On the simple formula for SPS TMCI

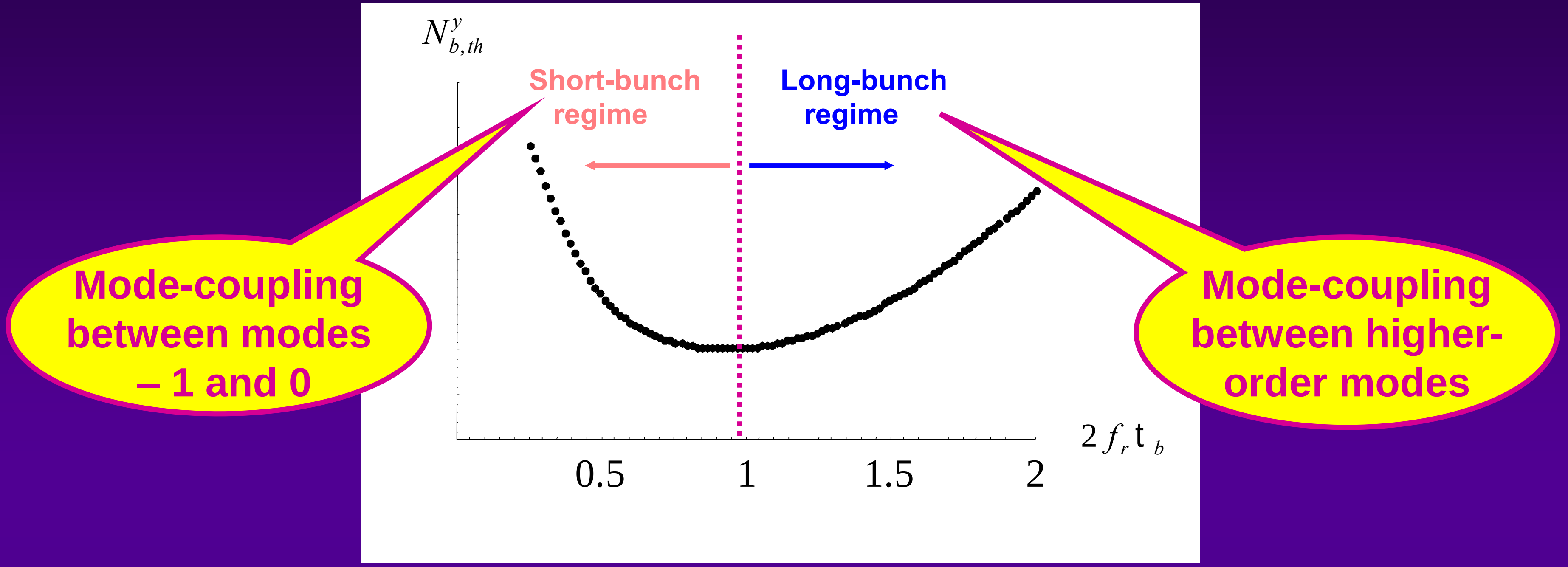
E. Métral

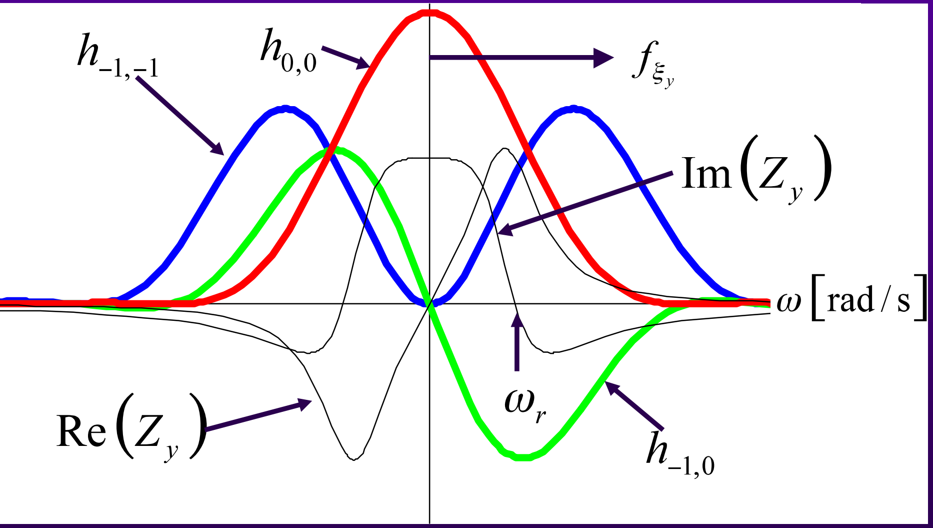
- **See http://cds.cern.ch/record/550194/files/p211.pdf => Several (5, seemingly diverse) formalisms lead to ~ same formula**
  - **1) TMCI (B. Zotter or even simpler, see after)**
  - **2) BBU**
  - **3) Fast blow-up**
  - **4) Post head-tail**
  - **5) Quasi coasting-beam approach using peak values of bunch current and momentum spread**

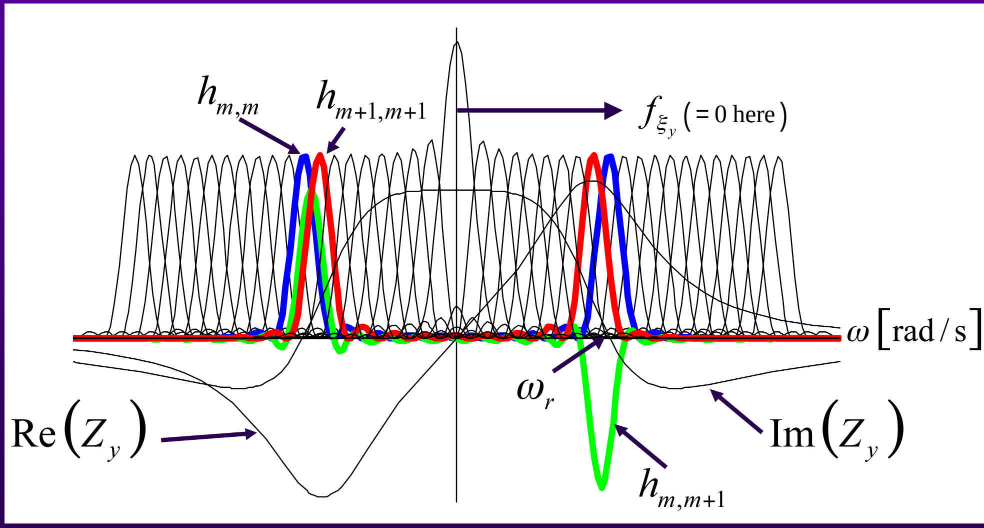
- **"Simple" TMCI approach (see after)**
  - **Reminder: Radial modes are the important ones**

Clarification following recent discussion with A. Burov et al. (FNAL workshop)

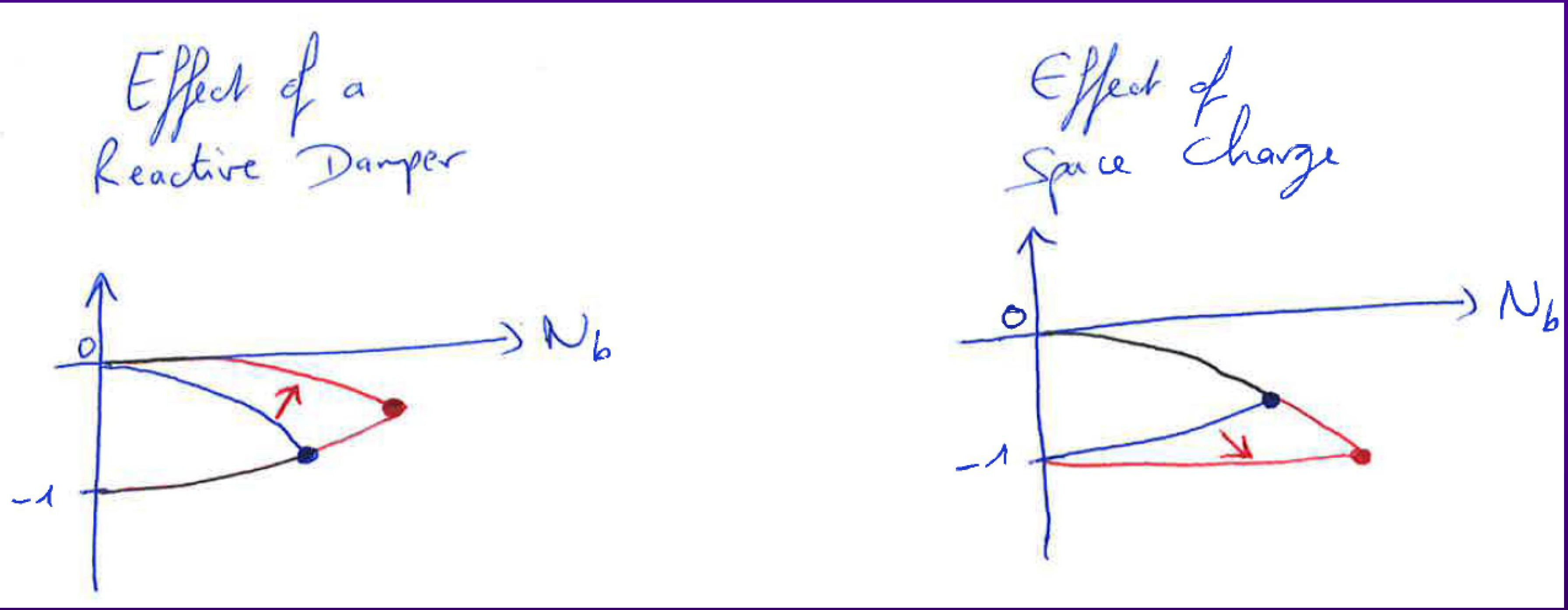
BROAD-BAND IMPEDANCE WITH NEITHER SC NOR ReaD







- “Short-bunch” regime => (HL-) LHC case
  - Both ReaD & SC are expected to be beneficial
    - ReaD => Shifts mode 0 up
    - SC => Shifts mode – 1 down



Still under discussion

- **"Long-bunch" regime => SPS case**
  - **Both ReaD & SC "*are expected*" to have no / small effect**
    - **ReaD => Modifies only (main) mode 0 and not the others (where the mode-coupling occurs)**
    - **SC => Modifies all the modes (except 0) similarly**

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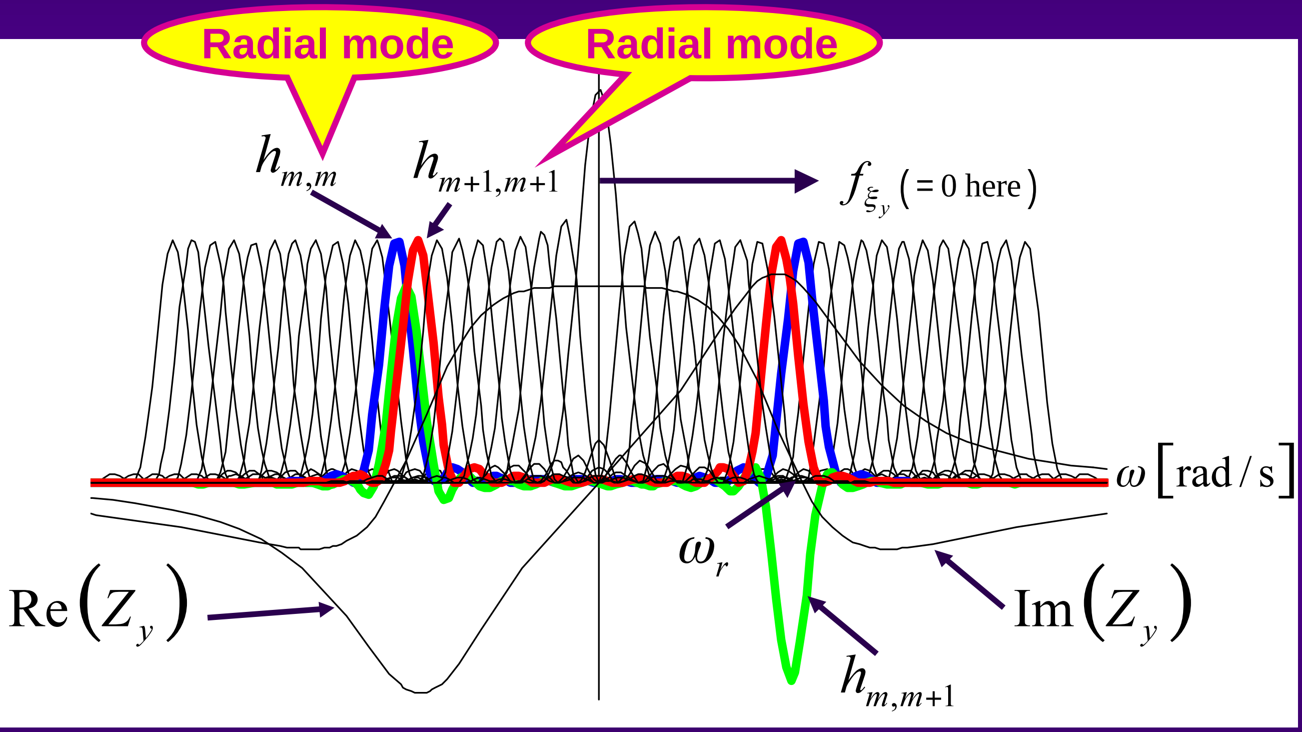
=> $$N^{y}_{b,th} \propto \mu \left| h \right| Q_y \, e_L$$

Coasting-beam result with peak values

And SC should have no effect for coasting beams (zero chroma)

ROLE OF AZIMUTHAL AND / OR RADIAL MODES

- **In the "simple" TMCI approach, what are important are the RADIAL MODES (*q*), as these are the ones defining the bunch spectrum (overlapping ~ the maximum of the real part of the impedance)**



- **The spectrum of mode *mq* is peaked at** $f_q \approx \frac{q+1}{2\,\tau_b}$ **and extends** $\sim \pm\, \tau_b^{-1}$

  $q \equiv m + 2\,k \qquad 0 \leq k < +\infty$

- There are ***q* nodes** on these "**standing-wave**" patterns

=> Similar situation for modes $m_1 q$ and $m_2 q$ (i.e. even for different azimuthal modes, as long as the radial one is the same)!