Precision Calculation on Jet Substructure

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DESY

HARPS Workshop
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Presentation Plan

Presentation Plan

Motivation

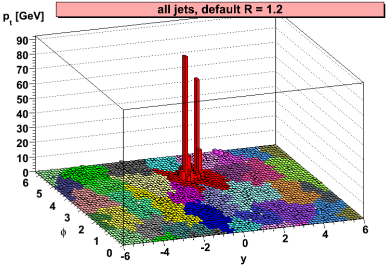
- Connection between **what can be measured** and **what can be calculated**
- We can achieve precise calculation of **measurable observables** with robust **error bars**
- **Grooming** is fundamental to eliminate
  $\rightarrow$ Non-perturbative effects in theoretical calculation
  $\rightarrow$ Underlying Event effects in experiments.
  $\rightarrow$ modified MassDrop Tagger (mMDT) and SoftDrop (SD)
- **Jet mass** is one of the simplest observables

Recent works

- Talk based on **jet mass + grooming** at **LO + (N)LL** for modified MassDrop Tagger and SoftDrop
  Marzani, LS, Soyez (17)
- See also: LO + (N)NLL using SCET
  Frye, Larkoski, Schwartz, Yan (16)
- Inclusive jet production with SCET
  Kang, Lee, Liu, Ringer (18)
- Experimental measures: CMS and ATLAS
  CMS-PAS-SMP-16-010
  ERN-EP-2017-231

Soft Drop (and modified MassDrop Tagger)

- Removes **soft and large-angle radiation**
  Butterworth, Davison, Rubin, Salam (2008)
  Dasgupta, Fregoso, Marzani, Salam (2013)
  Larkoski, Marzani, Soyez, Thaler (2014)



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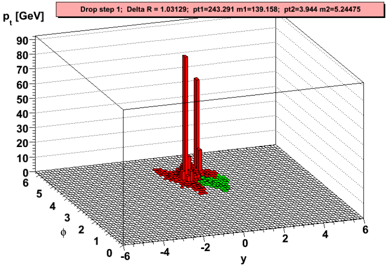
1. Break jet into two $j \to j_1 + j_2$
   using C/A algorithm
2. Check condition
   $\frac{\min(p_{t,1}, p_{t,2})}{(p_{t,1}+p_{t,2})} > z_{\text{cut}} \left(\frac{\theta_{12}}{R}\right)^{\beta}$



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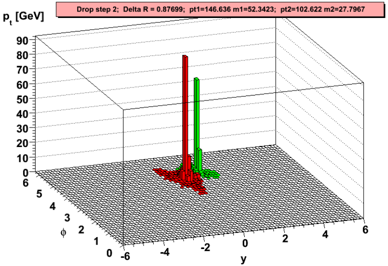

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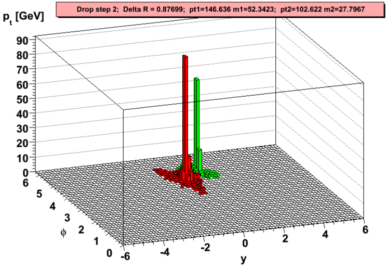
4. If passes, stop recursion



mMDT is equivalent to Soft Drop with $\beta = 0$

Cuts for dijets events

CMS	ATLAS
anti-k_t, $R = 0.8$	anti-k_t, $R = 0.8$
$p_{t,\mathrm{lead}}, p_{t,\mathrm{sublead}} > 200$ GeV	$p_{t,\mathrm{lead}} > 500$GeV
$\|y\| < 2.4$	$\|y\| < 2.5$
$p_{t,\mathrm{lead}} < (1.3/0.7) p_{t,\mathrm{sublead}}$	$p_{t,\mathrm{lead}} < 1.5 p_{t,\mathrm{sublead}}$
$\Delta\phi_{lead,sub} > \pi/2$	-
mMDT ($\beta = 0$), $z_{\mathrm{cut}} = 0.1$	mMDT ($\beta = 0$), $z_{\mathrm{cut}} = 0.1$
	SD ($\beta \leq 4$), $z_{\mathrm{cut}} = 0.1$

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$p_{t,\text{lead}}$ cut + symmetry condition is enough to select dijets.

Presentation Plan

Our accuracy for mMDT

- Our accuracy → **LL matched with NLO**

$$\sigma \overset{\text{FO}}{=} \sigma_{\text{LO}} + \alpha_s \delta_{\text{NLO}} + \dots$$
$$\overset{\text{LL}}{=} \sigma_{\text{LL}} \simeq \sigma_{\text{LL,LO}} + \alpha_s \delta_{\text{LL,NLO}} + \dots$$

- For mMDT leading contribution is single-log

$$\sigma_{\text{LL}} \ni \alpha_s^n \log(p_t/m)^n f_n(z_{\text{cut}})$$

→ includes α_s up to 1-loop and hard-collinear emissions

- Consider finite z_{cut} contributions
- Two options for p_t bins:
  1. Ungroomed momentum $p_{t,\text{jet}}$ **preferred version**
  2. Groomed momentum $p_{t,\text{mMDT}}$ **collinear unsafe**

Structure of LL calculation

- Resummation in the **boosted regime**, consider the variable

$$\rho = \frac{m^2}{p_{t,\text{jet}}^2 R^2} \ll 1.$$

- In practice, we want a results for each mass bin

$$\frac{\Delta\sigma}{\Delta m}(m_1, m_2; z_{\text{cut}}, p_{t1}, p_{t2}) = \frac{1}{m_2 - m_1}\int_{p_{t1}}^{p_{t2}} dp_t \frac{d\sigma^{\text{inclu}}}{dp_t} \Sigma(m; z_{\text{cut}}, p_t)\Big|_{m_1}^{m_2}.$$

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- Finite z_{cut} contributions have a **nontrivial flavor structure**

$$\Sigma(m; z_{\text{cut}}, p_t) = \exp\begin{pmatrix} -R_q - R_{q\to g} & R_{g\to q} \\ R_{q\to g} & -R_g - R_{g\to q} \end{pmatrix} \begin{pmatrix} f_q \\ f_g \end{pmatrix},$$

- R_x are single-log Sudakov corresponding to different decays.

Resummed results $p_{t,\text{jet}}$ case

$$R_q = C_F \int_0^1 \frac{d\theta^2}{\theta^2} \int_0^1 dz\; p_{gq}(z) \frac{\alpha_s}{\pi} \Theta\left(z_{\text{cut}} < z < 1 - z_{\text{cut}}\right) \Theta(z\theta^2 > \rho),$$

$$R_g = C_A \int_0^1 \frac{d\theta^2}{\theta^2} \int_0^1 dz\; p_{xg}(z) \frac{\alpha_s}{\pi} \Theta\left(z_{\text{cut}} < z < 1 - z_{\text{cut}}\right) \Theta(z\theta^2 > \rho),$$

$$R_{q\to g} \;=\; C_F \int_0^1 \frac{d\theta^2}{\theta^2} \int_0^1 dz\; p_{gq}(z) \frac{\alpha_s}{\pi} \Theta\left(1 - z < z_{\text{cut}}\right) \Theta(z\theta^2 > \rho),$$

$$R_{g\to q} = T_R n_f \int_0^1 \frac{d\theta^2}{\theta^2} \int_0^1 dz\; p_{qg}(z) \frac{\alpha_s}{\pi} \left[\Theta\left(1 - z < z_{\text{cut}}\right) + \Theta\left(z < z_{\text{cut}}\right)\right] \Theta(z\theta^2 > \rho).$$

Fixed coupling approximation

- For $\rho > z_{\text{cut}}$, no grooming effects
- For $\rho < z_{\text{cut}}$:

$$R_i = \frac{\alpha_s C_R}{\pi} \times \left[\log\left(\frac{1}{\rho}\right) \log\left(\frac{1}{z_{\text{cut}}}\right) + \ldots \right] + \frac{\alpha_s C_R}{\pi} \log\left(\frac{z_{\text{cut}}}{\rho}\right) \pi_i(z_{\text{cut}})$$

$$R_{i\to j} = \frac{\alpha_s C_R}{\pi} \pi_{i\to j}(z_{\text{cut}})$$

- Functions $\pi_x(z_{\text{cut}})$ contains only finite z_{cut} terms

  See backup for full results.

Fixed order calculation

- Fixed order (NLO) valid in $\rho \sim 1$ region;
- Used NLOJet++ with the parton distribution set CT14;
- Cluster jets with anti-k_t implemented in FastJet;
- Use mMDT implemented in fjcontrib.

Matching

- Additive matching does not work:
  $\rightarrow$ LL calculation misses constant α_s^2 terms, matched result tends to a constant at small ρ
  $\rightarrow$ Requires precise FO results in the small ρ tail, numerically complicated
- "Naive" multiplicative matching :

$$\sigma_{\mathrm{NLO+LL,naive}} = \sigma_{\mathrm{LL}} \frac{\sigma_{\mathrm{NLO}}}{\sigma_{\mathrm{LL,NLO}}}$$

Problem : $\rightarrow$ $\sigma_{\mathrm{(LL,)NLO}}$ may turn negative at small ρ.

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  Problem : $\rightarrow$ $\sigma_{\mathrm{(LL,)NLO}}$ may turn negative at small ρ.
- **Alternative** : expansion in α_s does not change the result at our accuracy

$$\sigma_{\mathrm{NLO+LL,naive}} = \sigma_{\mathrm{LL}} \frac{\sigma_{\mathrm{LO}} + \alpha_s \delta_{\mathrm{NLO}}}{\sigma_{\mathrm{LL,LO}} + \alpha_s \delta_{\mathrm{LL,NLO}}}$$

Matching

- Our **alternative multiplicative matching**

$$\sigma_{\mathrm{NLO+LL}} = \sigma_{\mathrm{LL}} \left[\frac{\sigma_{\mathrm{LO}}}{\sigma_{\mathrm{LL,LO}}} + \alpha_s \left(\frac{\delta_{\mathrm{NLO}}}{\sigma_{\mathrm{LL,LO}}} - \sigma_{\mathrm{LO}} \frac{\delta_{\mathrm{LL,NLO}}}{\sigma_{\mathrm{LL,LO}}{}^2} \right) \right] .$$

Matching

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- LL endpoint matched to (N)LO

$$\log\left(\frac{1}{\rho}\right) \to \log\left(\frac{1}{\rho} - \frac{1}{\rho_{\mathrm{max},i}} + e^{-B_q}\right),$$

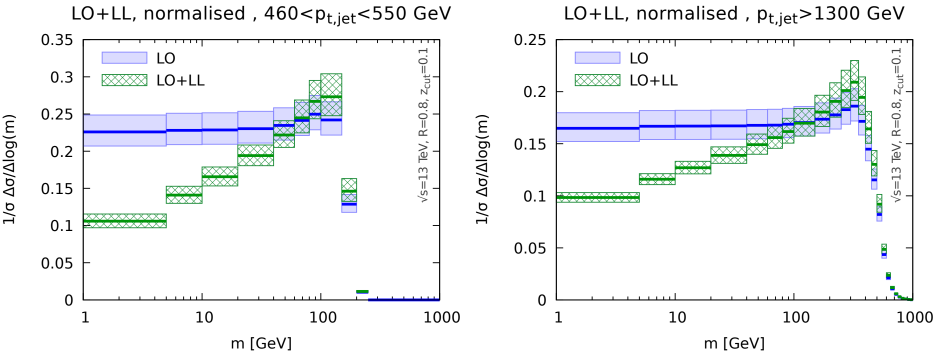
where $\rho_{\mathrm{max,NLO}} = 0.44974$ and $\rho_{\mathrm{max,LO}} = 0.279303$, for $R = 0.8$.

- Normalization to (N)LO x-section.

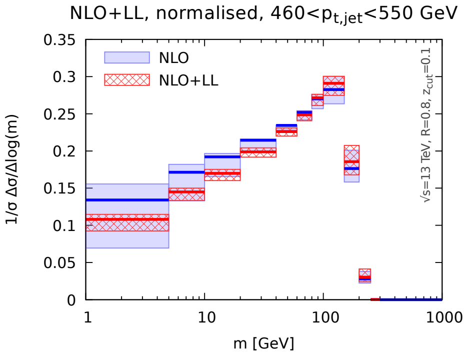
Uncertainties

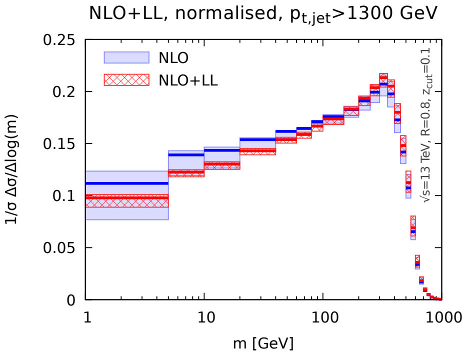
- Vary μ_R and μ_F (7-point scale variation) around $p_{t,\text{jet}}R$
  Cacciari, Frixione, Mangano, Nason, and Ridolf, 2004
- Vary μ_Q around $p_{t,\text{jet}}R$
- (Optional) Vary matching scheme (use also R and log-R)
  (minor effects)
- Vary α_s freezing scale (minor effects)

Perturbative results at LO + LL



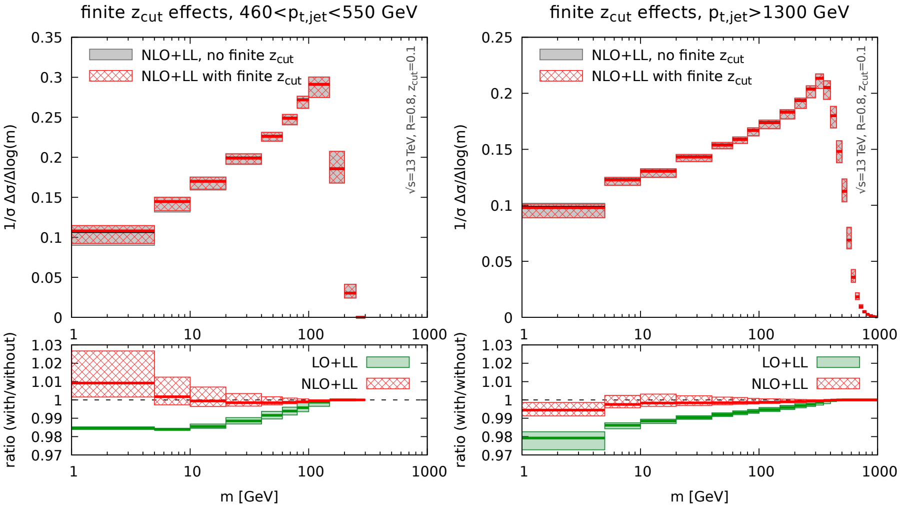
Perturbative results at NLO + LL





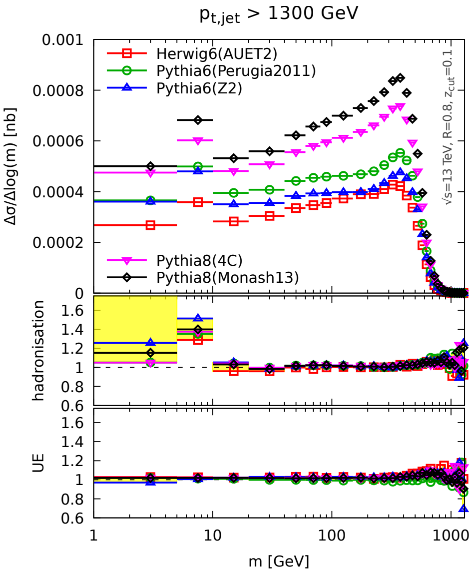
- Going from LO → NLO has large impact in uncertainties;
- Smaller effects from resummation.

Impact of finite z_{cut} effects



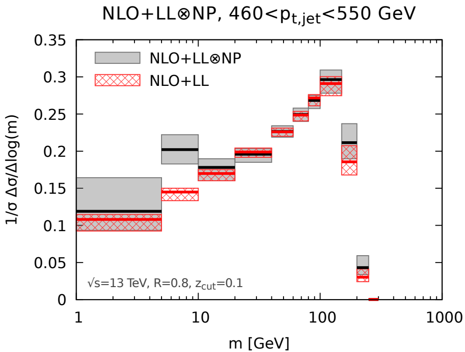
- For the $p_{t,\text{jet}}$ option, finite z_{cut} effects are small.

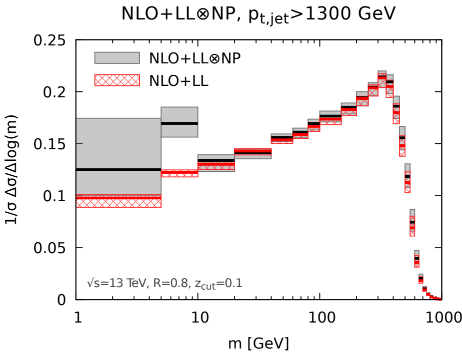
Non-perturbative corrections



- Extract NP corrections from different generators and tunes;
- Average of corrections as a multiplicative factor;
- Envelope as uncertainty;
- Added quadratically to perturbative uncertainty.

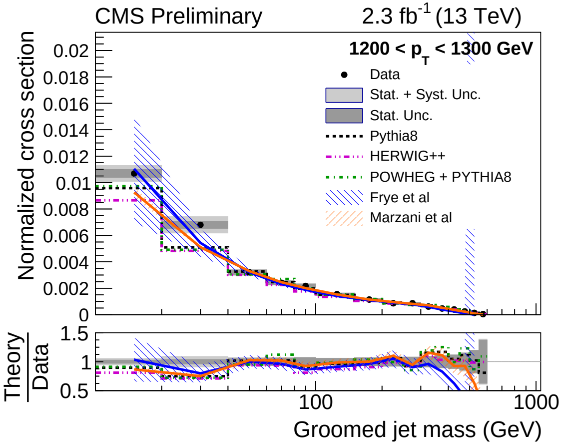
Final results LL + NLO





- Relatively small NP corrections above $m = 10\text{GeV}$.

Comparison to experiment



- Good agreement with experimental measurements.

  Plot from CMS-PAS-SMP-16-010

Presentation Plan

What happens if we consider $p_{t,\mathrm{mMDT}}$ bins instead of $p_{t,\mathrm{jet}}$ bins ?

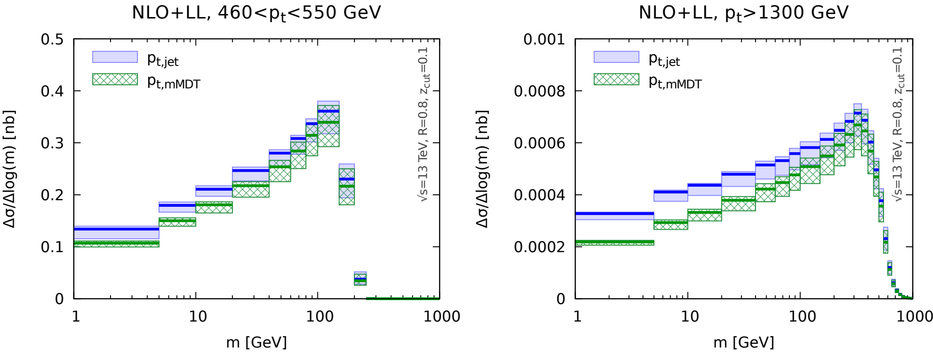
Collinear unsafety

- $\dfrac{d\sigma}{dp_{t,\text{mMDT}}}$ is **collinear unsafe**, but remains **Sudakov safe**;
- Example : bin [1000 : 1100]GeV, jet at $p_{t,\text{jet}} = 1010\text{GeV}$
  Emission of a parton at 20GeV
  $\rightarrow$ if **real** $\rightarrow p_{t,\text{mMDT}} = 990\text{GeV} \rightarrow$ not in bin
  $\rightarrow$ if **virtual** $\rightarrow p_{t,\text{mMDT}} = 1010\text{GeV} \rightarrow$ in bin
- No constraints over emission angle $\rightarrow$ collinear divergence;
- For a fixed mass $\rho \propto \theta$, mass naturally cuts the angle
  $\rightarrow$ finite, but comes with LL contributions.

Resummation at LL for $p_{t,\mathrm{mMDT}}$ variant

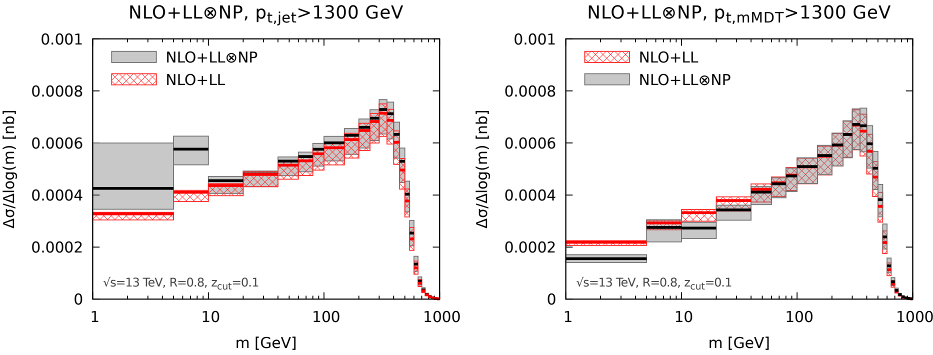
- Equations produce a sequence of angle-ordered parton branchings
  $\rightarrow$ Stops at a given maximum value $t_{\max}$
- Impose the mMDT condition by searching the first emission that satisfies $z_{\mathrm{cut}} < z < 1 - z_{\mathrm{cut}}$
  $\rightarrow$ eliminate emissions that fail
  $\rightarrow$ uses remaining emissions to reconstruct $p_{t,\mathrm{mMDT}}$
- Investigate remaining emissions to find the one that dominates mass
  $\rightarrow$ valid at LL

Comparison $p_{t,\text{jet}}$ vs. $p_{t,\text{mMDT}}$



- Normalization is ill-defined due IRC unsafety
  $\rightarrow$ we present x-sections;
- **Sizable pure finite z_{cut} effects**
  $\rightarrow$ difference in $p_{t,\text{jet}}$ vs. $p_{t,\text{mMDT}}$ is purely due to finite-z_{cut}

Comparison $p_{t,\text{jet}}$ vs. $p_{t,\text{mMDT}}$



- $p_{t,\text{mMDT}}$ is slightly more **resilient against NP effects**
- Theoretically difficult to extend to higher accuracies
- **We encourage the use of $p_{t,\text{jet}}$**, $p_{t,\text{mMDT}}$ still an interesting option

Presentation Plan

What changes for $\beta > 0$ case

Our accuracy – extended to SD ($\beta > 0$)

- Leading contribution now is **double-logarithm**
- Our accuracy is **NLL + NLO**
  $\rightarrow$ includes α_s up to 2-loops (CMW scheme) and multiple emissions
- Finite z_{cut} contributions are power corrections
- Matching requires flavor separation of σ_{jet} at LO and NLO, and of $d\sigma/dm$ at LO
- Multiplicative matching has flaws
  $\rightarrow$ we are using the envelope of log-R and R scheme.

NLL calculation

- For each $p_{t,\text{jet}}$ bin, the integrated distribution is

$$\Sigma_{\text{NLL}}(\rho; p_{t1}, p_{t2}) = \int_{p_{t1}}^{p_{t2}} dp_t \sum_i \frac{d\sigma^{(i)}_{\text{incl,LO}}}{dp_t} \frac{e^{-R_i(\rho)-\gamma_E R_i'(\rho)}}{\Gamma(1+R_i'(\rho))},$$

- The **radiator** behaves like
  Fixed coupling, no hard-collinear terms – full result in backup

$$R_i(\rho) = \frac{\alpha_s C_R}{\pi} \frac{1}{2+\beta} \left[2\beta \log\left(\frac{1}{\rho}\right)^2 + 2\log\left(\frac{1}{\rho}\right) \log\left(\frac{1}{z_{\text{cut}}}\right) - \log\left(\frac{1}{z_{\text{cut}}}\right)^2 \right]$$

- We changed how to express the hard-collinear contributions

$$P_i(z) = 2C_i \left(\frac{1}{z} + B_i\right) \to P_i(z) = \frac{2C_i}{z} \Theta\left(z < e^{-B_i}\right)$$

Advantage: Well-defined positive distribution $\to$ matching is easier

Disadvantage: Introduces artificial NNLL terms

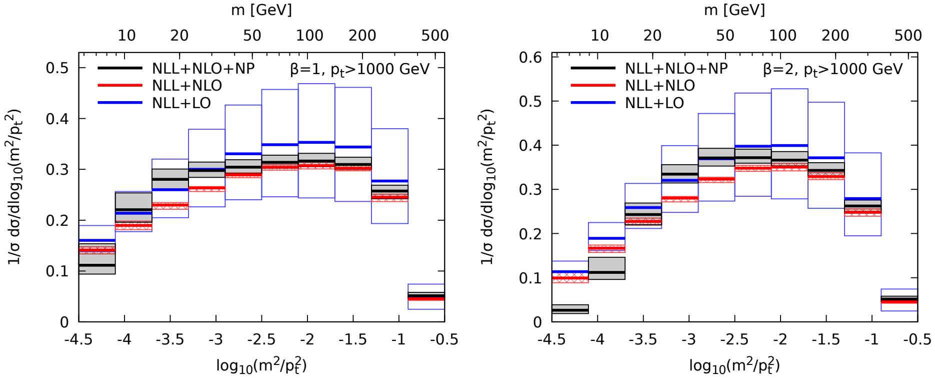
Matching

- $2 \to 3$ events at LO and NLO using NLOJet++
- Need flavor separation in LO for matching
  $\to$ used a patch to NLOJet++
  Banfi, Salam, Zanderighi (10)
- Used flavor sensitive log-R matching

$$\Sigma_{\mathrm{NLL+NLO}}(\rho) = \left[\sum_i \Sigma_{\mathrm{NLL}}^{(i)} \exp\left(\frac{\Sigma_{\mathrm{LO}}^{(i)} - \Sigma_{\mathrm{NLL,LO}}^{(i)}}{\sigma_{\mathrm{incl,LO}}^{(i)}} \right) \right]$$

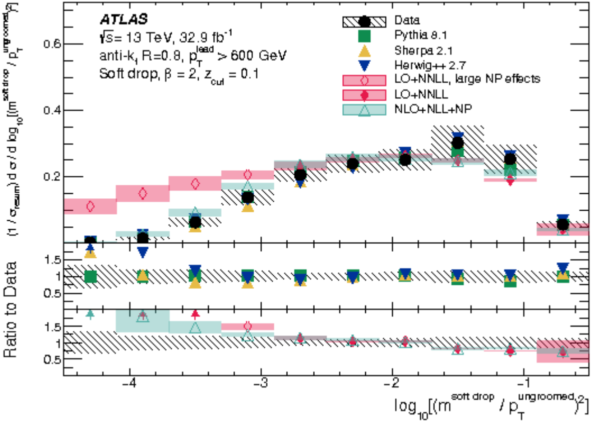
$$\times \exp\left(\frac{\bar{\Sigma}_{\mathrm{NLO}} - \Sigma_{\mathrm{NLL,NLO}}}{\sigma_{\mathrm{incl,LO}}} - \sum_i \frac{(\Sigma_{\mathrm{LO}}^{(i)})^2 - (\Sigma_{\mathrm{NLL,LO}}^{(i)})^2}{\sigma_{\mathrm{incl,LO}}^{(i)} \sigma_{\mathrm{incl,LO}}} \right).$$

Final results NLL + (N)LO



- Uncertainty decreases when going LO $\rightarrow$ NLO.
- **NP effects** increase for **large** β

Comparison to experiment



- Good agreement with experimental measurements.
  Plot from CERN-EP-2017-231

Presentation Plan

NP effects

- Estimate **non-perturbative effects** using a **theoretical approach**
  $\rightarrow$Calculation done for mMDT Dasgupta, Fregoso, Marzani, Salam (13)
- Two sources of NP effects: Hadronization and Underlying Events
- Take into account two effects in the final cross-section
  - **Mass** of the jet is affected by NP effects
  - Cause a **shift in the transverse momentum**, alters the SD condition

$$\frac{1}{\sigma}\frac{d\sigma}{dm}\bigg|_{\text{NP}} = \int dm_p \int_0^1 dz_p p_x(z_p) \frac{1}{\sigma}\frac{d\sigma}{dm}\bigg|_{\text{P}} \delta\left(m - m_p - \delta m\right) \Theta\left[z_p + \delta z - z_{\text{cut}}\theta_m^{\beta}\right]$$

NP effects

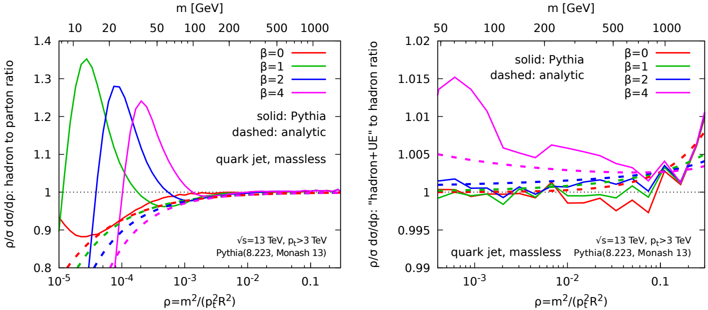
Hadranization

- Mass shift is $\delta m^2 = C_R \Lambda_{\text{had}} p_{t,\text{jet}} R_{\text{eff}}$ and p_t shift is $\delta p_{t,\text{jet}} = C_A \frac{\Lambda_{\text{had}}}{R_{\text{eff}}}$, with $R_{\text{eff}} = \frac{m}{p_{t,\text{jet}} \sqrt{z(1-z)}}$
- Finally corrections are of order $\sim \frac{\Lambda_{\text{had}}}{p_{t,\text{jet}}} \left(\frac{p_{t,\text{jet}}}{m} \right)^{\frac{2+2\beta}{2+\beta}}$

Underlying Events

- Mass shift is $\delta m^2 = \Lambda_{\text{UE}} p_{t,\text{jet}} R_{\text{eff}}^2$ and p_t shuft is $\delta_{pt} = \frac{1}{2} \Lambda_{\text{UE}} p_{t,\text{jet}} R_{\text{eff}}^4$
- Finally corrections are of order $\sim \frac{\Lambda_{\text{UE}}}{p_{t,\text{jet}}} \left(\frac{p_{t,\text{jet}}}{m} \right)^{\frac{2\beta-4}{2+\beta}}$
- Both **NP effects increases with** β, as expected.

NP effects



- Region $\Lambda \ll m \ll p_{t,\text{jet}}$ captures main features: **Increases with β** and $p_{t,\text{jet}}$ global trend.
- Peak in hadronization effects is not captured by the model
- Analytical approach is useful for understanding, but not useful for direct comparison with the data.

Presentation Plan

Conclusion

Many interesting theoretical insights

- mMDT vs. SD produces different log structure
- $p_{t,\mathrm{mMDT}}$ for binning is not recommended (harder to resum, despite more robust against NP effects)
- Finite z_{cut} effects small in typical $z_{\mathrm{cut}} \sim 0.1$ range
- Increasing LO to NLO has large impact in uncertainties
- In practice, practical complications in matching procedure
- Quantitative understanding of NP effects still not known, but can predict most main trends

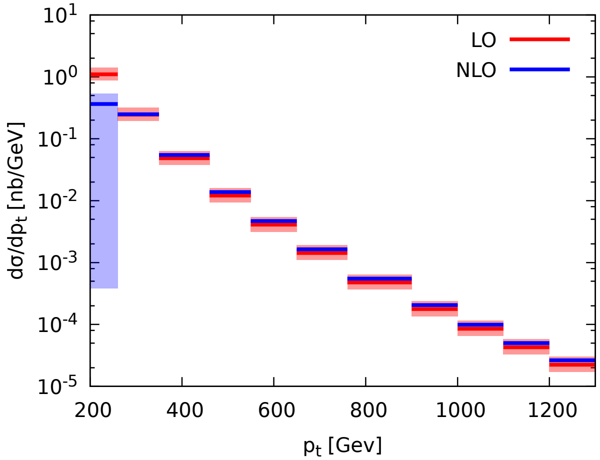
Conclusion

- Precise analytical calculations in jet substructure can be directly compared with measurements
  $\rightarrow$ Successful collaboration between theoretical end experimental communities
  $\rightarrow$ Important verification for both
  $\rightarrow$ Cross-check between different theoretical approaches

- For the future:
  $\rightarrow$ Systematical improvement to higher accuracies
  $\rightarrow$ Expanding to other processes

Backup slides

Instability of NLO contribution for mMDT



Resummed results $p_{t,\text{jet}}$ case

$$
\begin{aligned}
R_q &= C_F \mathcal{R}_q(\rho; z_{\text{cut}})\,\Theta(\rho < e^{B_q}) + C_F\,\mathcal{I}(\rho; z_{\text{cut}})\,\pi_q(z_{\text{cut}})\,\Theta(\rho < z_{\text{cut}}),\\
R_g &= C_A \mathcal{R}_g(\rho; z_{\text{cut}})\,\Theta(\rho < e^{B_g}) + C_A\,\mathcal{I}(\rho; z_{\text{cut}})\,\pi_g(z_{\text{cut}})\,\Theta(\rho < z_{\text{cut}}),\\
R_{q\to g} &= C_F\,\mathcal{I}(\rho; z_{\text{cut}})\,\pi_{q\to g}(z_{\text{cut}})\,\Theta(\rho < z_{\text{cut}}),\\
R_{q\to g} &= n_f\,T_R\,\mathcal{I}(\rho; z_{\text{cut}})\,\pi_{g\to q}(z_{\text{cut}})\,\Theta(\rho < z_{\text{cut}}),
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{R}_i(\rho; z_{\text{cut}}) = \frac{1}{2\pi\alpha_s\beta_0^2}\Big[& W\big(1 + 2\alpha_s\beta_0 B_i\big) - W\big(1 + 2\alpha_s\beta_0 \log(z_m)\big)\\
& + 2W\big(1 + \alpha_s\beta_0 \log(\rho z_m)\big) - 2W\big(1 + \alpha_s\beta_0(\log(\rho) + B_i)\big)\Big],
\end{aligned}
$$

$$
\mathcal{I}(\rho; z_{\text{cut}}) = \int_\rho^{z_{\text{cut}}} \frac{dx}{x}\,\frac{\alpha_s(x p_t R)}{\pi} = \frac{1}{\pi\beta_0}\log\left(\frac{1 + \alpha_s\beta_0\log(z_{\text{cut}})}{1 + \alpha_s\beta_0\log(\rho)}\right),
$$

with $W(x) = x\log(x)$, $z_m = \max(z_{\text{cut}}, \rho)$, $B_q = -\frac{3}{4}$,

Resummed results $p_{t,\text{jet}}$ case

$$\pi_q(z_{\text{cut}}) = \log(1 - z_{\text{cut}}) + \frac{3z_{\text{cut}}}{2},$$

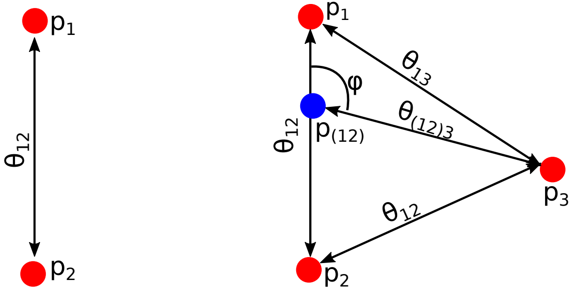
$$\pi_g(z_{\text{cut}}) = \log(1 - z_{\text{cut}}) + 2z_{\text{cut}} - \frac{z_{\text{cut}}^2}{2} + \frac{z_{\text{cut}}^3}{3} - \frac{n_f T_R}{C_A}\left(z_{\text{cut}} - z_{\text{cut}}^2 + \frac{2z_{\text{cut}}^3}{3}\right),$$

$$\pi_{q\to g}(z_{\text{cut}}) = -\log(1 - z_{\text{cut}}) - \frac{z_{\text{cut}}}{2} - \frac{z_{\text{cut}}^2}{4},$$

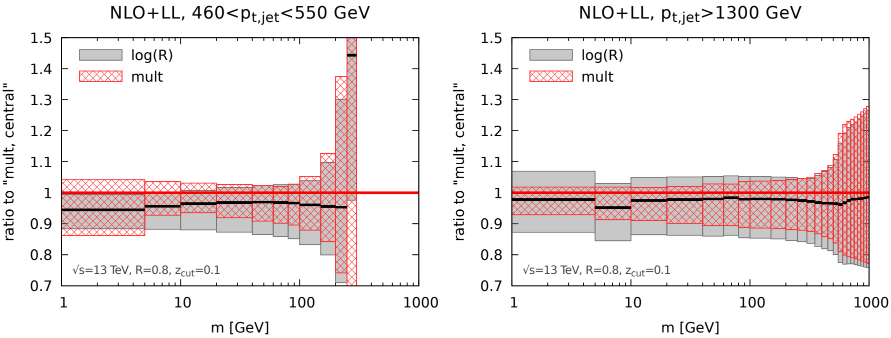
$$\pi_{g\to q}(z_{\text{cut}}) = z_{\text{cut}} - z_{\text{cut}}^2 + \frac{2z_{\text{cut}}^3}{3}.$$

Endpoint ρ_{max}

Determine ρ_{max} $\rightarrow$ find configurations with maximal mass for LO (left) and NLO (right).

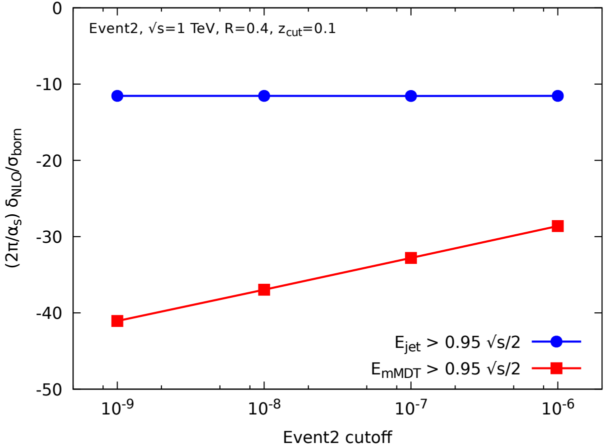


$p_{t,\mathrm{jet}}$ option : matching options



$$\Sigma^{\text{log-}R}_{\text{NLO+LL}} = \Sigma_{\text{LL}} \exp\left[\alpha_s\left(\Sigma^{(1)} - \Sigma^{(1)}_{\text{LL}}\right) + \alpha_s^2\left(\Sigma^{(2)} - \Sigma^{(2)}_{\text{LL}}\right) - \frac{\alpha_s^2}{2}\left(\Sigma^{(1)^2} - \Sigma^{(1)^2}_{\text{LL}}\right)\right].$$

Collinear unsafety $p_{t,\mathrm{mMDT}}$ case



Fixed-order calculation $p_{t,\mathrm{mMDT}}$ case

- At LO, emission pass z_{cut} cut to have a non-vanishing mass. Same result as $p_{t,\mathrm{jet}}$ case.

$$\rho \frac{d\sigma^{\mathrm{LL,LO}}}{d\rho}(\rho; z_{\mathrm{cut}}, p_{t1}, p_{t2}) = \int_{p_{t1}}^{p_{t2}} dp_{t,\mathrm{jet}} \left[\sigma_q(p_{t,\mathrm{jet}})R'_q + \sigma_g(p_{t,\mathrm{jet}})R'_g\right],$$

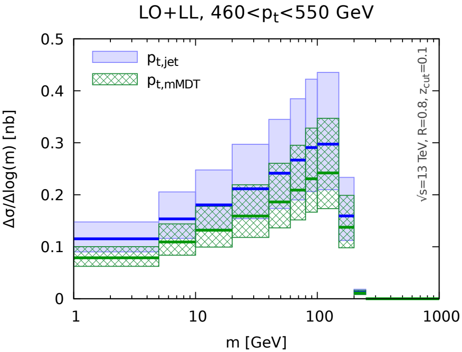
- At NLO, the measured $p_{t,\mathrm{mMDT}}$ must still fall inside the bin
  Only C_F^2 terms

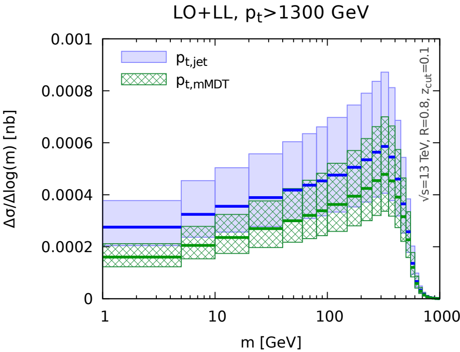
$$\rho \frac{d\sigma^{\mathrm{LL,NLO},C_F^2 a}}{d\rho} = \int_{p_{t1}}^{p_{t2}} dp_{t,\mathrm{jet}}\, \sigma_q(p_{t,\mathrm{jet}})R'_q\Big[-R_q - R_{q\to g}\Big]$$
$$- \int_{p_{t1}}^{\min\left[p_{t2}, \frac{p_{t1}}{1-z_{\mathrm{cut}}}\right]} dp_{t,\mathrm{jet}}\, \sigma_q(p_{t,\mathrm{jet}})\; R'_q\; \frac{\alpha_s C_F}{\pi} \log\frac{1}{\rho} \int_{1-\frac{p_{t1}}{p_{t,\mathrm{jet}}}}^{z_{\mathrm{cut}}} dz_1\, p_{gq}(z_1)$$

- Additional contributions from case $p_{t,\mathrm{jet}} > p_{t2}$, but $p_{t,\mathrm{mMDT}} < p_{t2}$

$$\rho \frac{d\sigma^{\mathrm{LL,NLO},C_F^2 b}}{d\rho} = \int_{p_{t2}}^{\frac{p_{t2}}{1-z_{\mathrm{cut}}}} dp_{t,\mathrm{jet}}\, \sigma_q(p_{t,\mathrm{jet}})\; R'_q\; \frac{\alpha_s C_F}{\pi} \log\frac{1}{\rho} \int_{1-\frac{p_{t2}}{p_{t,\mathrm{jet}}}}^{z_{\mathrm{cut}}} dz_1\, p_{gq}(z_1)$$

Comparison $p_{t,\text{jet}}$ vs. $p_{t,\text{mMDT}}$ – with LO matching





- Normalization is ill-defined due IRC unsafety
  $\rightarrow$ we present x-sections;
- **Sizable pure finite z_{cut} effects**
  $\rightarrow$ difference in $p_{t,\text{jet}}$ vs. $p_{t,\text{mMDT}}$ is purely due to finite-z_{cut}

Resummed results SD case

$$R_i(\rho) = \frac{C_i}{2\pi\alpha_s\beta_0^2}\left\{\left[W(1-\lambda_B) - \frac{W(1-\lambda_c)}{1+\beta} - 2W(1-\lambda_1) + \frac{2+\beta}{1+\beta}W(1-\lambda_2)\right]\right.$$
$$-\frac{\alpha_s K}{2\pi}\left[\log(1-\lambda_B) - \frac{\log(1-\lambda_c)}{1+\beta}\frac{2+\beta}{1+\beta}\log(1-\lambda_2) - 2\log(1-\lambda_1)\right]$$
$$\left.+\frac{\alpha_s\beta_1}{\beta_0}\left[V(1-\lambda_B) - \frac{V(1-\lambda_c)}{1+\beta} - 2V(1-\lambda_1) + \frac{2+\beta}{1+\beta}V(1-\lambda_2)\right]\right\}$$

$$\lambda_c = 2\alpha_s\beta_0\log(1/z_{\text{cut}}), \quad \lambda_\rho = 2\alpha_s\beta_0\log(1/\rho), \quad \lambda_B = 2\alpha_S\beta_0 B_i$$
$$\lambda_1 = \frac{\lambda_\rho + \lambda_B}{2}, \quad \lambda_2 = \frac{\lambda_c + (1+\beta)\lambda_\rho}{2+\beta},$$
$$W(x) = x\log(x), \quad V(x) = \frac{1}{2}\log^2(x) + \log(x)$$