SO(3) family symmetry and axions

Mario Reig

arXiv: 1805.08048; MR**,** J.W.F. Valle, F. Wilczek

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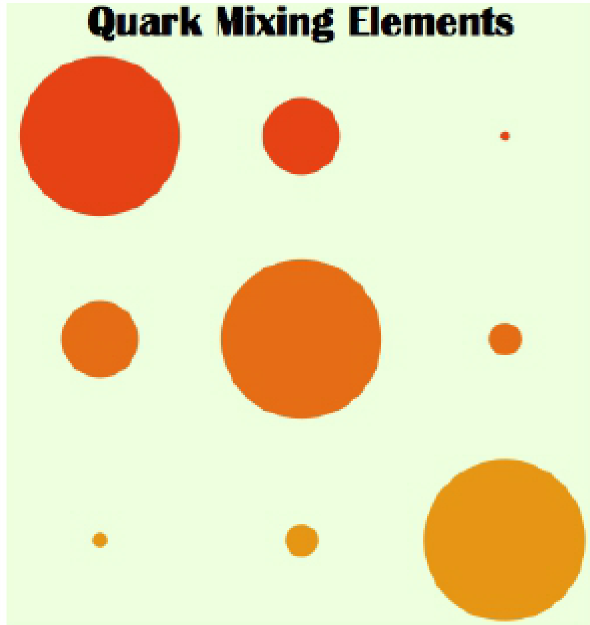
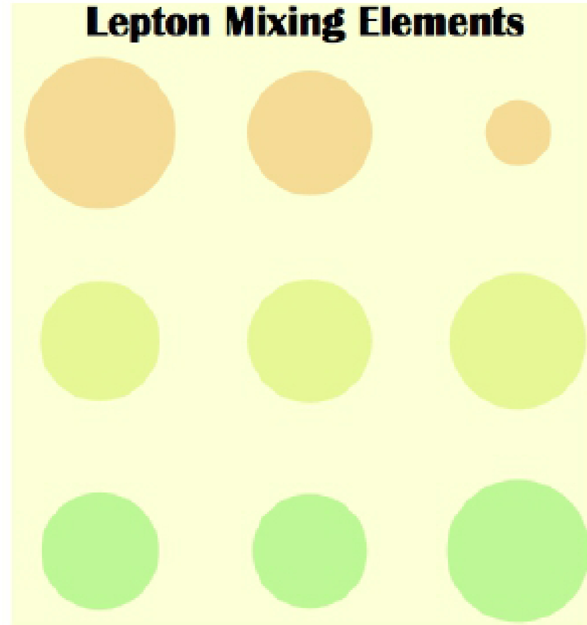
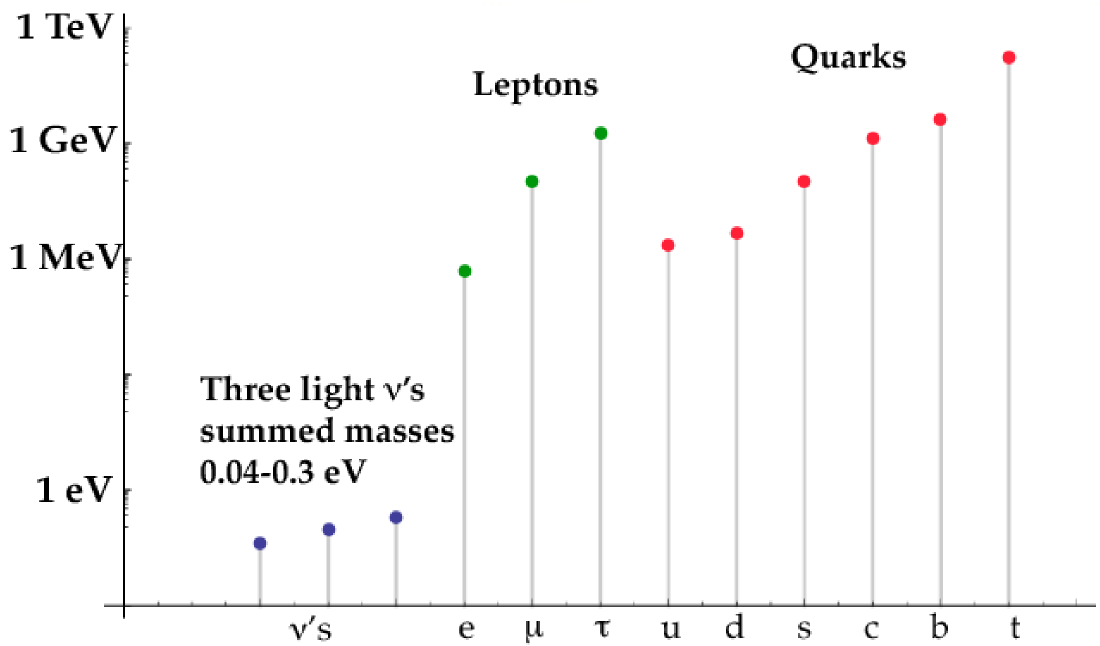
Flavour and DM 2018, Karlsruhe

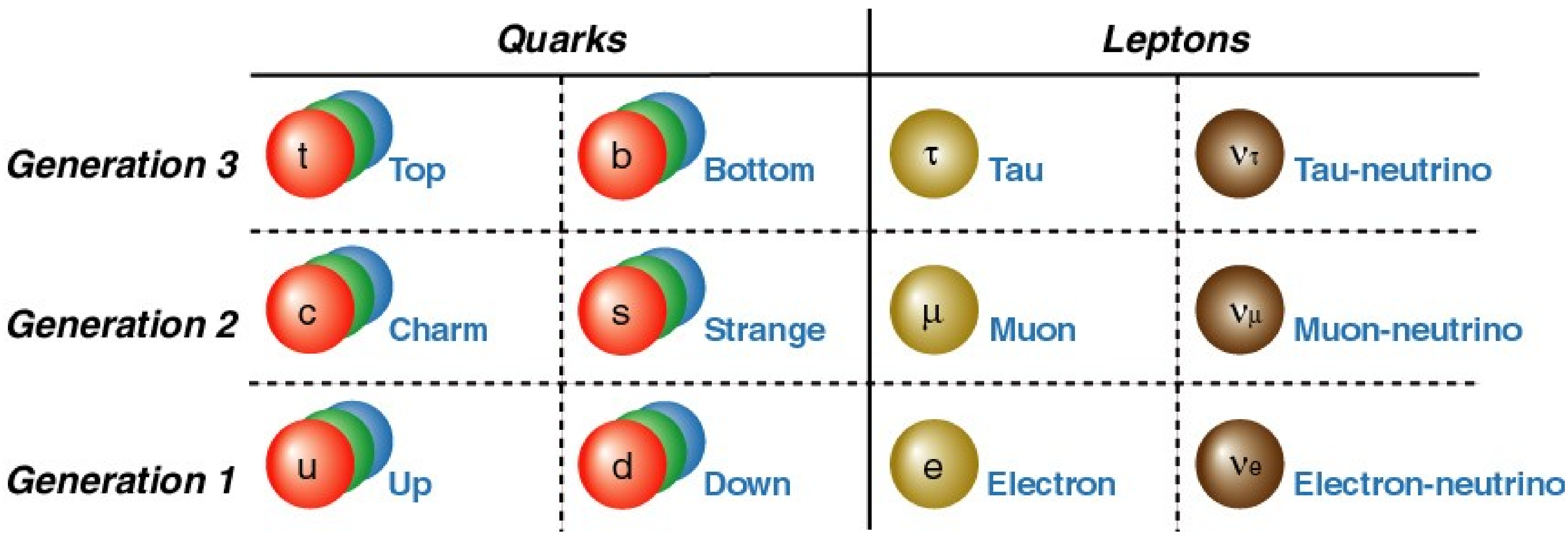
THE FLAVOR PUZZLE







WHY THREE FAMILIES???



Understanding flavor using symmetries

- **Fermion masses and mixings are explained using four different mechanisms:** (Fritzsch and Xing, ‘99)

  - Texture zeros

  - Family symmetries

  - Radiative mechanisms

  - Seesaw mechanisms

- **From the theoretical point of view all this mechanisms rely on symmetry (and its breaking!) arguments.**

- **A huge number of possibilities arise to describe 3 families…**

  A_4, S_3, T_7, $U(1)_{FN}$, $\Delta(27)$, SO(3), SU(3), ...

A different question...

Strong CP problem

$$\Delta\mathcal{L} = \frac{g^2\bar{\theta}}{16\pi^2}\epsilon^{\alpha\beta\gamma\delta}G^a_{\alpha\beta}G^a_{\gamma\delta}$$

$$\theta = \bar{\theta} + \text{Arg}[\text{det}\, M_q]$$

- Requires a huge conspiracy between EW and QCD sectors:

$$|\theta| \leq 10^{-10}$$

(From neutron EDM)

Solution: the axion!

(Peccei, Quinn, '77 ; Wilczek '78 ; Weinberg '78)

- Introduce a (spontaneously broken) chiral U(1) symmetry.

Nambu-Goldstone field: **the axion**

$$a(x) \to a(x) + \alpha\, f_{PQ}$$

- QCD anomaly induces an effective coupling to gluons

$$\Delta\mathcal{L} = \left(\theta + \frac{a(x)}{f_{PQ}}\right) \frac{g^2}{16\pi^2} \epsilon^{\alpha\beta\gamma\delta} G^a_{\alpha\beta} G^a_{\gamma\delta}$$

- Non-perturbative effects induce a potential for the axion, minimized for

$$\langle a \rangle = -\frac{\theta}{f_{PQ}}$$

- Axion mass:

$$m_a = 57 \left(\frac{10^{11}\text{GeV}}{f_A}\right) \mu\,\text{eV}$$

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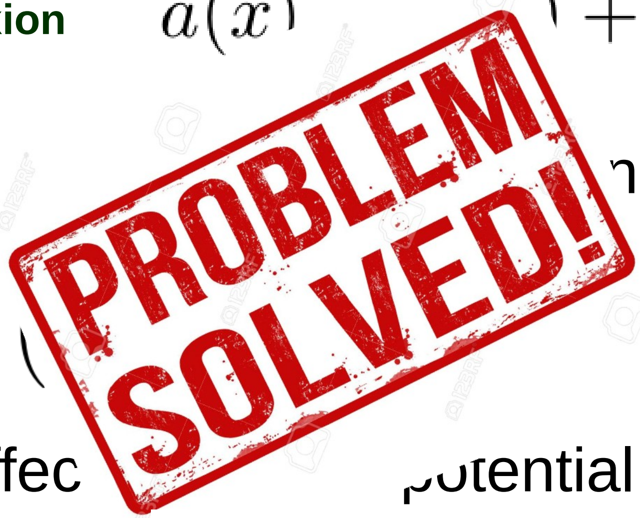

$$\Delta\mathcal{L} = (\ldots {}^{a}_{\nu\beta} G^{a}_{\gamma\delta}$$

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PQWW axion and invisible axion

- Original axion models with f_a at EW scale ruled out soon.

H^u , H^d
quarks carry PQ

Exotic quarks with PQ
quarks don't carry PQ

- Invisible axion models DFSZ & KSVZ: PQ broken by SM singlet condensate

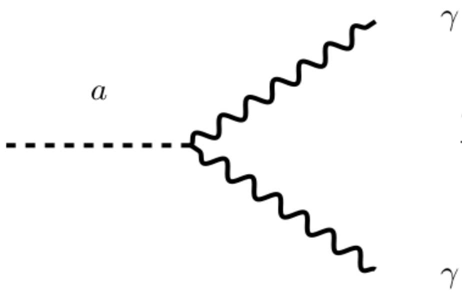
(Dine, Fischler, Sredniki, 1981; Zhitnitsky, '80 / Kim, 1979; Shifman, Vainshtein, Zakharov, '80)

- Invisible axion is a good DM candidate

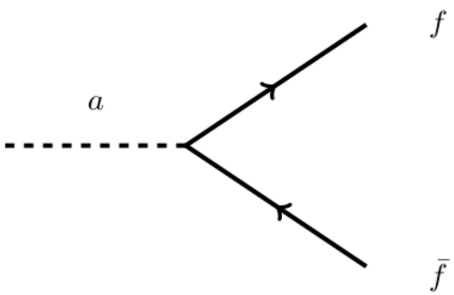
(Preskill, Wise, Wilczek; Abott, Sikivie; Dine, Fischler, '81)

Axion couplings to matter (DFSZ case)

- Quarks carry PQ. We add two higgs doublets



$$\frac{\alpha_{EM}}{8\pi}\frac{(E/N-1.92(4))}{f_a/N}F_{\mu\nu}\tilde{F^{\mu\nu}}$$



$$\frac{m_e}{f_a}a\bar{e}i\gamma_5 e$$

- Helioscopes and haloscopes to constrain axion-photon coupling.

- Strong constraints from stellar cooling

$$f_a \geq 4\times 10^9 \text{ GeV}$$

(Raffelt, 06)

FLAVORED AXIONS (FAMILONS)

(see talk by F. Björkeroth)

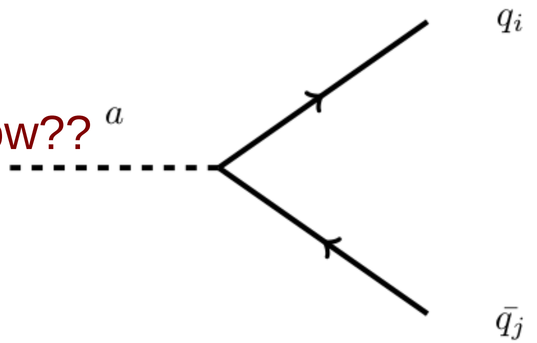
- What if PQ symmetry is part of the flavour group?
- Familons mediate rare decays, leading to strong constraints

$$K^+ \rightarrow \pi^+ + a$$

$$f_a \geq 4 \times 10^{11}\text{GeV}$$

Closing the axion DM window??

Expected bound NA62



(Calibbi, Goertz, Redigolo, Ziegler, Zupan '16; Björkeroth, Chun, King, '18)

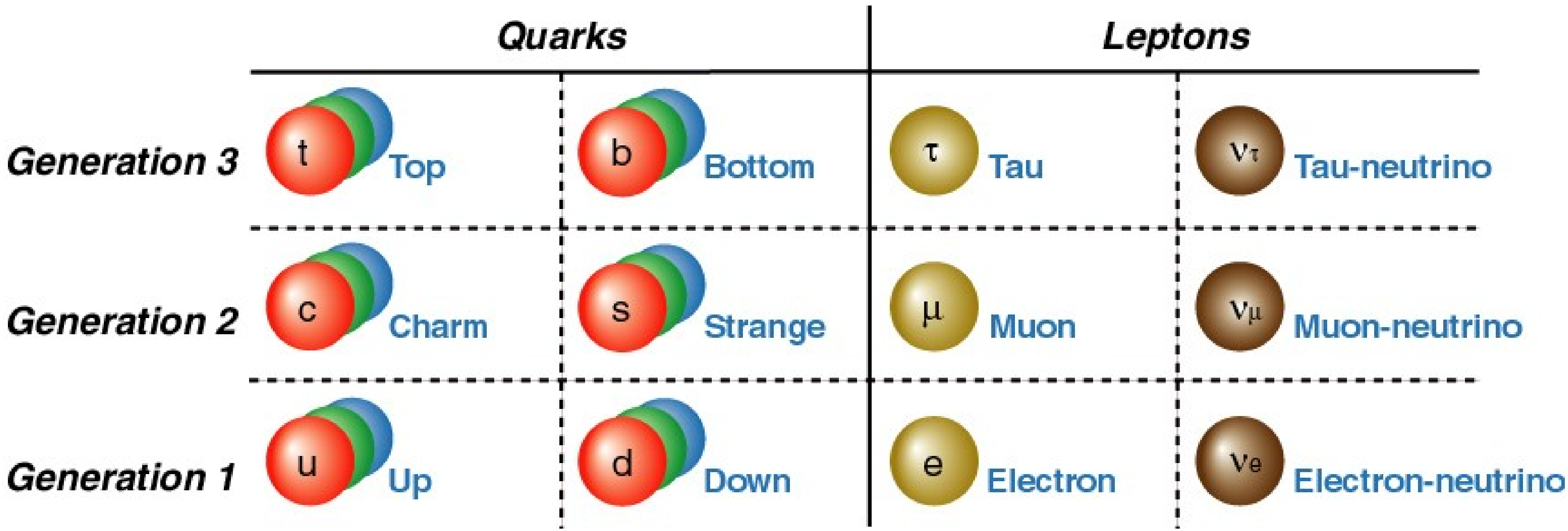
- A particular example: axiflavon/flaxion

$$U(1)_{FN} \equiv U(1)_{PQ} \qquad \mathcal{L} = a^u_{ij}\ Q_i U^c_j\ H\ (\Phi/\Lambda)^{[q]_i+[u]_j} + \ldots$$

(Calibbi, Goertz, Redigolo, Ziegler, Zupan / Ema, Hamaguchi, Moroi, Nakayama; '16)

BACK TO FLAVOR...

BACK TO FLAVOR… THE ROOT OF THE PROBLEM



SO(3) as a gauge family symmetry: The THREEfold way*

The famous question of "Who ordered the muon?" has now been escalated to "Why does Nature repeat herself?" (Wilczek & Zee, 1978)

- The model:

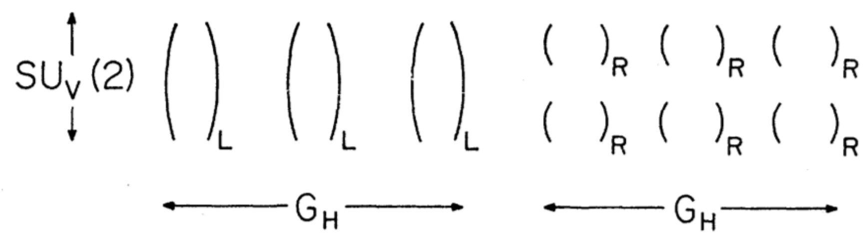


FIG. 1. Multiplet structure of the standard SU(2)⊗U(1) model with six quarks. We propose to gauge the horizontal group G_H.

- PREDICTIONS: CKM & MASS RELATIONS

$$R_L{}^C = \begin{pmatrix} 1 & -(m_d/m_s)^{1/2} + (m_u/m_c)^{1/2} & -(m_d m_s)^{1/2}/m_b + (m_u m_c)^{1/2}/m_t \\ (m_d/m_s)^{1/2} - (m_u/m_c)^{1/2} & 1 & 0 \\ (m_d m_s)^{1/2} m_b - (m_u m_c)^{1/2}/m_t & 0 & 1 \end{pmatrix}$$

$$m_e m_\mu / m_\tau{}^2 = m_d m_s / m_b{}^2 = m_u m_c / m_t{}^2$$

*: Not to confuse with Dyson's threefold way

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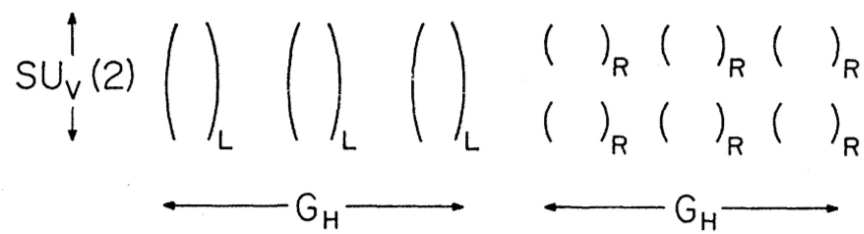

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$V_{cb}=0$

$$m_e m_\mu / m_\tau{}^2 = m_d m_s / m_b{}^2 = m_u m_c / m_t{}^2$$

M_{top}=15 GeV

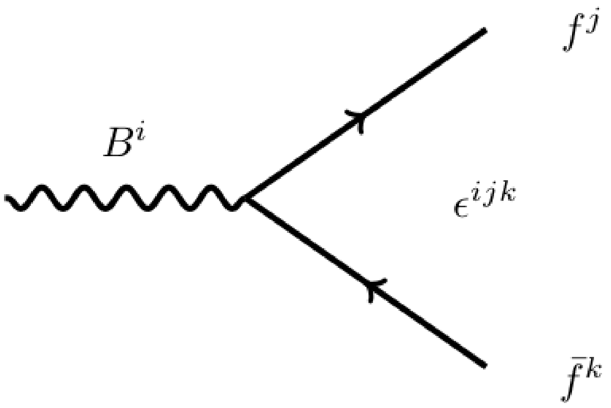
*: Not to confuse with Dyson's threefold way

Problems of the original threefold way

- A very light top quark: **M_{top}=15 GeV**

- Bottom decaying mainly to up through charged current: **V_{cb}=0**

- SO(3) broken at EW scale: unacceptable FCNC



Opportunities of the original threefold way

- A very light top quark: **M_{top}=15 GeV**

→ **up-type and down-type Higgses: PQ symmetry!**

- Bottom decaying mainly to up through charged current: **V_{cb}=0**

- SO(3) broken at EW scale: unacceptable FCNC

→ **Break SO(3) with a SM singlet: seesaw mechanism!**

The threefold way, revamped

- Extend the SM with a gauged, horizontal SO(3) symmetry.
- Use the PQ mechanism to solve strong CP problem, avoiding the wrong top quark mass prediction.
- Link flavor, PQ and lepton number symmetry breaking through the vev of a SM singlet.

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The Model

	q_L	u_R	d_R	l_L	e_R	ν_R	$\mathbf{5}^u$	$\mathbf{5}^d$	$\mathbf{3}^u$	$\mathbf{3}^d$	σ	ρ
$SU(3)_c$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$SU(2)_L$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$
$U(1)_Y$	$\frac{1}{6}$	$\frac{2}{3}$	-$\frac{1}{3}$	-$\frac{1}{2}$	-1	0	-$\frac{1}{2}$	$\frac{1}{2}$	-$\frac{1}{2}$	$\frac{1}{2}$	0	0
$SO(3)_F$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{5}$	$\mathbf{5}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{5}$	$\mathbf{1}$
$U(1)_{PQ}$	1	-1	-1	1	-1	-1	2	2	2	2	2	2

- **Fermions come in triplets.**
- **A duplicated Higgs, up and down-type, sector is introduced.**
- **An SM singlet, σ, breaks SO(3) and PQ at high E.**

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Mass hierarchies

arXiv: 1805.08048

- Let fermions be in SO(3) triplets, f~3.
- Because of product rules, 3x3=1+3+5, we can use a singlet, triplet or five-plet scalar to generate fermion masses.
- 3 and 5 are particularly interesting:

$$M \sim \bar{f}_i(\epsilon^{ijk}\langle \mathbf{3}_j\rangle + \langle \mathbf{5}_{ik}\rangle) f_k\,, \qquad \langle \mathbf{3}\rangle = \begin{pmatrix} b \\ 0 \\ 0 \end{pmatrix} \quad \langle \mathbf{5}\rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -a & 0 \\ 0 & 0 & a \end{pmatrix}$$

$$M = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -a & b \\ 0 & -b & a \end{pmatrix} \to m_1 = 0\,, m_2 = a-b\,, m_3 = a+b$$

In this context the 1st generation fermions are massless and the CKM is the identity

arXiv: 1805.08048

Emergence of the CKM matrix

- **To generate 1st generation fermion mass and CKM we take perturbations around the minimum.**

$$\langle \tilde{\mathbf{5}} \rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -a & \epsilon_1 \\ 0 & \epsilon_1 & a \end{pmatrix}, \quad \langle \tilde{\mathbf{3}} \rangle = \begin{pmatrix} b \\ 0 \\ \epsilon_2 \end{pmatrix}$$

- **Perturbations change the mass matrix:**

$$\tilde{M} = \begin{pmatrix} 0 & \epsilon_2 & 0 \\ -\epsilon_2 & -a & b+\epsilon_1 \\ 0 & -b+\epsilon_1 & a \end{pmatrix}$$

- **Generate 1st gen. Mass & quark mixing:**

$$m_1 \sim \frac{\epsilon_2^2}{m_2} \qquad \sin\theta_C \approx \sqrt{\frac{m_d}{m_s}} - \sqrt{\frac{m_u}{m_c}}, \qquad V_{cb} \approx \frac{\epsilon_1^u}{2a^u} - \frac{\epsilon_1^d}{2a^d},$$

Seesaw-like formula for 1st generation
Mixing angles as function of q masses

$$V_{ub} \approx \frac{\sqrt{m_d m_s}}{m_b} - \frac{\sqrt{m_u m_c}}{m_t},$$

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Duplicated Higgs sector: mass relations and the axion

- In the absence of vector-like quarks, the $U(1)_{PQ}$ can only be anomalous if there is a duplicated Higgs sector:

$$[SU(3)_C]^2 \times U(1)_{PQ} \neq 0 \leftrightarrow \exists\ H_u \ \&\ H_d$$

- A relation between down-type quarks and charged leptons appear due to SO(3) symmetry.

$$\frac{m_\tau}{\sqrt{m_e m_\mu}} \approx \frac{m_b}{\sqrt{m_d m_s}}$$

(also in: Morisi, Peinado, Shimizu, Valle, ‘11)

- The relation with up-type quarks is avoided thanks to the duplicated Higgs sector.

$$\frac{m_\tau}{\sqrt{m_e m_\mu}} \neq \frac{m_t}{\sqrt{m_u m_c}}$$

Non-trivial flavor-axion connection

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PQ breaking: DM & seesaw

- **The QCD axion is a good Dark Matter candidate with**

$$\Omega h^2 \approx 0.18\,\theta_i^2 \left(\frac{f_a}{10^{12}\text{GeV}}\right),$$

- **Misalignment angle, θ_i, of order O(0.1-1) makes the PQ scale, f_a to coincide with the seesaw scale: 10^{12}-10^{15} GeV.**

- **Recall that right handed neutrino mass, PQ & SO(3) breaking are related:**

$$\sigma \sim (\mathbf{1}, \mathbf{1}, 0, \mathbf{5}), \;\; \mathcal{L}_M = y_M \bar{\nu}_R^c \sigma \nu_R \,,$$

- **A connexion between axion-neutrino mass emerges**

(Also present in SMASH; Ballesteros et al., '16)

$$m_a \sim (\Lambda_{QCD} m_\pi / v^2) m_\nu$$

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Flavor protection

- **Pseudo-Goldstone bosons or axions coupled to flavor lead to strong constraints, mainly coming from** $k^+ \to \pi^+ a$ **.**

- **This constraints the PQ breaking scale to be** $f_a \geq 4 \times 10^{11}\text{GeV}$

(Celis, Fuentes-Martin, Serodio, 2014; Calibbi, Goertz, Redigolo, Ziegler, Zupan 2016, and many more!).

- **SO(3) symmetry ensures universal PQ charges for all families.**

- **SO(3) gauge familons contribute to** $K^0 - \bar{K}^0$ **and** $k^+ \to \pi^+ l^- l^+$ **however, this processes are suppressed by** f_a^2 **. This constraints the PQ scale to be** $f_a \geq 4 \times 10^7\text{GeV}$ **safely within the limits.**

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CONCLUSIONS

- **PQ symmetry, and the axion, offer attractive possibilities for flavor model building**

- **$SO(3)_F$ turns out to be a very compelling and predictive symmetry to describe flavor.**

- **Interesting flavor-axion-neutrino connection appears in the $SO(3)_F$ theory.**

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CONCLUSIONS

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THANK YOU!!

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Back-up slides

How to derive the mass relation

$$m_{u,d,e} = \frac{(y_{1,3,5}\epsilon_2^{u,d})^2}{2m_{c,s,\mu}}$$
$$m_{c,s,\mu} = y_{2,4,6}k^{u,d} - y_{1,3,5}v^{u,d}$$
$$m_{t,b,\tau} = y_{2,4,6}k^{u,d} + y_{1,3,5}v^{u,d}$$

Assumption: (exp. input) $m_{2nd} \ll m_{3rd}$ → $\frac{y_6}{y_5} \approx \frac{y_4}{y_3}$

$$\frac{y_6 k^d}{y_5 \epsilon_2^d} \approx \frac{y_4 k^d}{y_3 \epsilon_2^d} \to \frac{y_6 k^d}{y_5 \epsilon_2^d} + \frac{v^d}{\epsilon_d^2} \approx \frac{y_4 k^d}{y_3 \epsilon_2^d} + \frac{v^d}{\epsilon_2^d}$$

$$\frac{y_6 k^d + y_5 v^d}{\sqrt{2} y_5 \epsilon_2^d} \approx \frac{y_4 k^d + y_3 v^d}{\sqrt{2} y_3 \epsilon_2^d} \to \frac{y_6 k^d + y_5 v^d}{\sqrt{\frac{(y_5 \epsilon_2^d)^2 m_\mu}{2m_\mu}}} \approx \frac{y_4 k^d + y_3 v^d}{\sqrt{\frac{(y_3 \epsilon_2^d)^2 m_s}{2m_s}}}$$

$$\frac{m_\tau}{\sqrt{m_e m_\mu}} \approx \frac{m_b}{\sqrt{m_d m_s}}$$

CP violation in the quark sector

- CP violation observables are proportional to the Jarlskog invariant, *J.*
- CP violation arises from perturbations around minimum

$$\mathcal{J} = \frac{|V_{us}||V_{ud}||V_{ub}||V_{cb}||V_{tb}|}{(1-|V_{ub}|^2)} \sin\delta_{CKM}$$

$$\theta_C \approx \sqrt{\frac{m_d}{m_s}} - \sqrt{\frac{m_u}{m_c}} \qquad |V_{ub}| \approx \frac{\sqrt{m_d m_s}}{m_b} - \frac{\sqrt{m_u m_c}}{m_t}$$

$$|V_{cb}| = \frac{\epsilon_1^u}{2k^u} - \frac{\epsilon_1^d}{2k^d}$$

The Model

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$SU(2)_L$	**2**	**1**	**1**	**2**	**1**	**1**	**2**	**2**	**2**	**2**	**1**	**1**
$U(1)_Y$	$\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{2}$	-1	0	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	0	0
$SO(3)_F$	**3**	**3**	**3**	**3**	**3**	**3**	**5**	**5**	**3**	**3**	**5**	**1**
$U(1)_{PQ}$	1	-1	-1	1	-1	-1	2	2	2	2	2	2

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Quantum gravity and flavor symmetries

- **The absence of interaction terms forbidden by symmetry does not distinguish between global and local symmetries.**
- **In order to avoid problems with quantum gravity effects such as wormhole tunneling or black-hole evaporation, global symmetries should be gauged.** (L.M. Krauss, F. Wilczek, 1989)
- **The apparent HUGE number of possibilities to understand flavor using symmetries is reduced to a small number of continuous symmetries.**
- **Concerning family symmetries, only SU(3) and SO(3) appear as good candidates to describe 3 chiral families.** (F. Wilczek, A. Zee, 1979)

Flavor symmetries and unification

- **Constraints to flavor symmetries arise if we require compatibility with unification.**
- **Family SU(3) symmetry is quantumly inconsistent with minimal content of GUTs.**
- **Example: SO(10)xSU(3) with fermions in the (16,3) representation suffers the triangle $[SU(3)]^3$ anomaly.**
- **This leave us with only**

  one possibility...

  SO(3) symmetry