM and compare to QCD (result not yet available)
  - previous work at one loop: Dawson, Lewis & Zeng; Neill; Harlander, Neumann, Ozeren, Wiesemann
  - higher-dimensional operators also studied as corrections to the Standard Model (Buchmuller & Wyler '85 and MANY more!)

Increasing difficulty:

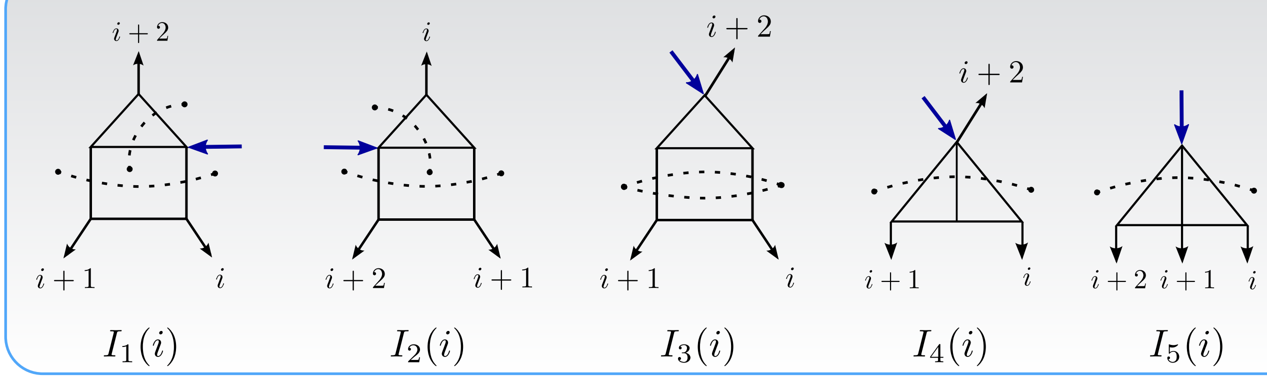
- Operators with 3 fields: approach the problem with increasing difficulty
- Protected: $\mathrm{Tr}\,(X^3), \mathrm{Tr}\,(X\,\{Y\,, Z\})$
- Non-protected: $O_B := \mathrm{Tr}\,(X\,[Y\,, Z])$
  - mixes with $\mathrm{Tr}\,(\psi\psi)$, part of SU(2|3) sector in N=4 SYM
- Non-protected: $\mathrm{Tr}\,F^3$ is descendant
  - descendant of "Konishi" $K \sim \mathrm{tr}(\bar{X}X + \bar{Y}Y + \bar{Z}Z)$

3-point form factor of $\mathrm{Tr}\, X^3$ at 2 loops

(AB, Penante, Travaglini, Wen)

$$F_3(1,2,3) := \langle X(p_1)\, X(p_2)\, X(p_3)\, |\mathrm{Tr} X^3\, |0\,\rangle$$

- Two-loop result expressed in terms of planar integrals



- Remainder very simple! But very different from $\mathrm{Tr}\, X^2$

$$\begin{aligned}\mathcal{R}^{(2)}_{3,3} := & -\frac{3}{2}\,\mathrm{Li}_4(u) + \frac{3}{4}\,\mathrm{Li}_4\left(-\frac{uv}{w}\right) - \frac{3}{2}\log(w)\mathrm{Li}_3\left(-\frac{u}{v}\right) + \frac{1}{16}\log^2(u)\log^2(v)\\ & + \frac{\log^2(u)}{32}\Big[\log^2(u) - 4\log(v)\log(w)\Big] + \frac{\zeta_2}{8}\log(u)[5\log(u) - 2\log(v)]\\ & + \frac{\zeta_3}{2}\log(u) + \frac{7}{16}\zeta_4 + \text{permutations}\,(u,v,w)\end{aligned}$$

maximal degree of transcen-dentality

Non-protected operators

(AB, Kostacinska, Penante, Travaglini)

- First example: descendant of Konishi in SU(2|3) sector

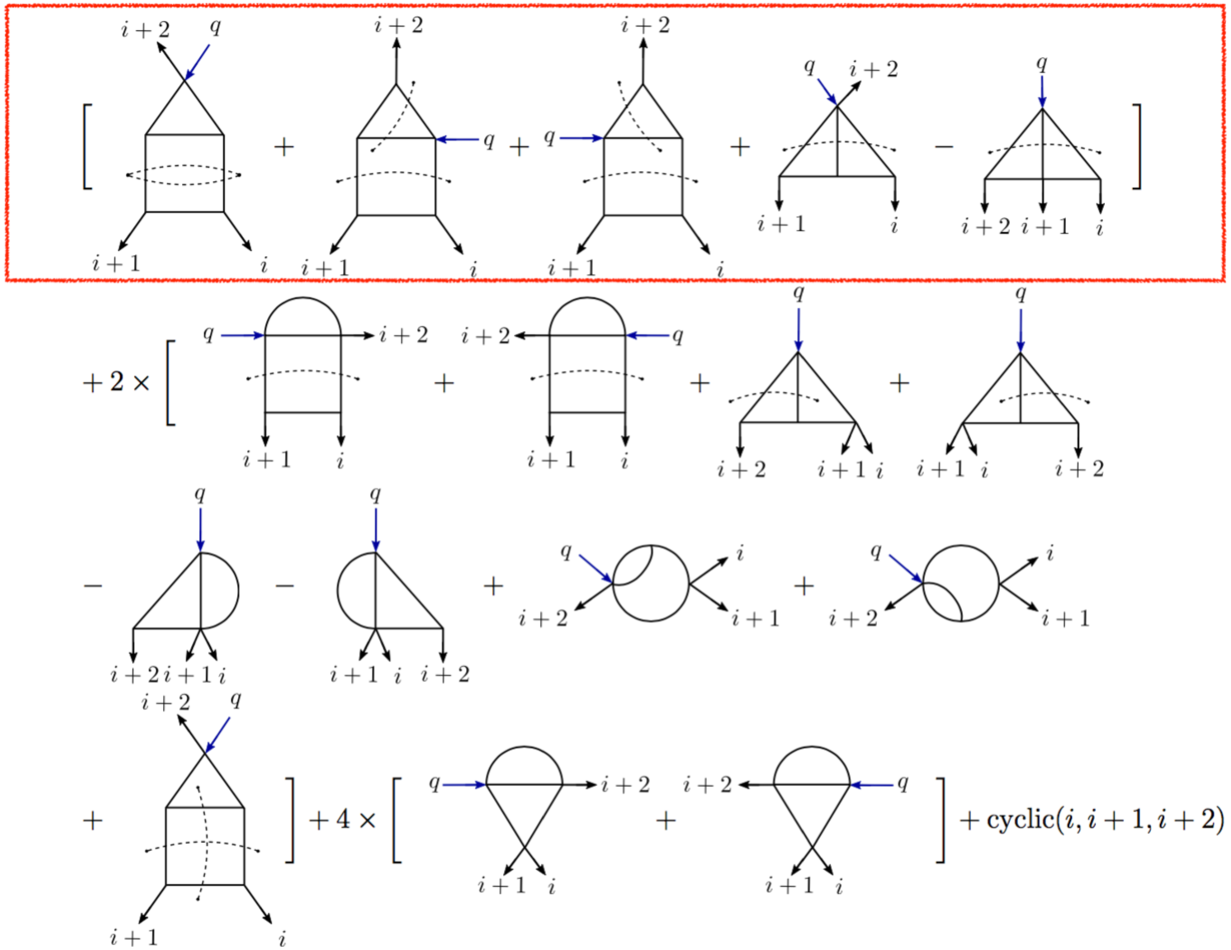
$$\mathcal{O}_K \;=\; \mathrm{Tr}(X[Y,Z]) - \frac{gN}{8\pi^2}\mathrm{Tr}(\psi\psi)$$

- $\mathrm{Tr}\,(X\,[Y\,,\,Z])$ not protected and mixes with $\mathrm{Tr}\,(\psi\psi)$

- 2-loop form factor has IR and UV divergences
  - Renormalise and resolve mixing to obtain correct anomalous dimension $\gamma_K = 12a - 48a^2 + \mathcal{O}(a^3)$
  - Use BDS to extract UV/IR finite remainder

- Admire the result and look for novel structures

- the combined result in terms of integral functions



Same as $\mathrm{Tr}(X^3)$

- numerators indicated by dotted lines
- remaining integrals: UV divergent, transcendentality < 4
- BDS-remainder

$$\mathcal{R}^{(2)}_{\mathcal{O}_{X[Y,Z]}} = \mathcal{R}^{(2)}_{\mathrm{BPS}} + \mathcal{R}^{(2)}_{\mathrm{offset}}$$

- Novel part: $\mathcal{R}^{(2)}_{\rm offset} = \sum_{I=0}^{3} \mathcal{R}^{(2)}_{\rm offset,I}$
- by decreasing degree of transcendentality:

$$\mathcal{R}^{(2)}_{\rm offset;3} = 2\Big[\mathrm{Li}_3(u) + \mathrm{Li}_3(1-u)\Big] - \frac{1}{2}\log^2(u)\log\frac{vw}{(1-u)^2} + \frac{2}{3}\log(u)\log(v)\log(w) + \frac{2}{3}\zeta_3 + 2\,\zeta_2\log(-q^2) + \text{perms}\,(u,v,w)$$

$$\mathcal{R}^{(2)}_{\rm offset;2} = -12\Big[\mathrm{Li}_2(1-u) + \mathrm{Li}_2(1-v) + \mathrm{Li}_2(1-w)\Big] - 2\log^2(uvw) + 36\,\zeta_2$$

$$\mathcal{R}^{(2)}_{\rm offset;1} = -12\,\log(uvw) - 36\,\log(-q^2)$$

$$\mathcal{R}^{(2)}_{\rm offset;0} = 126$$

- Transcendentaliy 4 terms "universal": appear in 2-loop FF's in SU(2) and SL(2) sectors (Loebbert, Nandan, Sieg, Wilhelm, Yang)
- Lower Degree Terms: intriguing relation to those of SU(2)/SL(2) FF's! (shuffling, permutations)
- Signs of universal building blocks of general FF's

Two-loop results for $\mathrm{Tr}\, F^3$

- Compare remainders for the two form factors:

$\langle g^+ g^+ g^+ |\, \mathrm{Tr} F^3_{\mathrm{ASD}} \,|\, 0 \,\rangle$ in any theory (even without supersymmetry)

$\langle X X X |\, \mathrm{Tr} X^3 \,|\, 0 \,\rangle$ in N=4 SYM

- Maximally transcendental parts agree!

$$\mathcal{R}^{(2)}_{F^3_{\mathrm{ASD}}}\Big|_{\mathrm{MAX\ TRANS}} = \mathcal{R}^{(2)}_{\mathrm{BPS}} = -\frac{3}{2}\,\mathrm{Li}_4(u) + \frac{3}{4}\,\mathrm{Li}_4\left(-\frac{uv}{w}\right) - \frac{3}{2}\log(w)\,\mathrm{Li}_3\left(-\frac{u}{v}\right) + \frac{1}{16}\log^2(u)\log^2(v)$$
$$+ \frac{\log^2(u)}{32}\Big[\log^2(u) - 4\log(v)\log(w)\Big] + \frac{\zeta_2}{8}\log(u)\Big[5\log(u) - 2\log(v)\Big]$$
$$+ \frac{\zeta_3}{2}\log(u) + \frac{7}{16}\,\zeta_4 + \mathrm{perms}\,(u, v, w)\,.$$

- BPS operators in N=4 SYM compute (parts of) FFs in QCD!
- Next: sub-leading transcendentality terms

Translation of $\mathrm{Tr}\, F^3$ to N=4

- Translating the operator $\mathrm{Tr}\, (F_{\mathrm{ASD}}{}^3)$ to N=4 language leads to the Konishi supermultiplet $K \sim \mathrm{tr}(\bar{X}X + \bar{Y}Y + \bar{Z}Z)$
- Acting with 8 supercharges gives

$$\mathcal{O}_S \sim Tr(F^3_{\mathrm{ASD}}) + g\, Tr(F^2_{\mathrm{ASD}}\phi\bar{\phi}) + g\, Tr(F_{\mathrm{ASD}}\phi F_{\mathrm{ASD}}\bar{\phi})$$
$$+ g\, Tr(F_{\mathrm{ASD}}\psi\psi\phi) + g\, Tr(F_{\mathrm{ASD}}\psi\phi\psi) + g\, Tr(\psi\psi\psi\psi)\ ,$$

- O(g) terms give additional contact terms contributing from n=4 (which is needed in cuts), e.g.:

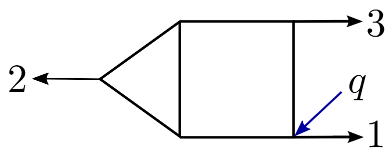
$$F^{(0)}(1^+, 2^+, 3^{\phi^{12}}, 4^{\phi^{34}}; q) = -\frac{1}{2}\frac{[12]}{[34]}([13][24] + [14][23]) + \frac{1}{6}[12]^2$$

- MHV super form factor for full stress tensor multiplet is known (Chicherin, Sokatchev) — extract relevant component
- At two loops a new structure appears $\dfrac{1}{\epsilon}\dfrac{1}{uvw}$
- Renormalisation involves mixing with operator $\sim q^2 Tr(F^2_{ASD})$

The remainder

- Remainder contains two types of terms:
  - purely transcendental: 4 (already discussed), 3, 2, 1 and 0
  - new feature: multiplied by a rational prefactor, e.g. $u/v,\ u/w,\ v/w$

- Calculation in N=4 done, N=2 and 1 (using mostly 4D cuts)
  - maximally transcendental part is universal since all extra integrals have lower transcendentality



- Final N=4 result extremely simple
  - Tests
    - UV: Reproduce expected 2-loop anomalous dimension of Konishi
    - IR-divergences exponentiate as expected
- N=2 and 1
  - Calculation more involved; Still: remainders R differ only slightly
  - Running coupling+operator mixing: first renormalise form factors…
  - … and compute Catani's remainder to remove IR divergences

- Transcendentality $3, 2, 1, 0$ parts of the N=4 SYM result for $\mathcal{O}_S$:

$$\mathcal{R}^{(2)}_{\mathcal{K},3}\Big|_{\rm pure} = \mathrm{Li}_3(u) + \mathrm{Li}_3(1-u) - \frac{1}{4}\log^2(u)\log\left(\frac{vw}{(1-u)^2}\right) + \frac{1}{3}\log(u)\log(v)\log(w) + \zeta_2\log(u) - \frac{5}{3}\zeta_3 + \text{perms}\,(u,v,w)$$

$$\mathcal{R}^{(2)}_{\mathcal{K};3}\Big|_{u/w} = \Big[-\mathrm{Li}_3\left(-\frac{u}{w}\right) + \log(u)\mathrm{Li}_2\left(\frac{v}{1-u}\right) - \frac{1}{2}\log(1-u)\log(u)\log\left(\frac{w^2}{1-u}\right) + \frac{1}{2}\mathrm{Li}_3\left(-\frac{uv}{w}\right) + \frac{1}{2}\log(u)\log(v)\log(w) + \frac{1}{12}\log^3(w) + (u\leftrightarrow v)\Big] + \mathrm{Li}_3(1-v) - \mathrm{Li}_3(u) + \frac{1}{2}\log^2(v)\log\left(\frac{1-v}{u}\right) - \zeta_2\log\left(\frac{uv}{w}\right)$$

$$\mathcal{R}^{(2)}_{\mathcal{K};2}\Big|_{\rm pure} = -\mathrm{Li}_2(1-u) - \log^2(u) + \frac{1}{2}\log(u)\log(v) - \frac{13}{2}\zeta_2 + \text{perms}\,(u,v,w)$$

$$\mathcal{R}^{(2)}_{\mathcal{K};2}\Big|_{u^2/w^2} = \mathrm{Li}_2(1-u) + \mathrm{Li}_2(1-v) + \log(u)\log(v) - \zeta_2$$

$$\mathcal{R}^{(2)}_{\mathcal{K};1} = \left(-4 + \frac{v}{w} + \frac{u^2}{2vw}\right)\log(u) + \text{perms}\,(u,v,w)\,, \qquad \mathcal{R}^{(2)}_{\mathcal{K};0} = 7\left(12 + \frac{1}{uvw}\right)$$

More surprises…

- pure terms of result for Konishi almost identical to result for $\mathrm{Tr}(X[Y, Z])$
- remainder "densities" of form factors in the $SU(2)$ sector of N=4 reappear as building blocks of the non-pure terms
    - Surprising!
    - Results much more structured than expected. Connections to integrability?
- Hints at universal building blocks?
- Results for N<4 and recent results for pure Yang-Mills confirm structural similarities (Q. Jin, G. Yang)

Summary

- Form factors in N=4 SYM
  - Share simplicities of amplitudes, but no dual conformal symmetry
  - Related to Higgs + gluons amplitudes in QCD in effective field theory approach
  - N=4 SYM computes the most complicated part of the remainder including terms of lower transcendentally
- Systematise (understand!) the connection between Higgs amplitudes in QCD and form factors in N=4 SYM

Further open questions

- Reinforce links with integrability
  - Dual conformal symmetry of amplitudes implies Yangian symmetry of dilatation operator *D* *(proof via form factors)*
  - Can extract *D* from form factors, e.g. SU(2|3)/SU(2) sector at 2 loops; complete 2-loop dilatation operator?
- Hidden symmetries responsible for simple results?
  How is dual conformal symmetry of amplitudes realised?
  (AB, Bianchi, Panerai, Travaglini)
- Apply modern methods like BCJ duality, Steinmann relations, cluster adjacency…

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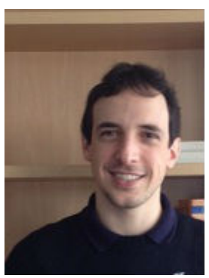

Lorenzo Bianchi plus Gang Chen from 10/2018

PhD students

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From 10/2018

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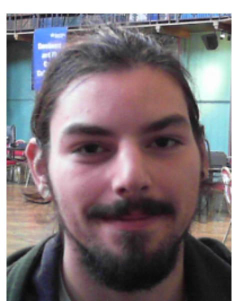

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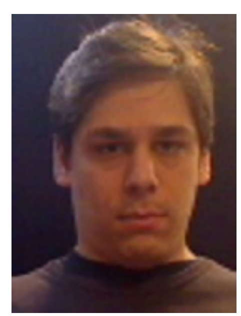

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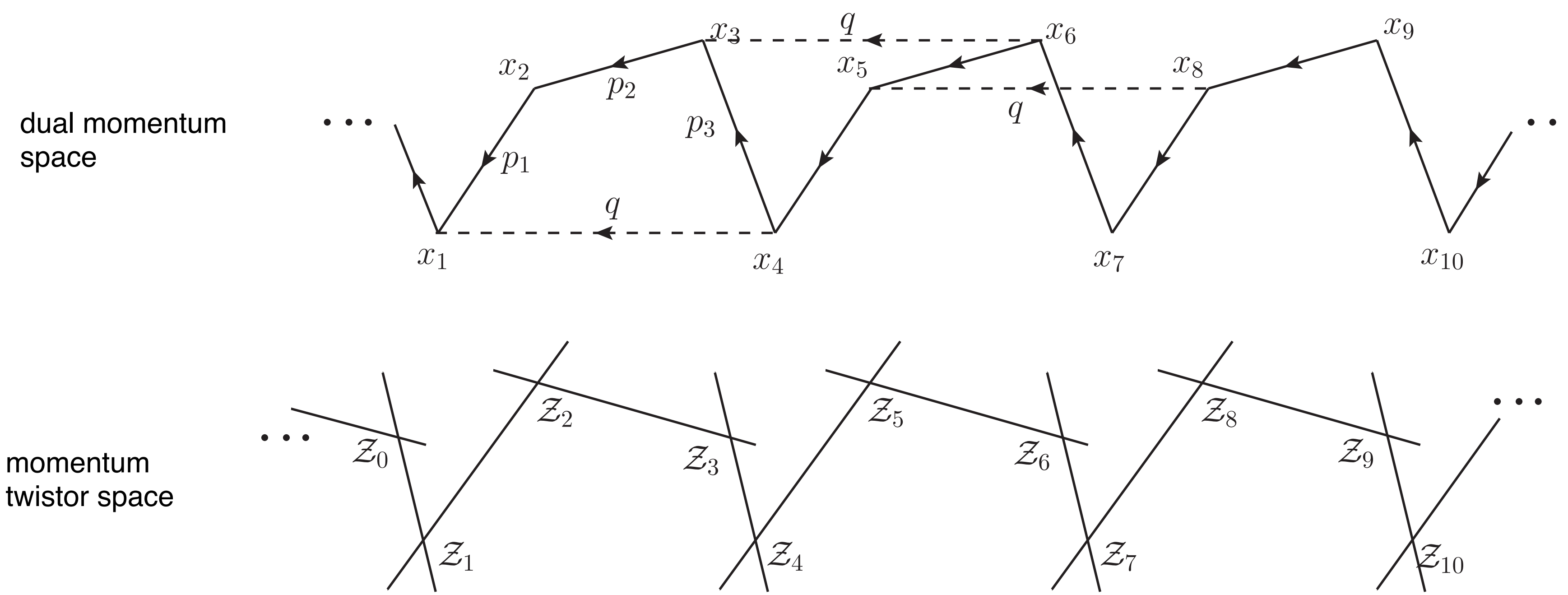
Random comments

(work in progress with Bianchi, Panerai, Travaglini)

- BDS ansatz is powerful also for two-loop form factors, but why? There is no (obvious) dual conformal symmetry (DCS) to explain further simplicities
- Let's take a closer look for chiral part of stress-tensor multiplet which contains $\mathrm{Tr}\,(X^2)$
- For various reasons it is useful to write kinematics in dual momentum space or momentum twistor space e.g. for n=3 (three on-shell legs):
- Periodic Wilson line

Periodic Wilson lines



$$\mathcal{Z}_i = (\lambda_i, x_i \cdot \lambda_i) = (\lambda_i, x_{i+1} \cdot \lambda_i) \qquad p_i = x_i - x_{i+1}$$

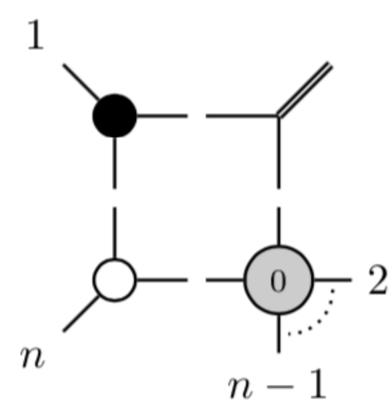
Tree level: dual ~~super~~conformal symmetry

- We can use BCFW recursion relations
- all-line shifts = MHV diagrams give for NMHV form factors
  (with Gurdogan, Mooney, Travaglini, Yang)

$$F^{(0)}_{NMHV} = F^{(0)}_{MHV} \sum_{i=1}^{n} \sum_{j=i+2}^{i+n-1} [*, i-1, i, j-1, j]$$

dual superconformal invariant

- two-line shifts produce a sum of box-coefficients (BCFW bridge) (Bork)
  - most of these are dual superconformal R-invariants
  - but special cases are C x R-invariant where C is only dual conformal but breaks SUSY

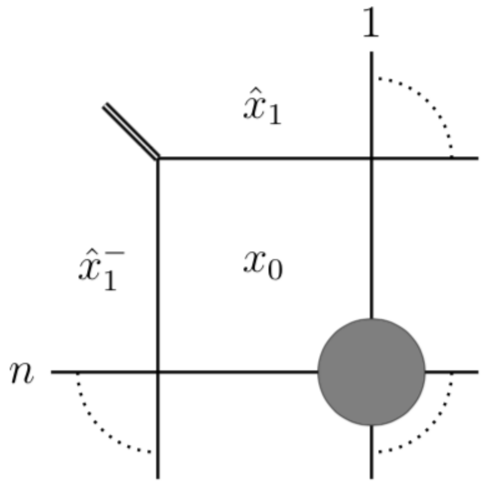


One-loop recursion

- Interestingly one can extend the BCFW recursion to one-loop integrands of FF's (probably also higher loops)
- All-line shift = MHV loop diagrams
- 2-line shifts = Forward Limit of tree-level FF's
  - E.g. 1-loop MHV = forward limit of tree-level NMHV with two extra legs

$$F^{(1)}_{MHV} = \int d^4 l d^4 \eta_l F^{(0)}_{NMHV}(\hat{1}, 2, \ldots, \hat{n}, \ell, -\ell)$$

  - Shown for all-line shifts in generality and checked in several cases for the 2-line shift
  - For amplitudes only gets single residues, for FF's there are special topologies that give two residue contributions



Dual conformal symmetry

- The loop-level recursion gives hope that dual conformal symmetry is realised somehow
- We have checked that finite parts of MHV 1-loop FF's obey expected anomalous dual conformal Ward identities (like amplitudes)

$$K^{\mu} \left. \frac{F^{(1)}_{MHV}}{F^{(0)}_{MHV}} \right|_{fin} = 2 \sum_{i=1}^{n} p_i^{\mu} \log \left(\frac{x^2_{i,i+2}}{x^2_{i-1,i+1}} \right)$$

- This requires to rewrite momentum variables in term of region momenta according to the know box expansion