Power Corrections for Fixed-Order Subtractions

Frank Tackmann

Deutsches Elektronen-Synchrotron

NLP Workshop, Nikhef

Subtractions.

Starting Point.

$$\sigma(X) = \int \mathrm{d}\mathcal{T}_N \frac{\mathrm{d}\sigma(X)}{\mathrm{d}\mathcal{T}_N} = \underbrace{\int^{\mathcal{T}_{\mathrm{cut}}} \mathrm{d}\mathcal{T}_N \frac{\mathrm{d}\sigma(X)}{\mathrm{d}\mathcal{T}_N}}_{\equiv\, \sigma(X, \mathcal{T}_{\mathrm{cut}})} + \int_{\mathcal{T}_{\mathrm{cut}}} \mathrm{d}\mathcal{T}_N \frac{\mathrm{d}\sigma(X)}{\mathrm{d}\mathcal{T}_N}$$

- $\sigma(X)$: generic N-jet cross section
  - At LO$_N$: $\sigma^{\mathrm{LO}}(X) = \int \mathrm{d}\Phi_N \, B_N(\Phi_N)\, X(\Phi_N)$
  - X: All defining Born-level measurements/cuts
  - Φ_N: Born-level phase-space
- $\mathcal{T}_N$: physical IR-safe N-jet resolution variable

  $$\mathcal{T}_N(\Phi_N) = 0 \qquad \mathcal{T}_N(\Phi_{\geq N+1}) > 0 \qquad \mathcal{T}_N(\Phi_{\geq N+1} \to \Phi_N) \to 0$$

- $\mathrm{d}\sigma(X)/\mathrm{d}\mathcal{T}_N$: differential $\mathcal{T}_N$ spectrum
  - At LO$_N$: $\dfrac{\mathrm{d}\sigma^{\mathrm{LO}}(X)}{\mathrm{d}\mathcal{T}_N} = \sigma^{\mathrm{LO}}(X)\, \delta(\mathcal{T}_N)$
  - $\mathcal{T}_N > 0$ defines an IR-safe physical N+1-jet cross section

Subtractions.

Add and subtract $\sigma^{\rm sub}(\mathcal{T}_{\rm off}) = \sigma^{\rm sub}(\mathcal{T}_{\rm cut}) + \int_{\mathcal{T}_{\rm cut}}^{\mathcal{T}_{\rm off}} d\mathcal{T}_N \frac{d\sigma^{\rm sub}}{d\mathcal{T}_N}$

$$\sigma = \sigma(\mathcal{T}_{\rm cut}) + \int_{\mathcal{T}_{\rm cut}} d\mathcal{T}_N \frac{d\sigma}{d\mathcal{T}_N}$$

$$= \underbrace{\sigma^{\rm sub}(\mathcal{T}_{\rm off})}_{\text{NNLO}_N} + \underbrace{\int_{\mathcal{T}_{\rm cut}} d\mathcal{T}_N \left[\frac{d\sigma}{d\mathcal{T}_N} - \frac{d\sigma^{\rm sub}}{d\mathcal{T}_N}\theta(\mathcal{T} < \mathcal{T}_{\rm off})\right]}_{\text{NLO}_{N+1}} + \underbrace{\sigma(\mathcal{T}_{\rm cut}) - \sigma^{\rm sub}(\mathcal{T}_{\rm cut})}_{\text{neglect}}$$

- Subtractions $\sigma^{\rm sub}(\mathcal{T}_{\rm cut})$ and $d\sigma^{\rm sub}/d\mathcal{T}_N$
  - Have to reproduce leading singular limit of $\sigma(\mathcal{T}_{\rm cut})$ and $d\sigma/d\mathcal{T}_N$ such that we can neglect $\Delta\sigma(\mathcal{T}_{\rm cut}) \equiv \sigma(\mathcal{T}_{\rm cut}) - \sigma^{\rm sub}(\mathcal{T}_{\rm cut})$ for $\mathcal{T}_{\rm cut} \to 0$

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$$
\begin{aligned}
\sigma &= \sigma(\mathcal{T}_{\rm cut}) + \int_{\mathcal{T}_{\rm cut}} d\mathcal{T}_N \frac{d\sigma}{d\mathcal{T}_N} \\
&= \sigma^{\rm sub}(\mathcal{T}_{\rm off}) + \int_{\mathcal{T}_{\rm cut}} d\mathcal{T}_N \left[\frac{d\sigma}{d\mathcal{T}_N} - \frac{d\sigma^{\rm sub}}{d\mathcal{T}_N}\theta(\mathcal{T} < \mathcal{T}_{\rm off})\right] + \sigma(\mathcal{T}_{\rm cut}) - \sigma^{\rm sub}(\mathcal{T}_{\rm cut}) \\
&= \underbrace{\sigma^{\rm sub}(\mathcal{T}_{\rm cut})}_{\text{NNLO}_N} + \underbrace{\int_{\mathcal{T}_{\rm cut}} d\mathcal{T}_N \frac{d\sigma}{d\mathcal{T}_N}}_{\text{NLO}_{N+1}} + \underbrace{\sigma(\mathcal{T}_{\rm cut}) - \sigma^{\rm sub}(\mathcal{T}_{\rm cut})}_{\text{neglect}}
\end{aligned}
$$

- Subtractions $\sigma^{\rm sub}(\mathcal{T}_{\rm cut})$ and $d\sigma^{\rm sub}/d\mathcal{T}_N$
  - Have to reproduce leading singular limit of $\sigma(\mathcal{T}_{\rm cut})$ and $d\sigma/d\mathcal{T}_N$ such that we can neglect $\Delta\sigma(\mathcal{T}_{\rm cut}) \equiv \sigma(\mathcal{T}_{\rm cut}) - \sigma^{\rm sub}(\mathcal{T}_{\rm cut})$ for $\mathcal{T}_{\rm cut} \to 0$
- $\mathcal{T}_{\rm off}$ is a priori arbitrary and exactly cancels
  - Determines $\mathcal{T}_N$ range over which subtraction acts *differentially* in $\mathcal{T}_N$
  - Setting $\mathcal{T}_{\rm off} = \mathcal{T}_{\rm cut}$ reduces it to a global subtraction (aka slicing)

Power Expansion.

Expand cross section in powers of $\tau_N \equiv \frac{\mathcal{T}_N}{Q}$ and $\tau_{\rm cut} \equiv \frac{\mathcal{T}_{\rm cut}}{Q}$

(where Q is a typical hard scale whose precise choice is irrelevant for now)

$$\frac{d\sigma}{d\tau_N} = \frac{d\sigma^{(0)}}{d\tau_N} + \frac{d\sigma^{(2)}}{d\tau_N} + \frac{d\sigma^{(4)}}{d\tau_N} + \cdots$$

$$\sigma(\tau_{\rm cut}) = \sigma^{(0)}(\tau_{\rm cut}) + \sigma^{(2)}(\tau_{\rm cut}) + \sigma^{(4)}(\tau_{\rm cut}) + \cdots$$

- Leading-power (singular) terms

$$\frac{d\sigma^{\rm sing}}{d\tau_N} \equiv \frac{d\sigma^{(0)}}{d\tau_N} \sim \delta(\tau_N) + \left[\frac{\ln^{n-1}\tau_N}{\tau_N}\right]_+$$

$$\sigma^{\rm sing}(\tau_{\rm cut}) \equiv \sigma^{(0)}(\tau_{\rm cut}) \sim \ln^n \tau_{\rm cut}$$

  - Plus distributions encode real-virtual cancellation of IR singularities

- Subleading-power (nonsingular) terms

$$\tau_N \frac{d\sigma^{(2k)}}{d\tau_N} \sim \mathcal{O}(\tau_N^k) \qquad \sigma^{(2k)}(\tau_{\rm cut}) \sim \mathcal{O}(\tau_{\rm cut}^k)$$

Putting Everything Together.

$$\sigma = \underbrace{\sigma^{\rm sub}(\tau_{\rm cut})}_{\text{NNLO}_N} + \underbrace{\int_{\tau_{\rm cut}} d\tau_N \frac{d\sigma}{d\tau_N}}_{\text{NLO}_{N+1}} + \underbrace{\Delta\sigma(\tau_{\rm cut})}_{\text{neglect}}$$

Subtractions have to satisfy

$$\sigma^{\rm sub}(\tau_{\rm cut}) = \sigma^{(0)}(\tau_{\rm cut})\,[1 + \mathcal{O}(\tau_{\rm cut})]$$

such that neglecting $\Delta\sigma(\tau_{\rm cut})$ only misses $\mathcal{O}(\tau_{\rm cut})$ power-suppressed terms

$$\Delta\sigma(\tau_{\rm cut}) = \sigma(\tau_{\rm cut}) - \sigma^{\rm sub}(\tau_{\rm cut}) = \sigma^{(2)}(\tau_{\rm cut}) + \cdots \sim \mathcal{O}(\tau_{\rm cut})$$

Tradeoff: Lowering $\tau_{\rm cut}$ ...

- ... reduces size of missing power corrections $\Delta\sigma(\tau_{\rm cut})$
- ... increases numerical cancellations between first two terms
  - Requires numerically more precise calculation of $d\sigma/d\tau_N$ in a region where the N+1-jet NLO calculation quickly becomes much less stable
  - Computational cost increases substantially

Pros and Cons.

Key advantages

- All IR-singular contributions are projected onto physical observable $\mathcal{T}_N$
  - Subtractions are given by singular limit of a physical cross section
  - For a suitable observable can be systematically computed using a factorization theorem
  - Also allows computing power corrections, giving significant improvements
  - Simpler structure and fewer subtraction terms
- Nonsingular contributions are immediately given in terms of existing lower-order Born+1-jet calculations

Potential drawbacks

- Subtractions are nonlocal (i.e. not point-by-point in real emission phase space)
  - Phase-space slicing in $\mathcal{T}_N$ = global (maximally nonlocal) subtraction
- In practice, it is a question of numerical stability whether this is a disadvantage or not
  - Naively expect larger numerical cancellations (since they happen later)
  - Most relevant is numerical stability of real-virtual and double-real matrix elements in deep unresolved limit which are always needed regardless of subtraction method

Resolution Variables for Physical Subtractions.

In principle, any IR-sensitive resummable variable could be used

In fact, in the context of resummation, the singular terms are routinely obtained as a "by-product" of the resummation and used as subtraction to get the nonsingular terms.

Other variables used as subtractions for NNLO calculations

- Color-singlet production: q_T subtractions utilize q_T of color-singlet system [Catani, Grazzini '07]
  - Very sucessfully applied to Higgs, Drell-Yan, and essentially any combination of diboson production
    [Catani et al. '07, '09, '11; Ferrera, Grazzini, Tramontano '11, '14; Cascioli et al. '14; Gehrmann et al. '14; Grazzini, Kallweit, Rathlev, Torre '13, '15; several more implementations]
  - Primarily used as global subtraction (as far as I know)
- Top-quark decay rate: inclusive jet mass (global) [Gao, Li, Zhu '12]
- $e^+e^- \to t\bar{t}$: Total radiation energy (global) [Gao, Zhu '14]

Resolution Variables for Physical Subtractions.

N-jettiness event shape is explicitly designed as N-jet resolution variable with simplest possible factorization/resummation properties [Stewart, FT, Waalewijn '10]

- Differential 0-jettiness subtractions are implemented in GENEVA Monte Carlo (basis of its NNLO+NNLL′+PS matching) [Alioli et al. '13, '15]
- Global 0-jettiness
  - Drell-Yan and Higgs [Gaunt, Stahlhofen, FT, Walsh '15]
  - VH, diphoton [Campbell, Ellis, Li, Williams '16]
  - NNLO color-singlet in MCFM 8 [Boughezal et al. '16]
- Global 1-jettiness
  - $pp \to V/H+j$ [Boughezal, Focke, Liu, Petriello + Campbell, Ellis, Giele '15, '16]
  - $pp \to \gamma+j$ [Campbell, Ellis, Williams '16]

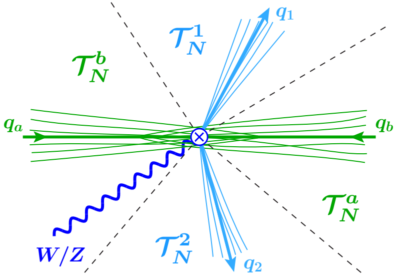
N-Jettiness.

N-Jettiness Event Shape.

[Stewart, FT, Waalewijn, '10]

$$\mathcal{T}_N = \sum_k \min\Big\{\frac{2q_a\cdot p_k}{Q_a}, \frac{2q_b\cdot p_k}{Q_b}, \frac{2q_1\cdot p_k}{Q_1}, \frac{2q_2\cdot p_k}{Q_2}, \dots, \frac{2q_N\cdot p_k}{Q_N}\Big\}$$
$$\equiv \mathcal{T}_N^a + \mathcal{T}_N^b + \mathcal{T}_N^1 + \cdots + \mathcal{T}_N^N$$

- Partitions phase space into N jet regions and 2 beam regions
- $Q_{a,b}$, Q_j determine distance measure
  - Geometric measures: $Q_i = 2\rho_i E_i$
- Massless born reference momenta q_i



$$q_{a,b} = x_{a,b}\frac{E_{\rm cm}}{2}(1, \pm\hat{z}), \quad q_j = E_j(1, \vec{n}_j)$$

Their choice corresponds to choosing an (IR-safe) Born projection

- Does not affect leading-power structure and resummation
- Part of N-jettiness definition and does affect power-suppressed terms

All-order Singular Structure.

$$\frac{\mathrm{d}\sigma^{\mathrm{sing}}(X)}{\mathrm{d}\tau_N} = \int \mathrm{d}\Phi_N \, \frac{\mathrm{d}\sigma^{\mathrm{sing}}(\Phi_N)}{\mathrm{d}\tau_N} \, X(\Phi_N)$$

$$\frac{\mathrm{d}\sigma^{\mathrm{sing}}(\Phi_N)}{\mathrm{d}\tau_N} = \mathcal{C}_{-1}(\Phi_N)\,\delta(\tau_N) + \sum_{m\geq 0} \mathcal{C}_m(\Phi_N)\,\mathcal{L}_m(\tau_N)$$

$$= \sum_{n\geq 0} \Big[\mathcal{C}_{-1}^{(n)}(\Phi_N)\,\delta(\tau_N) + \sum_{m=0}^{2n-1} \mathcal{C}_m^{(n)}(\Phi_N)\,\mathcal{L}_m(\tau_N)\Big] \Big(\frac{\alpha_s}{4\pi}\Big)^n$$

- Singular only depend on Born phase space $\Phi_N \equiv \{q_i, \lambda_i, \kappa_i\}$
  - Subtractions are FKS-like in this respect
- Integrated subtractions

$$\sigma^{\mathrm{sing}}(\Phi_N, \tau_{\mathrm{cut}}) = \mathcal{C}_{-1}(\Phi_N) + \sum_{m\geq 0} \mathcal{C}_m(\Phi_N)\,\frac{\ln^{m+1}(\tau^{\mathrm{cut}})}{m+1}$$

  - $\mathcal{C}_{-1}(\Phi_N)$ contains finite remainder of N-parton virtuals
  - At LO: $\mathcal{C}_{-1}^{(0)}(\Phi_N) = B_N(\Phi_N)$

Factorization Theorem.

[Stewart, FT, Waalewijn, '09, '10]

$$\frac{\mathrm{d}\sigma^{\mathrm{sing}}(\Phi_N)}{\mathrm{d}\mathcal{T}_N} = \int \mathrm{d}t_a\, B_a(t_a, x_a, \mu) \int \mathrm{d}t_b\, B_b(t_b, x_b, \mu) \biggl[\prod_{i=1}^{N} \int \mathrm{d}s_i\, J_i(s_i, \mu)\biggr]$$

$$\times\, \vec{C}^\dagger(\Phi_N, \mu)\, \widehat{S}_\kappa\Bigl(\mathcal{T}_N - \frac{t_a}{Q_a} - \frac{t_b}{Q_b} - \sum_{i=1}^{N} \frac{s_i}{Q_i}, \{\hat{q}_i\}, \mu\Bigr) \vec{C}(\Phi_N, \mu)$$

- All functions are IR finite and have an operator definition in SCET
- To obtain subtraction coefficients simply FO expand and collect terms
- Simplifying features of N-jettiness
  - No dependence on jet algorithm (jet clustering, jet radius, etc.)
  - No recoil effects from soft radiation
  - No additional $\vec{p}_T$ dependence or convolutions, no rapidity divergences
  - Overlap between soft and collinear contributions vanishes in pure dim. reg.

$\Rightarrow$ Become particularly useful at subleading power

Power Corrections.

Moult, Rothen, Stewart, FT, Zhu arXiv:1612.00450, arXiv:1710.03227

Ebert, Moult, Stewart, FT, Vita, Zhu, arXiv:1807.10764

Missing Power Corrections.

There is an important caveat

- Power suppression gets weaker at higher orders in α_s due to stronger log enhancement

$$\sigma^{(2)}(\tau_{\mathrm{cut}}) = \sum_{n=0} \sigma^{(2,n)}(\tau_{\mathrm{cut}}) \Big(\frac{\alpha_s}{4\pi}\Big)^n$$

$$\sigma^{(2,n)}(\tau_{\mathrm{cut}}) = \tau_{\mathrm{cut}} \sum_{m=0}^{2n-1} A_m^{(2,n)} \ln^m \tau_{\mathrm{cut}}$$

$\Rightarrow$ Dominant missing terms at $\mathcal{O}(\alpha_s^n)$ scale as

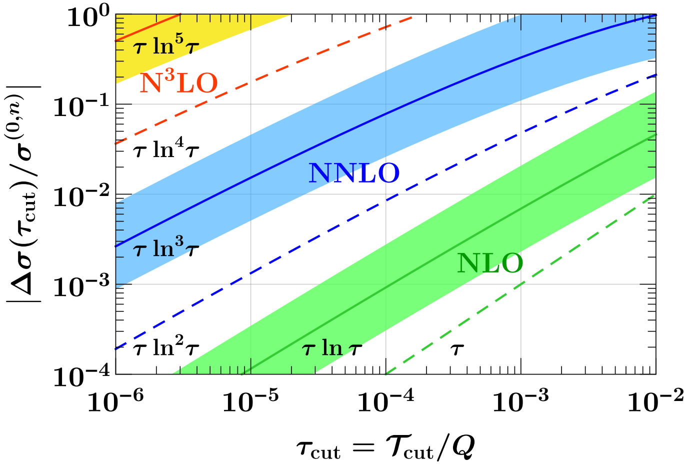
$$\Delta\sigma(\tau_{\mathrm{cut}}) \sim \alpha_s^n \, \tau_{\mathrm{cut}} \, \ln^{2n-1} \tau_{\mathrm{cut}}$$

- Can use this to get a rough order of magnitude estimate of their size by taking $A^{(2,n)} = \sigma^{(0,n)} \times [1/3, 3]$
- Works quite well for most cases we have checked

Estimating Size of Missing Power Corrections.

Simple estimate of $\Delta\sigma(\tau_{\mathrm{cut}})$ at N^{n}LO

- relative to full N^{n}LO coefficient

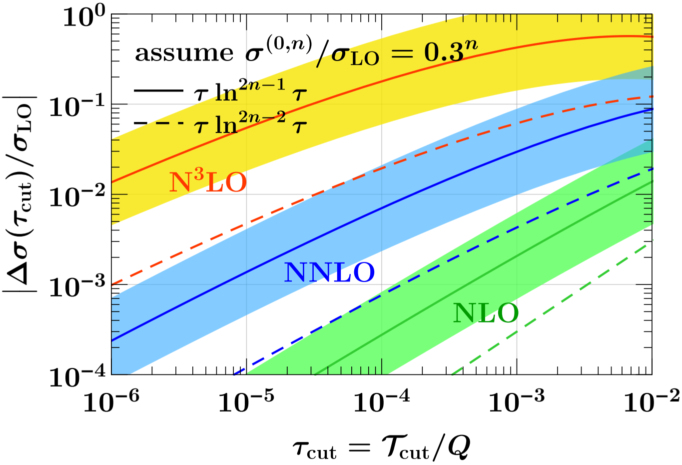


Typical values in current implementations are in $\tau_{\mathrm{cut}} \simeq 10^{-5} \dots 10^{-3}$ range

Estimating Size of Missing Power Corrections.

Simple estimate of $\Delta\sigma(\tau_{cut})$ at N^{n}LO

- relative to σ_{LO}, assuming a 30% correction at each α_s order

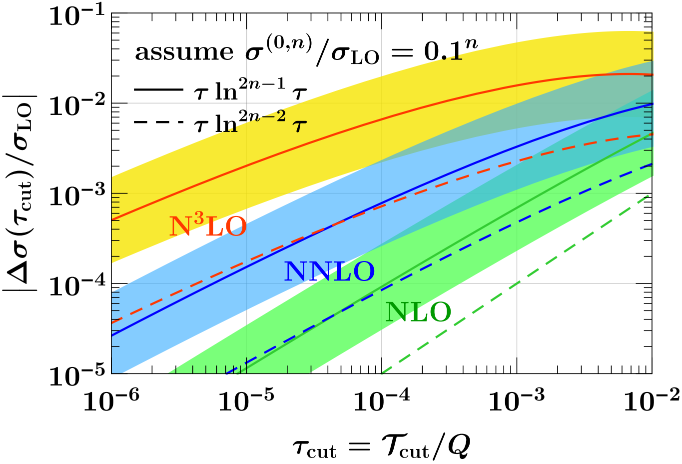


Typical values in current implementations are in $\tau_{cut} \simeq 10^{-5} \ldots 10^{-3}$ range

Estimating Size of Missing Power Corrections.

Simple estimate of $\Delta\sigma(\tau_{\rm cut})$ at N^nLO

- relative to $\sigma_{\rm LO}$, assuming a 10% correction at each α_s order



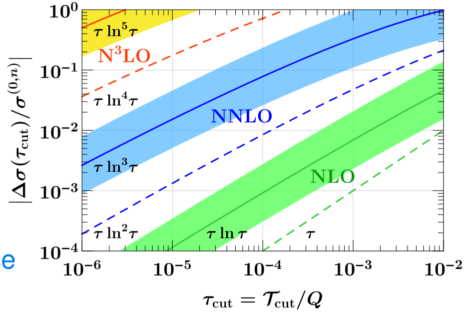
Typical values in current implementations are in $\tau_{\rm cut} \simeq 10^{-5} \ldots 10^{-3}$ range

Improving the Subtractions.

Recall

$$\sigma^{\rm sub}(\tau_{\rm cut}) = \sigma^{\rm sing}(\tau_{\rm cut})\,[1 + \mathcal{O}(\tau_{\rm cut})]$$

$$\Delta\sigma(\tau_{\rm cut}) = \sigma(\tau_{\rm cut}) - \sigma^{\rm sub}(\tau_{\rm cut})$$



Calculate dominant power corrections and include them in $\sigma^{\rm sub}(\tau_{\rm cut})$ to reduce size of missing $\Delta\sigma(\tau_{\rm cut})$ terms

[Moult, Rothen, Stewart, FT, Zhu '16, '17; Ebert, Moult, Stewart, FT, Vita, Zhu '18]

[Boughezal, Liu, Petriello '16; Boughezal, Isgro, Petriello'18]

- Each log term can potentially give an order of magnitude improvement
  - Even the LL next-to-leading power (NLP) terms are very interesting
- Many things that could be ignored at leading power start to matter at subleading power.
  - Choice of N-jettiness measure, Born measurement, ...

SCET at Subleading Power.

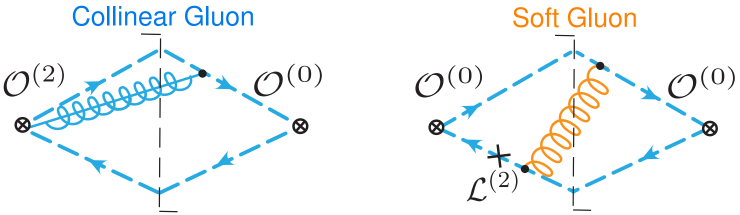
SCET provides manifest organization of sources of power corrections

- Insertions of subleading SCET Lagrangian
  - Corrects dynamics of propagating soft and collinear particles
- Subleading hard-scattering operators
  - SCET helicity operator basis extended to subleading power
    [Feige, Kolodrubetz, Moult, Stewart; Moult, Vita, Stewart '17]
- Subleading corrections to the measurement/observable

At FO, we don't actually need a full NLP factorization theorem

- Sufficient to perform FO calculation with SCET organizing it into contributions from hard, collinear, and soft
  - Typically easiest to expand the (known) full-theory amplitudes in terms of λ using soft and collinear momentum scaling
  - Easier and safer than expanding full-theory calculation directly in $\mathcal{T}_N/Q$

Simplest Example: Subleading Thrust at NLO.



$$\frac{1}{\sigma_0}\frac{d\sigma^{(2,1)}}{d\tau} = 8C_F\Big[\Big(\frac{1}{\epsilon} + \ln\frac{\mu^2}{Q^2\tau}\Big) - \Big(\frac{1}{\epsilon} + \ln\frac{\mu^2}{Q^2\tau^2}\Big)\Big] = 8C_F \ln\tau\,,$$

- Result gives directly (no additional expansions) the NLP contribution
- $1/\epsilon$ poles must cancel between collinear and soft contributions
  - In SCET these are UV poles arising from the scale separation between different sectors
  - From full-theory point of view these are IR poles and must cancel because there are no nontrivial IR divergences at subleading power

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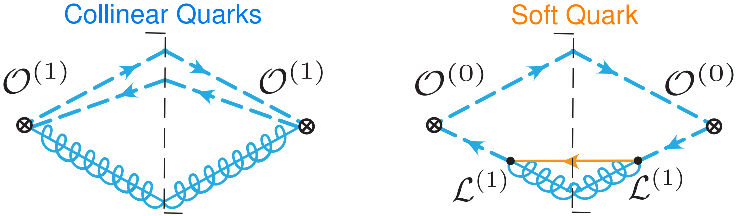


$$\frac{1}{\sigma_0}\frac{d\sigma^{(2,1)}}{d\tau} = 4C_F\Big[-\Big(\frac{1}{\epsilon} + \ln\frac{\mu^2}{Q^2\tau}\Big) + \Big(\frac{1}{\epsilon} + \ln\frac{\mu^2}{Q^2\tau^2}\Big)\Big] = -4C_F\ln\tau$$

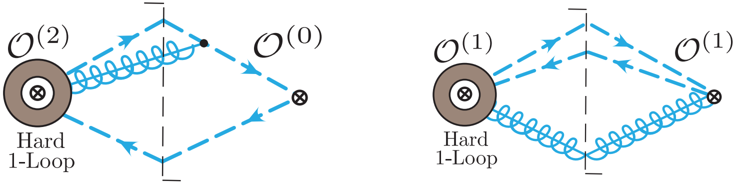
- Result gives directly (no additional expansions) the NLP contribution
- $1/\epsilon$ poles must cancel between collinear and soft contributions
  - In SCET these are UV poles arising from the scale separation between different sectors
  - From full-theory point of view these are IR poles and must cancel because there are no nontrivial IR divergences at subleading power

Subleading Thrust at NNLO.

Analogous cancellation of $1/\epsilon$ poles must happen at NNLO

- Yields nontrivial constraints (consistency relations) on the different contributions from hard, collinear, and soft sectors
  - Significantly reduces number of NNLO coefficients that must be calculated
  - Equivalently provides for powerful cross checks

- The LL NNLO result is determined by a single coefficient
  - hard-collinear (easiest) or collinear-soft or soft-soft

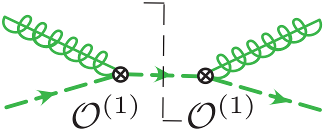


$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma^{(2,2)}}{\mathrm{d}\tau} = \Big[-32C_F^2 + 8C_F(C_F + C_A)\Big]\ln^3\tau + \cdots$$

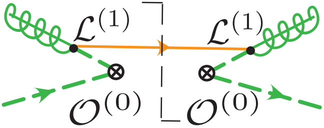
- New color structure compared to leading power from quark channel

0-Jettiness at NLP.

Collinear Quark



Soft Quark



In principle, we can "just" cross the thrust calculation

- Same cancellation of $1/\epsilon$ poles between different sectors and resulting consistency relations
- Quark and gluon channels turn into different incoming partonic channels

However, there are also important additional subtleties

- Dependence on Born measurement Φ_0
- Dependence on $\mathcal{T}_0$ definition

Treatment of Born Measurement.

For subtractions, we want to be differential in Φ_0

- Consider two choices $\Phi_0 \equiv \{Q, Y\}$ and $\Phi_0' \equiv \{q^+, q^-\}$
  - They are equivalent at Born level, and therefore at LP, but not beyond

$$q^{\mp} = \sqrt{q^2 + q_T^2}\, e^{\pm Y} = Q\, e^{\pm Y}\Big[1 + \mathcal{O}(\lambda^2)\Big]$$

  - $\dfrac{\mathrm{d}\sigma}{\mathrm{d}Q\,\mathrm{d}Y\,\mathrm{d}\mathcal{T}_0}$ and $\dfrac{\mathrm{d}\sigma}{\mathrm{d}q^+\mathrm{d}q^-\mathrm{d}\mathcal{T}_0}$ have *different* power corrections

- At NLP, must explicitly take into account specific Born measurement

$$Q^2 = q^2 = (p_a + p_b - k)^2\,, \qquad Y = \frac{1}{2}\ln\frac{q^-}{q^+} = \frac{1}{2}\ln\frac{p_a^- - k^-}{p_b^+ - k^+}$$

  - Easiest is to exactly solve Born measurement in terms of incoming momentum fractions

$$p_{a,b}^{\mp} = \zeta_{a,b} E_{\mathrm{cm}} = k^{\mp} + e^{\pm}\sqrt{Q^2 + k_T^2}$$

  - Corresponds to routing soft/collinear residual momenta k into incoming parton legs. Their expansion then yields derivatives of PDFs.

LL NLP Results for 0-Jettiness.

Results for coefficients of the partonic cross section

Here: $\delta_a \equiv \delta(\xi_a - x_a)$ and $\delta'_a \equiv x_a\, \delta'(\xi_a - x_a)$ and $\tau \equiv \mathcal{T}_0/Q$

- LL NLO

$$C_{q\bar q}^{(2,1)}(\xi_a, \xi_b) = 8C_F \left(\delta_a\delta_b + \frac{\delta'_a\delta_b}{2} + \frac{\delta_a\delta'_b}{2}\right) \ln\tau + \cdots$$

$$C_{qg}^{(2,1)}(\xi_a, \xi_b) = -2T_F\, \delta_a\delta_b\, \ln\tau + \cdots$$

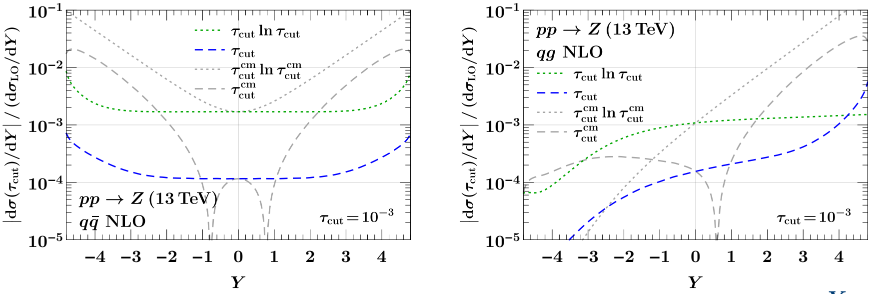
- LL NNLO

$$C_{q\bar q}^{(2,2)}(\xi_a, \xi_b) = -32C_F^2 \left(\delta_a\delta_b + \frac{\delta'_a\delta_b}{2} + \frac{\delta_a\delta'_b}{2}\right) \ln^3\tau + \cdots$$

$$C_{qg}^{(2,2)}(\xi_a, \xi_b) = 4T_F(C_F + C_A)\, \delta_a\delta_b\, \ln^3\tau + \cdots$$

  - Channels that exist at leading power contain derivatives of PDFs at NLP
  - qg channel already contributes at leading-log, in contrast to leading power

Dependence on $\mathcal{T}_0$ Definition.



General definition: $\mathcal{T}_0^x = \sum_i \min\Big\{\rho_x\, k_i^+,\, k_i^- / \rho_x\Big\}$ has $\lambda^2 = \dfrac{\mathcal{T}_0^x e^Y}{Q\rho_x}$

- leptonic: $\rho_{\rm lep} = e^Y$
  - Defines $\mathcal{T}_0^{\rm lep}$ in leptonic (Born) frame $\to$ uniform power expansion in $\mathcal{T}_0/Q$
  - $\mathcal{T}_0 \equiv \mathcal{T}_0^{\rm lep}(\Phi_0) \to$ requires to be differential in Φ_0
- hadronic: $\rho_{\rm cm} = 1$
  - Defines $\mathcal{T}_0^{\rm cm}$ in hadronic cm frame $\to$ power exp. deteriorates for large $|Y|$
  - Same effect is present for beam sectors for general $\mathcal{T}_N$

The slide added while you had coffee.

$$
\begin{aligned}
\frac{\mathrm{d}\sigma^{(2,1)}_{q\bar q}}{\mathrm{d}Q^2\,\mathrm{d}Y\,\mathrm{d}\mathcal{T}_0^x} = \hat\sigma^{\mathrm{LO}}(Q)\,\frac{\alpha_s C_F}{\pi}
&\times \Big[f_q(x_a) f_{\bar q}(x_b) \Big(\frac{e^Y}{Q\rho_x}\,\ln\frac{\mathcal{T}_0^x\rho_x}{Qe^Y} + \frac{\rho_x}{Qe^Y}\,\ln\frac{\mathcal{T}_0^x e^Y}{Q\rho_x} \Big) \\
&\quad + f'_q(x_a) f_{\bar q}(x_b)\,\frac{\rho_x}{Qe^Y}\,\ln\frac{\mathcal{T}_0^x e^Y}{Q\rho_x} \\
&\quad + f_q(x_a) f'_{\bar q}(x_b)\,\frac{e^Y}{Q\rho_x}\,\ln\frac{\mathcal{T}_0^x \rho_x}{Qe^Y} \Big]
\end{aligned}
$$

with $\quad x_a = \dfrac{Qe^Y}{E_{\mathrm{cm}}}\,, \qquad x_b = \dfrac{Qe^{-Y}}{E_{\mathrm{cm}}}$

- You cannot partially integrate the PDF derivatives because there is no integral when being fully differential in $\Phi_0 = \{Q, Y\}$
- At NLL, also qg channel will have $f'_q(x_a)$ and $f'_g(x_b)$

Numerics.

Numerical Results.

We can obtain the *complete* nonsingular contributions numerically

$$\frac{1}{\sigma_{\rm LO}}\frac{\mathrm{d}\sigma^{\rm nons}}{\mathrm{d}\ln\mathcal{T}_0} = \frac{1}{\sigma_{\rm LO}}\frac{\mathrm{d}\sigma}{\mathrm{d}\ln\mathcal{T}_0} - \frac{1}{\sigma_{\rm LO}}\frac{\mathrm{d}\sigma^{\rm sing}}{\mathrm{d}\ln\mathcal{T}_0}$$

- Use $V/H+j$ NLO$_1$ calculation from MCFM8 for $\mathrm{d}\sigma/\mathrm{d}\ln\mathcal{T}_0$
- Perform a χ^2 fit to (with $\tau \equiv \mathcal{T}_0/m_Z$ or $\tau \equiv \mathcal{T}_0/m_H$)

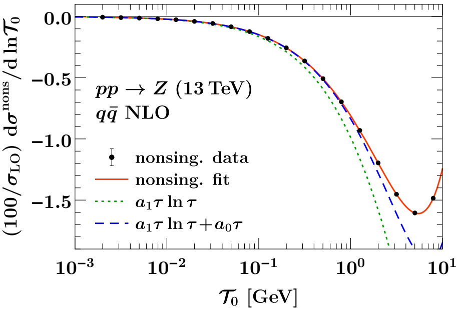
$$F_{\rm NLO}(\tau) = \frac{\mathrm{d}}{\mathrm{d}\ln\tau}\Big\{\tau\big[(a_1+b_1\tau+c_1\tau^2)\ln\tau + a_0 + b_0\tau + c_0\tau^2\big]\Big\}$$

$$F_{\rm NNLO}(\tau) = \frac{\mathrm{d}}{\mathrm{d}\ln\tau}\Big\{\tau\big[(a_3+b_3\tau)\ln^3\tau + (a_2+b_2\tau)\ln^2\tau + a_1\ln\tau + a_0\big]\Big\}$$

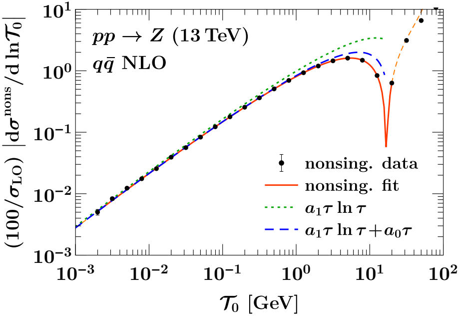
  - Requires high MC statistics to get precise enough nonsingular data to be able to distinguish different terms of similar shape
  - Important to include b_i, c_i coefficients in the fit to avoid biasing the fit result for the NLP a_i coefficients we are interested in
  - Important to carefully select fit range in $\mathcal{T}_0$ and validate fit stability

Drell-Yan at NLO.

$d\sigma/d\ln\mathcal{T}_0$ (linear scale)



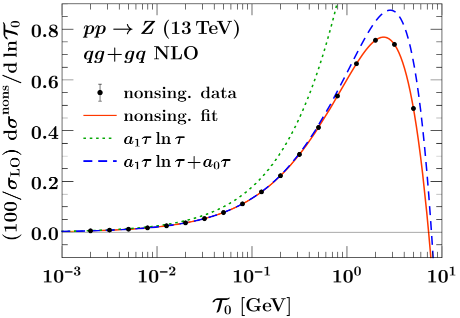
$d\sigma/d\ln\mathcal{T}_0$ (log scale)



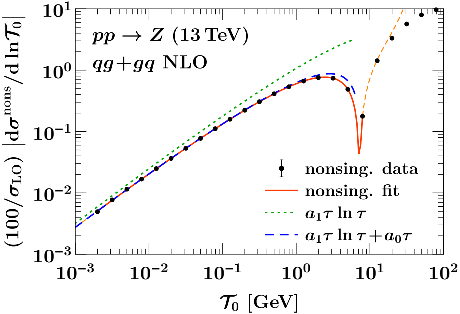
NLO	a_1 (LL)	a_0 (NLL)
$q\bar{q} \to Zg$ fitted	$+0.25366 \pm 0.00131$	$+0.13738 \pm 0.00057$
calculated	$+0.25509$	$+0.13708$
$qg \to Zq$ fitted	-0.27697 ± 0.00113	-0.40062 ± 0.00052
calculated	-0.27720	-0.40104

Drell-Yan at NLO.

$\mathrm{d}\sigma/\mathrm{d}\ln\mathcal{T}_0$ (linear scale)



$\mathrm{d}\sigma/\mathrm{d}\ln\mathcal{T}_0$ (log scale)



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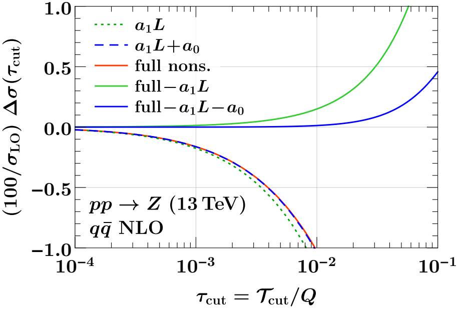
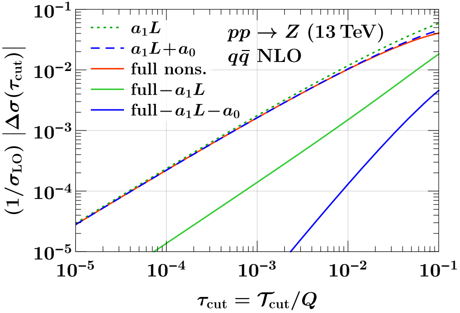
Drell-Yan at NLO.

$100\,\Delta\sigma(\tau_{\rm cut})/\sigma_{\rm LO}$ (linear scale)



$\Delta\sigma(\tau_{\rm cut})/\sigma_{\rm LO}$ (log scale)



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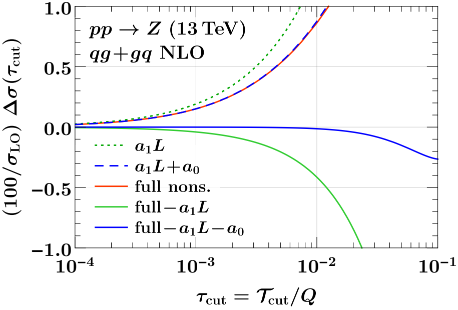

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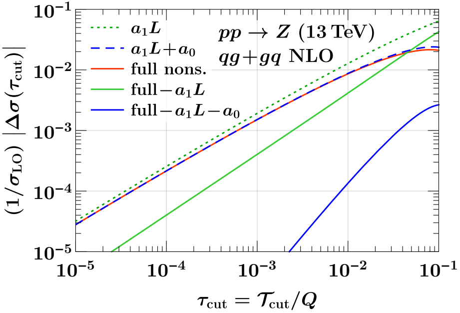

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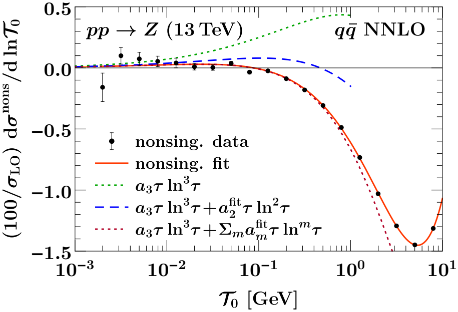
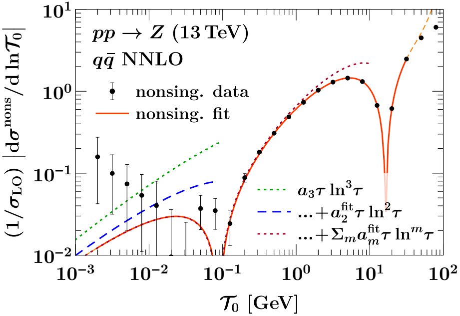


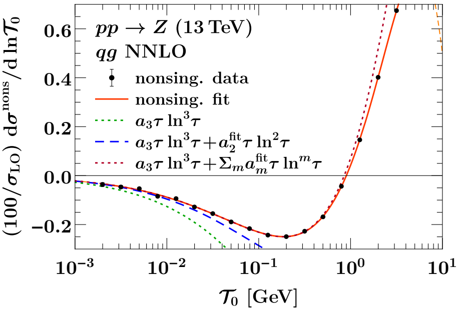
$\mathrm{d}\sigma/\mathrm{d}\ln\mathcal{T}_0$ (log scale)



NNLO	a_3 (LL)	a_2 (NLL)
$q\bar{q} \to Zg$ fitted	-0.01112 ± 0.00150	-0.04662 ± 0.00180
calculated	-0.01277	
$qg \to Zq$ fitted	$+0.02373 \pm 0.00247$	$+0.04234 \pm 0.00242$
calculated	$+0.02256$	

Drell-Yan at NNLO.

$d\sigma/d\ln\mathcal{T}_0$ (linear scale)



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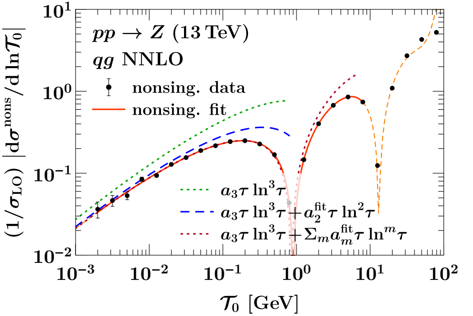

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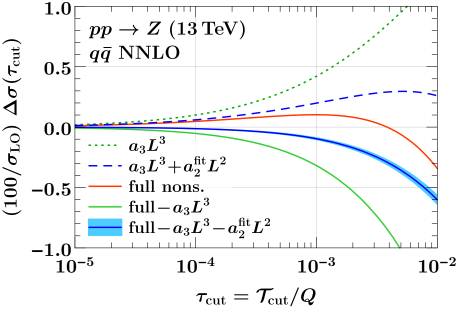
$100\,\Delta\sigma(\tau_{\mathrm{cut}})/\sigma_{\mathrm{LO}}$ (linear scale)



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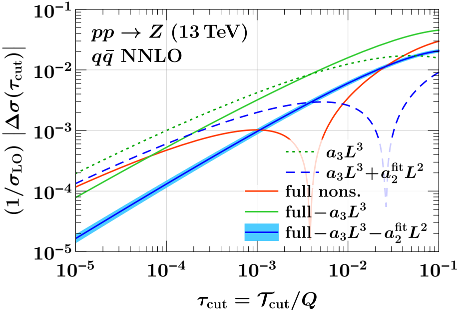

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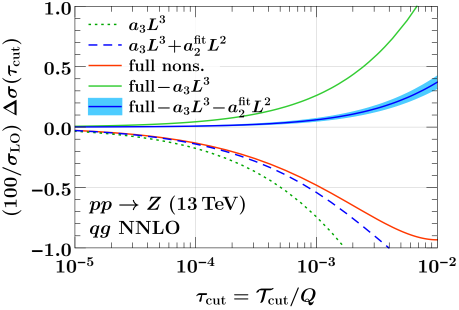

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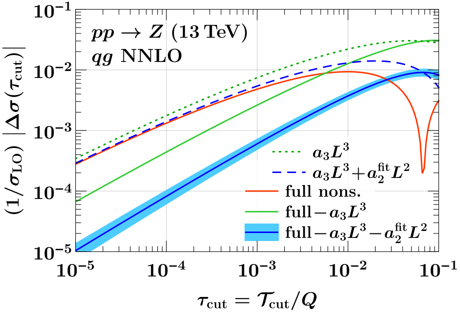

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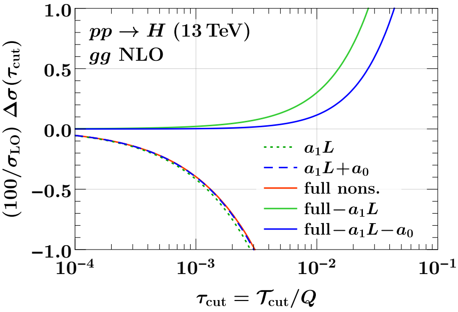
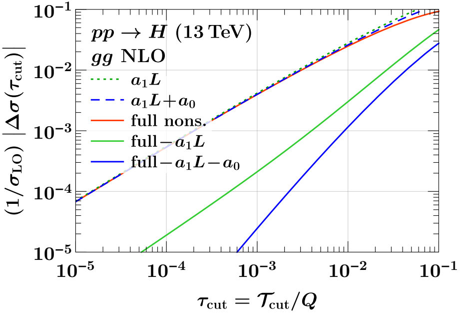
Higgs at NLO.

$100\,\Delta\sigma(\tau_{\rm cut})/\sigma_{\rm LO}$ (linear scale)



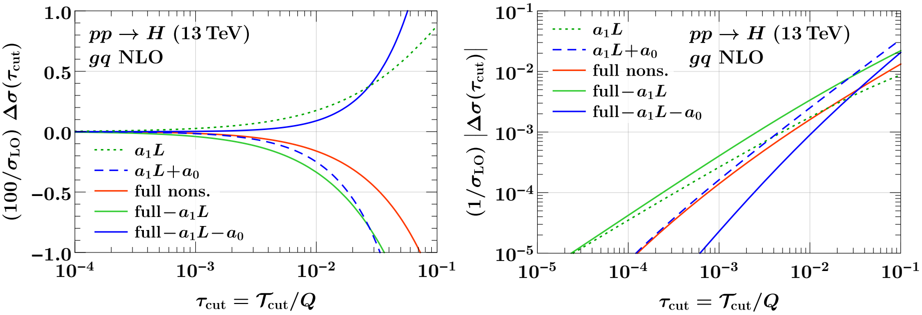
$\Delta\sigma(\tau_{\rm cut})/\sigma_{\rm LO}$ (log scale)



NLO	a_1 (LL)	a_0 (NLL)
$gg \to Hg$ fitted	$+0.60936 \pm 0.00600$	$+0.18241 \pm 0.00425$
calculated	$+0.60400$	$+0.18627$
$gq \to Hq$ fitted	-0.03733 ± 0.00066	-0.42552 ± 0.00032
calculated	-0.03807	-0.42576
$q\bar{q} \to Hg$ fitted	–	$+4.90060 \pm 0.00013$
calculated	–	$+4.90047$

Higgs at NLO.

$100\,\Delta\sigma(\tau_{cut})/\sigma_{LO}$ (linear scale) $\Delta\sigma(\tau_{cut})/\sigma_{LO}$ (log scale)



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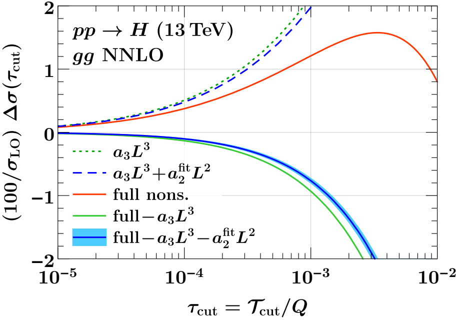
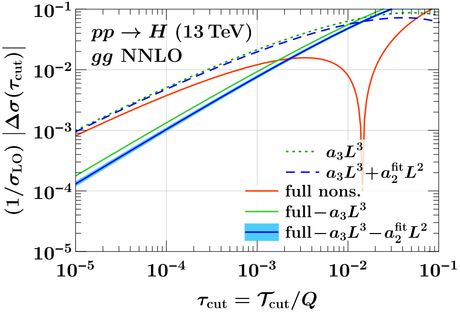


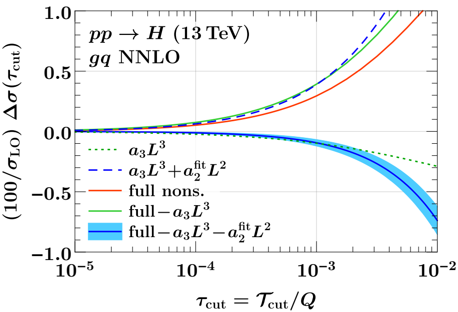
$\Delta\sigma(\tau_{\rm cut})/\sigma_{\rm LO}$ (log scale)

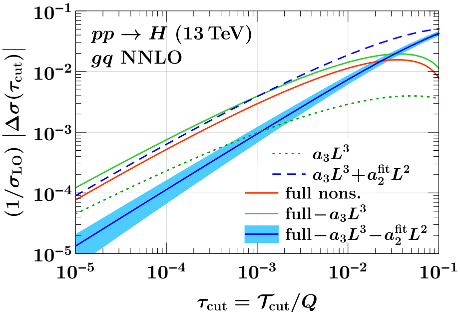


NNLO	a_3 (LL)	a_2 (NLL)
$gg \to Hg$ fitted	-0.05785 ± 0.00713	-0.03491 ± 0.00758
calculated	-0.06497	
$gq \to Hq$ fitted	$+0.00998 \pm 0.00509$	$+0.10193 \pm 0.00536$
calculated	$+0.00296$	
$qq' \to Hg$ fitted	–	-0.00159 ± 0.00037
calculated	–	

Higgs at NNLO.

$100\,\Delta\sigma(\tau_{cut})/\sigma_{LO}$ (linear scale) $\Delta\sigma(\tau_{cut})/\sigma_{LO}$ (log scale)





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calculated	–	

Summary.

Key advantages of employing physical resolution variable for subtraction

- Subtractions are given by singular limit of a physical cross section
- Integrated subtractions are given by matrix elements of operators
- In principle, can recycle existing Born+1 jet calculations

Main drawback are neglected $\Delta\sigma(\tau_{\mathrm{cut}})$ terms

- Is solved by computing and including NLP corrections
- Even the fixed-order LL terms give substantial improvement

Fixed-order are also the first step toward resumming

- to be continued in Gherardo's talk ...