Flavoured Axion Phenomenology

Fredrik Björkeroth[1]

in collaboration with:

Eung Jin Chun[2], Stephen F. King[3]

[1] INFN-LNF, [2] KIAS, [3] University of Southampton

Flavour and Dark Matter 2018, Karlsruhe, 24.09.18

Outline

This talk will cover

1. Theory
2. Phenomenology: hadrons
3. Phenomenology: leptons

This talk will cover

1. Theory
2. Phenomenology: hadrons
3. Phenomenology: leptons

Based on work in

PLB 777 (2018) 428-434 [1711.05741 [hep-ph]]
JHEP 1808 (2018) 117 [1806.00660 [hep-ph]]

In more detail:

1. Theory
   - Flavoured Peccei-Quinn symmetry
   - Axion couplings to the SM
   - A to Z with Pati-Salam: a UV-complete model

In more detail:

1. Theory
   - Flavoured Peccei-Quinn symmetry
   - Axion couplings to the SM
   - A to Z with Pati-Salam: a UV-complete model
2. Phenomenology: hadrons
   - Two-body meson (K, D, B) decays
   - Axion-meson mixing

Outline

In more detail:

1. Theory
   - Flavoured Peccei-Quinn symmetry
   - Axion couplings to the SM
   - A to Z with Pati-Salam: a UV-complete model
2. Phenomenology: hadrons
   - Two-body meson (K, D, B) decays
   - Axion-meson mixing
3. Phenomenology: leptons
   - Two-body decays $\ell_1 \to \ell_2 a$
   - Radiative decays $\ell_1 \to \ell_2 a \gamma$
   - $\mu \to 3e$
   - $\mu - e$ conversion

Outline

1. Theory
   - Flavoured Peccei-Quinn symmetry
   - Axion couplings to the SM
   - A to Z with Pati-Salam: a UV-complete model

The strong *CP* problem is of almost no consequence

[paraphrasing Michael Dine, talk 2015]

The strong CP problem is of almost no consequence

[paraphrasing Michael Dine, talk 2015]

Allowed term in QCD

$$\mathcal{L} \supset \bar{\theta} \frac{g^2}{32\pi^2} G_{\mu\nu} \tilde{G}^{\mu\nu}, \quad \bar{\theta} = \theta_{\rm QCD} + \arg \det \, M^u M^d$$

Values

- Measurement: neutron EDM [Pendlebury et al '15]

$$\bar{\theta} \lesssim 10^{-10}$$

- Naively: $\bar{\theta} \sim 1$
- Anthropically: $\bar{\theta} \sim 10^{-3}$ is fine [Dine, Draper '15]
- Exact strong CP ($\bar{\theta} = 0$) not necessary

The PQ solution

Ingredients in a standard PQ solution

- Global $U(1)_{PQ}$ symmetry with chiral anomaly
- Complex scalar field $\varphi \to \langle \varphi \rangle$ which breaks $U(1)_{PQ}$

Ingredients in a standard PQ solution

- Global $U(1)_{PQ}$ symmetry with chiral anomaly
- Complex scalar field $\varphi \to \langle\varphi\rangle$ which breaks $U(1)_{PQ}$

Archetypal "invisible axion" models

KSVZ

$$\mathcal{L} \supset \lambda\varphi\overline{Q}Q$$

- Add: heavy quarks Q
- Axion-ψ_{SM} coupling: loop level

DFSZ

$$\mathcal{L} \supset \lambda\varphi^2 H_u H_d$$

- Add: second Higgs doublet
- Axion-ψ_{SM} coupling: tree level

Non-standard PQ symmetry

$U(1)_{PQ}$ does not need to be put in by hand!

$\rightarrow$ *accidental* PQ symmetry

- From $\mathbb{Z}_N$ [Babu, Gogoladze, Wang '03, Dias, Pleitez, Tonasse '02, '04]
- In SUSY [Chun, Lukas '92]
- From gauge symmetry [Di Luzio, Nardi, Ubaldi '16]

$U(1)_{PQ}$ does not need to be put in by hand!
$\rightarrow$ *accidental* PQ symmetry

- From $\mathbb{Z}_N$ [Babu, Gogoladze, Wang '03, Dias, Pleitez, Tonasse '02, '04]
- In SUSY [Chun, Lukas '92]
- From gauge symmetry [Di Luzio, Nardi, Ubaldi '16]

It may arise from discrete flavour symmetries [FB, Chun, King '17]

- $U(1)_{PQ}$ is now connected to Yukawa structures
  $\rightarrow$ the axion is *flavoured*

$U(1)_{PQ}$ does not need to be put in by hand!
$\rightarrow$ *accidental* PQ symmetry

- From $\mathbb{Z}_N$ [Babu, Gogoladze, Wang '03, Dias, Pleitez, Tonasse '02, '04]
- In SUSY [Chun, Lukas '92]
- From gauge symmetry [Di Luzio, Nardi, Ubaldi '16]

It may arise from discrete flavour symmetries [FB, Chun, King '17]

- $U(1)_{PQ}$ is now connected to Yukawa structures
  $\rightarrow$ the axion is *flavoured*

Other recent developments in flavoured axions

- Flaxion [Ema, Hamaguchi, Moroi, Nakayama '16]
- Axiflavon [Calibbi, Goertz, Redigolo, Ziegler, Zupan '16]

A low scales, the relevant Lagrangian is

$$\mathcal{L} = \mathcal{L}_{\text{kin}} + \mathcal{L}_m + \mathcal{L}_\partial + \mathcal{L}_{\text{anomaly}}$$

where

$$\mathcal{L}_{\text{kin}} + \mathcal{L}_m = \frac{1}{2}(\partial_\mu a)^2 - \frac{1}{2} m_a^2 a^2 + \sum_{f=u,d,e} \bar{f}_i (\not{\partial} - m_i) f_i,$$

$$\mathcal{L}_\partial = -\frac{\partial_\mu a}{v_{PQ}} \sum_{f=u,d,e} \bar{f}_i \gamma^\mu (V_{ij}^f - A_{ij}^f \gamma_5) f_j,$$

$$\mathcal{L}_{\text{anomaly}} = \frac{\alpha_s}{8\pi} \frac{a}{f_a} G_{\mu\nu}^a \tilde{G}^{a\mu\nu} + c_{a\gamma} \frac{\alpha}{8\pi} \frac{a}{f_a} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

Focus on axion couplings to fermions

$$\mathcal{L}_\partial = -\frac{\partial_\mu a}{v_{PQ}} \sum_{f=u,d,e} \bar{f}_i \gamma^\mu (V^f_{ij} - A^f_{ij}\gamma_5) f_j,$$

where

$$V^f = \frac{1}{2}\left(U^\dagger_{Lf} x_{f_L} U_{Lf} + U^\dagger_{Rf} x_{f_R} U_{Rf}\right)$$
$$A^f = \frac{1}{2}\left(U^\dagger_{Lf} x_{f_L} U_{Lf} - U^\dagger_{Rf} x_{f_R} U_{Rf}\right)$$

- $x_{f_L} = \mathrm{diag}(x_{f_{L1}}, x_{f_{L2}}, x_{f_{L3}})$, $x_{f_R} = \mathrm{diag}(x_{f_{R1}}, x_{f_{R2}}, x_{f_{R3}})$
- U_{Lf} and U_{Rf} are unitary matrices: $Y^f_{\mathrm{diag}} = U^\dagger_{Lf} Y^f U_{Rf}$
- $V_{\mathrm{CKM}} = U^\dagger_{Lu} U_{Ld}$

$$V^f = \frac{1}{2}\left(U_{Lf}^\dagger x_{f_L} U_{Lf} + U_{Rf}^\dagger x_{f_R} U_{Rf}\right)$$
$$A^f = \frac{1}{2}\left(U_{Lf}^\dagger x_{f_L} U_{Lf} - U_{Rf}^\dagger x_{f_R} U_{Rf}\right)$$

Special cases

1. All generations couple equally: $x_{f_L}, x_{f_R} \propto I_3$

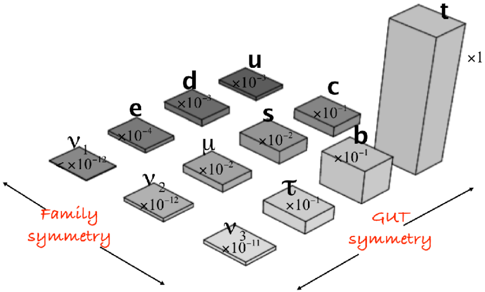
$$\begin{array}{rcl} V^f & = & \frac{1}{2}(x_{f_L} + x_{f_R})\mathbb{I}_3 \\ A^f & = & \frac{1}{2}(x_{f_L} - x_{f_R})\mathbb{I}_3 \end{array} \Rightarrow \text{no flavour violation!}$$

   In standard DFSZ it's always possible to arrange $x_{f_L} = -x_{f_R} \Rightarrow V^f = 0$

2. Anomaly-free: $x_{f_L} = x_{f_R}$
   $\rightarrow$ no chiral anomaly ($N_{DW} = 0$) $\rightarrow$ no PQ solution!

A unified model of flavour: "A to Z" [King '14, King, Di Bari '15]

- Pati-Salam gauge group $SU(4)_C \times SU(2)_L \times SU(2)_R$
- A_4 family symmetry
- $\mathbb{Z}_N$ family/shaping symmetries



[King, 1301.1340]

Can accommodate all quarks/leptons, masses + mixings

Pati-Salam $[SU(4)_C \times SU(2)_L \times SU(2)_R]$

- Left-handed fermions in

$$F_i \sim (4,2,1)_i = \begin{pmatrix} u_r & u_g & u_b & \nu \\ d_r & d_g & d_b & e \end{pmatrix}_i$$

- Right-handed fermions in

$$F_i^c \sim (\bar{4},1,2)_i = \begin{pmatrix} u_r^c & u_g^c & u_b^c & N^c \\ d_r^c & d_g^c & d_b^c & e^c \end{pmatrix}_i$$

$\mathbf{A}_4$

- Left-handed fermions in triplet

$$F \sim 3 = (F_1, F_2, F_3)$$

- Right-handed fermions in singlets

$$F_1^c,\ F_2^c,\ F_3^c \sim 1$$

Features

- Matter couples to ϕ_i: A_4 triplets + gauge singlets
- Yukawa couplings become dynamical:

$$y_{ij} F F^c H \to \frac{\langle \phi_i \rangle}{\Lambda} F_i F_j^c H$$

- $\mathcal{L}$ has accidental $U(1)_{PQ}$ with generation-dependent PQ charges

Field	PS	A_4	$U(1)_{PQ}$
F	$(4, 2, 1)$	3	0
$F^c_{1,2,3}$	$(\bar{4}, 1, 2)$	1	2,1,0
ϕ_i^f	1	3	...

- Physical axion = linear combination of ϕ_i^f phase fields

Predictions

$f_a \gtrsim 10^{12}$ GeV is close to cosmological upper bound

Dark matter axion? Mass: $m_a \sim 1 - 10\mu\text{eV}$

Axion couplings to matter predicted by Yukawa structures

Flavour violation via axion interactions

2. Phenomenology: hadrons
   - Two-body meson (K, D, B) decays
   - Axion-meson mixing
   - Axion-pion mixing

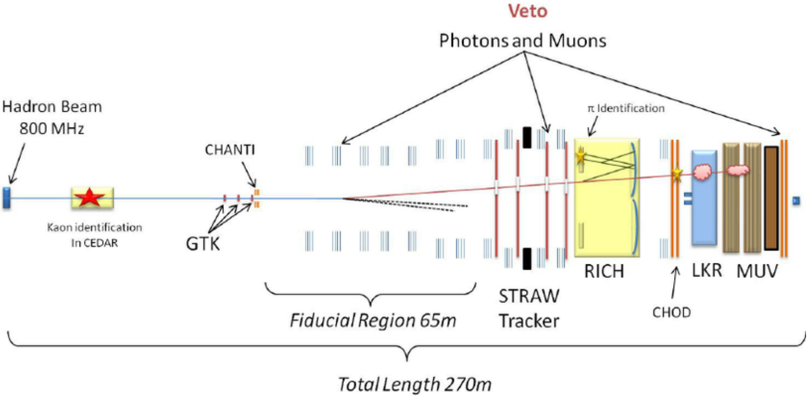
Decay: $P \to P' a$, where $P = (\bar{q}_P q')$, $P' = (\bar{q}_{P'} q')$.
Branching ratio

$$\mathrm{Br}(P \to P' a) = \frac{1}{16\pi\Gamma(P)} \frac{\left|V^f_{q_P q_{P'}}\right|^2}{v_{PQ}^2} m_P^3 \left(1 - \frac{m_{P'}^2}{m_P^2}\right)^3 |f_+(0)|^2 \,,$$

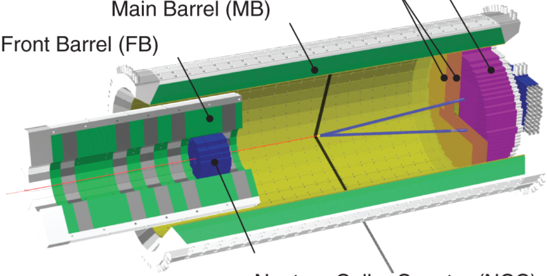
- $f_+(0)$ is a hadronic form factor
- Only unknown quantity is the ratio $|V^f|/v_{PQ}$
- Example: $K^+ \to \pi^+ a$ decay proceeds by $\bar{s} \to \bar{d} a$ with coupling strength $V^d_{sd} \equiv V^d_{21}$

Decay	$f_+(0)$
$K \to \pi$	1
$D \to \pi$	0.74(6)(4)
$D \to K$	0.78(5)(4)
$D_s \to K$	0.68(4)(3)
$B \to \pi$	0.27(7)(5)
$B \to K$	0.32(6)(6)
$B_s \to K$	0.23(5)(4)

- NA62 @ CERN SPS: $K^+ \to \pi^+ a$ ($K^+ \to \pi^+ \nu\bar{\nu}$)
  - Current status: one $\nu\bar{\nu}$ event! [R. Marchevski at Moriond '18]
  - Reach: $\mathrm{Br} \lesssim 1 \times 10^{-12}$ (by end of Run 3) $\to$ most promising probe!

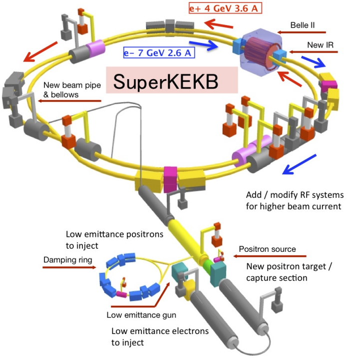


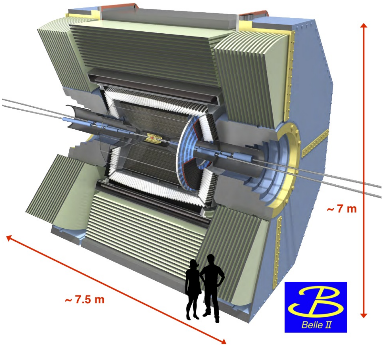
- KOTO @ J-PARC: $K_L^0 \to \pi^0 a$
  - Current status: taking data
  - Weak limit on $K_L^0 \to \pi^0 \nu\bar{\nu}$ from 2013 pilot dataset
  - Reach: $\mathrm{Br} < \mathcal{O}(10^{-8})$



- KLEVER @ CERN SPS: $K_L^0 \to \pi^0 a$
  - Current status: proposed (early stages) [Moulson 16']
  - Installed during LS3, taking data in 2026.
  - Reuses some of NA62 infrastructure; liquid Kr calorimeter

- Belle(-II): B and B_s decays
  - Current status: $\mathrm{Br} < \mathcal{O}(10^{-5})$ for $B^\pm \to K^\pm \nu\bar{\nu}$
  - Reach: possibly full OoM improvement with Belle-II





- What about D decays?

Define constant $\tilde{c}_{P\to P'}$, such that

$$\mathrm{Br}(P \to P' a) = \tilde{c}_{P\to P'} \left| V^f_{q_P q_{P'}} \right|^2 \left(\frac{10^{12}\ \mathrm{GeV}}{v_{PQ}} \right)^2$$

where

$$\tilde{c}_{P\to P'} = \frac{1}{16\pi\,\Gamma(P)} \frac{m_P^3}{(10^{12}\ \mathrm{GeV})^2} \left(1 - \frac{m_{P'}^2}{m_P^2} \right)^3 \left| f_+(0) \right|^2$$

Limits

Decay	Branching ratio	Experiment	$\tilde{c}_{P\to P'}$	v_{PQ}/GeV
$K^+ \to \pi^+ a$	$< 0.73 \times 10^{-10}$	E949 + E787	3.51×10^{-11}	$> 6.9 \times 10^{11} \lvert V_{21}^d \rvert$
	$< 0.01 \times 10^{-10}*$	NA62 (future)		$> 5.9 \times 10^{12} \lvert V_{21}^d \rvert$
	$< 1.2 \times 10^{-10}$	E949 + E787		
	$< 0.59 \times 10^{-10}$	E787		
$K_L^0 \to \pi^0 a$	$< 5 \times 10^{-8}$	KOTO	3.67×10^{-11}	$> 2.7 \times 10^{10} \lvert V_{21}^d \rvert$
$(K_L^0 \to \pi^0 \nu\bar{\nu})$	$(< 2.6 \times 10^{-8})$	E391a		
$B^\pm \to \pi^\pm a$	$< 4.9 \times 10^{-5}$	CLEO	5.30×10^{-13}	$> 1.0 \times 10^{8} \lvert V_{31}^d \rvert$
$(B^\pm \to \pi^\pm \nu\bar{\nu})$	$(< 1.0 \times 10^{-4})$	BaBar		
	$(< 1.4 \times 10^{-4})$	Belle		
$B^\pm \to K^\pm a$	$< 4.9 \times 10^{-5}$	CLEO	7.26×10^{-13}	$> 1.2 \times 10^{8} \lvert V_{32}^d \rvert$
$(B^\pm \to K^\pm \nu\bar{\nu})$	$(< 1.3 \times 10^{-5})$	BaBar		
	$(< 1.9 \times 10^{-5})$	Belle		
	$(< 1.5 \times 10^{-6})*$	Belle-II (future)		
$B^0 \to \pi^0 a$			4.92×10^{-13}	
$(B^0 \to \pi^0 \nu\bar{\nu})$	$(< 0.9 \times 10^{-5})$	Belle		$\gtrsim 2.3 \times 10^{8} \lvert V_{31}^d \rvert$
$B^0 \to K^0_{(S)} a$	$< 5.3 \times 10^{-5}$	CLEO	6.74×10^{-13}	$> 1.1 \times 10^{8} \lvert V_{32}^d \rvert$
$(B^0 \to K^0 \nu\bar{\nu})$	$(< 1.3 \times 10^{-5})$	Belle		
$D^\pm \to \pi^\pm a$	< 1		1.11×10^{-13}	$> 3.3 \times 10^{5} \lvert V_{21}^u \rvert$
$D^0 \to \pi^0 a$	< 1		4.33×10^{-14}	$> 2.1 \times 10^{5} \lvert V_{21}^u \rvert$
$D_s^\pm \to K^\pm a$	< 1		4.38×10^{-14}	$> 2.1 \times 10^{5} \lvert V_{21}^u \rvert$
$B_s^0 \to \overline{K}^0 a$	< 1		3.64×10^{-13}	$> 6.0 \times 10^{5} \lvert V_{31}^d \rvert$

Let us rotate away the anomaly term by

$$q \to e^{i\frac{\beta_q}{2}\frac{a}{f_a}\gamma_5} q, \qquad \beta_q = \frac{m_*}{m_q},$$

where $q = u, d, s$ and $m_*^{-1} = m_u^{-1} + m_d^{-1} + m_s^{-1}$. The axion-quark Lagrangian transforms as

$$\mathcal{L}_\partial \to \mathcal{L}'_\partial \supset -\frac{\partial_\mu a}{v_{PQ}} \left[\sum_{q=u,d,s} c_q \bar{q}\gamma^\mu\gamma_5 q + c_{sd}\bar{s}\gamma^\mu\gamma_5 d + c_{sd}^* \bar{d}\gamma^\mu\gamma_5 s \right],$$

where

$$c_u = A_{11}^u + N_{DW}\beta_u/2,$$
$$c_d = A_{11}^d + N_{DW}\beta_d/2,$$
$$c_s = A_{22}^d + N_{DW}\beta_s/2,$$
$$c_{sd} = A_{21}^d.$$

We can write this as kinetic mixing between axions and mesons:

$$\mathcal{L}_{aP}^{\text{eff}} = -\sum_P c_P \frac{f_P}{v_{PQ}} \partial_\mu a \partial^\mu P,$$

with

$$c_{\pi^0} = c_u - c_d, \qquad c_\eta = c_u + c_d - 2c_s$$

$$c_{\eta'} = c_u + c_d + c_s, \qquad c_{K^0} = c_{sd} = c^*_{\bar{K}^0}$$

Diagonalising the kinetic mixing,

$$a \to \frac{a}{\sqrt{1 - \sum_P \eta_P^2}}, \qquad P \to P + \frac{\eta_P a}{\sqrt{1 - \sum_P \eta_P^2}}$$

where

$$\eta_P \equiv \frac{c_P f_P}{v_{PQ}}$$

Meson mass difference

Meson mass splitting

$$(\Delta m_P)_{\text{axion}} \simeq |\eta_P|^2 m_P = |c_P|^2 \frac{f_{P^0}^2}{v_{PQ}^2} m_P.$$

System	$(\Delta m_P)_{\text{exp}}/\text{MeV}$	v_{PQ}/GeV
$K^0 - \overline{K}^0$	$(3.484 \pm 0.006) \times 10^{-12}$	$\gtrsim 2 \times 10^6 \lvert c_{K^0}\rvert$
$D^0 - \overline{D}^0$	$(6.25\,^{+2.70}_{-2.90}) \times 10^{-12}$	$\gtrsim 4 \times 10^6 \lvert c_{D^0}\rvert$
$B^0 - \overline{B}^0$	$(3.333 \pm 0.013) \times 10^{-10}$	$\gtrsim 8 \times 10^5 \lvert c_{B^0}\rvert$
$B_s^0 - \overline{B}_s^0$	$(1.1688 \pm 0.0014) \times 10^{-8}$	$\gtrsim 1 \times 10^5 \lvert c_{B_s^0}\rvert$

PDG [Patrignani et al '16]

Notes

- Assume central SM value
- Uncertainty dominated by theory; require $(\Delta m_P)_{\text{axion}} \lesssim (\Delta m_P)_{\text{exp}}$
- Possible improvements to $(\Delta m_K)_{\text{th}}$ from lattice soon [Bai, Christ, Sachrajda '18]

3. Phenomenology: leptons
    - Two-body decays $\ell_1 \to \ell_2 a$
    - Radiative decays $\ell_1 \to \ell_2 a \gamma$
    - $\mu \to 3e$
    - $\mu - e$ conversion

We define a total coupling

$$\left|C^e_{\ell_1\ell_2}\right|^2 = \left|V^e_{\ell_1\ell_2}\right|^2 + \left|A^e_{\ell_1\ell_2}\right|^2$$

Analogously to mesons, we have

$$\mathrm{Br}(\ell_1 \to \ell_2 a) = \tilde{c}_{\ell_1\to\ell_2} \left|C^e_{\ell_1\ell_2}\right|^2 \left(\frac{10^{12}\ \mathrm{GeV}}{v_{PQ}}\right)^2$$

where

$$\tilde{c}_{\ell_1\to\ell_2} = \frac{1}{16\pi\,\Gamma(\ell_1)} \frac{m^3_{\ell_1}}{(10^{12}\ \mathrm{GeV})^2} \left(1 - \frac{m^2_{\ell_2}}{m^2_{\ell_1}}\right)^3$$

We may also probe the angular distribution. For muons,

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}\cos\theta} \simeq \frac{|C^e_{21}|^2}{32\pi}\frac{m_\mu^3}{v_{PQ}^2}(1 - AP_\mu \cos\theta)$$

where

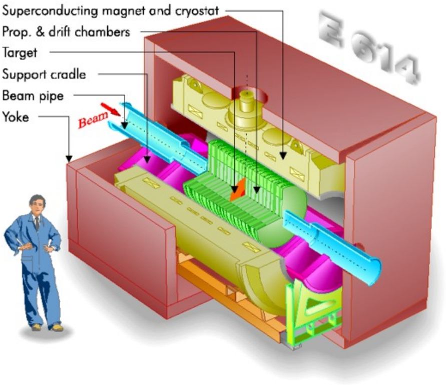
$$A = -\frac{2\mathrm{Re}[A^e_{21}(V^e_{21})^*]}{|C^e_{21}|^2}$$

Notes

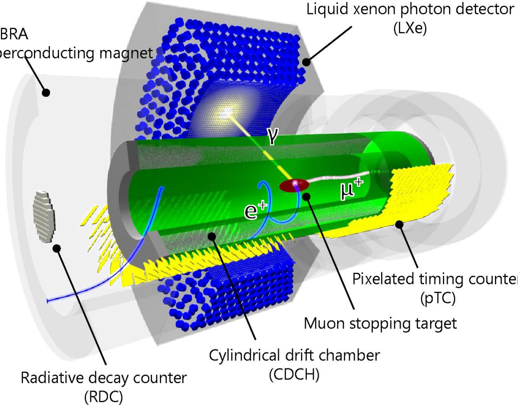
- Standard Model weak interactions are 'V-A' $\Leftrightarrow A = -1$
- Isotropic decays ($A = 0$) for $A^e_{21} = 0$ or $V^e_{21} = 0$.
- Strongest signal for 'V+A' (RH) interactions

- Jodidio *et al* @ TRIUMF [Jodidio *et al* '86]
  - Stopped μ^+ on metal foil
  - Assume isotropic decays ($A = 0$)

- TWIST @ TRIUMF [Bayes *et al* '14]
  - Sensitive to anisotropies
  - Limits for $A = 0$ not as good as TRIUMF



- MEG(-II) @ PSI
  - Searching for $\mu \to e\gamma$ in stopped μ^+
  - Status: MEG completed, MEG-II under construction
  - Reach: TBD



Decay	Branching ratio	Experiment	$\tilde{c}_{\ell_1 \to \ell_2}$	v_{PQ}/GeV
$\mu^+ \to e^+ a$	$< 2.6 \times 10^{-6}$	$(A = 0)$ Jodidio *et al*	7.82×10^{-11}	$> 5.5 \times 10^9 \lvert V^e_{21} \rvert$
	$< 2.1 \times 10^{-5}$	$(A = 0)$ TWIST		$> 1.9 \times 10^9 \lvert C^e_{21} \rvert$
	$< 1.0 \times 10^{-5}$	$(A = 1)$ TWIST		$> 2.8 \times 10^9 \lvert C^e_{21} \rvert$
	$< 5.8 \times 10^{-5}$	$(A = -1)$ TWIST		$> 1.2 \times 10^9 \lvert C^e_{21} \rvert$
	$\lesssim 5 \times 10^{-9} *$	Mu3e (future)		$\gtrsim 1 \times 10^{11} \lvert C^e_{21} \rvert$
$\tau^+ \to e^+ a$	$< 1.5 \times 10^{-2}$	ARGUS	4.92×10^{-14}	$> 1.8 \times 10^6 \lvert C^e_{31} \rvert$
$\tau^+ \to \mu^+ a$	$< 2.6 \times 10^{-2}$	ARGUS	4.87×10^{-14}	$> 1.4 \times 10^6 \lvert C^e_{32} \rvert$

Decays like $\ell_1 \to \ell_2 a\gamma$, in the limit $m_{\ell_2} = m_a = 0$, may be expressed

$$\frac{\mathrm{d}^2\Gamma}{\mathrm{d}x\,\mathrm{d}y} = \frac{\alpha \left| C^e_{\ell_1\ell_2} \right|^2 m^3_{\ell_1}}{32\pi^2 v^2_{PQ}} f(x,y)$$

where

$$f(x,y) = \frac{(1-x)(2-y-xy)}{y^2(x+y-1)}, \quad x = \frac{2E_{\ell_2}}{m_{\ell_1}}, \quad y = \frac{2E_\gamma}{m_{\ell_1}}$$

Kinematics and energy conservation fix

$$x, y \leq 1,\ x+y \geq 1,\ \cos\theta_{2\gamma} = 1 + \frac{2(1-x-y)}{xy}$$

Must consider

- Soft divergences
- Experimental cuts (e.g. $E_\gamma > 40$ MeV in MEG)

Decay	Branching ratio	Experiment
$\mu^+ \to e^+\gamma$	$< 4.2 \times 10^{-13}$	MEG
	$\lesssim 6 \times 10^{-14}*$	MEG-II (future)
$\tau^- \to e^-\gamma$	$< 3.3 \times 10^{-8}$	BaBar
$\tau^- \to \mu^-\gamma$	$< 4.4 \times 10^{-8}$	BaBar

Best limit on $\mu \to ef\gamma$ (for some scalar f)

- Crystal Box experiment [Bolton et al '88]
- $\mathrm{Br}(\mu \to ef\gamma) < 1.1 \times 10^{-9}$
- No assumptions on decay isotropy
- MEG-II should be more sensitive (full study needed)

Flavoured axion can mediate $\mu \to 3e$ through the $\mu e a$ vertex (t- and s-channel). To $\mathcal{O}(m_e^2)$, the branching ratio is

$$\begin{aligned}\mathrm{Br}(\mu^+ \to e^+ e^- e^+) &\approx \frac{m_e^2 m_\mu^3}{16\pi^3 \Gamma(\mu)} \frac{|A_{11}^e|^2 |C_{21}^e|^2}{v_{PQ}^4} \left(\ln \frac{m_\mu^2}{m_e^2} - \frac{15}{4} \right), \\ &\approx 1.43 \times 10^{-41} |A_{11}^e|^2 |C_{21}^e|^2 \left(\frac{10^{12}\ \mathrm{GeV}}{v_{PQ}} \right)^4\end{aligned}$$

- Experiment: Mu3e @ PSI
  - Status: under construction, taking data in 2019
  - Reach: $\mathrm{Br} < \mathcal{O}(10^{-16})$
  - 4 OoM improvement over SINDRUM (1987)
  - $v_{PQ} \gtrsim 10^6$ GeV

The same $\mu e a$ vertex can mediate $\mu - e$ conversion in nuclei

$$R_{\mu e}^{(A,Z)} \equiv \frac{\Gamma(\mu^- \to e^-(A,Z))}{\Gamma_{\mu^-\mathrm{cap}}^{(A,Z)}}$$
$$\sim \frac{m_\mu^5}{(q^2 - m_a^2)^2} \frac{(\alpha Z)^3}{\pi^2\, \Gamma_{\mu^-\mathrm{cap}}^{(A,Z)}} \frac{m_\mu^2 m_N^2}{v_{PQ}^4} |C_{21}^e|^2 |S_N^{(A,Z)} C_{aN}|^2$$

- Spin-dependent process [see Cirigliano '17]
  - not seen: $\mathcal{O}(1)$ form factors
- Relevant couplings: C_{21}^e and $g_{aN} = C_{aN} m_N / v_{PQ}$
  - C_{aN} is model-dependent, depends on diagonal charges
- Experiments
  - SINDRUM-II: current best limit $R_{\mu e}^{\mathrm{Au}} < 7 \times 10^{-13}$
  - Mu2e @ Fermilab and COMET @ J-PARC: under construction
  - Measure $R_{\mu e}^{\mathrm{Al}}$; both expected to reach 4 OoM improvement

Final comments

- Flavoured axions/ALPs have a rich phenomenology
  - Meson decays, charged lepton decays
  - Meson mixing
  - $\mu \to e$ conversion and $\mu \to 3e$

Final comments

- Flavoured axions/ALPs have a rich phenomenology
  - Meson decays, charged lepton decays
  - Meson mixing
  - $\mu \to e$ conversion and $\mu \to 3e$
- Model building
  - Discrete family symmetries and axions go well together
  - Flavour data fixes all axion couplings: very predictive!
  - In unified scenaries (e.g. A to Z), we can have correlations
  - A to Z model: no "new" scalars necessary, axion is a flavon

Final comments

- Flavoured axions/ALPs have a rich phenomenology
  - Meson decays, charged lepton decays
  - Meson mixing
  - $\mu \to e$ conversion and $\mu \to 3e$
- Model building
  - Discrete family symmetries and axions go well together
  - Flavour data fixes all axion couplings: very predictive!
  - In unified scenaries (e.g. A to Z), we can have correlations
  - A to Z model: no "new" scalars necessary, axion is a flavon
- Looking forward to new experiments coming online!
  - NA62
  - MEG-II
  - Belle-II
  - Mu3e

Thank You!

Backup slides

Full Yukawa/mass superpotential

$$W_F^{\text{eff}} = (F \cdot h_3)F_3^c + \frac{(F \cdot \phi_1^u)h_u F_1^c}{\langle \Sigma_u \rangle} + \frac{(F \cdot \phi_2^u)h_u F_2^c}{\langle \Sigma_u \rangle} + \frac{(F \cdot \phi_1^d)h_d F_1^c}{\langle \Sigma_{15}^d \rangle} + \frac{(F \cdot \phi_2^d)h_{15}^d F_2^c}{\langle \Sigma_d \rangle} + \frac{(F \cdot \phi_1^u)h_d F_1^c}{\langle \Sigma_d \rangle}$$

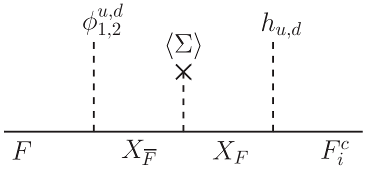
$$W_{\text{Maj}}^{\text{eff}} = \frac{\overline{H^c}\,\overline{H^c}}{\Lambda} \left(\frac{\xi^2}{\Lambda^2} F_1^c F_1^c + \frac{\xi}{\Lambda} F_2^c F_2^c + F_3^c F_3^c + \frac{\xi}{\Lambda} F_1^c F_3^c \right)$$

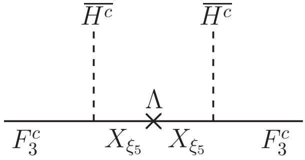
Notes

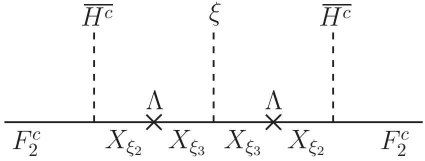
- $\overline{H^c} \sim (4,1,2)$ breaks $SU(4)_C \to SU(3)_C$, generates RHν masses
- $\Sigma \sim (1/15,1,1) \to \langle \Sigma \rangle \lesssim M_{\text{GUT}}$
- $\xi \sim (1,1,1) \to \langle \xi \rangle /\Lambda \sim 10^{-5}$

● 3rd family ● Up quarks ● Down quarks/charged leptons

Sample diagrams







Field	G_{PS}	A_4	$\mathbb{Z}_5$	$\mathbb{Z}_3$	$\mathbb{Z}'_5$	R	$U(1)_{PQ}$
F	$(4,2,1)$	3	1	1	1	1	0
$F^c_{1,2,3}$	$(\bar{4},1,2)$	1	$\alpha,\alpha^3,1$	$\beta,\beta^2,1$	$\gamma^3,\gamma^4,1$	1	$-2,-1,0$
$\overline{H^c}$	$(4,1,2)$	1	1	1	1	0	0
H^c	$(\bar{4},1,2)$	1	1	1	1	0	0
$\phi^u_{1,2}$	$(1,1,1)$	3	α^4,α^2	β^2,β	γ^2,γ	0	$2,1$
$\phi^d_{1,2}$	$(1,1,1)$	3	α^3,α	β^2,β	γ^2,γ	0	$2,1$
h_3	$(1,2,2)$	3	1	1	1	0	0
h_u	$(1,2,2)$	$1''$	α	1	1	0	0
h^u_{15}	$(15,2,2)$	1	α	1	1	0	0
h_d	$(1,2,2)$	$1'$	α^3	1	1	0	0
h^d_{15}	$(15,2,2)$	$1'$	α^4	1	1	0	0
Σ_u	$(1,1,1)$	$1''$	α	1	1	0	0
Σ_d	$(1,1,1)$	$1'$	α^3	1	1	0	0
Σ^d_{15}	$(15,1,1)$	$1'$	α^2	1	1	0	0
ξ	$(1,1,1)$	1	α^4	β^2	γ^2	0	2

Discrete $\mathbb{Z}_N$ symmetries

- $\mathbb{Z}_5$
  - Shaping symmetry of original A to Z model
  - Ensures CSD(4)
- $\mathbb{Z}_3$
  - Ensures PQ symmetry at renormalisable level
  - Forbids most off-diagonal terms in $Y^{d,e}$ (new!)
- $\mathbb{Z}_5'$
  - Protects PQ symmetry to sufficient order

Yukawa and mass matrices

$$Y^u = Y^\nu = \begin{pmatrix} 0 & b & \epsilon_{13} c \\ a & 4b & \epsilon_{23} c \\ a & 2b & c \end{pmatrix} \qquad Y^d = \begin{pmatrix} y_d^0 & 0 & 0 \\ B y_d^0 & y_s^0 & 0 \\ B y_d^0 & 0 & y_b^0 \end{pmatrix}$$

$$Y^e = \begin{pmatrix} -(y_d^0/3) & 0 & 0 \\ B y_d^0 & x y_s^0 & 0 \\ B y_d^0 & 0 & y_b^0 \end{pmatrix} \qquad M_R = \begin{pmatrix} M_1 & 0 & M_{13} \\ 0 & M_2 & 0 \\ M_{13} & 0 & M_3 \end{pmatrix}$$

Neutrino matrix after seesaw,

$$m^\nu = m_a \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + m_b e^{i\eta} \begin{pmatrix} 1 & 4 & 2 \\ 4 & 16 & 8 \\ 2 & 8 & 4 \end{pmatrix} + m_c e^{i\xi} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

PQ charges

$$W_F^{\text{eff}} \sim (\underset{0}{F} \cdot \underset{0}{h_3})\underset{0}{F_3^c} + (\underset{0}{F} \cdot \underset{2}{\phi_1^u})\underset{0}{h_u}\underset{-2}{F_1^c} + (\underset{0}{F} \cdot \underset{1}{\phi_2^u})\underset{0}{h_u}\underset{-1}{F_2^c}$$

$$+ (\underset{0}{F} \cdot \underset{2}{\phi_1^d})\underset{0}{h_d}\underset{-2}{F_1^c} + (\underset{0}{F} \cdot \underset{1}{\phi_2^d})\underset{0}{h_{15}^d}\underset{-1}{F_2^c} + (\underset{0}{F} \cdot \underset{2}{\phi_1^u})\underset{0}{h_d}\underset{-2}{F_1^c}$$

$$W_{\text{Maj}}^{\text{eff}} \sim \underset{0}{\overline{H^c}}\,\underset{0}{\overline{H^c}}\,(\underset{2}{\xi}\,\underset{2}{\xi}\,\underset{-2}{F_1^c}\underset{-2}{F_1^c} + \underset{2}{\xi}\,\underset{-1}{F_2^c}\underset{-1}{F_2^c} + \underset{0}{F_3^c}\underset{0}{F_3^c} + \underset{2}{\xi}\,\underset{-2}{F_1^c}\underset{0}{F_3^c})$$

Notes

- PQ symmetry realised also at renormalisable level
- Higgs sector completely neutral → no GUT-scale PQ breaking
- $U(1)_{PQ}$ assignments unique
- Third family is neutral

Breaking $U(1)_{PQ}$

- $\phi_i^f \to \langle \phi_i^f \rangle \sim v_{\phi_1^f}$ breaks all discrete symmetries *and* $U(1)_{PQ}$
- PQ-breaking scale

$$v_{PQ}^2 = (N_a f_a)^2 = \sum_{\phi} x_\phi^2 v_\phi^2$$

- Dominated by largest VEV: $\langle \phi_2^u \rangle$ (related to charm mass)

Axion

$$a = \frac{1}{v_{PQ}} \sum_{\varphi} x_\varphi v_\varphi a_\varphi$$

Domain wall number

$$N_a \equiv \left| 6x_F + 2\sum_i x_{F_i^c} \right| = \left| 6(0) + 2(-2 + -1 + 0) \right| = 6$$

Protecting the PQ symmetry

Consider terms like

$$\frac{\{\phi\}^n}{M_P^n} W$$

These generate a PQ-breaking axion mass

$$m_*^2 \sim m_{3/2}^2 \frac{v_{PQ}^{n-2}}{M_P^{n-2}}$$

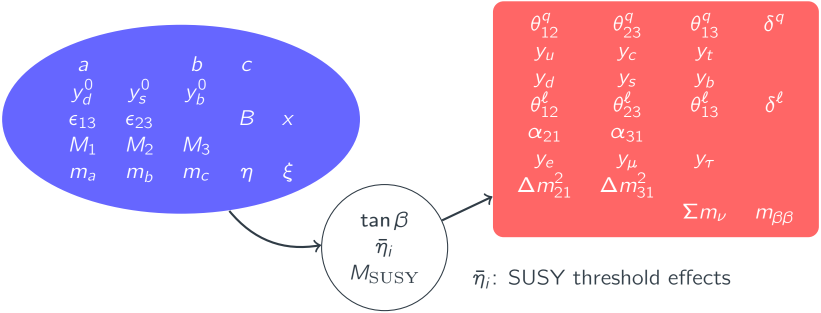
[Holman et al '92]
[Kamionkowski, March-Russell '92]
[Barr, Seckel '92]

We require $m_*^2/m_a^2 < 10^{-10}$, where

$$m_a^2 \approx m_\pi^2 \frac{f_\pi^2}{f_a^2}$$

To protect our solution, we forbid all PQ-violating terms like $\{\phi\}^n$ up to $n = 7$ (or $dim = 10$)!

Fitting to quark and lepton mixing data



$\bar{\eta}_i$: SUSY threshold effects

Simple MCMC

- Minimise χ^2 to find best fit

$$\chi^2 = \sum_i \left(\frac{P(x_i) - \mu_i}{\sigma_i} \right)^2$$

- Calculate 95% credible intervals (hpd)

Measured values run up to M_{GUT} (assuming MSSM) [Antusch, Maurer '13]

Leptons

Observable	Data		Model	
	Central value	1σ range	Best fit	Interval
θ^ℓ_{12} /°	33.57	$32.81 \to 34.32$	32.88	$32.72 \to 34.23$
θ^ℓ_{13} /°	8.460	$8.310 \to 8.610$	8.611	$8.326 \to 8.882$
θ^ℓ_{23} /°	41.75	$40.40 \to 43.10$	39.27	$37.35 \to 40.11$
δ^ℓ /°	261.0	$202.0 \to 312.0$	242.6	$231.4 \to 249.9$
y_e $/10^{-5}$	1.004	$0.998 \to 1.010$	1.006	$0.911 \to 1.015$
y_μ $/10^{-3}$	2.119	$2.106 \to 2.132$	2.116	$2.093 \to 2.144$
y_τ $/10^{-2}$	3.606	$3.588 \to 3.625$	3.607	$3.569 \to 3.643$
Δm^2_{21} $/10^{-5}\,\mathrm{eV}^2$	7.510	$7.330 \to 7.690$	7.413	$7.049 \to 7.762$
Δm^2_{31} $/10^{-3}\,\mathrm{eV}^2$	2.524	$2.484 \to 2.564$	2.540	$2.459 \to 2.616$
m_1 /meV			0.187	$0.022 \to 0.234$
m_2 /meV			8.612	$8.400 \to 8.815$
m_3 /meV			50.40	$49.59 \to 51.14$
$\sum m_i$ /meV		< 230	59.20	$58.82 \to 60.19$
α_{21}			10.4	$-38.0 \to 70.1$
α_{31}			272.1	$218.2 \to 334.0$
$m_{\beta\beta}$ /meV			1.940	$1.892 \to 1.998$

We set $\tan\beta = 5$, $M_{\mathrm{SUSY}} = 1$ TeV and $\bar{\eta}_b = -0.24$

Quarks

Observable	Data		Model	
	Central value	1σ range	Best fit	Interval
θ^q_{12} /°	13.03	12.99 → 13.07	13.04	12.94 → 13.11
θ^q_{13} /°	0.1471	0.1418 → 0.1524	0.1463	0.1368 → 0.1577
θ^q_{23} /°	1.700	1.673 → 1.727	1.689	1.645 → 1.753
δ^q /°	69.22	66.12 → 72.31	68.85	63.00 → 75.24
y_u /10^{-6}	2.982	2.057 → 3.906	3.038	1.098 → 4.957
y_c /10^{-3}	1.459	1.408 → 1.510	1.432	1.354 → 1.560
y_t	0.544	0.537 → 0.551	0.545	0.530 → 0.558
y_d /10^{-5}	2.453	2.183 → 2.722	2.296	2.181 → 2.966
y_s /10^{-4}	4.856	4.594 → 5.118	4.733	4.273 → 5.379
y_b	3.616	3.500 → 3.731	3.607	3.569 → 3.643

We set $\tan\beta = 5$, $M_{\mathrm{SUSY}} = 1$ TeV and $\bar{\eta}_b = -0.24$

Input parameters

Parameter	Value
$a\,/10^{-5}$	$1.246\,e^{4.047i}$
$b\,/10^{-3}$	$3.438\,e^{2.080i}$
c	-0.545
$y_d^0\,/10^{-5}$	$3.053\,e^{4.816i}$
$y_s^0\,/10^{-4}$	$3.560\,e^{2.097i}$
$y_b^0\,/10^{-2}$	3.607
$\epsilon_{13}\,/10^{-3}$	$6.215\,e^{2.434i}$
$\epsilon_{23}\,/10^{-2}$	$2.888\,e^{3.867i}$
B	$10.20\,e^{2.777i}$
x	5.880

Parameter	Value
m_a /meV	3.646
m_b /meV	1.935
m_c /meV	1.151
η	2.592
ξ	2.039

Constrained sequential dominance (CSD) [King '99, '00, '02]

- SD originally devised for neutrinos:
  1) $N_{\mathrm{atm}} \to$ atmospheric mass m_{ν_3} and mixing $\theta_{23} \sim 45^\circ$
  2) $N_{\mathrm{sol}} \to$ solar mass m_{ν_2} and solar+reactor mixing θ_{12}, θ_{13}
  3) N_{dec}, if present, nearly decoupled from theory $\to m_{\nu_1} \ll m_{\nu_{2,3}}$

CSD(n) with two neutrinos:

$$Y^\nu = \begin{pmatrix} 0 & b & * \\ a & nb & * \\ a & (n-2)b & * \end{pmatrix}, \qquad M_R \sim \mathrm{diag}(M_{\mathrm{atm}}, M_{\mathrm{sol}}, M_{\mathrm{dec}})$$

$$\begin{aligned} m^\nu &= v^2 Y^\nu M_R^{-1} (Y^\nu)^T \\ &= m_a \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + m_b \begin{pmatrix} 1 & n & n-2 \\ n & n^2 & n(n-2) \\ n-2 & n(n-2) & (n-2)^2 \end{pmatrix} + m_c \begin{pmatrix} * & * & * \\ * & * & * \\ * & * & * \end{pmatrix} \end{aligned}$$

- In unified scenario, CSD is extended to the quarks!
- Consider $n = 4$ [King '13]. With Y^d diagonal,

$$Y^u = Y^\nu = \begin{pmatrix} 0 & b & * \\ a & 4b & * \\ a & 2b & * \end{pmatrix}$$

- To first approximation, Cabibbo angle

$$\theta_{12}^q \approx \frac{Y_{12}^u}{Y_{22}^u} \approx \frac{1}{4}$$

- This is compellingly close to the true value $\theta_{12}^q \approx 0.227$.

- CSD(4) achieved by A_4 triplet flavons ϕ
- Flavons acquire VEVs with particular alignments:

$$\langle \phi_1^u \rangle = v_{\phi_1^u}(0,1,1), \qquad \langle \phi_1^d \rangle = v_{\phi_1^d}(1,0,0)$$
$$\langle \phi_2^u \rangle = v_{\phi_2^u}(1,4,2), \qquad \langle \phi_2^d \rangle = v_{\phi_2^d}(0,1,0)$$

- Example: first-generation up-type quarks

$$W \supset \frac{(F \cdot \phi_1^u) h_u F_1^c}{M} \to v_u \frac{v_{\phi_1^u}}{M} \left(F_1 F_2 F_3\right) \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} F_1^c$$

- Alignments can be fixed by A_4 and orthogonality arguments, implemented by a superpotential

Driving axion VEVs

$$W_{\text{driving}} = P^{u,d}_{1,2}\left(\bar{\phi}^{u,d}_{1,2}\phi^{u,d}_{1,2} - M^2\right) + P_\xi\left(\bar{\xi}\xi - M^2\right),$$

Field	G_{PS}	A_4	$\mathbb{Z}_5$	$\mathbb{Z}_3$	$\mathbb{Z}'_5$	R	$U(1)_{PQ}$
$\phi^u_{1,2}$	$(1,1,1)$	3	α^4,α^2	β^2,β	γ^2,γ	0	2, 1
$\phi^d_{1,2}$	$(1,1,1)$	3	α^3,α	β^2,β	γ^2,γ	0	2, 1
ξ	$(1,1,1)$	1	α^4	β^2	γ^2	0	2
$\bar{\phi}^u_{1,2}$	$(1,1,1)$	3	α,α^3	β,β^2	γ^3,γ^4	0	$-2,-1$
$\bar{\phi}^d_{1,2}$	$(1,1,1)$	3	α^2,α^4	β,β^2	γ^3,γ^4	0	$-2,-1$
$\bar{\xi}$	$(1,1,1)$	1	α	β	γ^3	0	-2

Yukawa matrices can be diagonalised by bi-unitary matrices $V^{u,d}_{L,R}$, $U^e_{L,R}$

$$Y^{u,\mathrm{diag}} = V^u_L Y^u (V^u_R)^\dagger,$$
$$Y^{d,\mathrm{diag}} = V^d_L Y^d (V^d_R)^\dagger,$$
$$Y^{e,\mathrm{diag}} = U^e_L Y^e (U^e_R)^\dagger.$$

We transform the fields by

$$Q \to (V^u_L)^\dagger Q,$$
$$d^c \to (V^d_R)^\dagger d^c,$$
$$u^c \to (V^u_R)^\dagger u^c.$$

Then $Y^u \to Y^{u,\mathrm{diag}}$, $Y^d \to V_{\mathrm{CKM}} Y^{d,\mathrm{diag}}$, where $V_{\mathrm{CKM}} = V^u_L (V^d_L)^\dagger$.