Light Z' Physics in $U(1)_X$ SM Extensions

Karlsruhe Institute of Technology

F. Correia[1] & S. Fajfer[2]

[1] Technische Universität Dortmund
[2] Jožef Stefan Institute

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Overview

Motivation - Discrepancies in the MeV regime

- Puzzle at low energies: $(g_\mu - 2)$ and Proton Radius Puzzle (Bound)

- Future Experiments (1608.03591):
  1. LHCb Run 3 (2021-2023) search for Dark Photons via $D^* \to D^0 A' (A' \to e^+ e^-)$;
  2. Mu3e Phase II (2018 -): muon decay channel $\mu \to e \nu_e \nu_\mu (A' \to e^+ e^-)$ for $10 < m_A[MeV] < 80$.
  3. DarkLight (2018 -): Electrons scattered off hydrogen gas to on-shell dark photons in $10 < m_A[MeV] < 100$.
  4. VEPP-3 (proposal): Positron beam on hydrogen gas target for $e^+ e^- \to \gamma A'$;
  5. E36 (J-PARC): $K_{\mu 2ee}$ decays.

- Dark Photons vs. Z': What are the consequences of axial-vector couplings and new decay modes?

Proton Radius Puzzle

Estimation Comparison between a prediction (theoretical) and measurement of the Lamb shift in muonic and atomic Hydrogen.

Prediction

$$\Delta E|^{l}_{the} = \delta E^{l}_{a} + \delta E^{l}_{b} + \cdots + \lambda^{l}\langle r_p^2\rangle|_l \tag{1}$$

where $l = \mu, e$. At leading order λ^l is given by

$$\lambda^l = \frac{2\alpha}{3a_l^3 n^3}\left(\delta_{P0} - \delta_{S0}\right) \tag{2}$$

where $n = 2$ for $2P - 2S$ and $a_l = (\alpha m_{lp})^{-1}$ is the Bohr radius of the system with reduced mass m_{lp}.

Proton Radius

$$\Delta E|^{l}_{the} = \Delta E|^{l}_{exp}; \qquad \Delta E|^{\mu}_{exp} = 202.3706(23)\ \text{meV} \tag{3}$$

At the theory side

$$\Delta E|^{\mu}_{the} = 206.0336(15) + 0.0332(20) - 5.2275\langle r_p^2\rangle \tag{4}$$

Discrepancy

$$\sqrt{\langle r_p^2\rangle|^0_\mu} = 0.84087(39)\ \text{fm} \tag{5a}$$

$$\sqrt{\langle r_p^2\rangle|^0_e} = 0.8758(77)\ \text{fm} \qquad \text{CODATA-2010} \tag{5b}$$

Guiding Principles

- **Minimality** : Introducing the minimal set of new degrees of freedom;
- **Non-Universality** : Selected puzzles as a signal of favored flavors;
- **Standard Model** features :
  1. Preserve fermion representations;
  2. Cancellation of anomalies per generation;
- **Low-Energy Phenomenology** (1103.0721):
  1. Interactions νe or νN not stronger than G_F;
  2. Absent of fundamental electrically charged particles with $m_p < 100$(GeV);
  3. QED and particle physics at the MeV.

Anomalies Requirement

- $U(1)_X^3$
- $U(1)_Y U(1)_X^2$
- $U(1)_Y^2 U(1)_X$
- $SU(2)^2 U(1)_X$
- $SU(3)^2 U(1)_X$
- $grav^2 U(1)_X$

Solutions per generation

$$X_D = 2X_Q - X_U, \quad X_L = -3X_Q, \quad X_l = -2X_Q - X_U, \quad X_\chi = X_U - 4X_Q$$

Anomalies vs Fermion Masses

From Yukawas, it is possible to generate fermion masses and forbid FCNCs by preserving only one Higgs doublets once

$$X_{L_i} - X_{l_i} - X_0 = 0; \quad X_{Q_i} - X_{U_i} + X_0 = 0; \quad X_{Q_i} - X_{D_i} - X_0 = 0 \tag{6}$$

which are satisfied by the anomaly solution. Moreover, one should demand

$$X_{L_i} - X_{l_i} = X_{L_j} - X_{l_j}; \tag{7}$$

in order all charged fermions are massive. However, by imposing only the above condition, the mass matrix is block-diagonal, what is excluded by the CKM structure. We must additionally impose

$$X_{L_i} - X_{l_j} = X_{L_j} - X_{l_j}; \tag{8}$$

which fills two more entries. Nevertheless, both conditions combined imply

$$X_{L_i} = X_{L_j}; \qquad X_{l_i} = X_{l_j}; \tag{9}$$

hence breaking the non-universality requirement.

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$$SM \otimes U(1_X)$$

- Second Generation of Right-Handed fields;
- Two-Higgs Doublet Model;
- Scalar Singlet: Breaking residual $U(1)$;
- Phenomenology of light neutral gauge boson: Remaining fields around the decoupling limit;

Particle Content

- Three vector fields W^μ from $SU(2)_L$; One vector B^Y_μ from $U(1)_Y$ and B^X_μ from $U(1)_X$;
- Three independent coupling constants g, g_Y, g_X apart from a kinetic mixing term κ;
- Three generations of Weak Isospin doublets:

$$(L_L)_i = \begin{pmatrix} \nu_i \\ e_i \end{pmatrix}_L \qquad (Q_L)_i = \begin{pmatrix} u_i \\ d_i \end{pmatrix}_L \tag{10}$$

  with $i = 1, 2, 3$;
- Right-Handed $SU(2)_L$ singlets: $\chi_R, l_{iR}, u_{iR}, d_{iR}$;
- Y hypercharges:

$$Y_L = -\frac{1}{2}; \quad Y_Q = \frac{1}{6}; \quad Y_l = -1; \quad Y_\nu = 0; \quad Y_\chi = 0; \quad Y_u = \frac{2}{3}; \quad Y_d = -\frac{1}{3} \tag{11}$$

- X hypercharges:

$$X_L = 0; \quad X_Q = 0; \quad X_{e2} = 1; \quad X_\chi = -1; \quad X_{u2} = -1; \quad X_{d2} = 1 \tag{12}$$

  with the remaining RH fields uncharged.
- Higgs doublets ϕ^0, ϕ^X and singlet s:

$$Y^{\phi^0} = Y^{\phi^X} = \frac{1}{2}; \quad X^{\phi^0} = 0; \quad X^{\phi^X} = -1; \quad Y^s = 0 \quad X^s = 1 \tag{13}$$

On the Kinetic Mixing

$$\mathcal{L} \supset -\frac{1}{4}\mathbf{W}^{\mu\nu}\cdot\mathbf{W}_{\mu\nu} - \frac{1}{4}B^{Y\mu\nu}B^{Y}_{\mu\nu} - \frac{1}{4}B^{X\mu\nu}B^{X}_{\mu\nu} + \frac{\epsilon}{2}B^{Y\mu\nu}B^{X}_{\mu\nu}$$

Field redefinition:

$$B^Y_\mu \to B^Y_\mu + \epsilon B^X_\mu \tag{14}$$

or

$$\mathcal{L}_{k.m.} \supset -\frac{1}{2}(B^Y_\mu + \epsilon B^X_\mu)\hat{\mathcal{O}}^{\mu\nu}(B^Y_\nu + \epsilon B^X_\nu) - \frac{1}{2}B^X_\mu \hat{\mathcal{O}}^{\mu\nu} B^X_\nu + \epsilon\, B^X_\mu \hat{\mathcal{O}}^{\mu\nu}(B^Y_\nu + \epsilon B^X_\nu) \tag{15}$$

such that, up to order $\mathcal{O}(\epsilon)$,

$$\mathcal{L}_{k.m.} \supset -\frac{1}{2}B^Y_\mu \hat{\mathcal{O}}^{\mu\nu} B^Y_\nu - \frac{1}{2}B^X_\mu \hat{\mathcal{O}}^{\mu\nu} B^X_\nu + \mathcal{O}(\epsilon^2) \tag{16}$$

i.e. the crossed terms vanishes and the mixing effect is converted into the Covariant Derivative:

$$D_\mu \to D_\mu = \partial_\mu - ig\mathbf{W}_\mu\cdot\tau - ig_Y B^Y_\mu Y^p - i(\kappa Y^p + g_X X^p)B^X_\mu \tag{17}$$

where $\epsilon g_Y \equiv \kappa$.

Couplings and Masses

- Gauge Boson masses:

$$D_\mu \phi^p = \left[\partial_\mu - ig(W^+ \mathbb{I}_+ + W^- \mathbb{I}_-) - ig\tau_3 W^3_\mu - ig_Y Y^p B^Y_\mu - i(\kappa Y^p + g_X X^p) B^X_\mu\right] \phi^p$$

$$D_\mu s = (\partial_\mu - ig_X X^s B^X_\mu) s$$

with hypercharges assignment

$$Y^{\phi^0} = Y^{\phi^X} = \frac{1}{2}; \quad X^{\phi^0} = 0; \quad X^{\phi^X} = -1; \quad Y^s = 0 \quad X^s = 1 \tag{18}$$

Therefore the presence of $\phi_0^\dagger \phi_X s$ is allowed in the scalar potential.

- Scalars:

$$\phi_0 = \begin{pmatrix} \varphi_0^+ \\ \frac{v_0 + H_0 + i\chi_0}{\sqrt{2}} \end{pmatrix}; \qquad \phi_X = \begin{pmatrix} \varphi_X^+ \\ \frac{v_X + H_X + i\chi_X}{\sqrt{2}} \end{pmatrix}; \qquad s = \frac{v_s + H_s + i\chi_s}{\sqrt{2}} \tag{19}$$

-

$$\mathbb{M}^0 = \frac{v^2}{8} \begin{pmatrix} g^2 & -gg_Y & g(2g_X c_\beta^2 - \kappa) \\ -gg_Y & g_Y^2 & -g_Y(2g_X c_\beta^2 - \kappa) \\ g(2g_X c_\beta^2 - \kappa) & -g_Y(2g_X c_\beta^2 - \kappa) & 4[g_X^2 \frac{\bar{v}^2}{v^2} - g_X \kappa c_\beta^2] + \kappa^2 \end{pmatrix} \tag{20}$$

where

$$v^2 \equiv (v_0^2 + v_X^2), \qquad \bar{v}^2 \equiv (v_s^2 + v_X^2), \qquad c_\beta^2 = \frac{v_X^2}{v^2} \tag{21}$$

Couplings and Masses

- Mixing Matrix:

$$\mathbb{V} = \begin{pmatrix} s_\theta c_\phi & -s_\theta s_\phi & -c_\theta \\ s_\phi & c_\phi & 0 \\ c_\theta c_\phi & -c_\theta s_\phi & s_\theta \end{pmatrix} \tag{22}$$

- Photon Couplings:

$$e\mathbb{Q} = g s_\phi \tau^3 + g_Y c_\phi Y \tag{23}$$

and by applying it to the standard fields, it can be extracted

$$g s_\phi = g_Y c_\phi = e \tag{24}$$

- Z Couplings:

$$g_Z = c_\theta g_Z^{SM} + s_\theta(\kappa Y + g_X X) \tag{25}$$

- X Couplings:

$$g_R = s_\theta g_Z^{SM} - c_\theta(\kappa Y + g_X X) \tag{26}$$

where $g_Z^{SM} = \frac{g}{c_\phi}(\tau_3 - s_\phi^2 \mathbb{Q})$.

Couplings and Masses

- Z Couplings:

$$g_Z = c_\theta g_Z^{SM} + s_\theta(\kappa Y + g_X X)$$

- X Couplings:

$$g_R = s_\theta g_Z^{SM} - c_\theta(\kappa Y + g_X X)$$

where $g_Z^{SM} = \frac{g}{c_\phi}(\tau_3 - s_\phi^2 \mathbb{Q})$.

- Neutral Vector Masses: If $g_X, \kappa \ll \bar{g}$

$$m_Z^2 \to \frac{v^2}{4}\bar{g}^2, \qquad m_X^2 \to \frac{v^2}{4} a_1 \tag{27}$$

where

$$\bar{g}^2 = g_Y^2 + g^2\,, \qquad a_1 = 4\left[g_X^2 \frac{\bar{v}^2}{v^2} - g_X \kappa c_\beta^2\right] + \kappa^2$$

- Mixing Angle:

$$s_\theta \approx \frac{|2 g_X c_\beta^2 - \kappa|}{\bar{g}} \left[1 - \frac{m_X^2}{m_Z^2}\right]^{-1} \tag{28}$$

Fermion Gauge Interactions

$$\mathcal{L}_{kin} \supset i\left[\overline{L}_{\alpha L}\not{D}L_{\alpha L} + \overline{Q}_{\alpha L}\not{D}Q_{\alpha L} + \bar{l}_{\alpha R}\not{D}l_{\alpha R} + \overline{d}_{\alpha R}\not{D}d_{\alpha R} + \overline{u}_{\alpha R}\not{D}u_{\alpha R} + \overline{\chi}_R\not{D}\chi_R\right] \quad (29)$$

with $\alpha = 1, 2, 3$, $\beta = 1, 2$. The Covariant Derivative in terms of the mass eigenstates can be written like

$$D_\mu = \partial_\mu - ig(W^+\mathbb{I}_+ + W^-\mathbb{I}_-) - ie\mathbb{Q}A_\mu - ig_Z Z_\mu - ig_R X_\mu \quad (30)$$

Flavor Violating processes in both Z and X interactions are exclusive to RH sector. Defining the vector of fermion fields $f = (f_1, f_2, f_3)$ and rotating the system to the mass basis, $f_R \to V_{fR}f'_R \equiv V_{fR}f_R$, the general currents depending on the X charges can be fully separated via:

$$\mathcal{L}_{kin} \supset -c_\theta g_X\left[\overline{u}_R\mathbb{F}^U\gamma^\mu u_R + \overline{d}_R\mathbb{F}^D\gamma^\mu d_R + \bar{l}_R\mathbb{F}^l\gamma^\mu l_R\right]X_\mu \quad (31)$$

apart from the s_θ-dependent universal contribution. The matrices

$$\mathbb{F}^f \equiv V^\dagger_{fR}\mathbb{X}^f V_{fR}, \qquad \text{where} \qquad (\mathbb{X}^f)_{ij} \equiv X^f\delta_{2i}\delta_{2j} \quad (32)$$

or

$$(\mathbb{F}^f)_{ij} = X^f(V^\dagger_{fR})_{i2}(V_{fR})_{2j} \quad (33)$$

summarizes the amount of flavor violation and fermion non-universality in the model.

Fermion Gauge Interactions

$$(\mathbb{F}^f)_{ij} = X^f (V_{fR}^\dagger)_{i2} (V_{fR})_{2j} \quad (34)$$

- By unitarity the trace of $\mathbb{F}^f$ is equal to X^f:

$$\begin{aligned} \mathrm{Tr}[\mathbb{F}^f] &= \mathrm{Tr}[V_{fR}^\dagger \mathbb{X}^f V_{fR}] \\ &= \mathrm{Tr}[\mathbb{X}^f] \\ &= X^f \end{aligned} \quad (35)$$

- In the scenario where flavor is aligned to mass eigenstates, i.e. when the absolute value of diagonal elements of V_{fR} are larger than the non-diagonal ones, the flavor violating processes also will favor second generation in the final state.

$$|\mathbb{F}^f| \equiv X^f \begin{pmatrix} |V_{fR}|_{21}^2 & |V_{fR}|_{21}|V_{fR}|_{22} & |V_{fR}|_{21}|V_{fR}|_{23} \\ |V_{fR}|_{21}|V_{fR}|_{22} & |V_{fR}|_{22}^2 & |V_{fR}|_{22}|V_{fR}|_{23} \\ |V_{fR}|_{21}|V_{fR}|_{23} & |V_{fR}|_{22}|V_{fR}|_{23} & |V_{fR}|_{23}^2 \end{pmatrix} \quad (36)$$

Non-Universality

- Diagonal Currents: Vector and Axial-Vector Couplings

$$\mathcal{L} \supset \frac{1}{2}\,\bar{f}\,\gamma_\mu(g_V^f + g_A^f\gamma^5)\,f\,Z^\mu + \frac{1}{2}\,\bar{f}\,\gamma_\mu(x_V^f + x_A^f\gamma^5)\,f\,X^\mu \tag{37}$$

- Lepton Couplings: By replacing electric charges and hypercharges:

$$x_V^l = g\frac{s_\theta}{c_\phi}\left(-\frac{1}{2} + 2s_\phi^2\right) + c_\theta\kappa\frac{3}{2} - c_\theta g_X\mathbb{F}_{ii}^l \tag{38a}$$

$$x_A^l = g\frac{s_\theta}{c_\phi}\left(\frac{1}{2}\right) + c_\theta\kappa\frac{1}{2} - c_\theta g_X\mathbb{F}_{ii}^l \tag{38b}$$

$$x_V^\nu = -x_A^\nu = g\frac{s_\theta}{c_\phi} + c_\theta\kappa \tag{38c}$$

Parameter Space - $U(1)_X$

Parameter Space P: Initially the set P is given by

$$P := [c_\beta, \kappa, g, g_Y, g_X, v_X, v_0, v_s, \mathbb{F}] \tag{39}$$

To reproduce the Electroweak interactions, both g and g_Y can be solved in terms of the remaining elements. The m_W and m_Z pole mass can, in addition, solve v_0 and v_X. However, in the asymptotic limit both masses depends only on v such that it may be convenient to preserve c_β in the analysis. Finally, the scale v_s can be *replaced* by m_X. We end up with a five-dimensional parameter space, namely

$$P := [c_\beta, \kappa, g_X, m_X, \mathbb{F}] \tag{40}$$

The kinetic mixing variable is independent and can be replaced by the new mixing angle. Accordingly, there must be a region for κ where the SM Z interactions are exactly reproduced, i.e. $s_\theta = 0$.

Constraints

Most stringent from previous work (PRD 94, 115023 (2016)):

- ρ parameter;
- Parity Non-Conserving Processes;
- Proton Puzzle in the $U(1)_X$;
- χ Fermion - Mixing Energy Considerations;
- Kaon Leptonic Decays $K_{\mu 2ee}$;
- $(g_e - 2)$.

ρ Parameter

The ρ parameter is a quantity defined by three observables, namely m_W, m_Z and the weak mixing angle through the expression

$$\rho = \frac{m_W^2}{m_Z^2 c_w^2} \tag{41}$$

In the SM these parameters are connected by a natural relation and results in $\rho = 1$ at tree-level. In order to verify how the parameter will escape from the unity, in first approximation we can rewrite the Z mass

$$m_Z^2 \approx \frac{v^2}{4}\bar{g}^2\left(1 + s_\theta^2\right) \tag{42}$$

where the X_μ light mass condition. It follows that

$$\rho_X^{tree} \approx c_\theta^2 \tag{43}$$

which cannot touch the central value of the experimental measurement

$$\rho \in 1.00040(24) \tag{44}$$

At two sigmas we can demand $0.99992 < c_\theta^2 \leq 1$ or

$$s_\theta^2 < 8 \cdot 10^{-5} \tag{45}$$

Parity Non-Conserving Observables

LEP phenomenology should be repeated.

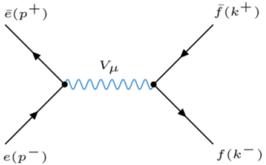


Figure: Forward-Backward Asymmetries in $e^+e^- \to \bar{f}f$ are important tests for axial-vector couplings. The model would predict non-universality in for $f = \mu$ and $f = \tau$. Here $V = \gamma, Z, X$.

The Forward-Backward Asymmetry is defined like

$$A(\theta) \equiv \frac{d\sigma(\theta) - d\sigma(\pi - \theta)}{d\sigma(\theta) + d\sigma(\pi - \theta)} \tag{46}$$

Here we will focus on the energy region distant of both Z and X peaks, i.e. $2m_\mu \ll \sqrt{s} \ll m_Z$ and we must compute the generic diagram of Fig.1 for $V = \gamma, Z, X$. For convenience, the generic vertex is written like

$$\bar{f}fV_\mu : \quad ie\gamma_\mu(v_f^V - a_f^V\gamma_5) \tag{47}$$

For instance, $(v_f^\gamma, a_f^\gamma) = (-q_f, 0)$ where $q_e = -1, q_u = \frac{2}{3}, q_d = -\frac{1}{3}$.

Parity Non-Conserving Observables

The amplitude can then be expressed like

$$\mathcal{M}_V = \frac{e^2}{s - m_V^2}[\bar{v}(p^+)\gamma^\mu(v_e^V - a_e^V\gamma_5)u(p^-)][\bar{u}(k^-)\gamma_\mu(v_f^V - a_f^V\gamma_5)v(k^+)] \quad (48)$$

with $|\mathcal{M}|^2 = |\sum_{V=\gamma,Z,X}\mathcal{M}_V|^2$.

$$A(\theta) \approx \frac{[d\sigma^{\gamma Z}(\theta) + d\sigma^{\gamma X}(\theta)] - [d\sigma^{\gamma Z}(\pi-\theta) + d\sigma^{\gamma X}(\pi-\theta)]}{d\sigma^{\gamma}(\theta) + d\sigma^{\gamma}(\pi-\theta)} \quad (49)$$

In the CM reference frame it results in

$$A(\theta) \approx \frac{8sc_\theta|\mathbf{k}|\sqrt{s}}{4c_\theta^2|\mathbf{k}|^2 + 4m_f^2 + s}\left[\frac{a_e^X a_f^X}{s - m_X^2} + \frac{a_e^Z a_f^Z}{s - m_Z^2}\right] \quad (50)$$

Here c_θ is the scattering angle, and $\mathbf{k}$ the 3-momenta of the products. In the region $\sqrt{s} \gg m_\mu$ the contribution from X exchange can be represented by $\delta A^X(\theta) \propto \frac{a_e^X a_f^X}{s}$ or

$$A(\theta) \propto \left[\frac{a_e^X a_f^X}{s} - \frac{a_e^Z a_f^Z}{m_Z^2}\right] \quad (51)$$

Once $a^Z \sim g$ and $a^X \sim g_X$, where for instance $\sqrt{s} \sim \frac{m_Z}{10}$ the region $g_X \sim 10^{-1}g$ would be highly constrained.

Proton Puzzle in $U(1)_X$

The discrepancy can be accommodated

$$\Delta E|^l_{the} = \delta E^l_0 + \delta E^l_X + \lambda^l \langle r_p^2\rangle|^X_l \to \langle r_p^2\rangle|^X_\mu = \langle r_p^2\rangle|^X_e \tag{52}$$

The difference between the “X” and “0” frameworks can be expressed as a small deviation like

$$\langle r_p^2\rangle|^X_l = \langle r_p^2\rangle|^0_l - \delta^X_l \quad \text{where} \quad \delta^X_l \equiv \frac{\delta E^l_X}{\lambda^l} \tag{53}$$

In summary, a proton radius constraint is imposed by

$$\delta^X_e - \delta^X_\mu = \langle r_p^2\rangle|^0_e - \langle r_p^2\rangle|^0_\mu \tag{54}$$

The correction δ^X_l originates from a contribution to the Coulomb potential due to the exchange of a massive vector boson X_μ

$$V^l_X(r) = \frac{g_l g_p}{e^2}\frac{\alpha e^{-m_X r}}{r} \tag{55}$$

with a correspondent shift in $2P - 2S$

$$\begin{aligned}\delta E^l_X &= \int dr\, V^l_X(r)\left(|R_{21}(r)|^2 - |R_{20}(r)|^2\right) r^2 \\ &= -\frac{\alpha}{2a_l^3}\left(\frac{g_l g_p}{e^2}\right)\frac{f(a_l m_X)}{m_X^2}\end{aligned} \tag{56}$$

For $m_X > 10$ MeV we can take $f(x) = \frac{x^4}{(1+x)^4} \sim 1$.

Proton Puzzle in $U(1)_X$

A *Proton curve* is defined by

$$6\,\frac{g_p}{e^2}\,\frac{(g_e - g_\mu)}{m_X^2} = \langle r_p^2\rangle|_e^0 - \langle r_p^2\rangle|_\mu^0 \tag{57}$$

which in principle can be solved by an attractive force (i.e. $\text{sgn}g_p = -\text{sgn}g_l$) strongly coupled with muons. In the $U(1)_X$ framework, and under the limit where $f(x) \sim 1$, the sgn g_p must be opposite only to the non-universal part of the X^μ coupling. The couplings g_p and g_l are given by:

$$g_p = -c_\phi^2 \kappa; \qquad g_l = \frac{x_V^l}{2} \tag{58}$$

For simplicity $\mathbb{F}_{\tau\tau}$ may be taken zero such that $\mathbb{F}_{\mu\mu} + \mathbb{F}_{ee} = 1$, what reduces the Proton curve to

$$6\,\frac{g_p g_X}{e^2}\,\frac{2\mathbb{F}_{\mu\mu} - 1}{m_X^2} = 0.060(13)\ \text{fm}^2 \tag{59}$$

Missing Energy Considerations - $K_{\mu Y}$

$$\mathcal{L} \supset i\overline{\chi}_R \not{D} \chi_R - Y_s\, \overline{\chi_L}\chi_R s - Y_s^*\, \overline{\chi_R}\chi_L s^* \tag{60}$$

The Narrow-Width approximation is assumed to be valid in the region where $m_X > 2m_\chi$ i.e such that $e^+e^-, 3\bar{\nu}\nu$ and $\chi\bar{\chi}$ are the only directly accessible decay products of X_μ:

$$d\Gamma(K \to \mu\nu\chi\bar{\chi}) = \frac{1}{3} d\Gamma(K \to \mu\nu X) \mathrm{Br}(X \to \chi\bar{\chi}) \tag{61}$$

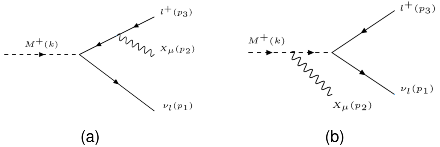


Figure: The Feynman diagrams contributing to M_{lY} in the $U(1)_X$ model.

Missing Energy Considerations - $K_{\mu Y}$

- Since χ_R is a singlet under the SM gauge group and $X_\chi = -1$, it follows

$$X_\mu \bar{\chi}\chi : i\frac{g_X}{2}\gamma_\mu(1+\gamma_5), \quad c_\theta \sim 1 \tag{62}$$

- Vector and Axial electron couplings

$$x_V^e = g_X\left[\frac{g}{c_\phi}\left(2s_\phi^2 - \frac{1}{2}\right)\frac{|2c_\beta^2 - n|}{\bar{g}} + \frac{3n}{2} - \mathbb{F}_{ee}\right] \tag{63}$$

$$x_A^e = g_X\left[\frac{g}{2c_\phi}\frac{|2c_\beta^2 - n|}{\bar{g}} + \frac{n}{2} - \mathbb{F}_{ee}\right] \tag{64}$$

$$x_V^\nu = -x_A^\nu = g_X\left[\frac{g}{c_\phi}\frac{|2c_\beta^2 - n|}{\bar{g}} + n\right] \tag{65}$$

- Missing Mass:

$$\begin{aligned}\Gamma_{K\mu Y} &= \Gamma_{K\mu\bar{\chi}\chi} + 3\Gamma_{K\mu\bar{\nu}\nu} \\ &= \frac{1}{3}\Gamma(K\to\mu\nu X)\left[\mathrm{Br}(X\to\chi\bar{\chi}) + 3\mathrm{Br}(X\to\nu\bar{\nu})\right]\end{aligned} \tag{66}$$

where the ' 3' factor accounts for three neutrino flavors.

Missing Energy Considerations - $K_{\mu Y}$

- Branching Ratio:

$$Br(X \to \bar{a}a) = \frac{|\mathcal{M}_{X\bar{a}a}|^2 \sqrt{\lambda(m_X^2, m_a^2, m_a^2)}}{\sum_l |\mathcal{M}_{X\bar{l}l}|^2 \sqrt{\lambda(m_X^2, m_l^2, m_l^2)}} \tag{67}$$

where $l = \chi, e, \nu_e, \nu_\mu, \nu_\tau$.

- Squared Amplitude:

$$|\mathcal{M}_{X\bar{l}l}|^2 = 4\left[2m_l^2\left({x_V^l}^2 - 2{x_A^l}^2\right) + m_X^2\left({x_V^l}^2 + {x_A^l}^2\right)\right] \tag{68}$$

- Since $x_V^\chi = x_A^\chi$

$$|\mathcal{M}_{X\bar{\chi}\chi}|^2 \propto [m_X^2 - m_\chi^2] \tag{69}$$

- Current Bound (PRD 8, 7 1973):

$$\frac{\Gamma_{K\mu Y}}{\Gamma_{K\mu\nu}} < 3.5 \times 10^{-6}, \qquad 90\%\ C.L. \tag{70}$$

in the interval

$$227.6 < m_Y(MeV) < 302.2 \tag{71}$$

Kaon Leptonic Decays - $K_{\mu 2ee}$

The analysis is similar to that of $\Gamma_{K\mu Y}$, now with

$$\frac{1}{3}\frac{\Gamma(K \to \mu\nu_\mu X)}{\Gamma_K}\text{Br}(X \to e^+e^-) < 3.1 \times 10^{-9} \tag{72}$$

for $145 < m_X(MeV) < 2m_\mu$. Note that the experimental value corresponds to the result of integrating the distribution $\frac{d\Gamma_{K\mu 2ee}}{dm_{ee}}$ for $m_{ee} > 145\text{MeV}$. The assumption that X_μ goes on-shell is the same to state that for a fixed $m_X = m_{ee}$ the contribution from $X \to ee$ will not exceed the uncertainty of the total $\Gamma_{K\mu 2ee}$. Therefore, by demanding the decay rate to be smaller than the experimental uncertainty we are already stating that no enhancement will be seen in this region $m_{ee} > 145 MeV$.

$(g_e - 2)$

The correction to a_e due to the presence of X_μ corresponds to a shift of the fine-structure constant:

$$d\alpha = 2\pi a_e^X \quad \rightarrow \quad \frac{d\alpha^{-1}}{\alpha^{-1}} = -\frac{2\pi a_e^X}{\alpha} \tag{73}$$

The r.h.s is the relative correction to the measurement of α^{-1} which should not exceed 0.5 ppb. The dipole function can be written like

$$a_e^X = \frac{m_e^2}{4\pi^2}\left[(x_V^e)^2 I_V(m_X^2) + (x_A^e)^2 I_A(m_X^2)\right] \tag{74}$$

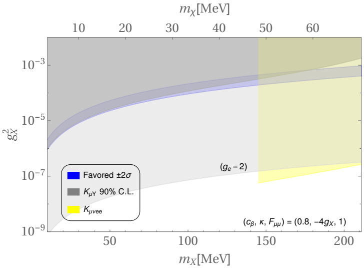
where

$$\begin{aligned} I_V(m_X^2) &= \int_0^1 dz \frac{z^2(1-z)}{[m_l^2 z^2 + m_X^2(1-z)]} \quad \overset{m_X \gg m_l}{\rightarrow} \quad \frac{1}{3m_X^2} \\ I_A(m_X^2) &= \int_0^1 dz \frac{z(1-z)(z-4) - \left(2\frac{m_l^2}{m_X^2}\right)}{[m_l^2 z^2 + m_X^2(1-z)]} \quad \overset{m_X \gg m_l}{\rightarrow} \quad -\frac{5}{3m_X^2} \end{aligned} \tag{75}$$

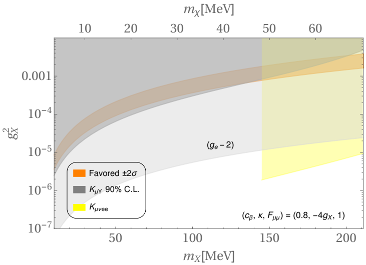
Since the limit $m_X \gg m_e$ is valid in our region we can set the bounding curve

$$f\left(\frac{m_e^2}{m_X^2}\right) \equiv \left(\frac{m_e^2}{m_X^2}\right)\frac{1}{6\pi\alpha}\left|(x_V^e)^2 - 5(x_A^e)^2\right| < 0.5\text{ppb} \tag{76}$$

Parameter Space facing Selected Process



(a)



(b)

Figure: The favored region for the proton radius anomaly explanation facing the selected bounds. Under the Narrow-Width approximation the vector X_μ decays into a lepton pair $\bar{l}l$ for $l = e, 3\nu, \tau$. Here $m_X = 3m_\chi$ while $\mathbb{F}_{\tau\tau} = 0$.

Parameter Space - Optimal

We must deal with the task of fixing a plane of a five-dimensional parameter space under the assumption that the model must explain, for instance, the proton puzzle. For that particular discrepancy one needs

$$\text{sgn} g_X = -\text{sgn}\kappa \tag{77}$$

In the examples depicted in the previous figures one can verify how stringent $(g-2)_e$ bounds are. A possible strategy to loose these lines is to look in their definition and work with the interference between vector and axial-vector couplings. For instance, in the region around the root

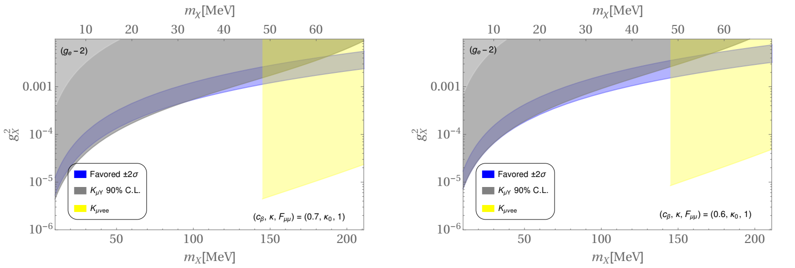
$$|(x_V^e)^2 - 5(x_A^e)^2| = 0 \tag{78}$$

for some fixed $\mathbb{F}$, the bound would be approximately absent. For instance, for $\mathbb{F}_{ee} = 0$ the solutions are

$$n \in \left[-\frac{7}{5}, \frac{3}{2}, 3\right] c_\beta^2 \tag{79}$$

for $\kappa = n g_X$. Hence, only one value can satisfy the condition of Eq.(77).

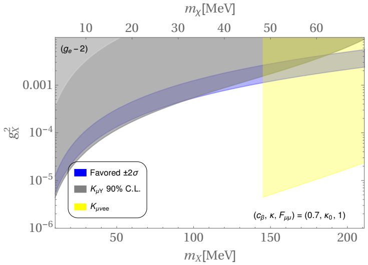
Parameter Space - Optimal



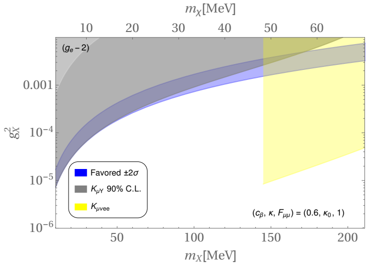
(a) (b)

Figure: Close to the root for the $(g_e - 2)$ bound one can reduce the discrepancy of the proton puzzle from 5σ to 2σ.

Forthcoming Bounds and Tests



(a)



(b)

Figure: Close to the root for the $(g_e - 2)$ bound one can reduce the discrepancy of the proton puzzle from 5σ to 2σ.

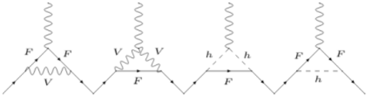
Meson Mixing : Tests on $\mathbb{F}$ in the quark sector;

$(g_\mu - 2)$: Long-Standing discrepancy facing Proton Curve;

$K_{\pi X}$: Bounds from $K_{\pi\bar{\nu}\nu}$;

Neutrino Trident Production : Clean tests for leptonic couplings.

$(g_\mu - 2)$



- X Boson Contribution

$$\mathcal{L} = \frac{1}{2}\sum_F \bar{\mu}[x_V\gamma^\rho + x_A\gamma^\rho\gamma^5]F\, X_\rho \tag{80}$$

- Neglecting flavor violating vertex, i.e. $F = \mu$.

$$[a_\mu]_a = \frac{m_\mu^2}{16\pi^2}\int_0^1 dz\, \frac{\left[x_V^2[(z-z^2)z] + x_A^2[(z-z^2)(z-4) - 2\frac{m_\mu^2}{m_X^2}z^3]\right]}{m_\mu^2 x^2 + m_X^2(1-x)} \tag{81}$$

In the very large Higgs mass assumption only $[a_\mu]_a$ contributes. However, for $c_\beta < .9$ it leads to negative sign to the dipole function, thus forbidding the explanation.

$(g_\mu - 2)$

We include the contributions from Light Higgs to the dipole function in the region where the asymptotic approximation to the integrals is still valid $m_h > 20 m_\mu$.

- General Yukawa Lagrangian

$$\mathcal{L}_Y = \sum_{h,F} \bar{\mu}[C_S + C_P \gamma_5] F\ h \tag{82}$$

- **Asymptotic Limit of the Integrals** : For $m_{h^+}, m_{h^0} >> m_\mu$

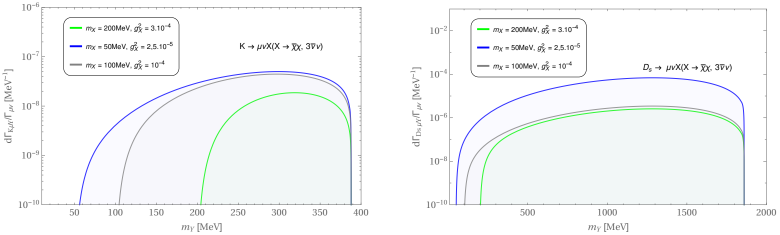
$$[a_\mu]_c \quad \rightarrow \quad \frac{m_\mu^2}{8\pi^2} (|C_S^+|^2 + |C_P^+|^2) \left(-\frac{1}{3}\right) \tag{83}$$

$$[a_\mu]_d^S \quad \rightarrow \quad \frac{m_\mu^2}{m_{h_0}^2} \frac{|C^0|_S^2}{8\pi^2} \left[\log\left[\frac{m_{h_0}^2}{m_\mu^2}\right] - \frac{7}{6}\right] \tag{84}$$

$$[a_\mu]_d^P \quad \rightarrow \quad \frac{m_\mu^2}{m_{h_0}^2} \frac{|C^0|_P^2}{8\pi^2} \left[\log\left[\frac{m_{h_0}^2}{m_\mu^2}\right] - \frac{11}{6}\right] \tag{85}$$

- Charged scalars cannot contribute to the correct sign;
- $c_\beta > .9$: Scalars allowed to stay in the decoupling region;
- For $c_\beta < .9$ (small v_X), light neutral scalars with $m_{h^0} \in (10 - 100) m_\mu$ are required to restore $g_\mu - 2$. Charged scalars are disfavored in the low-energy regime.

Forthcoming Bounds and Tests - $K_{\mu Y}$



(a) (b)

Figure: The Differential Decay Width $d\Gamma_{M\mu Y}$ scaled by $\Gamma_{\mu\nu}$ for $M = K, D_s$. The curves compare allowed and forbidden points in the optimal plot. In Fig.(b) the channel $D_s \to \tau\bar{\nu}_\tau(\tau \to \mu\nu_\tau\bar{\nu}_\mu)$ hide the distribution generated by $X \to \bar{\chi}\chi, 3\bar{\nu}\nu$.

Forthcoming Bounds and Tests - $M_{\mu 2ll}$

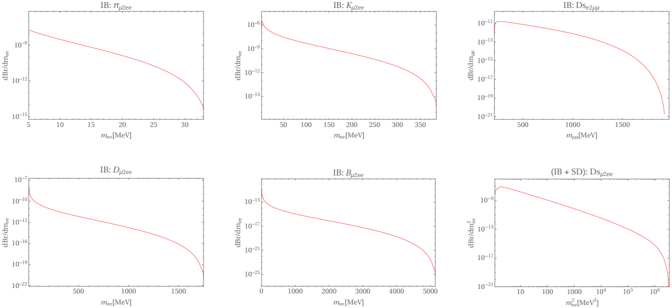


Figure: Differential Branching Ratio in terms of the dilepton invariant mass in the SM framework. Here the Inner Bremsstrahlung is considered to be dominant.

$$\mathrm{Br}(\pi_{\mu 2ee})_{IB} = 3.27 \cdot 10^{-5}; \quad \mathrm{Br}(K_{\mu 2ee})_{IB} = 2.48 \cdot 10^{-5}; \quad \mathrm{Br}(Ds_{\mu 2ee}) = 1.07 \cdot 10^{-6};$$

$$\mathrm{Br}(D_{\mu 2ee})_{IB} = 6.45 \cdot 10^{-8}; \quad \mathrm{Br}(B_{\mu 2ee})_{IB} = 1.66 \cdot 10^{-10}; \quad \mathrm{Br}(Ds_{e2\mu\mu}) = 5.46 \cdot 10^{-9};$$

$Ds_{\mu 2ll}$

$$Q_M \equiv \frac{\mathrm{Br}M_{\mu 2ee}[U(1)_X] - \mathrm{Br}M_{\mu 2ee}[SM]}{\mathrm{Br}M_{\mu 2ee}[SM]} \tag{86}$$

M	K	D	B
Q_M	$-2.1 \cdot 10^{-4}$	$1.8 \cdot 10^{-4}$	$7.8 \cdot 10^{-4}$

Table: Q_M from $(g_X^2, m_X) = (10^{-3}, 100)$ for Inner-Bremsstrahlung only. Here $(c_\beta, \kappa, F_{\mu\mu}) = (.6, n_1, 1)$.

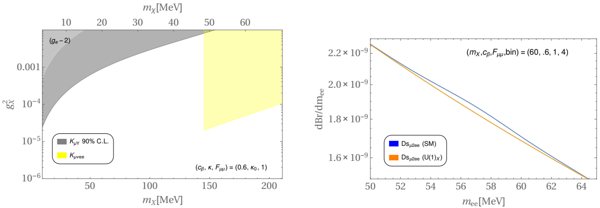


Figure: The parameter space for $\kappa = \frac{3}{2} c_\beta^2$ and the Differential Branching Ratio for $Ds_{\mu 2ee}$ for $(g_X^2, m_X) = (10^{-3}, 60)$. The *bin* $= 4$ MeV was chosen in order the Lorentzian and Gaussian arguments coincide around the X pole. Here $Q_M = 2.5 \cdot 10^{-4}$.

$Ds_{\mu 2ee}$

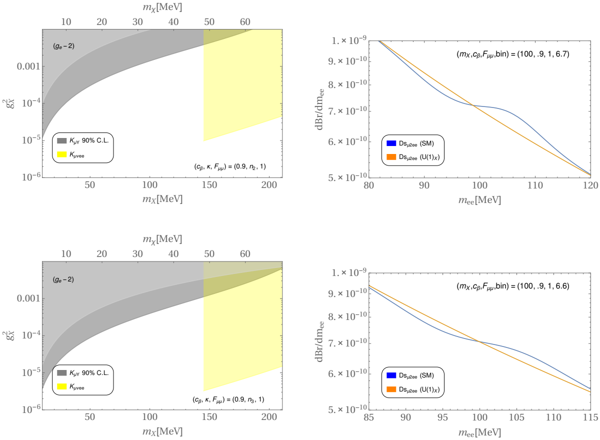


Figure: c_β can suppress the interference term from Meson X-Bremsstrahlung.

Conclusions

1. Light Z' and RH currents;
2. Dark Photons vs. Light Z': Axial vector couplings may provide a larger room in the parameter space;
3. Proton Puzzle must face $(g_\mu - 2)$;
4. Sensibility in Meson Leptonic Decays;