NMI (NEGATIVE-MASS INSTABILITY) & LMCI (LONGITUDINAL MODE-COUPLING INSTABILITY) FOR BUNCHED-BEAMS WITH GALACLIC

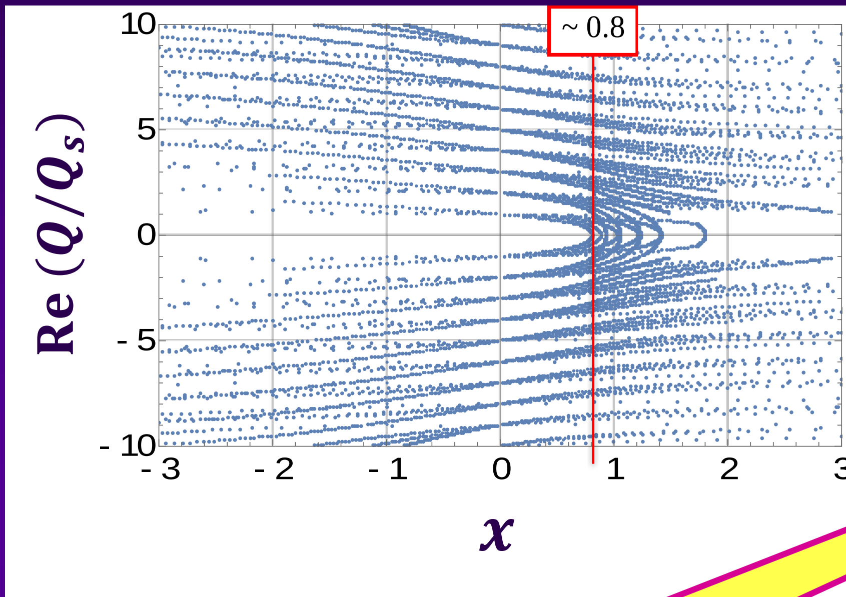
E. Métral

Many thanks for the on-going discussions with R. Baartman, A. Burov, A. Chao, Y.H. Chin, M. Migliorati, N. Mounet, K. Oide, C. Prior and G. Rumolo

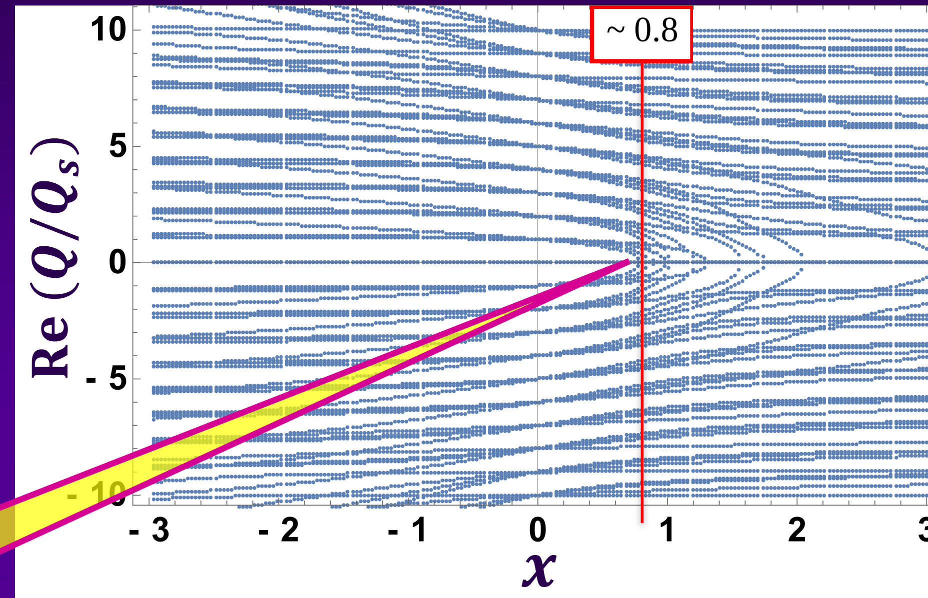
- **Reminder: See HSC section meeting on 14/05/18 (https://indico.cern.ch/event/725645/contributions/2992533/attachments/1648470/2635686/GALACLIC_EM_14-05-18.pdf)**
- **Some update**
- **Check: Can NMI really be explained by LMCI between azimuthal modes -1 and +1? => Was it already seen/predicted/checked in the past?**
- **Conclusion and next**

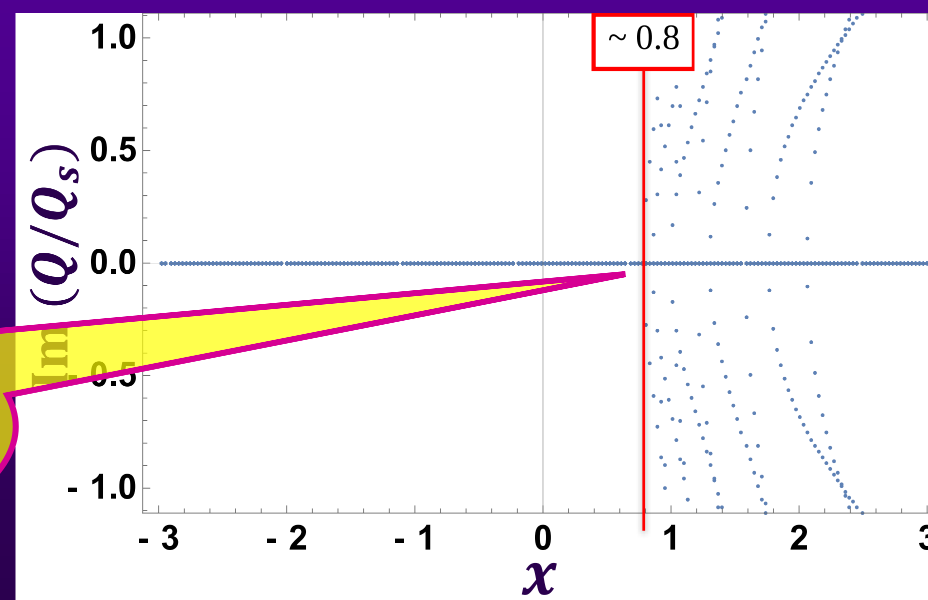
REMINDER: CONSTANT REACTIVE IMPEDANCE (1/3)

- **Laclare's formalism**



- **GALACLIC**





Mode-coupling between azimuthal modes m = - 1 and m = + 1

Contrary to transverse case,
there can be a (negative mass) instability:

1) > 0 inductive imped. BT

2) < 0 inductive imped. (SC) AT

=> Was not mentioned by Laclare1987

REMINDER: CONSTANT REACTIVE IMPEDANCE (2/3)

- **And $x \approx 0.8$ means Keil-Schnell-Boussard criterion (see below)**

- Case of a **Broad-Band resonator impedance** $f_r\,\tau_b = 2.8$
  - The threshold (mode-coupling) is reached when $\left|\varepsilon_{long}^{th}\right| \left|\dfrac{Z_l(p)}{p}\right|_{p=0} \approx 0.8$

    which can be re-written

$$\boxed{\left(\frac{\Delta p}{p_0}\right)^2_{\mathrm{FWHH}} \geq \frac{10}{3\,\pi} \frac{I_{b,peak}}{\beta^2 \left(E_{total}/e\right)\left|\eta\right|} \left|\frac{Z_l(p)}{p}\right|_0}$$

using $I_{b,peak} = \dfrac{3\,I_b}{2\,B}$ $\qquad \left(\dfrac{\Delta p}{p_0}\right)^2_{\mathrm{FWHH}} = \dfrac{\omega_s^2\,\tau_b^2}{2\,\eta^2}$

  - This is the **Keil-Schnell-Boussard criterion** (i.e. the Keil-Schnell criterion for coasting beams applied with peak values for bunched beams as proposed by Boussard). **Note that PWD leads to different thresholds below and above transition**

** No dependence on Q_s!*

*** The same formula can also be obtained by considering only the mode-coupling between the 2 adjacent modes overlapping the maximum of the resonator impedance*

REMINDER: CONSTANT REACTIVE IMPEDANCE (3/3)

- **As, with my previous approach (normalizing by the synchrotron tune Q_s which depends on intensity due to PWD), the intensity threshold is given by mode-coupling between -1 and +1, I would expect that the same threshold would be obtained by normalizing by the low-intensity synchrotron tune Q_{s0} because**

$$\frac{Q}{Q_{s0}} = \frac{Q}{Q_s} \times \frac{Q_s}{Q_{s0}}$$

It is going to 0 at the intensity threshold

… EXCEPT if this term goes to infinity…

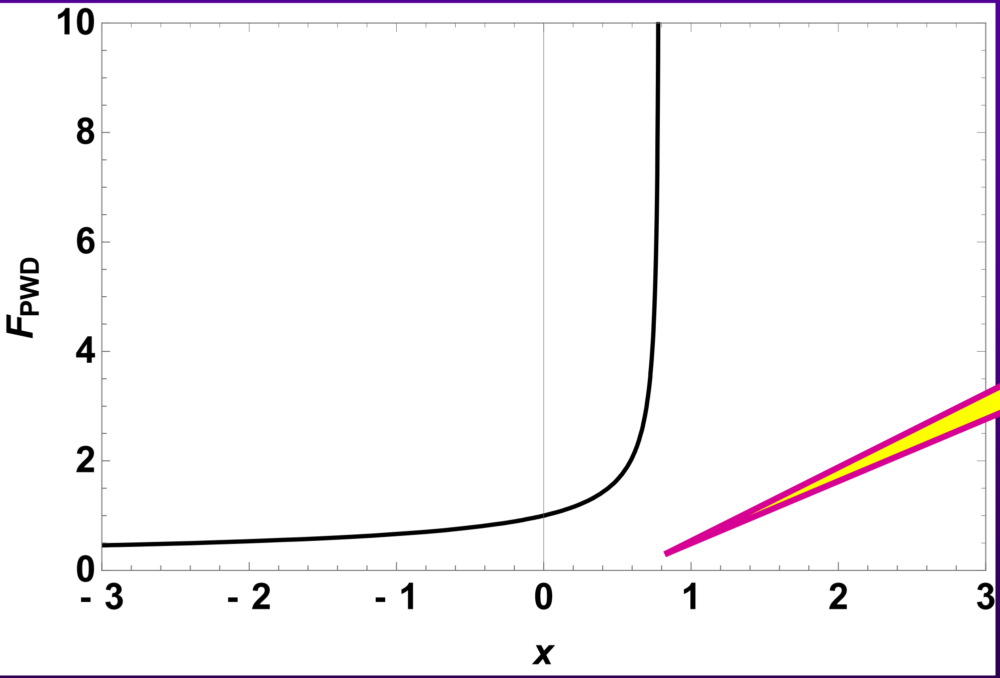
… And this is what happens…

SOME UPDATE (1/6)

- **Plotting my results wrt the low-intensity synchrotron tune Q_{s0} instead of the intensity-dependent Q_s, the following results are obtained (for the parabolic amplitude density considered)**

$$\frac{Q}{Q_{s0}} = \frac{Q}{Q_s} \times F_{PWD}$$

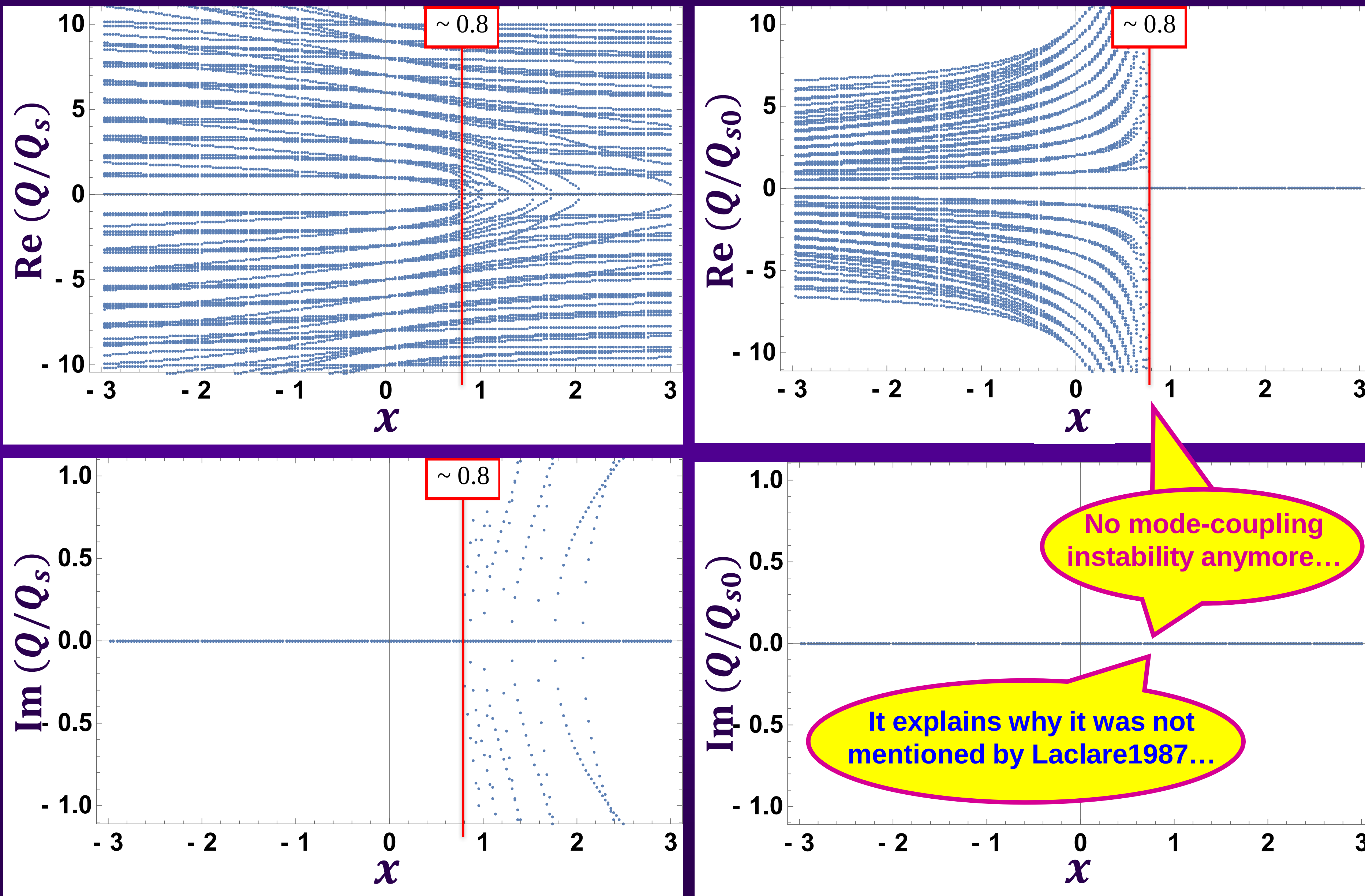
$$\boxed{F_{PWD} = \frac{Q_s}{Q_{s0}} = \frac{1}{\sqrt{1 - \frac{4}{\pi} x}}}$$



= ∞
when $x \approx 0.8$

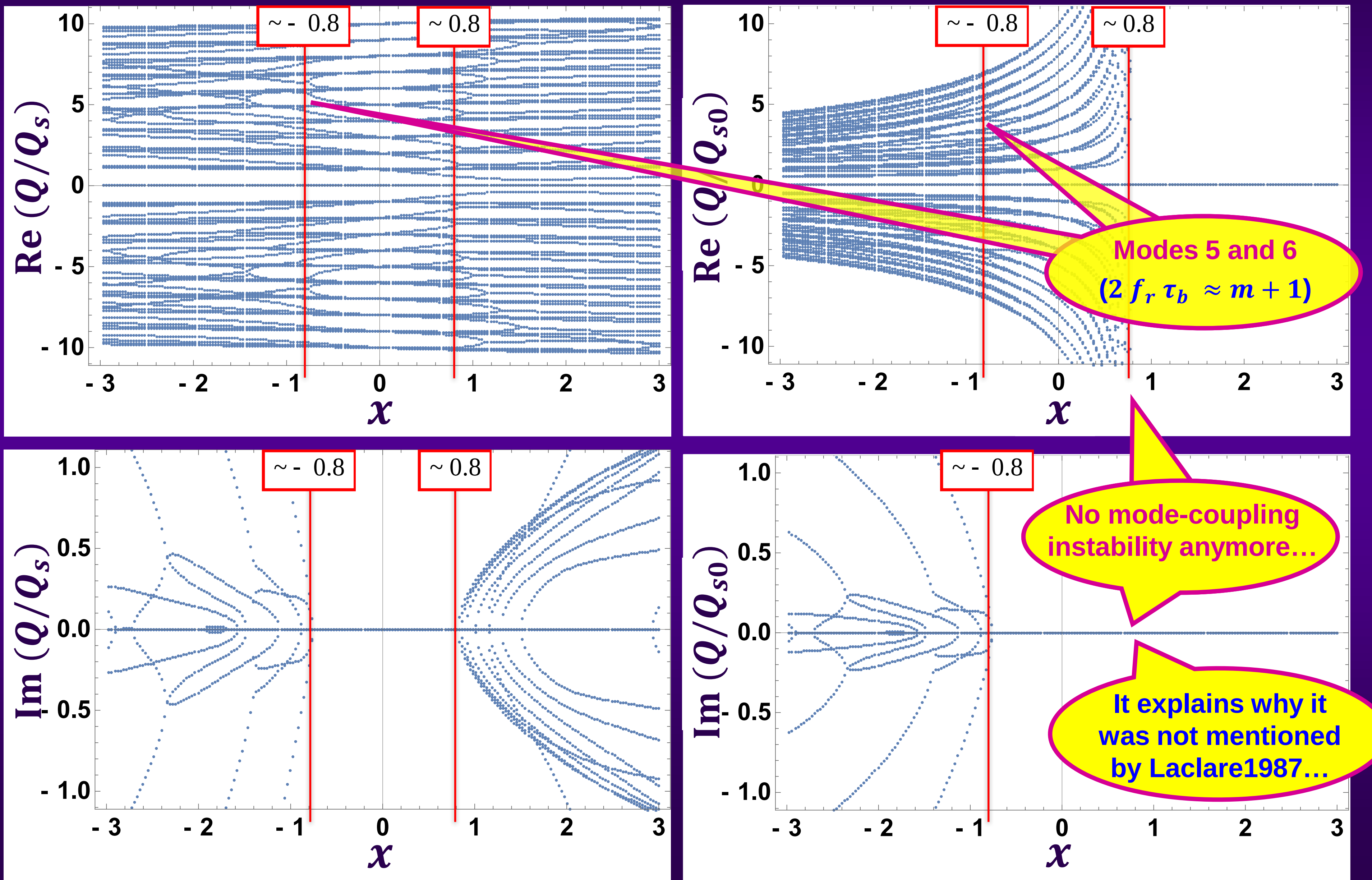
SOME UPDATE: CONSTANT REACTIVE IMPEDANCE (2/6)

- GALACLIC (using Q_s)
- GALACLIC (using Q_{s0})



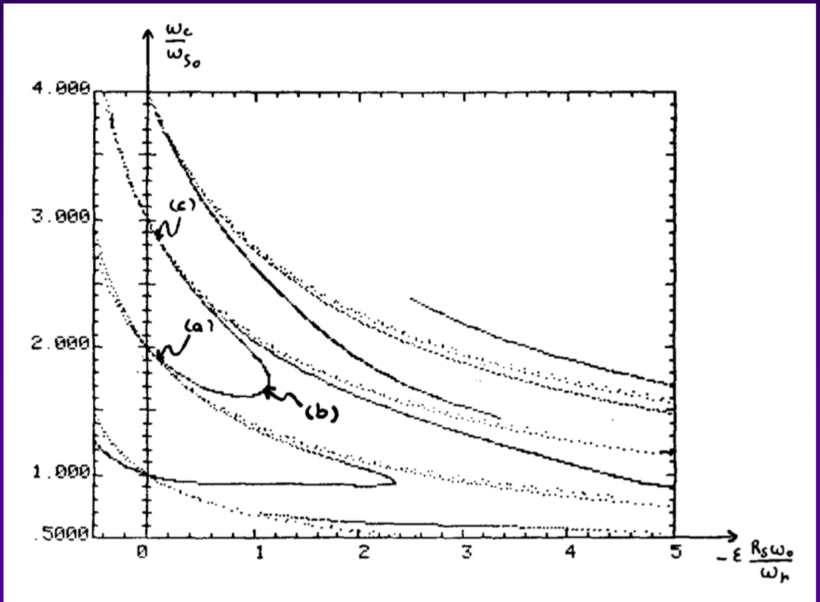
SOME UPDATE: BB IMPEDANCE $f_r \tau_b = 2.8$ (3/6)

- GALACLIC (using Q_s)
- GALACLIC (using Q_{s0})

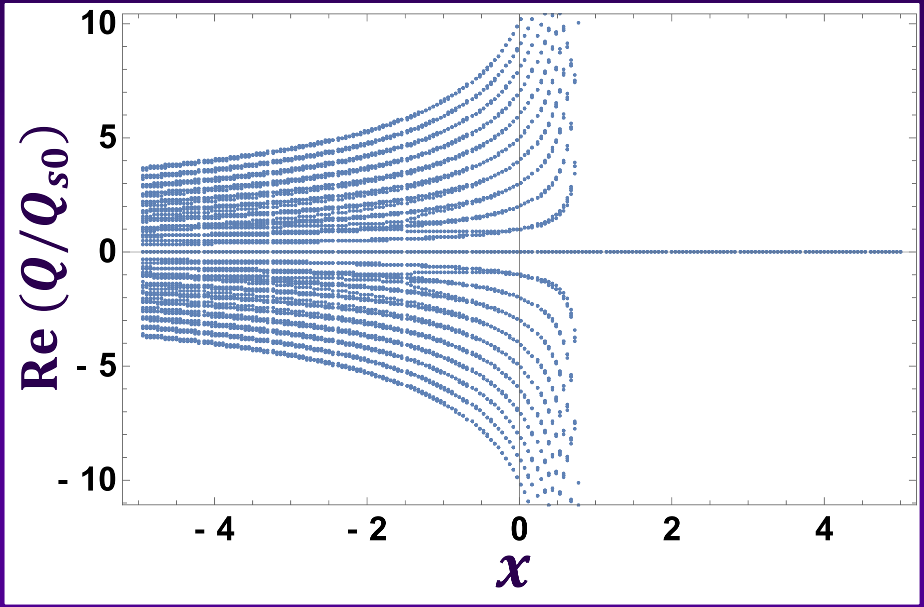


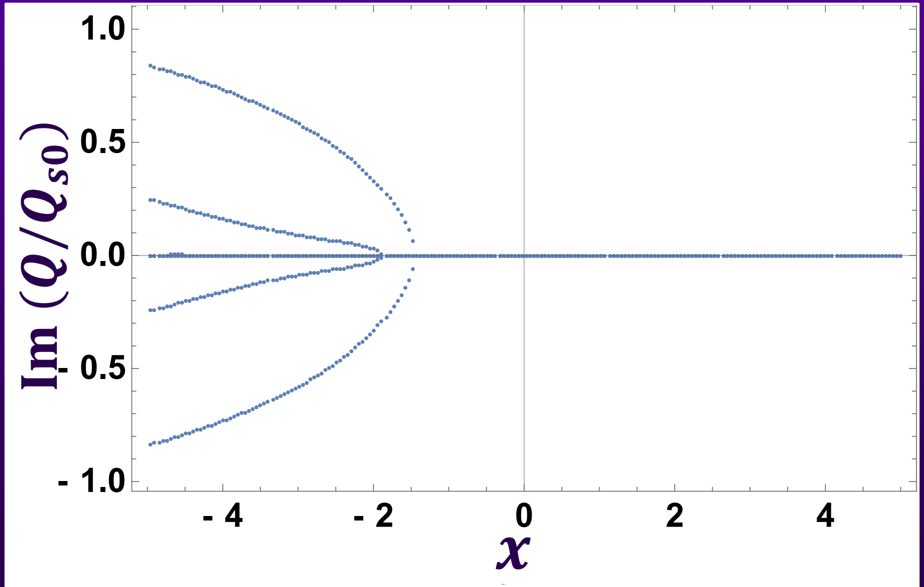
SOME UPDATE: BB IMPEDANCE $f_r \, \tau_b = 1.0$ (4/6)

- LACLARE1987
- GALACLIC (using Q_{s0})



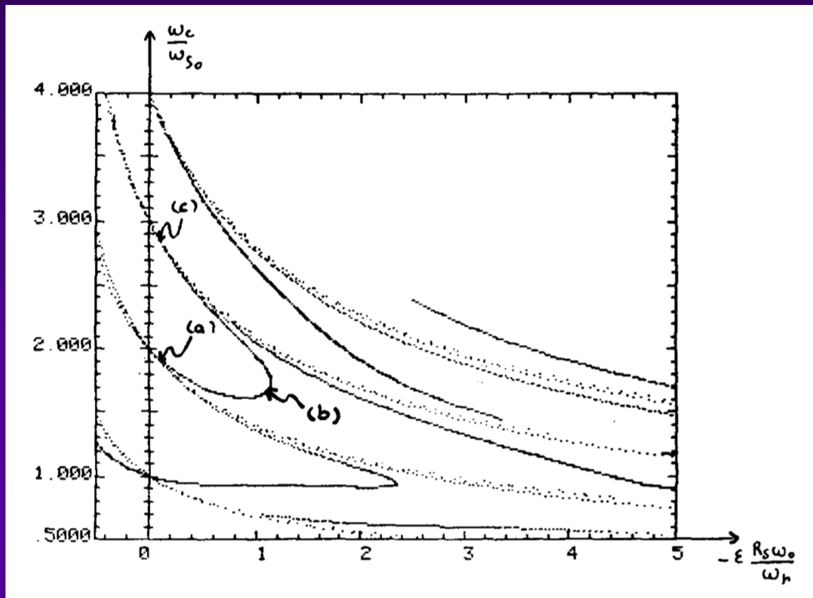
The example of Fig. 20 is a broad band with a bandwidth $\Delta f \cdot \tau_L = 1$





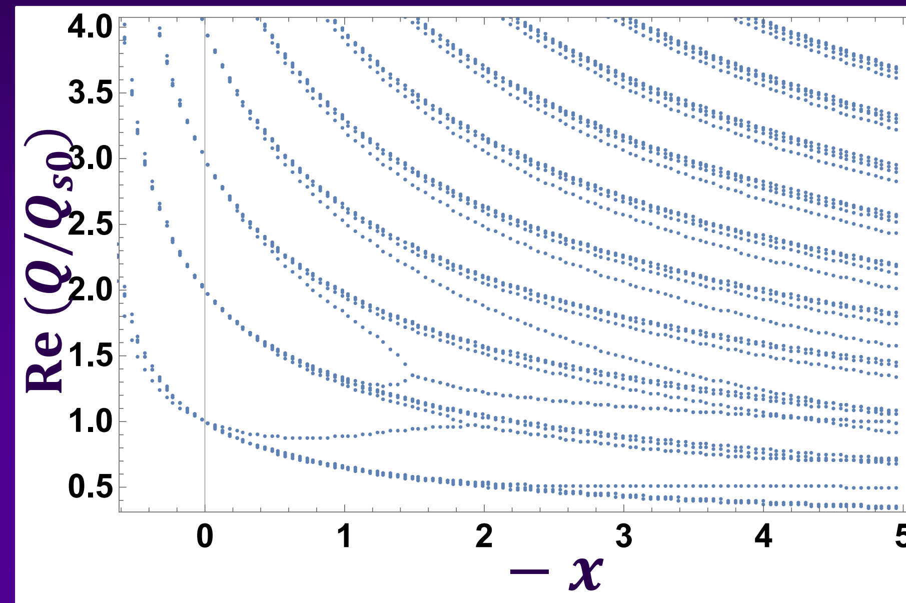
SOME UPDATE: BB IMPEDANCE $f_r \tau_b = 1.0$ (5/6)

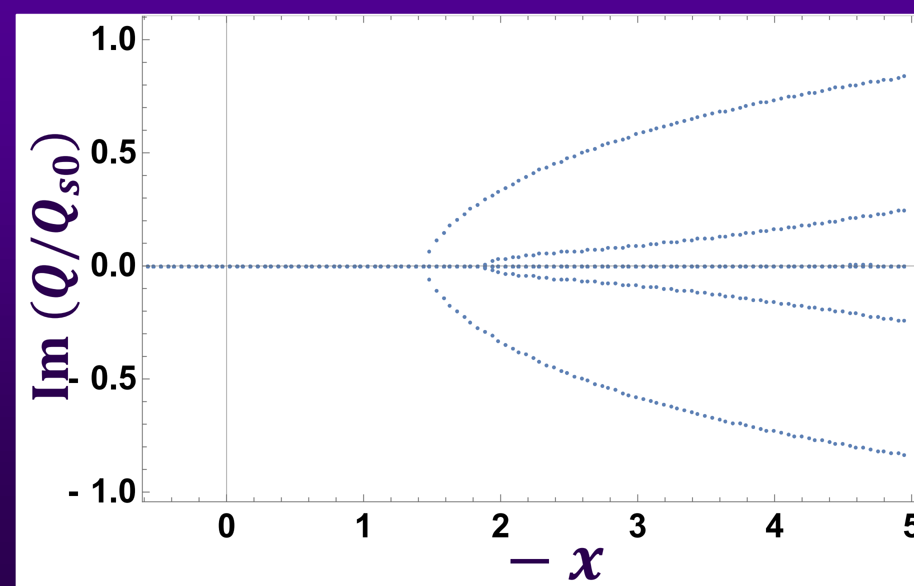
◆ **LACLARE1987**



The example of Fig. 20 is a broad band with a bandwidth $\Delta f \cdot \tau_L = 1$

◆ **GALACLIC (using Q_{s0})**

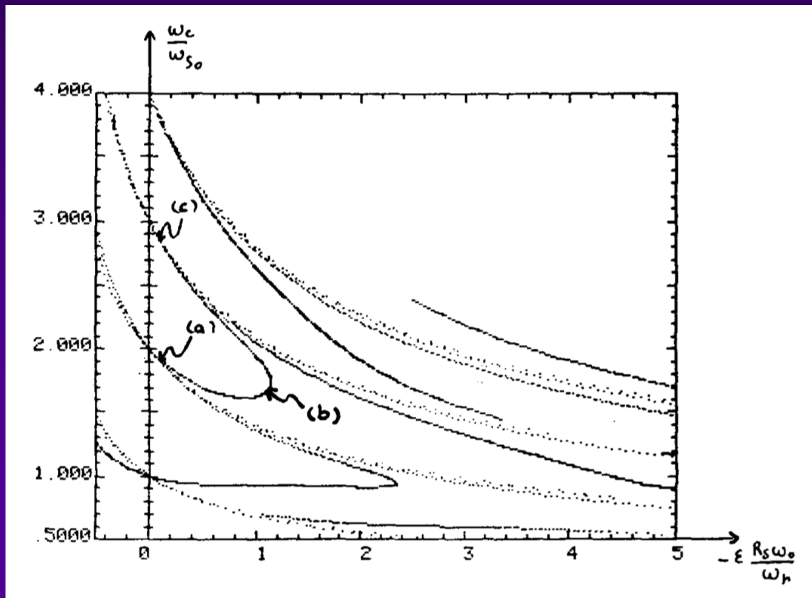




Close results (with in addition the imaginary part with GALACLIC)

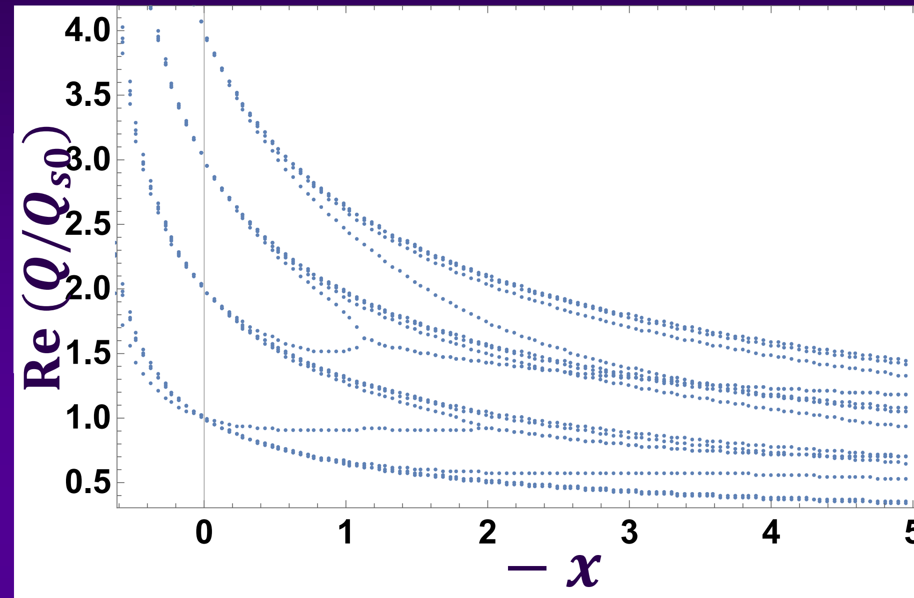
SOME UPDATE: BB IMPEDANCE $f_r \, \tau_b = 1.0$ (6/6)

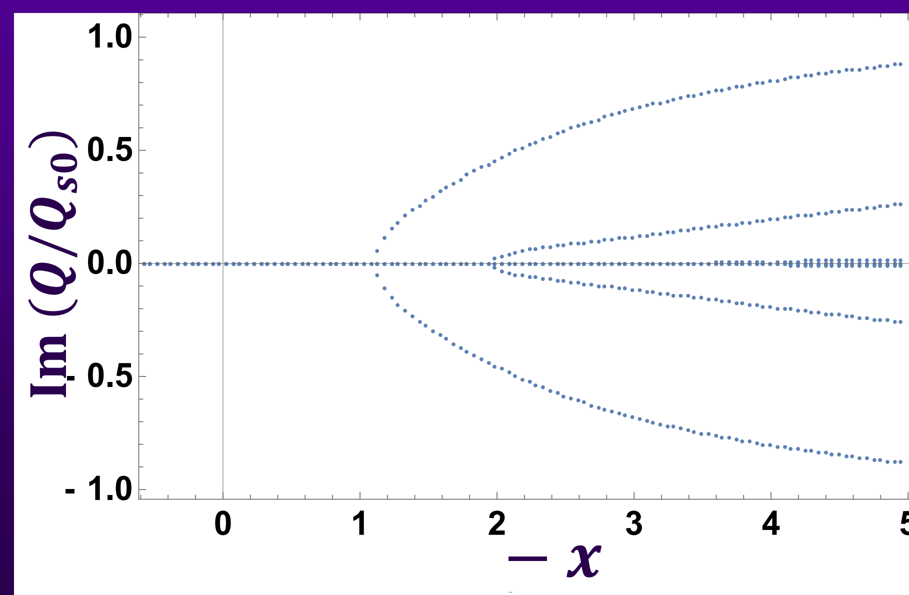
- **LACLARE1987**



The example of Fig. 20 is a broad band with a bandwidth $\Delta f \cdot \tau_L = 1$

- **GALACLIC (using Q_{s0})**





Even closer when I use $f_r \, \tau_b = 1.2$ …

CHECK: WHAT COULD BE FOUND IN PAST STUDIES? (1/4)

LONGITUDINAL INSTABILITIES OF BUNCHED BEAMS CAUSED BY SHORT-RANGE WAKE FIELDS

By

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A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

in

THE FACULTY OF GRADUATE STUDIES

DEPARTMENT OF PHYSICS

We accept this thesis as conforming to the required standard

THE UNIVERSITY OF BRITISH COLUMBIA

September 1995

Abstract

This thesis investigates the effects of short-range wake fields on the collective longitudinal motion of charged particle bunches in circular accelerators, especially the onset of instability. At high intensity, a short-range wake field can distort the bunch potential well and thereby change the stationary distribution. It is shown that if this is not taken into account, instability thresholds will be incorrectly predicted.

An integral equation derived from the linearized Vlasov equation is used to find the instability thresholds in the case of space-charge impedance alone for various distribution functions. The thresholds for instability caused by the coupling between the $m = \pm 1$ azimuthal modes have been obtained analytically for several common distributions. The criterion determining these thresholds appears to be the same as that for thresholds beyond which no stationary distribution can be found.

R. Baartman and B. Zotter [23] have shown that in the presence of space-charge interactions mode coupling will also cause instability for a few other types of distributions (so-called 'gaussian' and 'hollow beam'), but their calculations were done without taking into account potential well distortion, and thus the validity of these results is questionable. In this thesis we will try to study the problems self-consistently by including the effect of the potential well distortion.

[23] R. Baartman and B. Zotter, *Longitudinal Stability of Hollow Beams II: Mode-Coupling Theory*, TRIUMF Note TRI-DN-91-K177 (1991).

CHECK: WHAT COULD BE FOUND IN PAST STUDIES? (2/4)

value the eigenvalues ξ were found [23]. The results obtained in [24] for a 'hollow' distribution are shown in Fig. 11. Negative intensities correspond to space-charge impedance above transition (or inductive impedance below transition).

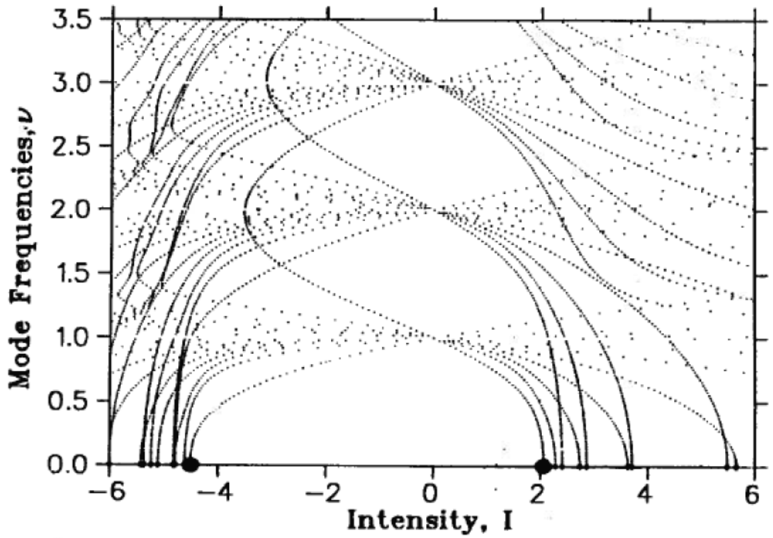

Figure 11: Coherent mode frequencies as a function of the intensity for a hollow beam distribution. Matrix size: 40 x 40.

[24] R. Baartman, S. Koscielniak, *Stability of Hollow Beams in Longitudinal Phase Space*, Particle Accelerators, **28**, 95 (1990).

We have also found the threshold for instability caused by coupling of $m = \pm 1$ azimuthal modes. Surprisingly, the criterion for this threshold is identical to that for the stationarity threshold.

CHECK: WHAT COULD BE FOUND IN PAST STUDIES? (3/4)

In Ref. [24], the case of a Gaussian bunch is also discussed

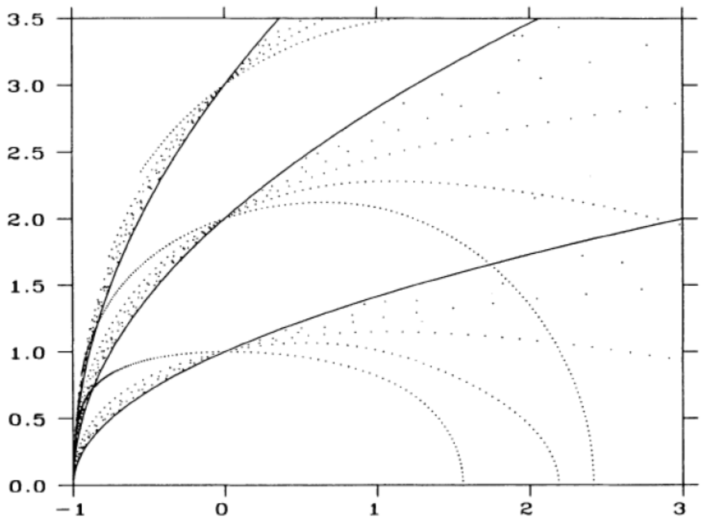

FIGURE 1 Coherent mode frequencies (dots) as a function of fractional shift in the square of the incoherent frequency (averaged over the bunch) for the case of a gaussian bunch. Solid lines are 1, 2, and 3 times the incoherent frequency. Matrix size: $\hat{p} = 10$.

focusing force is cancelled by space charge. Above transition, there is a dipole mode-coupling ($m = 1, -1$) instability threshold at $x \simeq 1.5$ (actually converges to 1.41 at high enough $\hat{p}$). This can be considered as the 'negative mass' instability for bunched beams; it is not generally observed because high intensity machines tend to be inductive wall dominated above transition and space charge dominated below.

CHECK: WHAT COULD BE FOUND IN PAST STUDIES? (4/4)

- **Other work**

LONGITUDINAL BUNCHED-BEAM INSTABILITIES IN A BARRIER RF SYSTEM

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0-7803-4376-X/98/$10.00 © 1998 IEEE 1545

Abstract

This paper deals with theoretical and numerical analysis of longitudinal instabilities in a barrier RF system. A theory was developed to formulate using the Vlasov equation and the synchrotron- energy mode expansion. The result can be expressed in a form of eigenvalue matrix. A simulation code ECLIPS (Evaluation Code for Longitudinal Instabilities in a Proton Synchrotron) was also developed. Both were applied to the JHF 50GeV proton synchrotron at injection. They show excellent agreements. The results demonstrate that the microwave and the negative mass instabilities in a bunched beam can be explained by mode-coupling instabilities.

Next, let us apply to a pure inductive impedance. The inductance is chosen to be equal to that of the resonator model at low frequency. The coasting beam theory predicts the excitation of negative mass instability. Figure 5 shows the coherent synchrotron mode frequencies and the growth rate versus the circulating current. Many synchrotron modes couple simultaneously with their negative mode partners (they are mirror images with respect to the Ω=0 line) at about 9A. The growth rate increases very rapidly after the mode-couplings. Figure 6

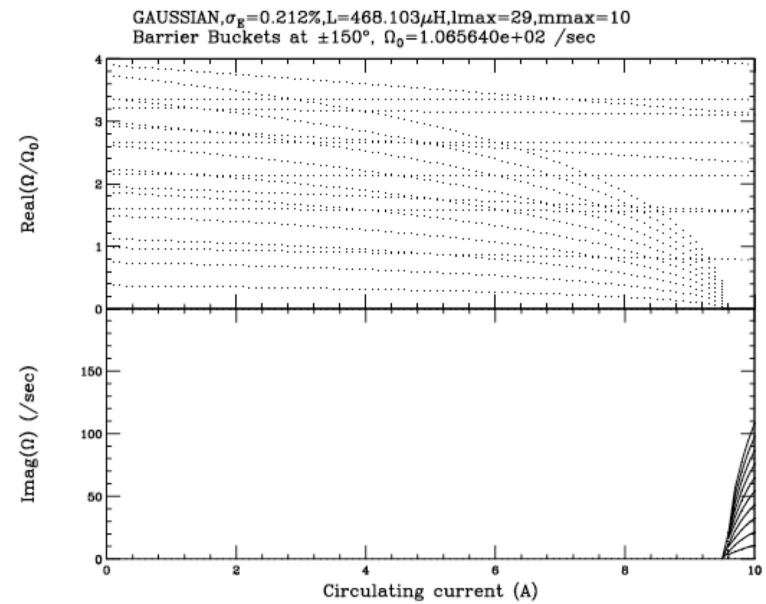

Figure 5: Coherent synchrotron mode frequencies and the growth rate for the inductive impedance (theory).

"In a barrier bucket system, NMI is expressed as mode-coupling between modes *m* and *–m*" (Y.H. Chin)

CONCLUSION AND NEXT

- ***REMINDER***
  - ***This approach is mainly for protons as synchrotron radiation and quantum fluctuations are not taken into account***
  - ***It is the simplest case, i.e. for a small bunch inside the RF bucket (i.e. there is no amplitude-dependent synchrotron tune)***
- **Laclare1987 results are recovered**
  - **No LMCI for 1) a constant reactive impedance and 2) a BB impedance below transition (when PWD is taken into account)**
  - **Similar result for a BB impedance above transition**
- **The threshold at $x \approx 0.8$ is given by**
  - **LMCI between azimuthal modes -1 and +1 (without including PWD effect)**
  - **Loss of steady state (from PWD) => See also http://inspirehep.net/record/921077/files/HEACC67_299-303.pdf from A.N. Lebedev, thanks AlexeyB!**
- **Other past studies?**
  - **In a barrier bucket system, NMI is expressed as LMCI between modes *m* and –*m* (YongHoC) => Treatment done including Landau damping**
  - **On-going discussions with many colleagues => Many thanks!**
- **Checks / benchmarks with other Vlasov solvers to be continued**
- **Tracking simulations to be done (on-going e.g. by MauroM)**