Shower-friendly corrections for well-separated jets

(work with N. Fischer, see **EPJC 77 (2017) no. 9, 601**)

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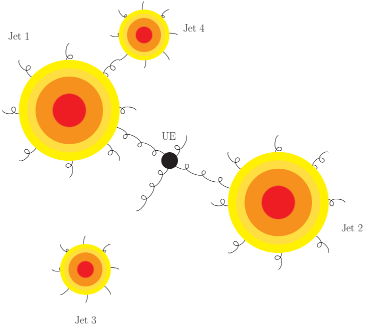
Stefan Prestel (Lund)

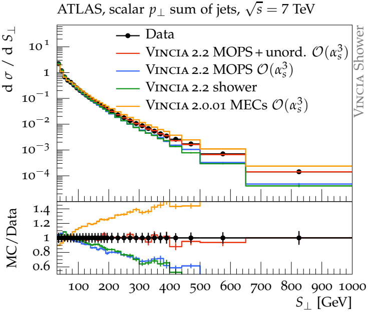
LHC measurements rely on detailed modelling of jets.



- Multijet production cross sections must be accurate
  $\Rightarrow$ Need fixed-order accurate jet rates
  $\Rightarrow$ Correct shower w/ fixed-order.
  But "jetty" LHC measurements are often far beyond the validity of the shower.
- Microscopic jet structure must be precise
  ...if we want to use jet substructure for precision measurements.
  $\Rightarrow$ Need fully differential understanding of all-order uncertainties.
- The transition should be smooth & with well-defined perturbative uncertainty.

For LHC experiments to accept new, improved showers, these need to be as good (or better) in describing hard well-separated jets.

Accurate description of background regions



Most tails necessary for searches/indirect measurements cannot be reached by an ordered PS – phase space too small!
PS reasoning in the tails potentially introduces large uncertainties from

- scale setting
- missing non-QCD evolution

...at LHC, scale setting tends to be a relevant issue.

Large non-shower region $\leftrightarrow$ large transition region ...where missing higher-order uncertainties mix with technical-parameter uncertainties :(

$\Rightarrow$ While current needs in searches are met by (NLO) merging schemes, develop unified method for both precision measurements + searches.

Let's look at the rate of the 1st emission from parton pair ab:

$$f_a(x_a, t(\tau')) \; f_b(x_b, t(\tau')) \int_{\tau}^{\tau'} \frac{d\bar{\tau}}{\bar{\tau}} \int_{x_i}^{1-\varepsilon} \frac{\mathrm{d}\zeta}{\zeta}$$

$$\Big[|\mathcal{M}_2(\Phi_2^a)|^2 \mathcal{J}_a \Pi_a(x_a, t_a(\bar{\tau}), t(\tau'))\, \Pi_b(x_b, t_a(\bar{\tau}), t(\tau'))\, \frac{\alpha_s(t_a)}{2\pi}\, P_{a'a}(z_a(\zeta))\, \frac{f_{a'}(x_a/z_a, t_a)}{f_a(x_a, t_a)}$$

$$+$$

$$|\mathcal{M}_2(\Phi_2^b)|^2 \mathcal{J}_b \Pi_a(x_a, t_b(\bar{\tau}), t(\tau'))\, \Pi_b(x_b, t_b(\bar{\tau}), t(\tau'))\, \frac{\alpha_s(t_b)}{2\pi}\, P_{b'b}(z_b(\zeta))\, \frac{f_{b'}(x_b/z_b, t_b)}{f_b(x_b, t_b)} \Big]$$

Now we can perform the shift

$$P_{i'i}(z_i) \to P_{i'i}(z_i) \frac{|\mathcal{M}_3(\Phi_3)|^2}{|\mathcal{M}_2(\Phi_2^a)|^2 P_{a'a}(z_a) + |\mathcal{M}_2(\Phi_2^b)|^2 P_{b'b}(z_b)}$$

$$\Rightarrow \qquad |\mathcal{M}_2(\Phi_2^a)|^2 P_{a'a}(z_a) + |\mathcal{M}_2(\Phi_2^b)|^2 P_{b'b}(z_b) = |\mathcal{M}_3(\Phi_3)|^2$$

Rate of 1st emission $\propto |\mathcal{M}_3|^2$; needs physical intermediate states.

Now let's look at it *as if we had precalculated* $|\mathcal{M}_3(\Phi_3)|^2$

$$
\begin{aligned}
& f_a(x_a, t(\tau'))\; f_b(x_b, t(\tau')) \int_\tau^{\tau'} \frac{d\bar\tau}{\bar\tau} \int_{x_i}^{1-\varepsilon} \frac{\mathrm{d}\zeta}{\zeta} \\
& |\mathcal{M}_3(\Phi_3)|^2 \Big[\mathcal{J}_a \Pi_a(x_a, t_a(\bar\tau), t(\tau'))\, \Pi_b(x_b, t_a(\bar\tau), t(\tau')) \frac{\alpha_s(t_a)}{2\pi} \frac{f_{a'}(x_a/z_a, t_a)}{f_a(x_a, t_a)} \\
& \frac{P_{a'a}(z_a(\zeta))|\mathcal{M}_2(\Phi_2^a)|^2}{|\mathcal{M}_2(\Phi_2^a)|^2 P_{a'a}(z_a) + |\mathcal{M}_2(\Phi_2^b)|^2 P_{b'b}(z_b)} \\
& + \\
& \mathcal{J}_b \Pi_a(x_a, t_b(\bar\tau), t(\tau'))\, \Pi_b(x_b, t_b(\bar\tau), t(\tau')) \frac{\alpha_s(t_b)}{2\pi} \frac{f_{b'}(x_b/z_b, t_b)}{f_b(x_b, t_b)} \\
& \frac{P_{b'b}(z_b(\zeta))|\mathcal{M}_2(\Phi_2^b)|^2}{|\mathcal{M}_2(\Phi_2^a)|^2 P_{a'a}(z_a) + |\mathcal{M}_2(\Phi_2^b)|^2 P_{b'b}(z_b)} \Big]
\end{aligned}
$$

$\Rightarrow$ Defines reweighting of x-section that recovers shower exactly.

$\Rightarrow$ Two histories contribute & need to be "mixed" in proportion.

$\Rightarrow$ Correct PS weight for an external phase-space point Φ_3 that was distributed according to $|\mathcal{M}_3(\Phi_3)|^2$:

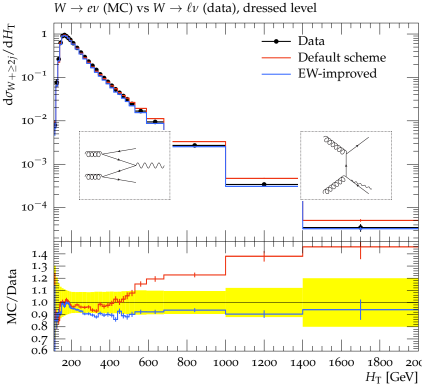
1. Calculate t_a, t_b, z_a, z_b and the probabilities $\mathcal{P}_a = P_{a'a}(z_a(\zeta))|\mathcal{M}_2(\Phi_2^a)|^2$ and $\mathcal{P}_b = P_{b'b}(z_b(\zeta))|\mathcal{M}_2(\Phi_2^b)|^2$
   The evaluation of $|\mathcal{M}_2|^2$ requires an explicit construction of Φ_2 with a clustering procedure that is an exact inverse of the shower splitting procedure.
2. Pick the "path" i with probability $\frac{\mathcal{P}_i}{\mathcal{P}_a+\mathcal{P}_b}$
3. Calculate the factors

$$\Pi_a(x_a, t_i, t')\,\Pi_b(x_b, t_i, t')\,\frac{\alpha_s}{2\pi}\,\frac{f_{i'}(x_i/z_i, t_i)}{f_i(x_i, t_i)}$$

   and multiply to the event weight.
   Knowlegde of Φ_2 allows using the PS to calculate $\Pi(x, t_i, t')$

Limitation 1

$\mathcal{P}$ have to be positive, which is not the case for sophisticated showers.
$\Rightarrow$ Fix: Pick according to $\frac{|\mathcal{P}_i|}{|\mathcal{P}_a|+|\mathcal{P}_b|}$ & apply weight $\frac{\mathcal{P}_i}{|\mathcal{P}_i|}\frac{|\mathcal{P}_a|+|\mathcal{P}_b|}{\mathcal{P}_a+\mathcal{P}_b}$



Limitation 2

Procedure only valid in the "PS shower phase space", i.e. regions fulfilling ordering constraints.
Improved (not removed) by larger phase-space coverage, e.g. from EW corrections, "inclusive clustering"

With this in mind, let's say we want

To not introduce new technical parameters
So that shower improvements in transition regions are not swamped by algorithmic uncertainties.

To describe emission with LO accuracy over complete phase space.
To have a fixed baseline accuracy (from which to improve) everywhere.

To treat events with no scale hierachies systematically.
So that their combination with the shower is straight-forward and smooth.

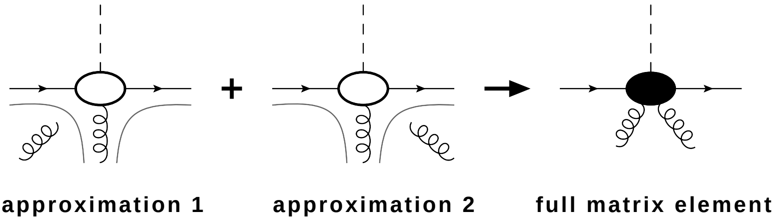
Of course,

we should never deteriorate the shower accuracy.

shower improvements should naturally be incorporated.

✓ Plain old ME corrections, provided "non-shower" events are treated carefully (benefit: PS cut-off only parameter)
✓ Unitarized MEPS; handling of "non-shower" evts not clear to me

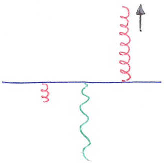
Matrix element corrections for the first emission



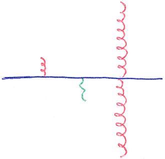
Sum up two paths, get full result, e.g. replacing the PS splitting kernels through

$$P_{i'i}(z_i) \to P_{i'i}(z_i)\frac{|\mathcal{M}_3(\Phi_3)|^2}{|\mathcal{M}_2(\Phi_2^a)|^2 P_{a'a}(z_a) + |\mathcal{M}_2(\Phi_2^b)|^2 P_{b'b}(z_b)}$$

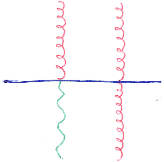
$$\Rightarrow \qquad |\mathcal{M}_2(\Phi_2^a)|^2 P_{a'a}(z_a) + |\mathcal{M}_2(\Phi_2^b)|^2 P_{b'b}(z_b) = |\mathcal{M}_3(\Phi_3)|^2$$



Showers require an ordering criterion to recover known anomalous dimensions given "standard" radiation functions.



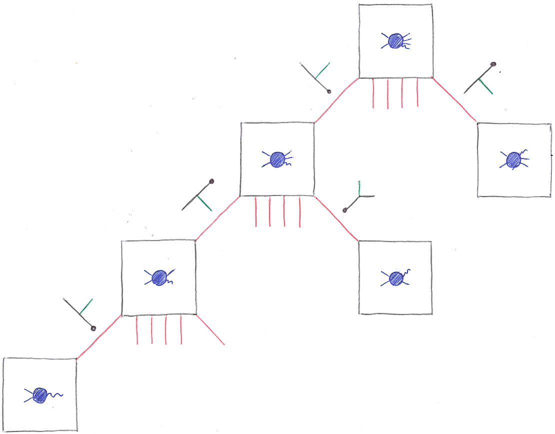
Showers require sensible starting/evolution conditions to recover (the arguments of) known logarithmic enhancements.



$\Rightarrow$ Can never fill complete phase space for an arbitrary (but fixed) multiplicity starting from an arbitrary (but fixed) lowest-multiplicity state.

However, the missing phase space can be filled by showering other non-shower states!

Successive ME corrections allow merging w/o a transition region
...but need to remember the mixing of shower & non-shower paths!



To describe jet rates in PS (ordered) phase space with LO accuracy, we need to knowing all ordered/unordered paths. Within this phase space, the Matrix-element corrections for Ordered Parton Showers (MOPS) are

$$P_n^i(\Phi_{n+1}/\Phi_n) \to P_n^i(\Phi_{n+1}/\Phi_n) \otimes \mathcal{R}_n(\Phi_{n+1})$$

where

$$\mathcal{R}_3 = \frac{\mathcal{M}_3}{\sum_k P_3^k \sum_j \Theta_{t_2^j > t_3^k} \mathcal{R}_2^j P_2^j \sum_i \Theta_{t_1^i > t_2^j} \mathcal{R}_1^i P_1^i \Theta_{t_{\text{fac}}^i > t_1^i} \mathcal{M}_0^i}$$

$$= \frac{\mathcal{M}_3}{\sum_k P_3^k \sum_j \Theta_{t_2^j > t_3^k} \frac{\mathcal{M}_2^k}{\sum_j P_2^j \sum_i \Theta_{t_1^i > t_2^j} \frac{\mathcal{M}_1^j}{\sum_i P_1^i \Theta_{t_{\text{fac}}^i > t_1^i} \mathcal{M}_0^i} P_1^i \Theta_{t_{\text{fac}}^i > t_1^i} \mathcal{M}_0^i} P_2^j \sum_i \cdots}$$

(shown for three emissions, general formula in paper)

The MOPS shower now needs to be complemented with the missing non-shower states, i.e. states that cannot be reached by any ordered sequence of shower splittings from an underlying state.

These cross sections can still be regularized by the PS cut-off – nothing else needed – and then precalculated & showered.

The treatment of non-shower contributions is crucial. We should have a sensible way to assess the perturbative uncertainty in these pieces!

$\Rightarrow$ Need an automatic, general and PS-friendly way to define scales for these configurations.

LO ME corrections ensure that the PS recovers the *coupling-stripped* LO MEs. Remembering the emission rate

$$f_a(x_a,t)\ f_b(x_b,t) \int \frac{d\bar\tau}{\bar\tau} \int \frac{\mathrm{d}\zeta}{\zeta}\, |\mathcal{M}_3(\Phi_3)|^2\, w_a\, \frac{\alpha_s(t_a)}{2\pi} \frac{P_{a'a}(z_a(\zeta))|\mathcal{M}_2(\Phi_2^a)|^2}{|\mathcal{M}_2(\Phi_2^a)|^2 P_{a'a}(z_a) + |\mathcal{M}_2(\Phi_2^b)|^2 P_{b'b}(z_b)} \\ + w_b\, \frac{\alpha_s(t_b)}{2\pi} \frac{P_{b'b}(z_b(\zeta))|\mathcal{M}_2(\Phi_2^b)|^2}{|\mathcal{M}_2(\Phi_2^a)|^2 P_{a'a}(z_a) + |\mathcal{M}_2(\Phi_2^b)|^2 P_{b'b}(z_b)} \Bigg]$$

and ignoring the all-order weights w_i, we see that the part $i = a, b$

$$\frac{P_{i'i}(z_i(\zeta))|\mathcal{M}_2(\Phi_2^i)|^2}{|\mathcal{M}_2(\Phi_2^a)|^2 P_{a'a}(z_a) + |\mathcal{M}_2(\Phi_2^b)|^2 P_{b'b}(z_b)} \quad \text{contributes with coupling } \alpha_s(t_i)$$

to the rate. The same result would be obtained by an "effective scale" choice

$$\alpha_s(t^{\text{eff}}) = \frac{\alpha_s(t_a)\, P_{a'a}(z_a(\zeta))|\mathcal{M}_2(\Phi_2^a)|^2 + \alpha_s(t_b)\, P_{b'b}(z_b(\zeta))|\mathcal{M}_2(\Phi_2^b)|^2}{|\mathcal{M}_2(\Phi_2^a)|^2 P_{a'a}(z_a) + |\mathcal{M}_2(\Phi_2^b)|^2 P_{b'b}(z_b)}$$

The effective scale defined by

$$\alpha_s(t^{\text{eff}}) = \frac{\alpha_s(t_a)\, P_{a'a}(z_a(\zeta))|\mathcal{M}_2(\Phi_2^a)|^2 + \alpha_s(t_b)\, P_{b'b}(z_b(\zeta))|\mathcal{M}_2(\Phi_2^b)|^2}{|\mathcal{M}_2(\Phi_2^a)|^2 P_{a'a}(z_a) + |\mathcal{M}_2(\Phi_2^b)|^2 P_{b'b}(z_b)}$$

has several good properties:

- $a)$ $t^{\text{eff}} \sim t_a$ if $P_{a'a} \gg P_{b'b}$ and similarly $t^{\text{eff}} \sim t_b$ if $P_{b'b} \gg P_{a'a}$, and smooth interpolation between the extremes
- $b)$ Incorporates all the dynamics of the splitting and all the dynamics of the underlying $\mathcal{M}$s
- $c)$ For no dominant "path", we find $t^{\text{eff}} \sim$ weighted average of t_i

$\Rightarrow$ Sensible process-independent scale-setting mechanism: Let the dynamics of $\mathcal{M}$s dictates the scale value.

Generalize to non-shower states: To assign (renormalization & PS starting) scales for non-PS states, we

- ◇ Require scale that captures maximal amount of dynamics.
- ◇ We directly let the ME dictate the preferred scale values:

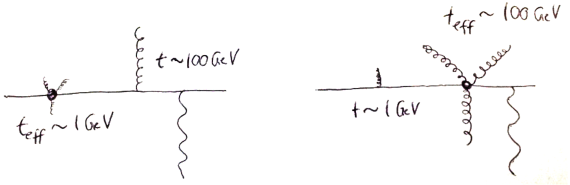
$$\alpha_s^{n+1}(t_{n+1}^{\mathsf{eff}}) = \frac{\sum\limits_i \alpha_s(t_{n+1}^i)\, P_{n+1}^i\, \alpha_s^n(t_n^{\mathsf{eff}\,i})\, \mathcal{M}_n^i}{\sum\limits_i P_{n+1}^i\, \mathcal{M}_n^i}$$

$\alpha_s(t\,^{\mathrm{eff}}_{\mathrm{B}+n})$ relies on all $\alpha_s(t\,^{\mathrm{eff}}_{\mathrm{B}+i<n})$. P_{n+1}^i are shower radiation functions, to smoothly map onto QCD evolution. $t\,^{\mathrm{eff}}$ can be extracted numerically by zero-finding.

Before we have a “physically” sensible calculation, note that

a) showered non-shower states overlap with non-shower states.

b) non-shower states can contain large scale hierarchies.



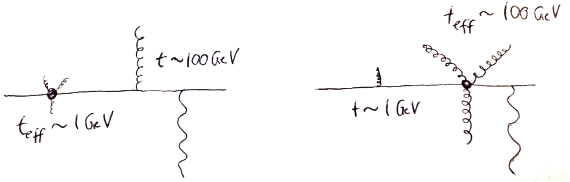
$\Rightarrow$ Need to remove overlap & introduce sensible suppression.

$a)$ showered non-shower states overlap with non-shower states.

Usually classified with “above or below merging scale”.
$\Rightarrow$ Cannot do this w/o introducing transition region!

Thus, define “non-shower” by effective scale and shower action:

- $+0$ particle states showered from t_{fac}
- $+1$ particle non-shower states defined by “all $t_{1,i} > t_{\text{fac}}$”
  and showered from probabilistically chosen $t_1 \in \{t_{1,i}\}$
- $+2$ particle non-shower states defined by “all $t_{2,i} > t_{1,i}$”
  and showered from calculated $t_{2,\text{eff}}$
- $+$n particle non-shower states defined by “no ordered paths
  && no ordered emission sequence $t_{k,\text{eff}} > t_{k+1} > \cdots > t_n$
  && no ordered sequence $t_{n-1,\text{eff}} > t_n$ if $\mathcal{S}_{n-1}$ is non-shower”
  and showered from probabilistically chosen $t_n \in \{t_{n,i}\}$ if $t_{n-1,\text{eff}} > t_n$
  and from $t_{n,\text{eff}}$ otherwise



b) non-shower states can contain large scale hierarchies.

We can now classify “hierachical events” by

$$t_{\mathsf{fac}} \gg t_{k,\mathsf{eff}}$$
$$t_{\mathsf{k,eff}} \gg t_n$$

which will induce Sudakovs $\Delta_0(t_{\mathsf{fac}}, t_{k,\mathsf{eff}})$ and/or $\Delta_k(t_{k,\mathsf{eff}}, t_n)$

- Parton-shower splittings are successively corrected to the LO matrix element for up to n emissions

  ...need to know unordered contributions to an ordered sequence; matrix element C++ code from e.g. MG5
- Non-shower corrections are defined as "not reachable by any shower sequence"

  Non-shower phase-space points precalculated (MG5+MadEvent)

Non-shower corrections require

- Definition of sensible, generalizable "effective scale"

  directly use full dynamics of LO matrix elements; maps smoothly onto scale setting in matrix element-corrected shower.
- Overlap removal & suppress residual hierachies by employing t_{eff}

The method contains the usual shower uncertainties:

- ◇ Parton-shower cut-off, scales in the shower
- ◇ Non-perturbative event generator modelling

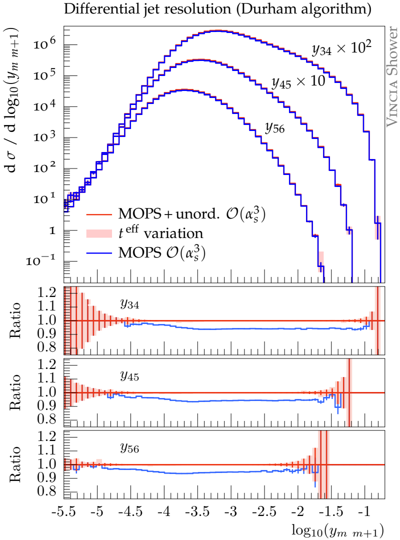
...just like in any other (ME+) shower prediction.

Only other uncertainty of the method: t_{eff} value, i.e. the *fixed-order* scale uncertainty of non-shower states.

$\Rightarrow$ Perturbative + perturbatively improvable.

$\Rightarrow$ Vary t_{eff} like any other perturbative parameter.

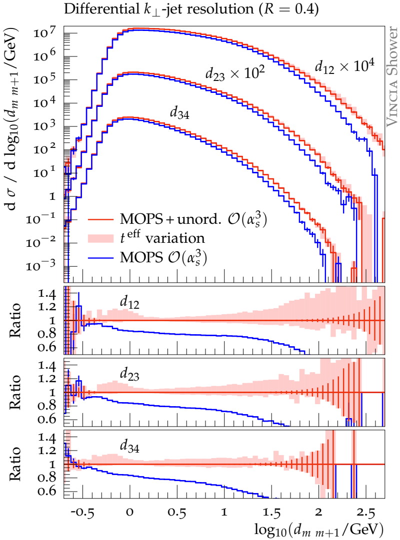
Vincia MOPS validation at LEP



VINCIA predictions for jet resolution measures $y_{m\ m+1}$ (Durham) for lepton collisions. ME corrections are applied for $\leq$ 3 emissions. The red band is obtained by varying the effective scale t^{eff} [GeV] in non-shower events by factors of two.

◇ MOPS approaches regular shower at small $y_{m\ m+1}$.
◇ Overall moderate, roughly constant effect from non-shower states.
◇ t^{eff} well-determined, thus uncertainty small.

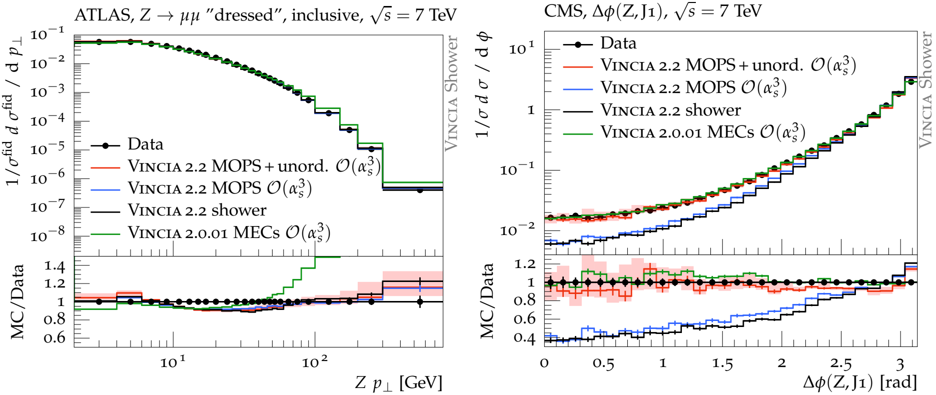
Vincia MOPS validation at LHC



VINCIA predictions for jet resolution measures $d_{m\,m+1}$ (longitudinally invariant $k_\perp$ jet algorithm with $R = 0.4$) for hadron collisions. ME corrections are applied for $\leq$ 3 emissions. The red band is obtained by varying the effective scale $t^{\,\text{eff}}$ [GeV] in non-shower events by factors of two.

◇ MOPS approaches regular shower at small $d_{m\,m+1}$.
◇ Effect from non-shower states significant.
◇ $t^{\,\text{eff}}$ less well-determined, thus uncertainty larger.
◇ $t^{\,\text{eff}}$ uncertainty at small $d_{m\,m+1}$ from competition of large α_s and Sudakov suppression.

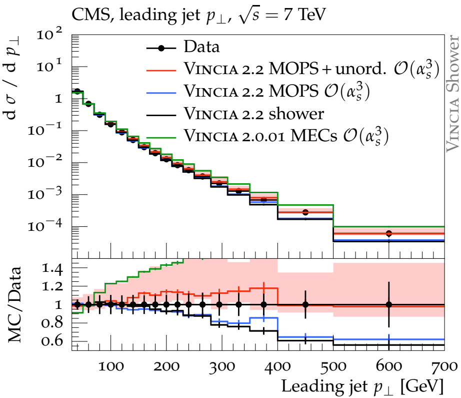
Vincia MOPS data comparisons at LHC

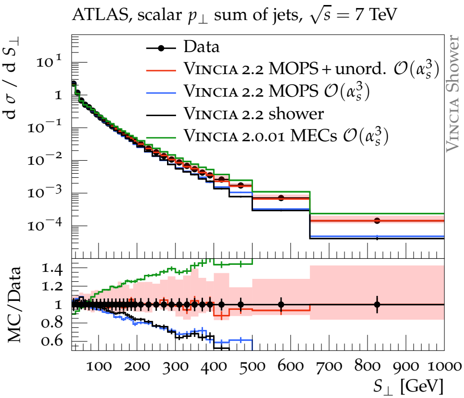


VINCIA 2.2 and VINCIA 2.0.01 predictions compared to ATLAS and CMS data. ME corrections are applied for ≤ 3 emissions. The red band is obtained by varying the effective scale in non-shower events by factors of two.

$\Rightarrow t^{\,\mathrm{eff}}$ relatively well-determined - observables dominated by shower states.

Vincia MOPS data comparisons at LHC





VINCIA 2.2 and VINCIA 2.0.01 predictions compared to ATLAS and CMS data. Predictions are rescaled to the experimental inclusive one-jet cross section. ME corrections are applied for ≤ 3 emissions. The red band is obtained by varying the effective scale in non-shower events by factors of two.

$\Rightarrow t^{\text{eff}}$ uncertainty large - observables dominated by non-shower states.

To extend the method to NLO, we need to

... include NLO clusterings in the construction of histories

... revise the definition of non-shower to include NLO paths

... have access to virtual MEs within the PS

... use NLO calculations in non-shower regions

Although this is a lot of work, it does not seem impossible if we have a sensible NLO shower.

Somewhat less satisfactory would be to rescale the zero-jet sample with $\overline{\mathrm{B}}_0/\mathrm{B}_0$ and all non-shower rates with $\overline{\mathrm{B}}_n/\mathrm{B}_n$ k-factors

- ◇ QCD calculations should describe soft/collinear partons, well-separated partons & anything in-between.
- ◇ Any calculation in the perturbative region should be assessed with well-defined perturbative (scale) uncertainties.
- ◇ We shouldn't deteriorate the PS upon including well-separated partons, and e.g. shun artificial "transistion regions".
- ✓ ME corrections for ordered PS are a useful blueprint.
- ✓ Allows to naturally supplement non-shower states w/o introducing technical parameters or "boolean" scale choices. Hopefully realistic uncertainties due to pert. scale defintion.
- ✓ One main ingredient is the definition of an "effective scale" based directly on perturbative information from LO MEs.
- $\Rightarrow$ Results at LO are quite encouraging.

COHERENT PARTON SHOWERS VERSUS MATRIX ELEMENTS – IMPLICATIONS OF PETRA/PEP DATA

Mats BENGTSSON and Torbjorn SJOSTRAND

Department of Theoretical Physics, University of Lund, Solvegatan 14A, S-223 62 Lund, Sweden

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In summary, we believe the time has come to put more emphasis on parton shower programs in the study of e^+e^- phenomenology There may be instances when the matrix element approach is the only valid one, e g for $\Lambda_{\overline{MS}}$ determinations, but the amount of "medium soft" gluon emission is probably underestimated in this approach, a

Thanks for your time!

We can use the full ME as splitting probability, but only bookkeep leading-color states.
We can distribute the full ME over leading-color configurations according to

$$|Mn|^2 = C^{MG}_{ii}|\mathcal{J}^{(i)}_n|^2 \to C^{MG}_{ii}|\mathcal{J}^{(i)}_n|^2 \frac{\sum_{j,k} C^{MG}_{jk}\mathcal{J}^{(j)}_n\mathcal{J}^{(k)*}_n}{\sum_j C^{MG}_{jj}|\mathcal{J}^{(j)}_n|^2}$$

where $\mathcal{J}^{(i)}_n$ are the entries in the color matrix supplied by MG5.

Thus, the matrix element for each colour structure gets a correction from the subleading colour part of the full matrix element that is proportional to the relative weight of that colour structure.
The sum over all colour flows reproduces the full colour-summed matrix element norm squared.

Note: MEC can again flip sign!

Use two step weighted Sudakov algorithm. Use weighted algorithm for accepting t, retain highest of all t, then use weighted algorithm again to accept state change according to **MEC**(P).

Histories: Choose with **MEC**$(|P|)$, retain analytical correction weight **MEC**(P)/**MEC**$(|P|)$

Effective scale: Use full P, but ensure to have reasonably large scale range to allow solving for effective scale with Newton solver.

ME corrections do not act on α_s values or their variations. Just make sure to retain all weights.

Same for enhancements: Make sure not to remove analytic correction factor for first weighed algorithm in the second weighting!